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Introduction

The Booster was initially designed for fast extraction (FEB) to the AGS via the BtA line. Subsequently, it was modified to allow for slow extraction (SEB) to NSRL using the R line. Recently, an attempt was made to fast extract the beam to the R line using the FEB kicker, located at F3, the SEB bump, and the SEB septum magnet at D6. The motivation for this was a proposed experiment at the end of the R line that would require a rapid burst of beam.

In order for this scenario to be feasible, the phase advance from the F3 kicker to the D6 septum needs to be such that a kick at F3 translates into a positive displacement at D6. With an orbit bump centered at the D6 septum magnet which moves the closed orbit towards the septum, the kick at F3 also needs to be strong enough to move the beam from the closed orbit on the inner side of the septum, to the other side of the septum. Assuming that the closed orbit is as close to the septum as possible without causing beam loss, a certain beam size there, and given that the septum is 15.2 mm thick, an estimate for the amount of displacement required can be calculated. Another constraint is that little or no loss occurs from where it is kicked to where it exits the septum.

In the typical slow extraction setup, a thin septum (0.76 mm thick), located at D3 is also used. The required displacement to get across this septum is less because it is thinner. This septum bends the beam on the outside of the septum away from the circulating beam so that when it reaches D6 it is further to the outside there. The question of whether to use the D3 septum or not arose when trying to decide on the approach. Although the trajectory passing through the D3 and then the D6 is correct for slow extracted beam, these elements were not designed for the extraction of fast extracted beam and so there is no reason to believe apriori that both septa are compatible with a beam kicked from F3. From past experience with FEB from the AGS into the SEB beam line, which has a similar configuration and did not use the thin septum similarly available there, it was tentatively decided to try extracting at D6 without trying to pass to the outside of the D3 septum.

The location of the septum that the beam is to be kicked across effects the optimal phase advance from F3. If the D3 septum is used, the optimal horizontal betatron tune is higher than it would be if the D6 septum was used because D3 is closer to F3. That is, if a maximum displacement at D6 rather than D3 was desired. For maximum displacement at D3 the optimal tune is about 4.8, and for maximum displacement at D6 the optimal tune is about 4.50.[1] The D3 septum delivers a bend of about 3 mrad.[2] The D3 septum scenario then involves kicking the beam so that it has maximum positive displacement near D3, thereby passing to the outside of the D3 septum when it is kicked, then using its 3 mrad kick to move the beam to the outside at the septum at D6. This scenario presupposes the use of an orbit bump which bumps the closed orbit towards both the D3 and D6 septa. This bump would be similar to the one typically used with slow extraction.

[1] "NSRL beam development studies plans:NSRL 7", Kevin Brown
[2] BAF Conceptual Design Report

In this scenario, the beam trajectory would have a maximum displacement at D3, and so would be at the peak of a betatron oscillation about the closed orbit.

The phase advance from D3 to D6 is roughly 90 degrees[3], so there would be little gain at D6 from any displacement at D3 associated with a betatron oscillation about the closed orbit, and only the displacement from the bend at D3 would remain. It is unclear whether this, by itself, is enough to kick the beam across the septum at D6. Which scenario (i.e.- maximum displacement at D3 or D6) is preferable probably depends, at least in part, on how strong the F3 kick is relative to the bend at D3 (~ 3 mrad). The strength of the F3 kick is not well established from beam-based measurements. For reasons that are probably less readily quantifiable, the choice also depends on which horizontal tune and type of bump is preferable. There is enough flexibility to produce a bump at D6 and not at D3 using the 5 backleg windings that are used to make the standard SEB bump, but there is little or no experience with that configuration. In any event, we started with a tune of about 4.70, and tried to make a bump at D6, but not at D3 as well.

The reason we started with a tune of 4.70 or so, and not one closer to 4.5, was because I incorrectly thought that a relatively high tune was optimal for displacement at D6, not D3. It is also true that this is a more natural operating point then near or below 4.50. Fe^{+20} beam was used with a typical NSRL extraction kinetic energy of 600 MeV per nucleon. The nominal extraction rigidity for FEB to the AGS is about 9.2 T*m for Au^{+32}, this Fe^{+20} kinetic energy corresponds to a rigidity of about 11.5 T*m. The Fe^{+20} beam intensity was about 1×10^{9} ions.

Non-beam based measurements of the F3 kicker strength give the field for a cap bank voltage of 35kV as 206g.[4] The magnet length is 2.286 m.[5] At a rigidity of 11.5 T*m this results in a kick of 4.087 mrad. The kicker was set to 33 kV, and assuming the kick scales linearly with voltage, 33 kV would correspond to 3.85 mrad. However, it is likely that the actual kick was below this because the kicker's stop charge was set to occur hundreds of milliseconds before it discharged, and the cap bank gradually loses its voltage if it is not charging. The stop charge should be set to occur just before the discharge for the maximum kick.

Initial FEB to R Line

Currents were put into the bump magnets for a bump only at D6, not at both D3 and D6 and a tune of 4.70.[6] Figure 1 shows the flag at the entrance to the D6 septum that was used to determine whether the beam was moved into the aperture of the D6 septum

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[3] With a tune of 4.80 the phase advance is 2π (0.25)=90 degrees, with a tune of 4.50 it is $2\pi(0.234)$=84 degrees.

[4] W.Zhang, R. Sanders, A. Soukas, and J. Tuozzolo, "An Overview of the Fast Injection-Extraction Kicker Systems of the Brookhaven AGS-Booster Complex", from the proceedings of the 1999 Particle Accelerator Conference, New York, 1999.

[5] W. Zhang, J. Bunicci, W. Frey, and A. Soukas, "Report on the Test and Measurement of the Fast Kikcer System" Booster Technical Note No. 133, Dec. 22, 1988.

[6] Currents provided by Lee Hammons

magnet. A light was illuminating the flag, and crop marks can be seen. The crop marks on the left indicate the position of the outside edge of the septum.[7]

After considerable tuning of the bump, the radius, and the horizontal tune, extracted beam was visible on the D6 flag (figure 2). Losses were visible on the D6, D7, and D8 loss monitors, but only near the time when the kicker fires. That is, there were none associated with the bump coming up.[8] Considerable tuning was required to kick the beam out cleanly (as viewed on the wall current monitor signal). The major stumbling block was losses at F6. Adjusting the above parameters was required to move the loss to the D6 area.

The final values for the setup were: A radial setpoint of 12 mm, a horizontal tune setting of 4.84, and bump currents of: C7= -200 A, D1= -80A, D4= 600A, D7= -200A, E1=600A. 600A is the maximum value for these supplies. The kicker had a setting of 33 kV.

In figure 1 the edge of the septum is indicated by the blue vertical line. The same vertical line is shown in figure 2 which shows that the vast majority of the beam is to the right of this. At this point the study ended, but later I realized that, although I had looked at many of the loss monitors around the ring, I could not confirm that no losses were visible anywhere except in the extraction region.

The beam instrumentation in the R line was not designed to measure fast extracted beam, so the efficiency cannot be determined from it. Figures 3, 4 and 5 show its response.

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[7] The actual flag video that appears on the computer screen is a mirror image (x -> -x) and the vertical and horizontal are flipped. Suffice it to say that in the picture above, which has been adjusted to account for this, left is towards the inside of the Booster and up is up. The beam is going into the flag. Kevin Brown, private communication.

[8] see http://www.cadops.bnl.gov/elog/graphics/rhic-pp_2005/Tue_Oct_18_2005_195221_8128.gif and http://www.cadops.bnl.gov/elog/graphics/rhic-pp_2005/Tue_Oct_18_2005_195607_8378.gif

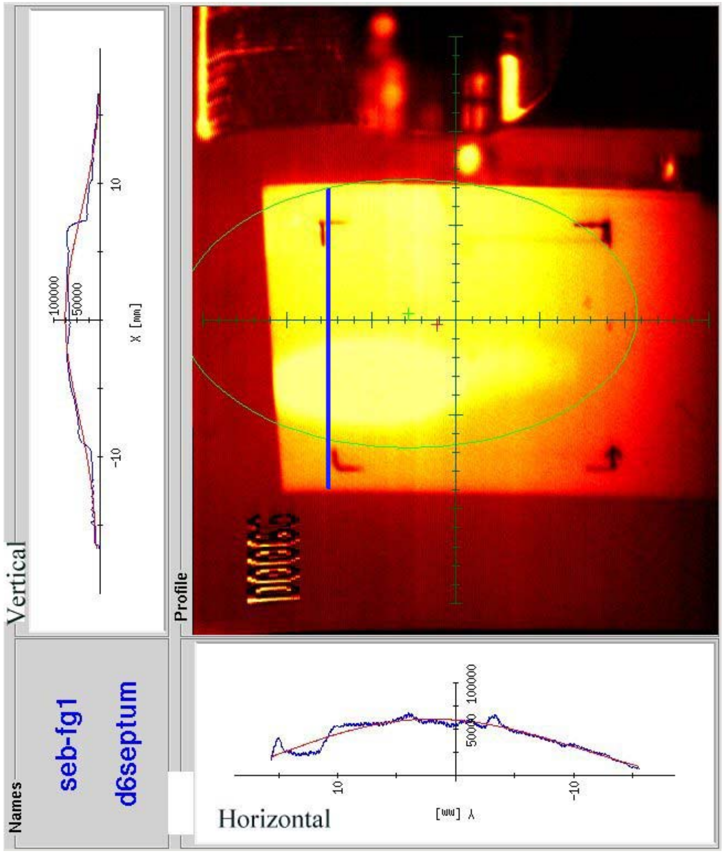

Figure 1: The D6 flag at the entrance to the D6 septum. The crop marks on the left mark the outside edge of the extraction septum. So, the blue vertical line indicates the approximate location of the outer edge of the D6 septum.

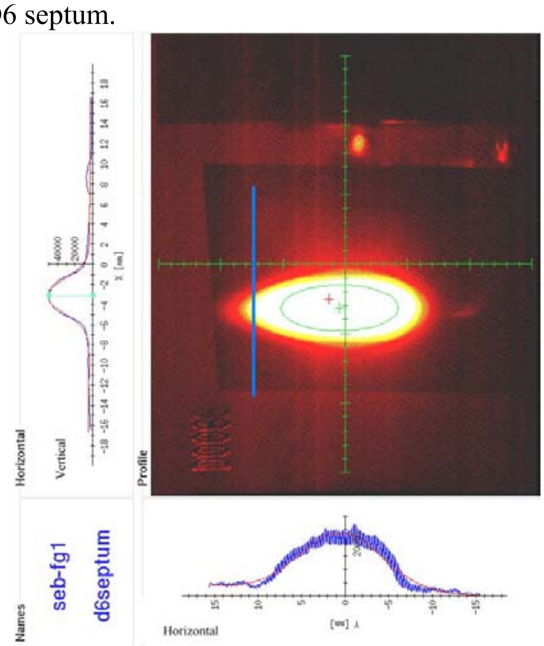

Figure 2: Fast extracted beam on the D6 flag. The blue line marks the outer edge of the D6 septum, as it was shown in figure 1.

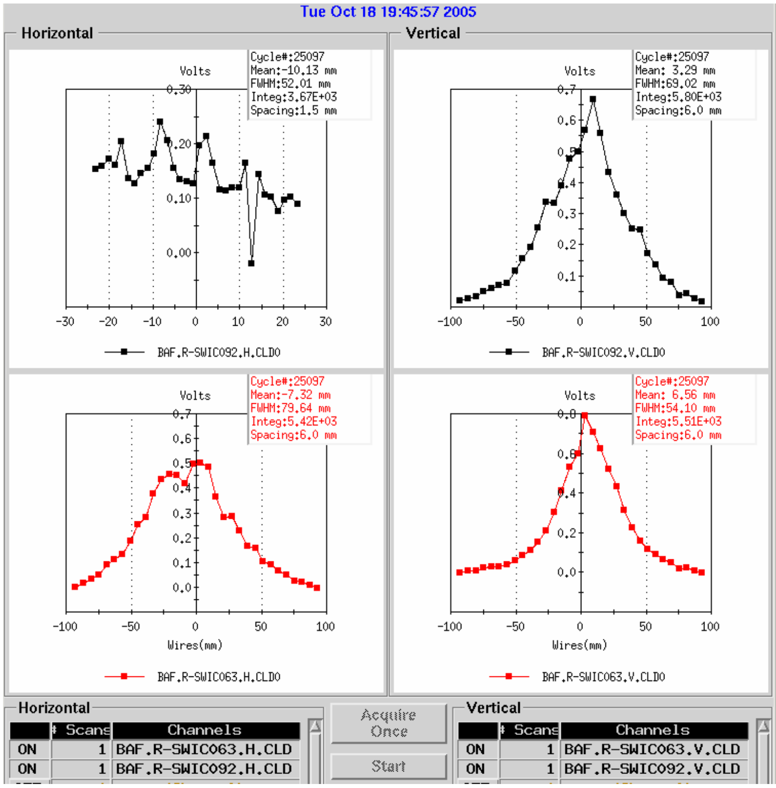

Figure 3: Fe+20 Fast extracted beam on SWICs in R line. Top is at 63ft, bottom is at 92 ft.

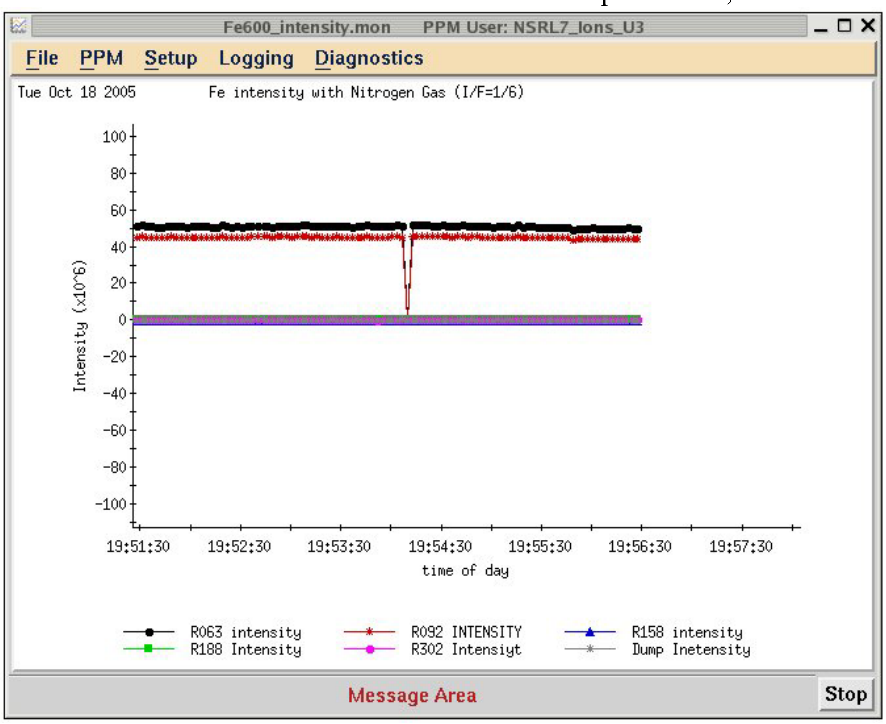

Figure 4: Fe^{+20} fast extracted beam intensity monitor on ion chambers at 63 and 92 ft. The calibration is incorrect for fast extracted beam.

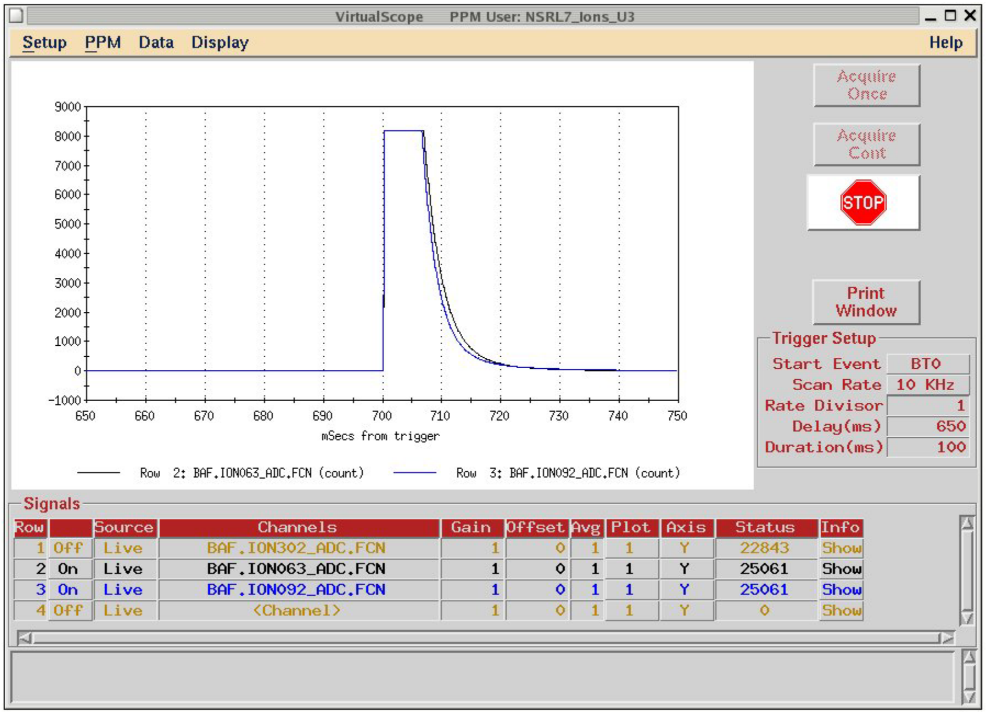

Figure 5: Time response of ion chambers at 63 and 92 feet in R line.

The Next FEB to R Line Experience

In order to confirm that there were no losses except in the extraction region, and in light of the expectation that the optimal tune for FEB at D6 should be significantly lower than the tune with which it was established, the FEB setup was restored a week or so later.[9] The setup was easily restored, and no losses were found outside the extraction region. This time the extraction septum was off, so no beam was visible in R line. This fact was not expected to effect the position on the flag, but could certainly effect the loss pattern of the beam after it is kicked and reaches the field region of the D6 septum magnet.

As in the previous case, all the losses occurred around the time of the kick, not when the bump comes up. However, a hardwired PUE signal showing extraction on a fast time scale showed unexpected behavior. Figure 6 shows the sum and difference signals from the horizontal PUE at B2 together with the F3 kicker current waveform. When the beam is kicked at F3 it will travel past B2 before reaching D6 providing it does not scrape beforehand. The figure shows that it does pass B2, but that it appears to do so a second time as well. There are 3 bunches, and this difference signal shows that they originally have a positive signal, then on the next turn they seem to disappear, then they reappear with a negative signal. The B2 sum signal is essentially the same on these three turns. This implies that they are kicked, pass B2 with a very small difference signal, go around the machine to F3 where they are kicked again, pass B2, and then are extracted at D6.

[9] Initial data was taken on October 18, 2005, the following data is from Oct. 27.

This is possible in part because, as can be seen in the figure, the duration of the F3 pulse's flattop is almost three times the revolution period. A negative signal is towards the outside.[10]

After some adjustment of the usual parameters, another picture of the D6 flag was made with the extracted beam. In this case, the beam was even further to the outside (figure 8). Apparently, completely clearing the extraction septum.

Upon noticing that the beam was likely extracted after two turns not one, and realizing that the tune was higher than expected for nominal (one turn) fast extraction, the tune was lowered below 4.50 and parameters were adjusted to obtain one turn FEB. The best situation arrived at, given the time constraints, was with a horizontal tune setting of 4.40, a radial setting of +7mm, and the following bump magnet settings: C7= 0A, D1= 100A, D4= 600A, D7= 200A, E1= 400A. The intensity was about 4-5e8 ions. In this case, about 15% of the beam is lost when the extraction bump comes up (figure 9). Losses coincident with this can be seen at D6, D8 and E1 (figure 10). Although it was possible to remove these losses, the position on the D6 flag moved significantly inward when this was done. Even in the state where the beam is furthest to the outside, as indicated on the flag, it appears to be far from clearing the septum (figure 11).

[10] See Nov. 3, 2005 1813 entry in NSRL 6 elog.

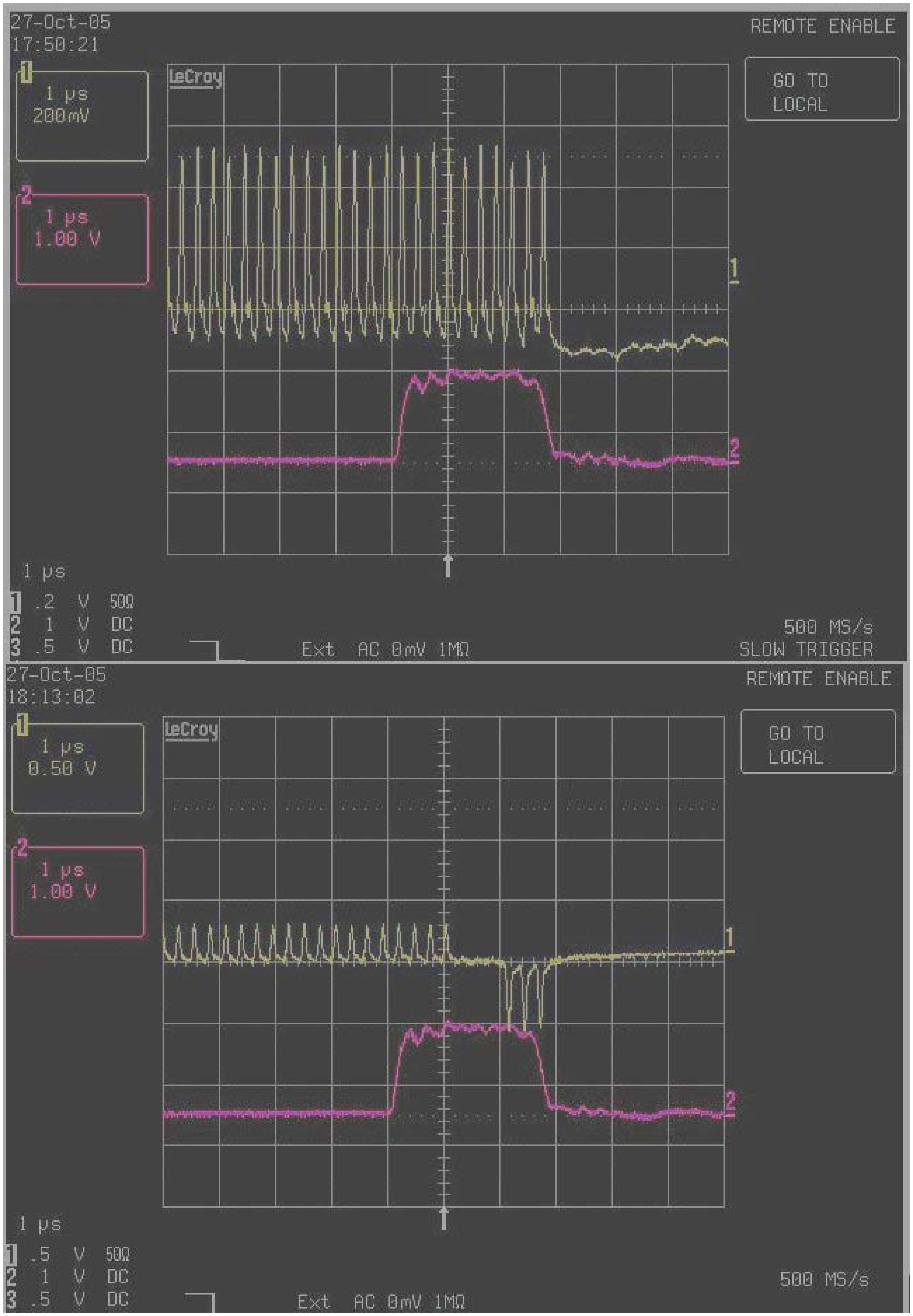

Figure 7: B2 PUE sum (top) and difference (bottom) signals at extraction time. The yellow signal is the PUE, the red signal is the kicker current.

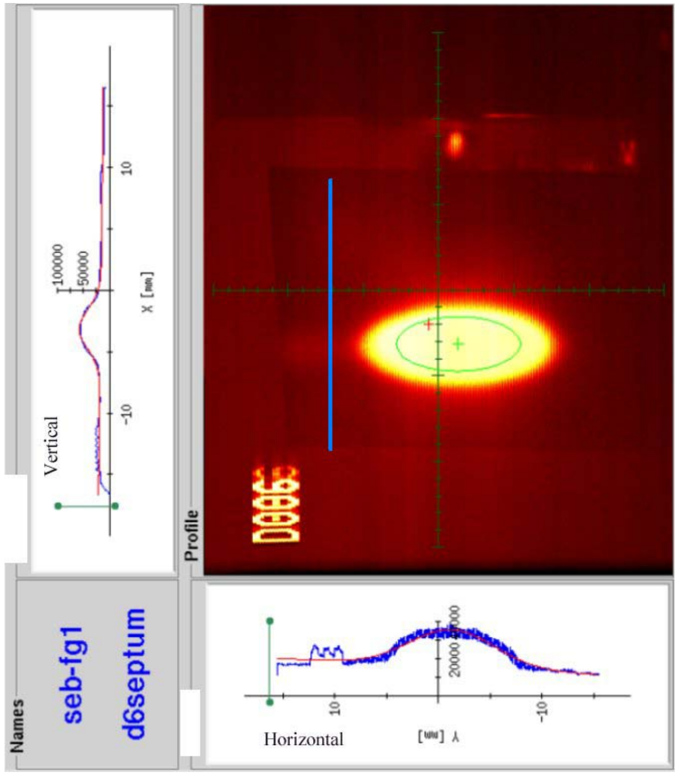

Figure 8: FEB on D6 flag, with horizontal tune setting of 4.86 (from Oct 27). Blue line is approximate position of outer edge of extraction septum. 1.1e9 ions in Booster.

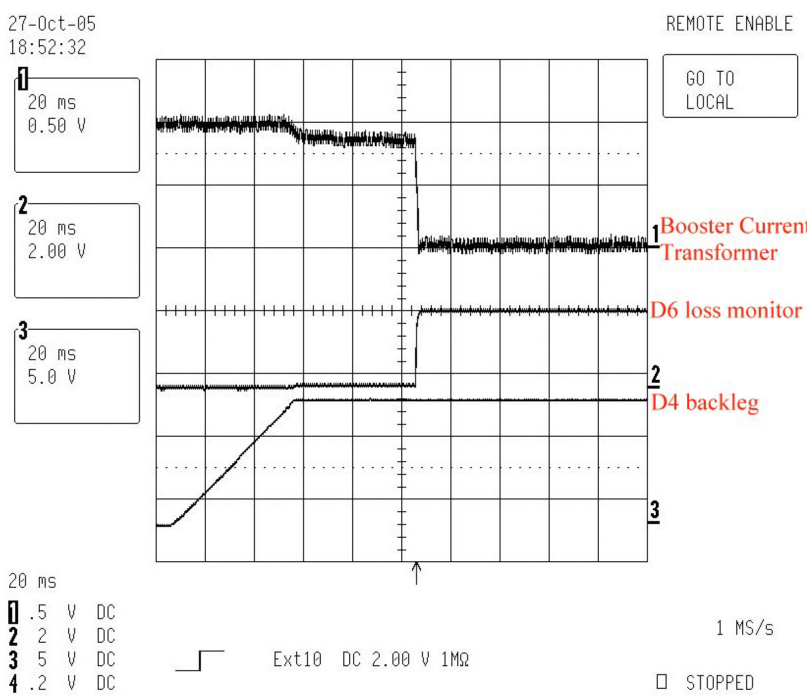

Figure 9: FEB with the horizontal tune set to 4.40. A beam loss of about 15% is apparent when the extraction bump comes up. The D4 backleg current illustrates the timing of the bump. Triggered by the kicker trigger.

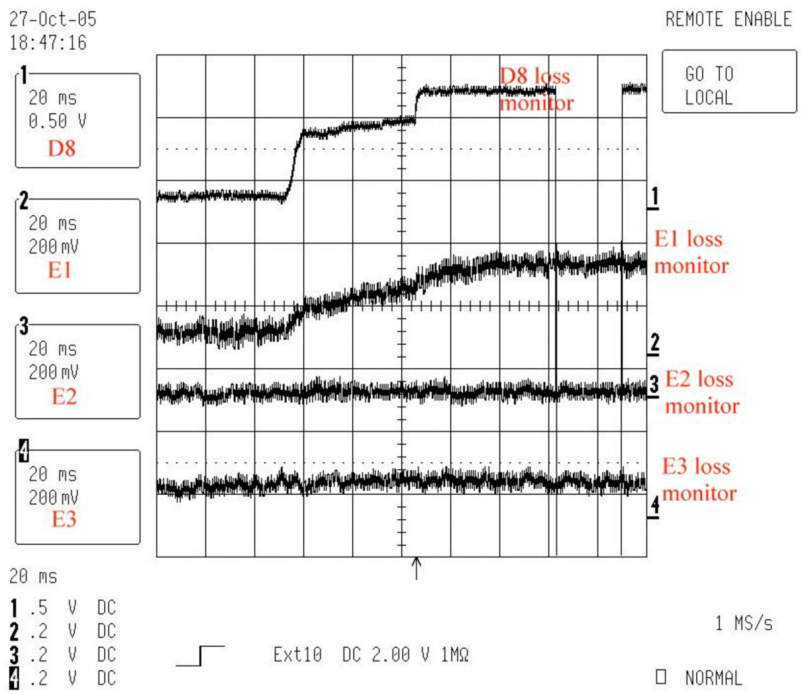

Figure 10: FEB with horizontal tune set to 4.40. Losses on D8 and E1 are apparent when the bump comes up (check bump timing in figure 9). Triggered by kicker trigger.

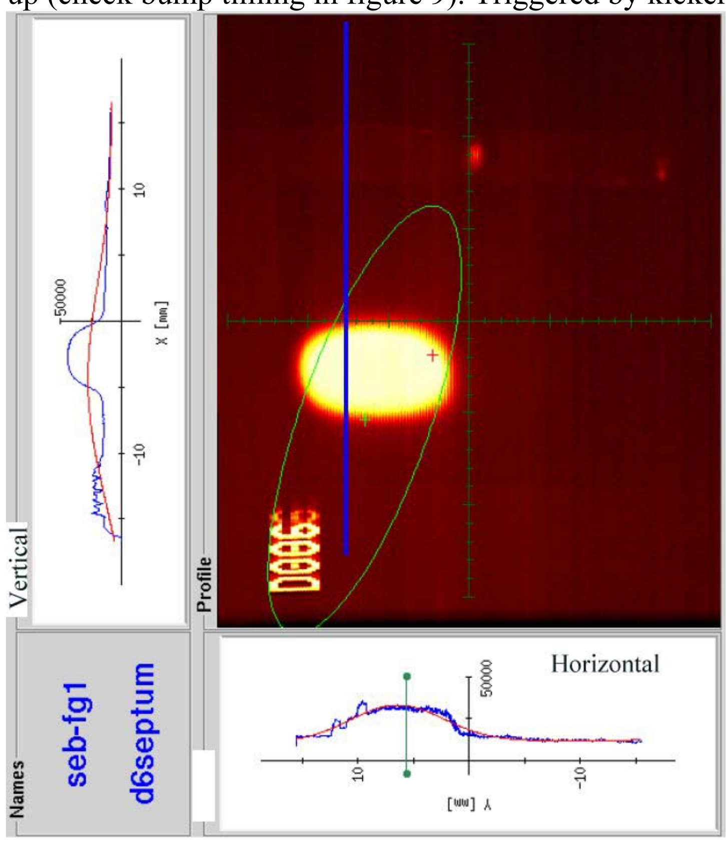

Figure 11: FEB on D6 flag, with horizontal tune setting of 4.40 (from Oct 27). The blue line shows the approximate position of outer edge of extraction septum. About 0.5e9 ions in Booster.

Analysis of 2-Kick FEB

Using a Booster MAD output file for a horizontal tune of 4.80 and no orbit distortions as a starting point a model of the 2-turn FEB to R line was developed using MathCad.[11] This model incorporates a bump at D6 derived from the currents used during the study, the dispersion orbit associated with the non-zero radius used there, a kick at F3, and an orbit taken at extraction energy without the bump on. Various other factors, such as the Booster aperture and the beam size are also taken into account.

Overview

The modeling process was divided into several steps:

1) Calculate the orbit with the bump at D6 made from the bump magnet currents that were used during the FEB study.
2) Calculate the dispersion orbit from the radius given by BoosterOrbit Display, and add this to the orbit with the D6 bump. Then, subtract the *dispersion orbit* found in 1) from the BPM data obtained from an orbit with no bump to get the *non-dispersion induced orbit*.
3) Finding a fit for the *non-dispersion induced orbit.*
4) Add the fit found in 3) to the orbit from 2) which includes the bump and *dispersion orbit*.
5) For this closed orbit, which is taken as starting from the C5 injection foil location (the MAD output file starting location), add a kick at F3, and propagate the resulting (x,x') to the C5 foil.
6) Take the value of *(x,x')* at the end of 5) and propagate it to F3, where it receives another kick of equal amplitude. Then propagate this to the end (the C5 foil).
7) Take the value of (x,x') at the end of 6) and propagate it to D6.
8) Adjust the value of the F3 kick, which is taken as a free parameter, to produce the observed position on the D6 flag.

The MAD file was made with a tune of 4.80. In order to model the orbit for tunes other than this the phase advance is scaled by Q/4.80, where Q is the tune for the orbit in question. As will be shown, 4.80 is close to the actual tune expected given the tune's set value and the change due to Booster's natural chromaticity and the calculated radius.

The Orbit with a Bump at D6

Determining the Radius and Horizontal Tune

In order to calculate the orbit with the bump at D6, the horizontal tune needs to be known. Using the data from Oct 27, the horizontal tune setting in the OpticsControl program is 4.86. Figure 12 shows the horizontal orbit without the bump and at extraction

[11] The vertical tune for this file was 4.95.

time (700 ms from BT0). The average horizontal position calculated in the program is 3.516 mm. Figure 13 is the horizontal orbit with the bump on. In this case, the horizontal average is 4.931 mm. The difference between them is likely due to the fact that the orbit is not evenly and adequately sampled.

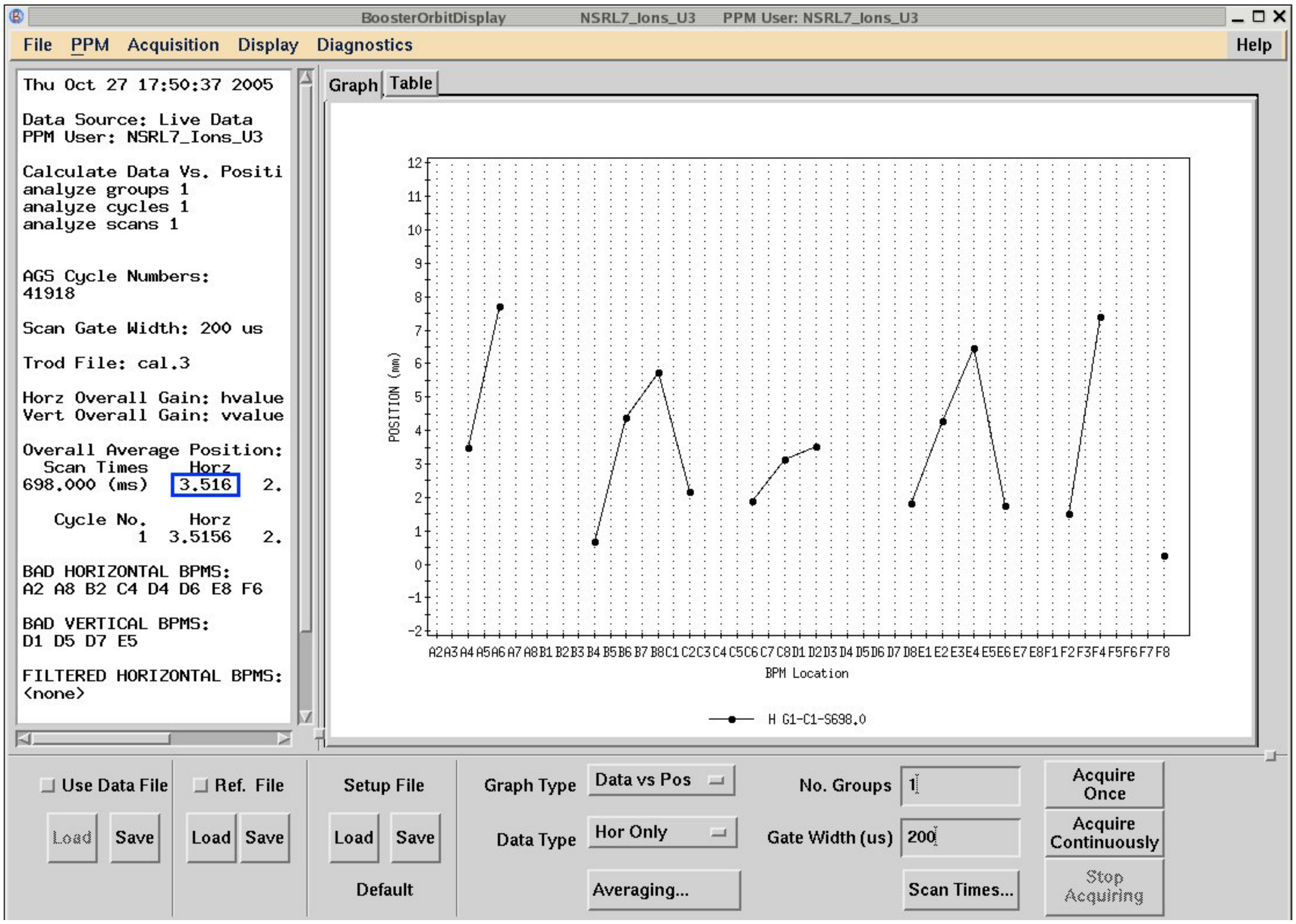

Figure 12: Raw horizontal orbit at extraction time with bump off. The average horizontal position is highlighted in blue.

The calibration of the orbit system is incorrect in these figures. The correct calibration is thought to be 3.64 times the values shown.[12] So, the radii in either case are 12.80 mm (no bump) and 17.9 mm (with bump). Ideally, one would like to use this average to obtain a measure of the radius by normalizing it to the average value of the dispersion. The average value of the dispersion (from the MAD output) is 1.555 m.[13] The average of the dispersions at the 16 BPMs is 1.992 m. If the bpms adequately sampled the orbit one would expect the following relationship to hold between the average radius and the average horizontal position on the BPMs,

[12] Actually, 4*0.91=3.64, K.S. Smith, private communication.
[13] Obtained by averaging the dispersion at all 751 locations in the MAD output.

$$R_{avg} = \frac{D_{avg}}{D_{BPM}} \bar{x}_{BPM} \qquad \text{Equation 1}$$

where R_{avg} is the beam radius, D_{avg} is the average Booster disperion, D_{BPM} is the average of the dispersions at the BPMs, and $\bar{x}_{BPM}$ is the average position at the BPMs. The smaller the orbit distortions at the BPMs are, the less important incomplete sampling of the orbit is in giving an accurate representation of the dispersion component of the orbit at the BPMs. So, taking the orbit without a bump seems more reasonable. This approach gives an R_{avg} of 0.781*12.80mm=10.0 mm.

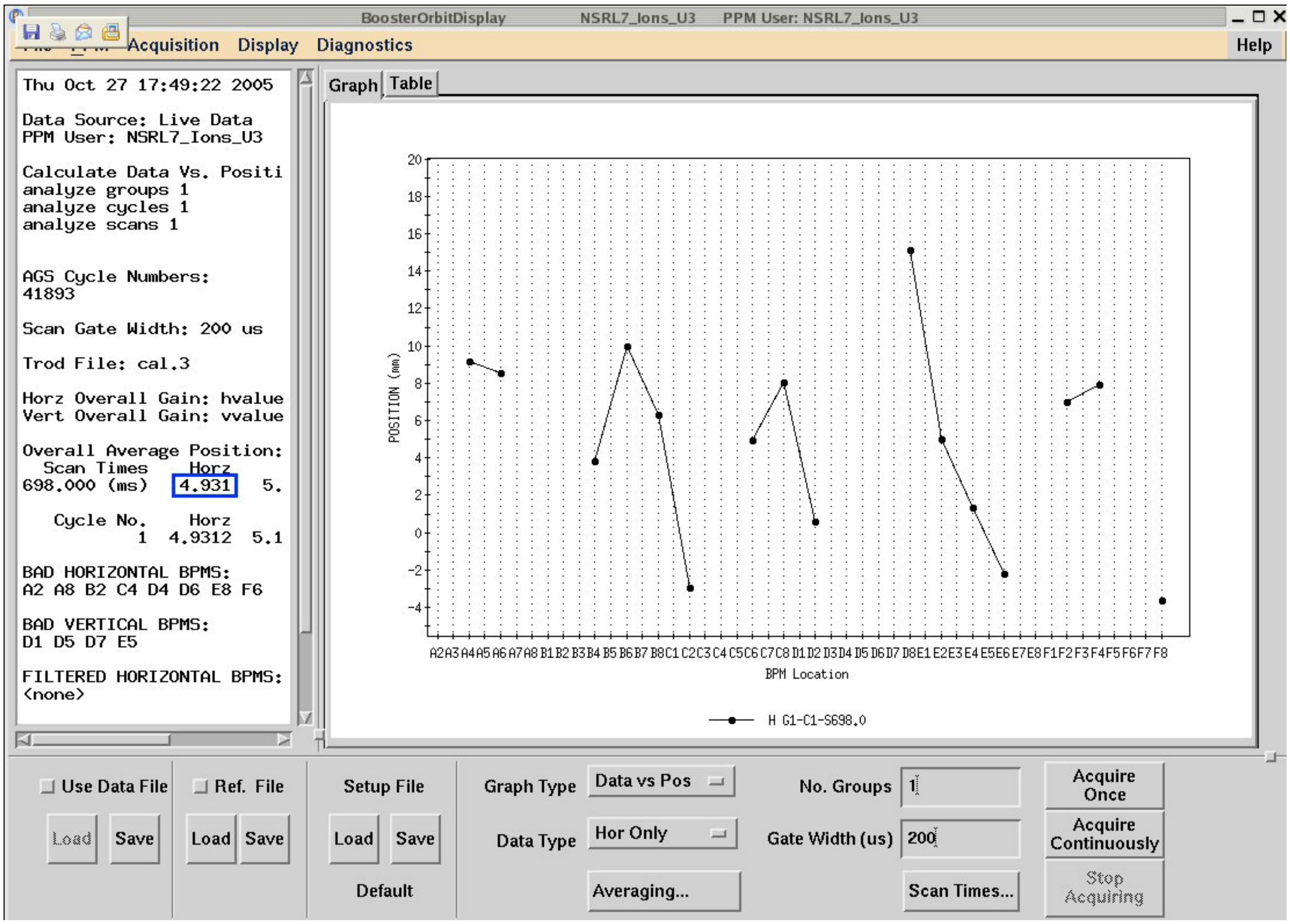

Figure 13: Raw horizontal orbit at extraction time with bump on. The average horizontal position is highlighted in blue.

The chromaticity sextupoles had zero current, and the OpticsControl program gives a horizontal chromaticity of –1.6 in this case. This results in a vertical tune shift of –0.05 from the set tune, or 4.81.

As a first pass, in order to compensate for the slight difference between the tune in the MAD output file and that calculated above, the phase advances $\psi(s-s_o)$ used in the calculations of (x,x') at all the locations in the ring given in the MAD output will be multiplied by the factor 4.81/4.80. 's' is the longitudinal position as given in the MAD output (which starts at the C5 injection foil location, s_o).[14] The Twiss parameters, $\alpha(s)$ and $\beta(s)$, will remain those that are found in the MAD output.

Determining the Bend Angles that Comprise the Bump

The bump is comprised of bends made by adding or subtracting current from the C7, D1, D4, D7, and E1 main magnet dipoles by powering the flat trim windings that are wrapped around each of these magnets. Each winding consists of 2 turns, whereas the main windings consist of 16 turns.[15] Since there are 36 main dipole magnets, each dipole gives a 10 degree, or $\pi/18$ radian bend when only the main windings are powered. The main magnet current, I_{mm}, was 3495 A in this case, and so the bend in millradians, produced by a current I_{trim} in one of the dipoles is,

$$\delta_{trim} = \frac{I_{trim}}{I_{mm}} \cdot \frac{\pi}{18} \cdot \frac{2}{16} \cdot 1000 \, {}^{mrad}\!/\!{}_{rad} = \frac{I_{trim}}{I_{mm}} \cdot \frac{\pi}{162} \cdot 1000 \, {}^{mrad}\!/\!{}_{rad} \qquad \text{Equation 2}$$

A negative current results in a bend to the inside. The following matrix gives the bend angles that comprise the bump,

$$\begin{pmatrix} \delta_{C7} \\ \delta_{D1} \\ \delta_{D4} \\ \delta_{D7} \\ \delta_{E1} \end{pmatrix} = \begin{pmatrix} -1.248 \\ -0.499 \\ +3.745 \\ -1.248 \\ +3.745 \end{pmatrix} mrad \quad , \quad \begin{pmatrix} I_{C7} \\ I_{D1} \\ I_{D4} \\ I_{D7} \\ I_{E1} \end{pmatrix} = \begin{pmatrix} -200 \\ -80 \\ +600 \\ -200 \\ +600 \end{pmatrix} Amps \qquad \text{Equation 3}$$

The location of the centers of these main magnets are given in the MAD output, and in the computation the bends are placed at these locations as 'kicks'.

Calculation of (x,x')

To calculate the $(x(s),x'(s))$ at each location s contained in the MAD output, first $(x(0),x'(0))$ is set to $(0,0)$. Then the ring is divided into 6 segments, (s_i,s_f), which are: $(0\ m, s_{C7-})$, (s_{C7+},s_{D1-}), (s_{D1+},s_{D4-}), (s_{D4+},s_{D7-}), (s_{D7+},s_{E1-}), $(s_{E1+}, 201.78\ m)$. Where '-' is just before the kick, '+' is just after the kick, and 0 m and 201.78 m are the beginning and end of the ring, respectively. For locations between s_i and s_f, $(x(s),x(s)')$ are given by the standard transfer matrix from s_i to s_f, namely,[16]

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[14] ψ is in units of 2π.
[15] C.J. Gardner, "Notes on the Acceleration of Iron Ions for the Booster Applications Facility", Dec. 29, 2002. http://www.cadops.bnl.gov/AGS/Operations/GardnerNotes/NsrlNotes/ironforbaf.pdf
[16] Edwards and Syphers, "An Introduction to the Physics of High Energy Accelerators", pg. 77

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = M \begin{pmatrix} x(s_i) \\ x'(s_f) \end{pmatrix}, \quad where\ M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \qquad \text{Equation 4}$$

where,

$$m_{11} = \sqrt{\frac{\beta(s)}{\beta(s_i)}}\left[\cos(2\pi\psi(s-s_i) + \sin(2\pi\psi(s-s_i)\right]$$

$$m_{12} = \sqrt{\beta(s)\beta(s_i)}\left[\sin(2\pi\psi(s-s_i)\right]$$

Equation 5

$$m_{21} = \frac{-\left[(1+\alpha(s_i)\alpha(s))\sin(2\pi\psi(s-s_i)) + (\alpha(s)-\alpha(s_i))\cos(2\pi\psi(s-s_i))\right]}{\sqrt{\beta(s)\beta(s_i)}}$$

$$m_{22} = \sqrt{\frac{\beta(s_i)}{\beta(s)}}\left[\cos(2\pi\psi(s-s_i)) + \alpha(s)\sin(2\pi\psi(s-s_i))\right]$$

At each bump magnet, the bend δ is added to *(x(s-),x'(s-))* to become *(x(s+),x'(s+))=(x(s-),x'(s-)+δ)*. The initial 'orbit' is formed this way, but it needs to have the same (x,x') at the beginning and end of the ring. The initial (x,x') is varied and the difference $(x(0)\text{-}x(201.78m)^2 + (x'(0)\text{-}x'(201.78m))^2$ is minimized to close the orbit. When this difference is minimized to less than 0.004, the resulting (x(0),x'(0)) is (12.642 mm, 3.7902 mrad).

Comparing the Modeled Bump Orbit with BPM data

A difference orbit was taken for the "bump-on minus bump-off" case.[17] Figure 14 plots the modeled bump and the data from the difference orbit. The different colored segments indicate the different segments in the computations described above. The chamber aperture is also shown.[18] Although the agreement is quite good, since there is some uncertainty in the actual value of the tune[19], the tune used in deriving the bump orbit was varied to minimize the rms error between the BPM data and the modeled bump orbit. Table I shows this rms error for several tunes. A tune of 4.77 gives the lowest rms error (1.82 mm), Q=4.81 gives an error that is somewhat larger (3.85 mm). The value of (x,x') where the orbit is closed is also shown in the table, it gives some indication of the size of

[17] Oct 27 2005 NSRL elog, 1752 entry (http://www.cadops.bnl.gov/elog/graphics/rhic-pp_2005/Thu_Oct_27_2005_175220_1325.gif)

[18] Adapted from A. Luccio, "Booster Chamber Aperture, Booster Technical Note #202, November 13, 1991. Several apertures have changed since this note. For example, the dump has been moved from D6 to B6, and the D3 and D6 septa have been installed. Several adjustments to the aperture contained in this note were made to reflect this. The apertures at D3 and D6 were obtained from NSRL slow extraction system notes "Thick Septum Magnet at D6" http://server.c-ad.bnl.gov/esfd/BAFbeam/d6table.html , and "Conceptual Design of the Booster Applications Facility (BAF)", October 1997, http://server.c-ad.bnl.gov/esfd/nsrl/operations/CDRfinalfigs.pdf .

[19] The actual value is uncertain because the tune was not measured, and the model used in the OpticsControl program for both the tune and chromaticity may not be exactly right.

the orbit distortion outside the bump region. The Q = 4.77 case gives significantly smaller values for (x,x') than the Q = 4.81 case. As the tune is reduced further, their values continue to get smaller but the rms error increases. Figure 15 shows both the Q=4.81 and the Q=4.77 orbits together with the BPM data.

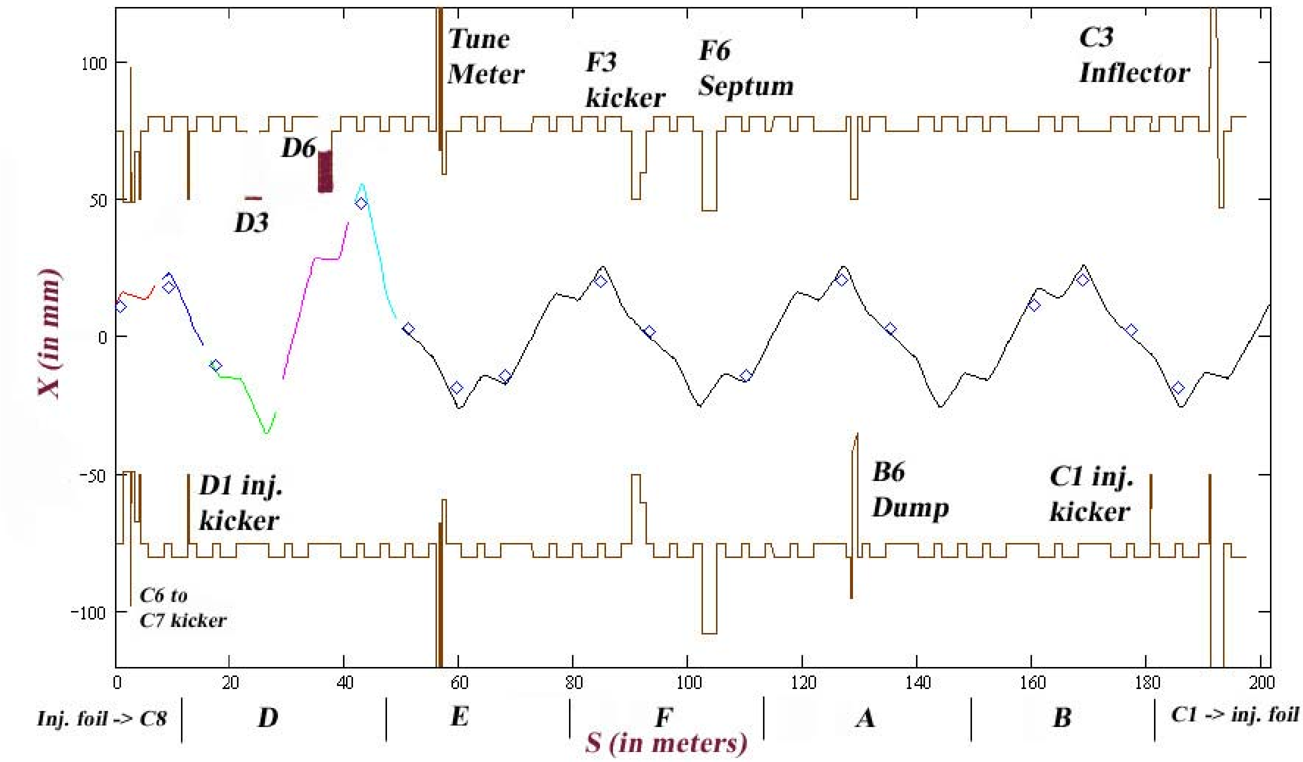

Figure 14: Modeled extraction bump with Q_h= 4.81, together with the "bump-on minus bump-off " difference orbit (points). Also shown is the Booster chamber aperture. The different colors indicate the different segments described in the text.

As can be seen from figure 4, the modeled bump seems to slightly overestimate the size of the bump and the residual orbit distortion as measured by the BPMs. If the overall strength of the bump magnets is reduced by a factor of 0.865, the error goes from 3.85 to 2.91. If the BPM data is scaled by a factor of 1.08, the error for Q=4.81 goes down from 3.85 to 3.25 mm. So, the improvement is greater with a tune adjustment, than with either an adjustment in the BPM calibration or in the bump strength. In general, the BPM positions were determined by eye from the graph, which probably introduced errors on the order of 0.1-0.2 mm.

Horizontal Tune	x(0), mm	x'(0), mm	Rms error (mm)
4.75	6.60	2.50	2.55
4.76	7.742	2.6958	2.05
4.77	8.64	2.886	1.82
4.78	9.68	3.097	1.97
4.79	10.80	3.328	2.50
4.80	11.80	3.559	3.17
4.81	12.64	3.7902	3.85
4.82	14.04	4.0843	5.02
4.83	15.80	4.44	6.53
4.84	17.80	4.84	8.27

Table I: Comparison of rms error between modeled orbit and BPM data as a function of the tune value used to calculate the bump orbit. The criterion for closing the orbit is the same as in the 4.81 case described above.

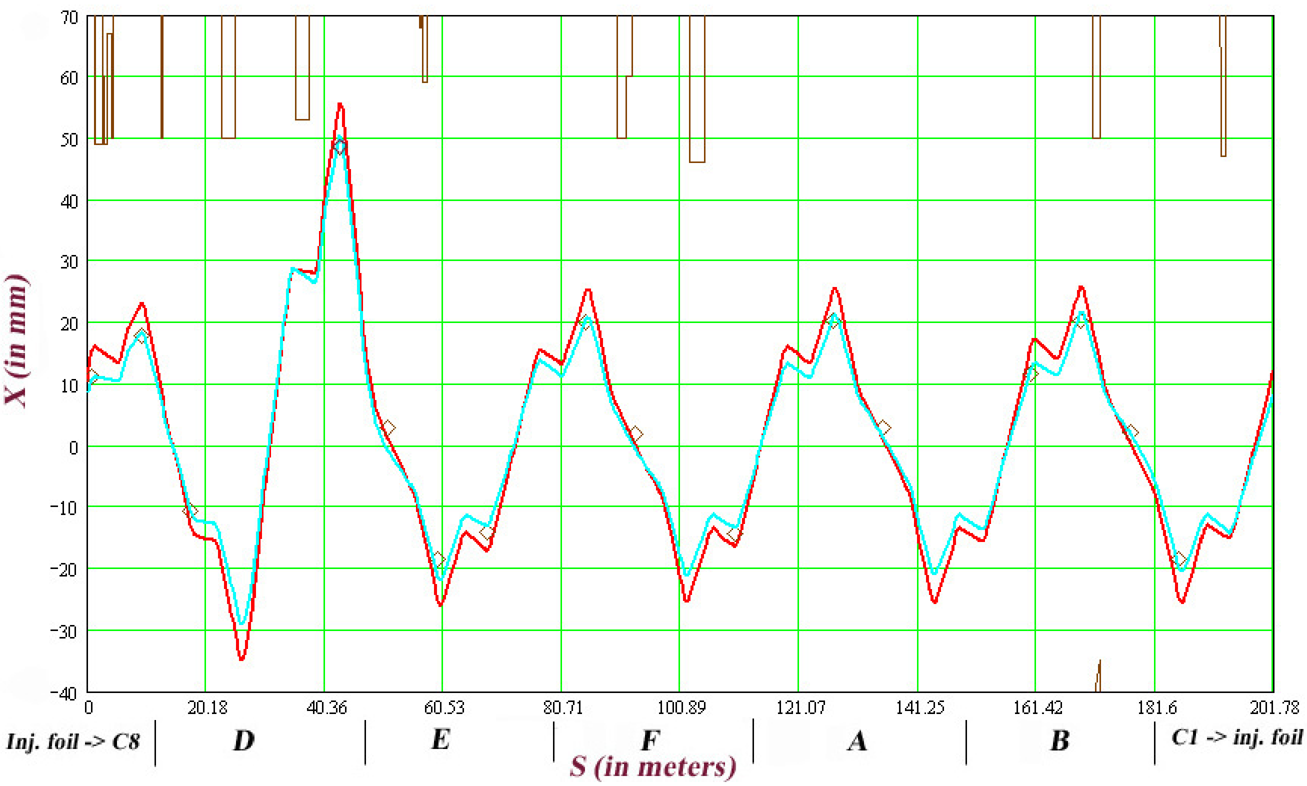

Figure 15: Bump Orbit with Q=4.77 (cyan) and Q=4.81 (red), together with BPM data for "bump-on minus bump-off" difference orbit.

The Dispersion Orbit

The dispersion (D) at each location in the ring is already given in the MAD output. So, the dispersion component of the orbit at a particular location is given by,

$$x_D(s) = \frac{D(s)}{D_{avg}} R_{avg} \ , \qquad \text{Equation 6}$$

where D_{avg} =1.555 m and R_{avg} = 10.0 mm.

Figure 16 shows the orbit with and without the dispersion component for a tunes of 4.77 and 4.81. The position at the s coordinate of the entrance to the D6 septum for either tune is nearly the same.

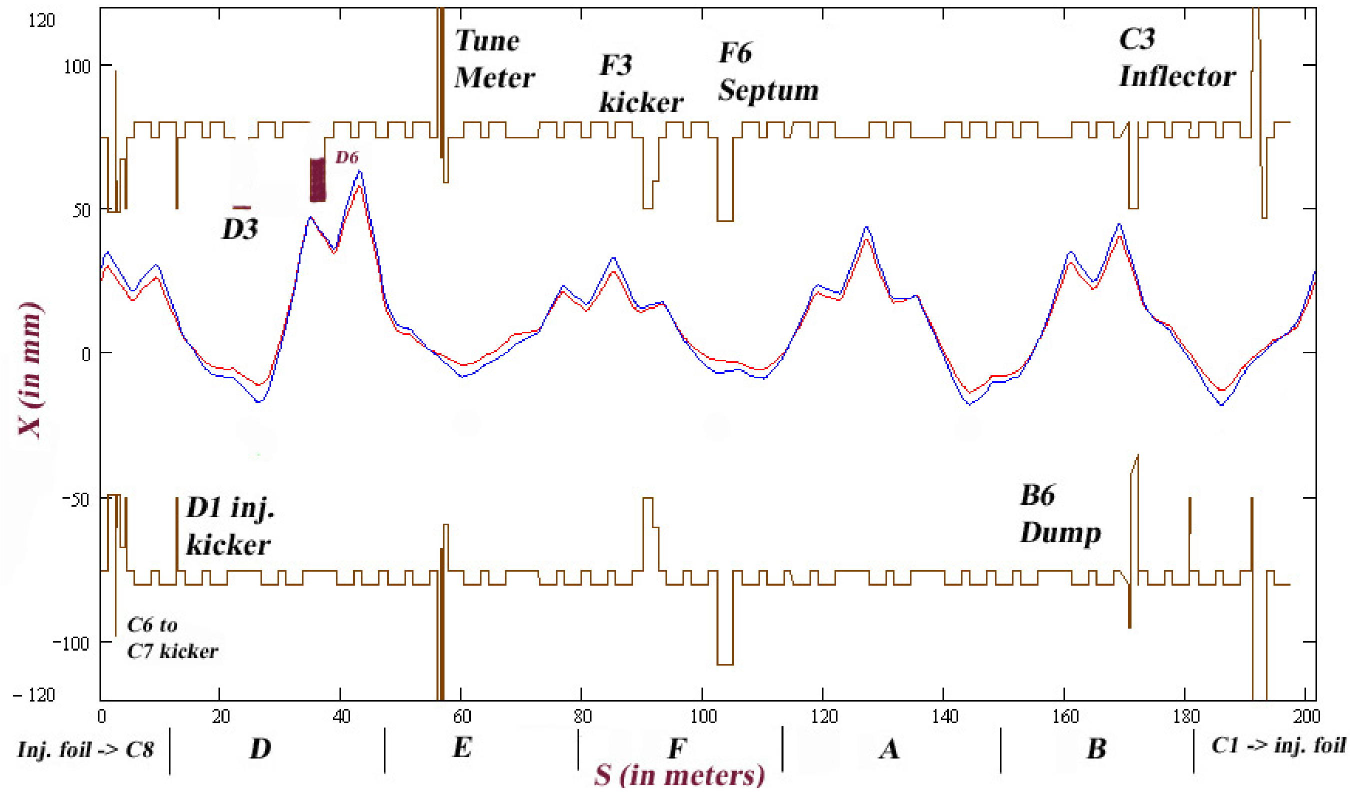

Figure 16: Modeled extraction bump plus dispersion orbit for Q=4.77 (red) and Q=4.81 (blue) (radius = 10.0 mm). Also shown is the Booster aperture.

Incorporating the Measured Orbit Distortion due to Dipole Field Errors into the Modeled Closed Orbit

The closed orbit as measured using the BoosterOrbitDisplay has two components: The component due to the effect of dispersion, and the component due to dipole field errors distributed around the ring. These two components are not measured separately, and are not independent of each other since when the radius is changed, the tune changes, and hence the non-dispersion part of the orbit also changes. In other words, the orbit that is measured when the beam is at zero radius is not the same as the component of the orbit that is not induced by dispersion when the radius is not zero. The approach used here to find the non-dispersion component is to subtract out the dispersion orbit described above from the data.

One of the aims of this paper is to try to produce a plausible extraction orbit, so it is important to find a fit to the bpm data that is a good representation of the orbit resulting from the dipole error field. Since, in this case, there are no dipole fields intentionally introduced to correct the extraction orbit, a reasonable expectation is that their distribution be random.

A formula that relates orbit distortions to the dipole errors that create them has the phase advance as the independent variable.[20] It has the following form,

$$x_{nd}(\psi) = \sqrt{\beta(\psi)} \sum_{n=0}^{\infty} \frac{Q^2}{Q^2 - n^2} \left[A_n \cos\left(2\pi n \frac{\psi}{Q}\right) + B_n \sin\left(2\pi n \frac{\psi}{Q}\right) \right] \quad , \quad \textbf{Equation 7}$$

where $x_{nd}(\psi)$ is the non-dispersion related component of the orbit, n is the harmonic number, ψ is the value of the phase advance in the MAD output for that s coordinate, A_n and B_n are the fourier coefficients of the dipole error field, Q is the tune in the MAD output, 4.80[21], $\beta(\psi)$ is the β function at ψ. This fitting is simplified by the fact that $\beta(\psi)$ is very nearly the same for each of the BPMs.[22]

Finding a Fit for the Data

Of course, the idea that because the dipole errors are random means that the magnitude of the dipole errors for each harmonic component is the same is not, in general, statistically valid. However, this may be a good approximation if there are many small and no large errors and the lattice is not prone to producing an error of one harmonic over another.[23] Regardless of these difficulties, and although on some level it's guesswork, the approach to fitting the data will use this idea because there is some reason to expect that the resulting fit will be consistent with the expected behavior of an orbit induced by random dipole errors. Specifically, the approach uses the assumption that the error field is evenly distributed over all harmonics between 1 and 9, and that harmonics greater than 9 are negligible. A small rms error would also suggest, though not conclusively, that the fit has some validity. A fit of the data with the same rms error which fit fourier harmonics and had no constraint associated with the response of the orbit harmonics to the tune would not be difficult to produce, but would be less likely to be a good representation of the actual orbit.

[20] E.D. Courant and H.S. Snyder, "The Theory of the Alternating Gradient Synchrotron", Ann. Phys. 3,1 (1958).
[21] Technically, ψ should be that associated with a tune of 4.81, and Q should be 4.81. However, since the value of ψ used for a tune different from 4.80 is just scaled by the appropriate factor it makes no difference if the above is modified to reflect the different tune.
[22] Although the β functions at the BPMs are not exactly the same, the effect on the fit of this distinction, as measured by the rms error, is negligible.
[23] Clearly, this is condition is not completely met. It is more than likely that there is one 'large' dipole error in the vicinity of D6.

Eleven parameters were varied iteratively to minimize the rms error. Nine of those are the phases of the nine harmonics, δ_n. The other two are the overall amplitude,C, and average value of x, called x_0. The rms error is given by

$$\sigma = \frac{\sqrt{\sum_{k=1}^{K}(Xf_k - Xm_k)^2}}{\sqrt{K}}, \qquad \textbf{Equation 8}$$

where k is the index that references the BPM, K (=16) is the total number of BPMs, Xf_k is the fitted value of x for the *kth* BPM, and Xm_k is the value of the modified data from the *kth* BPM. σ is minimized by varying Xf_k. Xf_k is given by the equation,

$$Xf_k = C\sqrt{\beta_{avg}}\sum_{n=1}^{9}\frac{Q^2}{Q^2-n^2}\cos\left(\frac{2\pi n\psi_k}{Q_{MAD}}+\delta_n\right)+x_0, \qquad \textbf{Equation 9}$$

where Q_{MAD} is the tune value in the MAD output, β_{avg} is the average value of the β function at the BPMs (12.738m), and C, δ_k, and x_0 are varied iteratively to minimize σ. When σ reached a value of 1.2 mm, the process was stopped.[24] Once values for these parameters are found, values for Xf at any ψ in the ring can be found using equation 9.

Figure 17 shows this fit in red, as well as an iterative fit without the tune constraint (σ=0.57 mm) in blue . The black lines show the locations of the D6 septum and the B2 PUE. The figure does not include the β function modulation, but the β functions at the B2 PUE and the entrance to the D6 septum are very close to the β function at the BPMs. So, differences in x at these locations are the same as they would be if it were modulated. It is clear from the figure that the choice of fit can effect the position at these two locations appreciably. In the case of D6, the orbit without the tune constraint is 3.5 mm further to the outside than the one with the tune constraint. In the B2 PUE case it is 7.9 mm further to the outside.

However, the fit using equation 9 does not address the fact that the x_0 component is in part the zeroth harmonic and in part a possible radial offset. In other words, a fit that includes a zeroth order component, includes the term[25],

$$\pm C^* [\beta_{avg}^{1/2} Q^2/(Q^2-0^2)] = \pm C^*\beta_{avg}^{1/2}.$$

Even though eq. 9 doesn't include this term, part of x_0 can be considered as composed of it. Given that C=0.277, it follows that $\pm C^*\beta_{avg}^{1/2} = \pm 0.949$ mm of x_0 is part of the complete fit that includes the zeroth order term. If it's negative it brings the average value of x in the fit closer to zero. Choosing it sign to be negative, the remaining 0.671 mm of x_0 can be treated as a radial offset. Consequently, a dispersion orbit for a radius of 9.329 mm is subtracted out of the raw BPM data, and the iterative fit is redone with x_0 in eq. 9 set to the initial fit amplitude ($-C^*\beta_{avg}^{1/2}$) and Q=4.81. The resulting fit

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[24] The values found for C, δ_k, and x_0 *are C=0.277,* δ_k *for k=1 to 9: (-0.22, -0.3, 0.51, 0.206, 0.103, 0.22, 0.49, 0.4, 0), and* x_0*=-1.62.*

[25] The sign of C is not known because, 'C' only represents the magnitude of the zeroth harmonic. In the case of other harmonics, the sign of C can effectively be switched by changing δ.

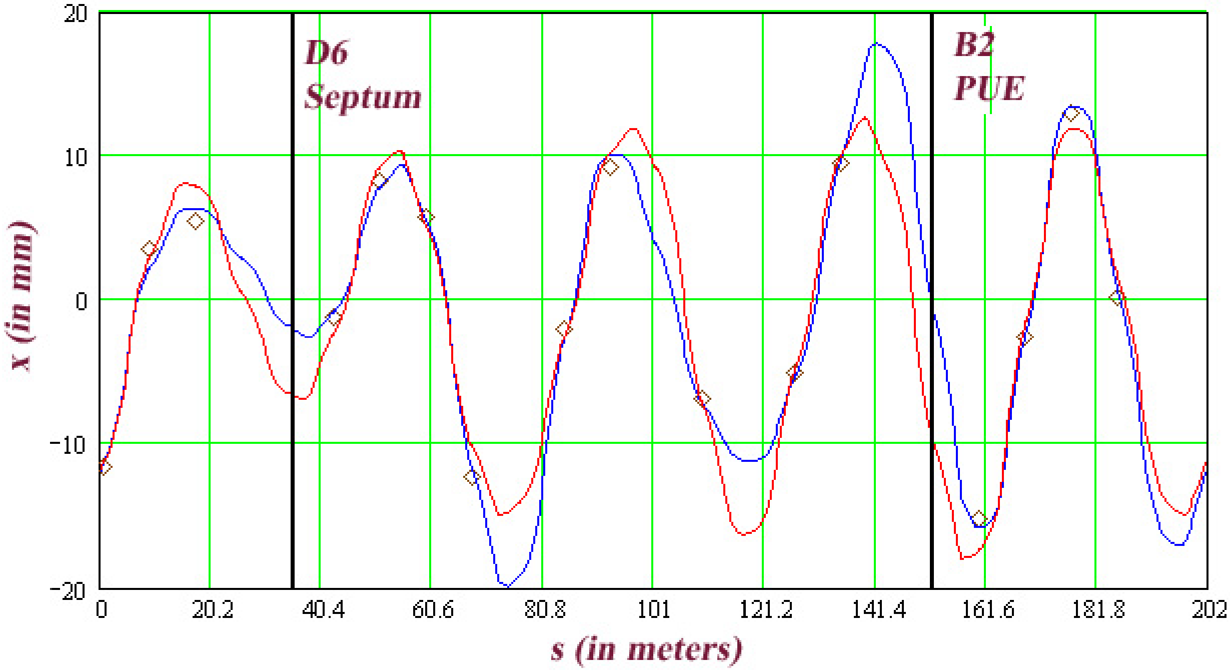

Figure 17: The fit of modified BPM data to equation 9 (red) and a fit without the tune constraint of eq. 9 (blue). Black lines are at the s coordinates of the entrance to the D6 septum and the B2 PUE. The data are with a dispersion orbit of 10.0mm removed, and has not been modulated by $\beta(\psi)^{0.5}$.

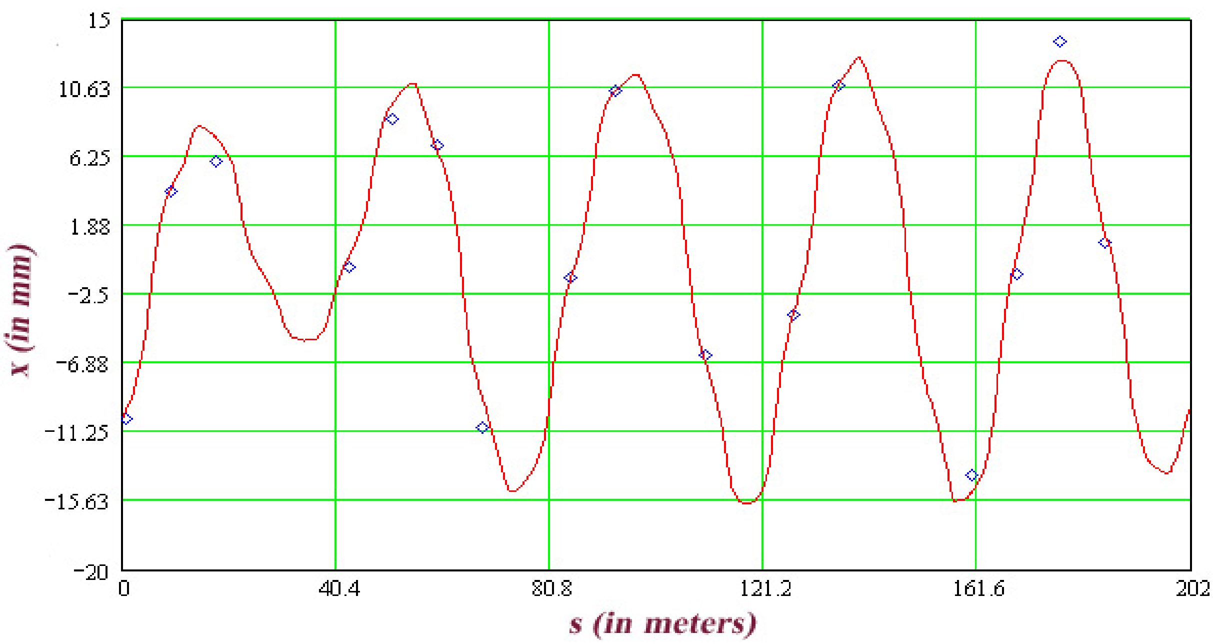

Figure 18: Final Modified BPM data and fit for Q=4.81. This fit does not include the $\beta(\psi)$ function modulation of eq. 7 (it follows eq. 9), and it does account for x_0. A dispersion orbit for a radius of 9.329 mm has been subtracted out of the BPM data.

has a σ of 0.857 mm. This fit is more plausible because it has a zeroth order component that scales with equation 7.[26] Figure 18 shows this final fit.

Figure 19 shows the extraction orbit with this fit included, as well as a dispersion orbit for a radius of 9.329 mm, the extraction bump, at tunes of 4.77 and 4.81. The figure also shows a closed orbit without the fit from the BPM data included, and a raw orbit that was taken with the bump on. The rms error between the orbit data and the Q=4.77 and Q=4.81 cases are 1.66 and 4.21 mm, respectively.

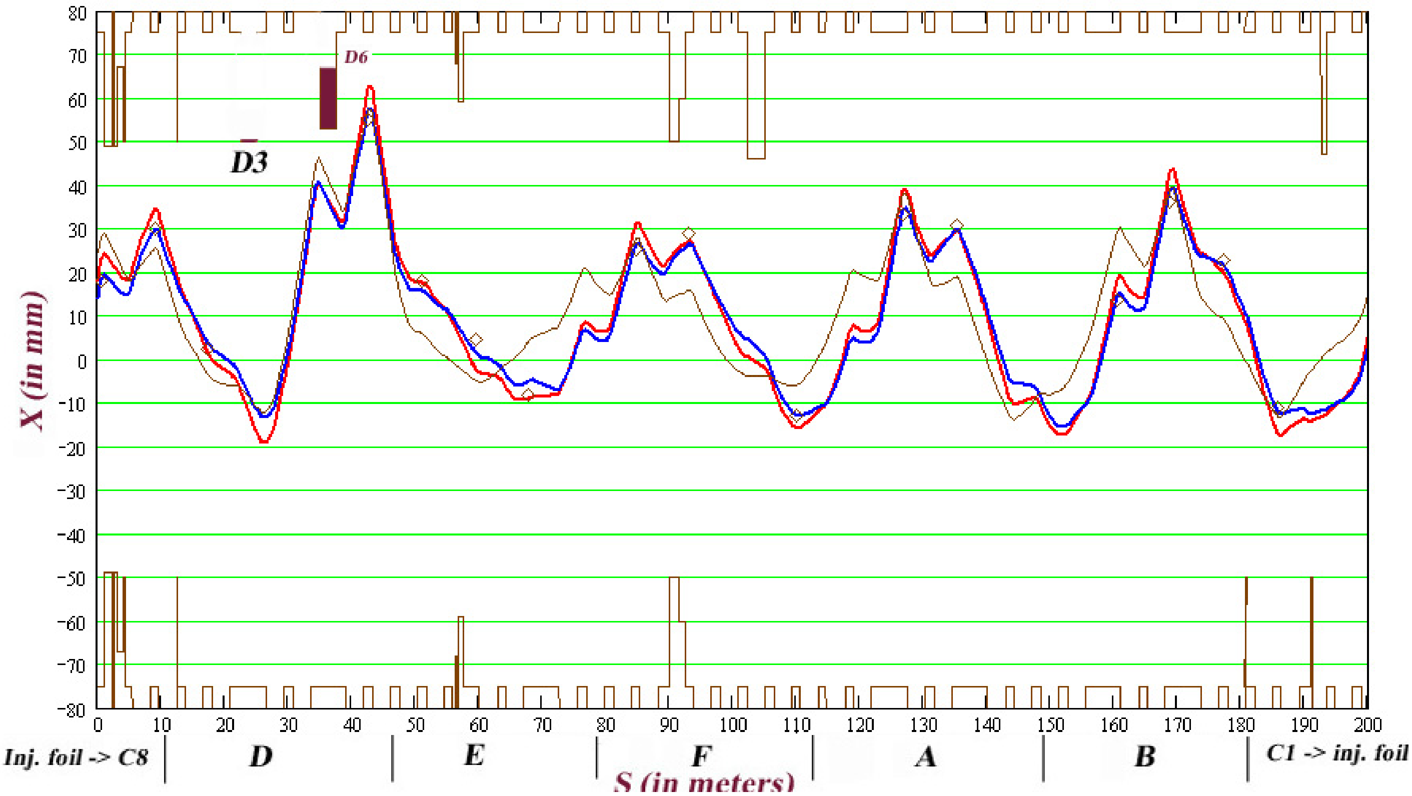

Figure 19: The extraction orbit with the dispersion (r=9.329mm), bump, and modified BPM data included for a tune of 4.81 (red) and 4.77 (blue). The brown curve is the closed orbit without the fit from the modified BPM data included. The brown points are BPM data from a raw extraction orbit.

The Kicks at F3

There are 4 modules in the F3 kicker that each deliver a kick, they are expected to be of equal amplitude. The kick is modeled in a similar way to how the extraction bump was modeled; just as there are 5 bump magnet kicks there are 4 kicker kicks[27], except here

[26] The values found for this fit *are* $C=0.2679$, δ_k *for* $k=1$ *to* 9: *(-0.205, -0.362, 0.492, 0.187, 0.111, 0.238, 0.434, 0.272, -0.177), and* $x_0=-C*\beta_{avg}^{1/2}=0.956$ mm.

[27] The β function ranges from 4.783 m at the first module (s=90.182m) to 8.697 m at the last module (s=91.831 m).

(x,x’) at the end of the ring given from propagating the 4 kicks from the kicker becomes the (x,x’) at the beginning of the ring on the next turn. This is propagated to the kicker, where it receives another 4 kicks, is propagated to the end of the ring again, and is then propagated from the beginning of the ring to the D6 septum. An initial value of 2.5 mrad is used for the kick. Figure 20 shows a trajectory (Q= 4.81) relative to the closed orbit on the three turns.

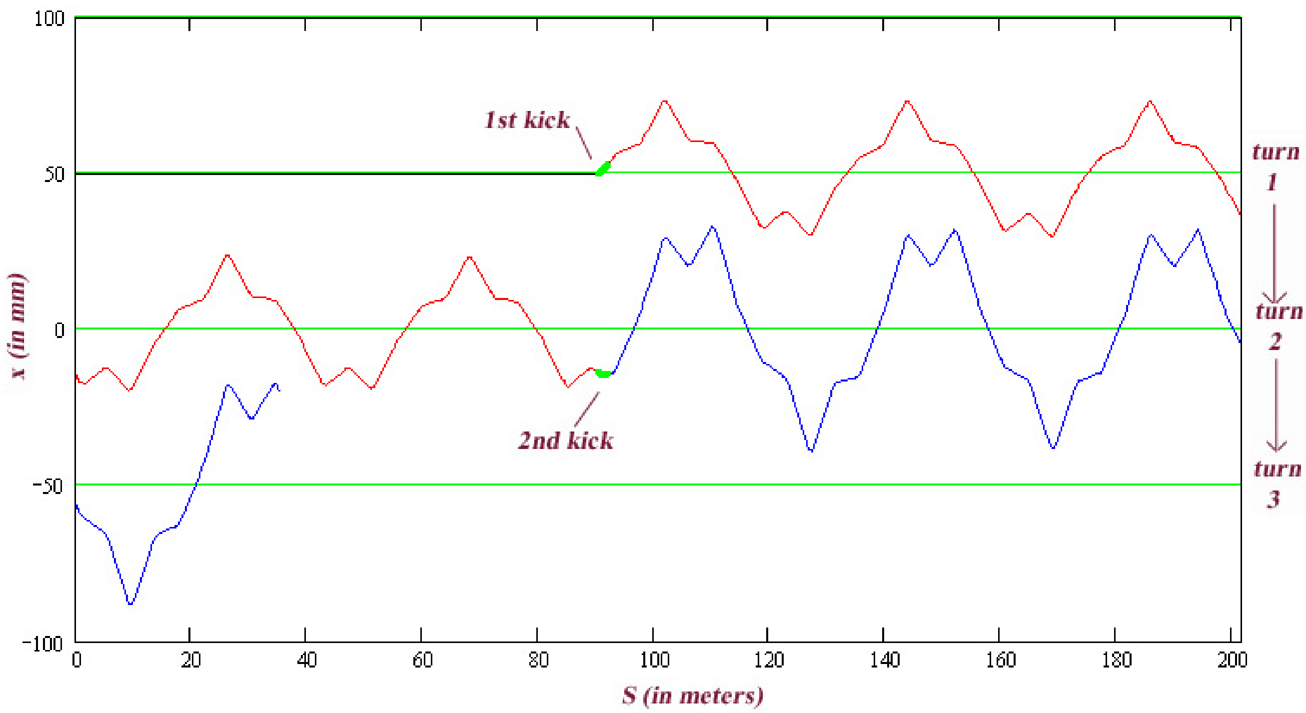

Figure 20: Model of kicks at F3. The red curve is the trajectory of the beam relative to the closed orbit after the first ‘kick’. The blue curve is that trajectory after the second ‘kick’. The location of the 2.5 mrad ‘kick’ is at about 91 meters and is shown in green. The tune is 4.81.

In addition to the BPM orbit, some information can be gleaned from the B2 PUE signal about the beam’s behavior after the kick. In particular, the expected position there can be compared with what was measured and shown in figure 7. In figure 7, positive voltage is towards the inside and zero voltage is likely to be near the centerline, but other than that there is no calibration. The closed orbit voltage is about 0.3 V, the voltage after the first turn is near 0 V, and the voltage after two kicks is about –0.55 V. The closed orbit at the B2 PUE (s=151.93 m) is expected to be –17.3 mm with Q = 4.81. To get the beam position to be zero after one kick, that kick needs to be about 4.8 mrad. Unfortunately such a kick causes the beam to scrape at D6 after the 1st kick and at F6 after the second kick. It also causes the position at B2 after the second kick to be considerably more than 0.55/0.3 times the closed orbit position (43.3 vs 31.9 mm).

So, there are problems with this scenario. First, the kick seems too large because it causes scraping at D6 and F6. Secondly, the agreement with what is seen at B2 isn’t particularly good. If a tune of 4.77 is tried instead, the closed orbit is –15.3 mm at B2, a kick of 3.6

mrad is required to bring the position to 0 mm after the first kick, and the position after the second kick is 30.2 mm (vs. the expected 28.1 mm). No scraping occurs after the second kick at F6 in this case, but some does occur at D6. Specifically, the position at D6 after the first kick would be 56.2 mm, and the inner edge of the septum is at 50 mm from R0.

Looking at it from another direction, what kick is required to give -0.55/.33 * x_{co} at B2 after the second kick? A kick of about 3.42 mrad for Q=4.77 and 3.88 mrad for Q=4.81. In the Q=4.81 case the position after the first kick is –3.2 mm and for Q=4.77 it is –0.8 mm. The 4.77 case matches the 1st kick position better, that is, it is closer to zero. But the D6 position after the first kick is still too large (55.4 mm).

Taking a step back, what is the kick required for the beam to pass from the Booster side of the septum to the position observed on the D6 flag? The D6 septum is 2.3 m in length, in the MAD output the end of the D6 half-cell is at 37.550 m, the septum's *s* coordinate is given as 36.34 m. This is likely its midpoint. Subtracting 1.15 m from 36.34 m gives 35.19 m. There is an entry in the MAD output at 35.13 m, this is very close to the entrance to the septum, which is where the D6 flag is located. The septum has an effective thickness of 15.2 mm, and its inner edge is 50 mm from R_0. The beam on the flag (figure 8) is about 13 mm wide and centered at 1.5 mm out from the flag's centerline. Therefore, the trajectory on the turn following the first kick needs to be 6.5 mm inside from the septum's outer edge if no loss is to occur there on that turn. The B2 PUE sum signal in figure 7 shows that no significant loss occurs at D6 on that turn. The beam's center on the flag is about 7 mm from the blue line in figure 8 that indicates the outer edge of the septum. That means the displacement due to the 2nd kick at the septum entrance needs to be greater than or equal to (6.5+15.2+7) mm = 28.7 mm.

The change in *x* at the entrance to the septum between the first and second kicks is a function of the tune. Table II shows that displacement for a kick of 3.42 mrad at different tunes. The maximum displacement occurs with a tune of 4.81. However, 4.77 has nearly the required displacement. So, Q=4.77, with a kick of 3.42 mrad meets the criteria pretty well, the modeled orbit is in good agreement with the BPMs and the B2 data is in good agreement with the expected behavior. The problem that remains is that the beam after the first kick will scrape at D6.

In order for the beam not to scrape at D6 after the first kick, the position of the beam at D6 after the first kick should be no further from R_0 than the position of the inside edge of the septum (50 mm) minus the width of the beam (6.5 mm), or 43.5 mm. The modeled position after the first kick is now 55.4 mm, so it would have to be 55.4 - 43.5 = 11.9 mm further to the inside to not scrape on this turn. This is a significant difference given the rms error (for Q=4.77) between the BPMs and the 'modeled' orbit (1.7 mm). However, it has been shown that discrepancies of this order can occur between different fits of the BPM data. As can be seen in figure 17 the modified BPM data fit without the tune constraint brings the closed orbit further to the outside by 3.5 mm, and the position at B2 varies by 7.9 mm between those two fits.

Tune	D6 C.O.	D6 after 1^{st} kick	D6 after 2^{nd} kick	D6 Diff.	B2 C.O.	B2 after 1^{st} kick	B2 after 1^{st} kick	σ
4.75	39.5	57.2	81.9	24.7	-14.5	0.9	27.6	2.1
4.76	40.0	56.5	83.0	26.5	-15.0	-0.0	27.8	1.7
4.77	40.1	55.4	83.3	27.9	-15.4	-0.9	27.9	1.7
4.78	40.2	54.4	83.4	29.0	-15.8	-1.9	27.7	2.0
4.79	40.4	53.3	83.1	29.8	-16.3	-2.9	27.3	2.7
4.80	40.3	52.0	82.2	30.3	-16.8	-3.9	26.7	3.5
4.81	40.0	50.4	80.7	30.3	-17.4	-5.0	25.8	4.2
4.82	40.1	49.2	79.3	30.1	-18.1	-6.2	24.6	5.4
4.83	40.4	48.2	77.7	29.5	-18.9	-7.6	23.0	6.9
4.84	40.8	47.3	75.8	28.5	-19.8	-9.0	21.1	8.7

Table II: D6 positions on closed orbit, after 1 kick (x_1), 2 kicks (x_2), and the difference (x_2-x_1) at the entrance to the D6 septum. Also, B2 position on closed orbit, after 1 kick, and after 2 kicks. σ is the rms error between the raw BPM orbit and the 'modeled' orbit at the BPM locations. All this is for a kick of 3.42 mrad.

Relative to R_0 the position on the flag, after the second kick, is (50+15.2+7) mm = 72.2 mm. A higher tune than 4.77 gives better agreement with this, and it also gives better agreement with the position after the first kick. For example, for a Q of 4.84, the position after the first kick (x_1) is 47.3 mm, still inside the Booster, and the position after the second kick (x_2) is 75.8 mm. However, the problem here is that the rms error between the modeled orbit and the BPM data is rather large (8.7 mm), and the B2 PUE data doesn't agree well with the model in that the position after the first kick is not close to zero, and the ratio $(x_2-x_1)/(x_{co}-x_1)$ is (21.1+9.0)/(-19.8+9.0)=-2.8 not -0.55/0.3=-1.8, where x_{co} is the B2 position on the closed orbit.

Revisiting the Modified BPM fit

In order to try to resolve the above inconsistency, a position at D6 was included in the fit, and it was set to be negative. If this fit conforms to equation 9 and the rms error and radius are reasonable, then there is no reason to prefer the prior fit to this one. The resulting fit has an rms error of 1.71 mm, and gives a radius of 8.48 mm. The rms error is larger than the other (0.85 mm), but is not unreasonable, and the radius is about a millimeter farther to the inside than the previous fit (9.33 mm) from the expected value of 10 mm.

This fit and the fit without D6 are shown in figure 21 for comparison. The position at B2 is largely unaffected, and that orbit around D6 with this fit is now more characteristic of the orbit around the rest of the machine. With this fit included in the modeled orbit (Q=4.77) the position after the first kick at D6 is 42.1 mm, which is far enough to the inside not to scrape. The positions at B2 are (x_{co}, x_1, x_2)=(-17.2, -2.7, 26.1) mm, and the rms error for this "modeled" orbit relative to the raw orbit with the bump on is 1.83 mm. The kick can be increased to bring the position after the first kick at B2 closer to zero, while also bringing the position on the D6 flag closer to the expected value (72.2 mm), and increasing the displacement between the first and second kicks. A kick of 3.58 mrad gives (x_{co}, x_1, x_2)=(-17.2, -2.0, 28.1) mm, has a D6 position after the first kick of 42.8 mm, and 72.1 mm after the second, with a displacement of 29.2 mm on the second kick at D6. All these values are reasonable. The ratio $(x_2-x_1)/(x_{co}-x_1)$ is -1.88 which close to the expected -1.8.

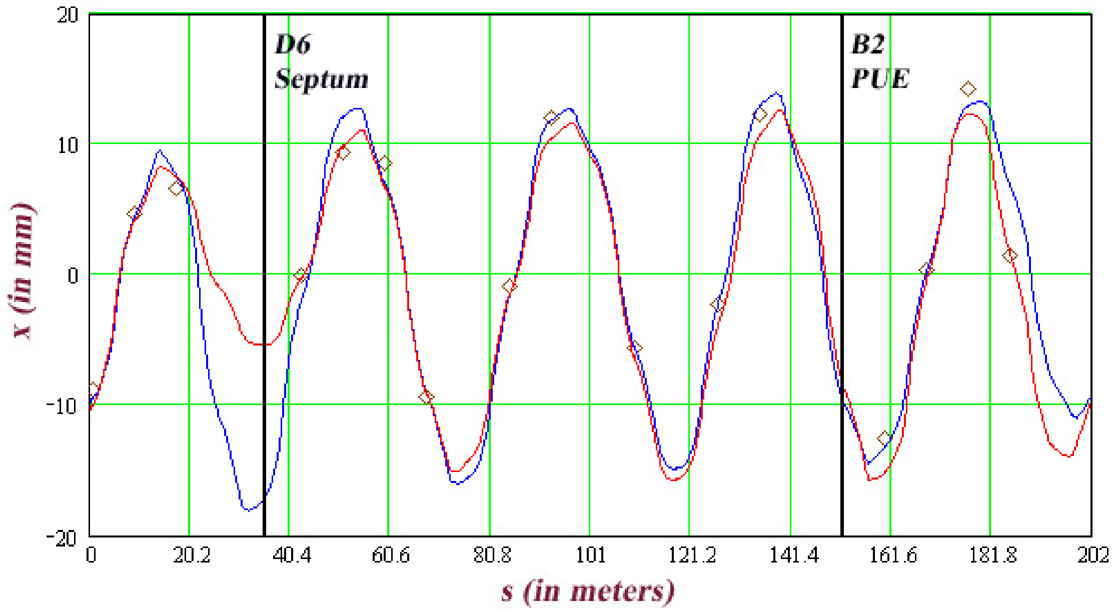

Figure 21: The previous fit of modified BPM data to equation 9 (red) and a fit with a constraint to make the position at D6 negative enough that it does not scrape after the first kick (blue). Black lines are at the s coordinates of the entrance to the D6 septum and the B2 PUE. The data are with a dispersion orbit of 8.48 mm removed, and has not been modulated by $\beta(\psi)^{0.5}$.

Figure 22 is a 'turn by turn' display of the different turns with the kicks superimposed onto the closed orbit, and figure 23 is a similar display, but without the turns offset from each other and with the apertures included. Also shown in these figures are the B2 positions expected from the trace in figure 7. That is, zero after the first kick, $-[0.3/(0.3+0.55)]*(x_2-x_{co})$ for the closed orbit, and $[0.55/(0.3+0.55)]*(x_2-x_{co})$ after the second kick. A raw orbit measurement with the bump on (shown in figure 13) is also included in this figure.

Summary

A consistent model for 2 kick FEB extraction to R line was developed using Twiss parameters, the dispersion function from a standard MAD output file, currents for the SEB bump magnets, orbit data from the BPMs, and turn by turn information from a PUE. However, this solution for FEB extraction is underdetermined for several reasons. First, the sampling of the orbit done by the OrbitDisplay program that is used to find orbit distortions related to dipole field errors is not adequate to determine the orbit with enough certainty at several key places, particularly D6, B2, and F6. That is, various solutions can be found, some of which are consistent with observations (like no scraping at D6 after 1st kick) and some of which that are not, and none of these potential solutions are obviously

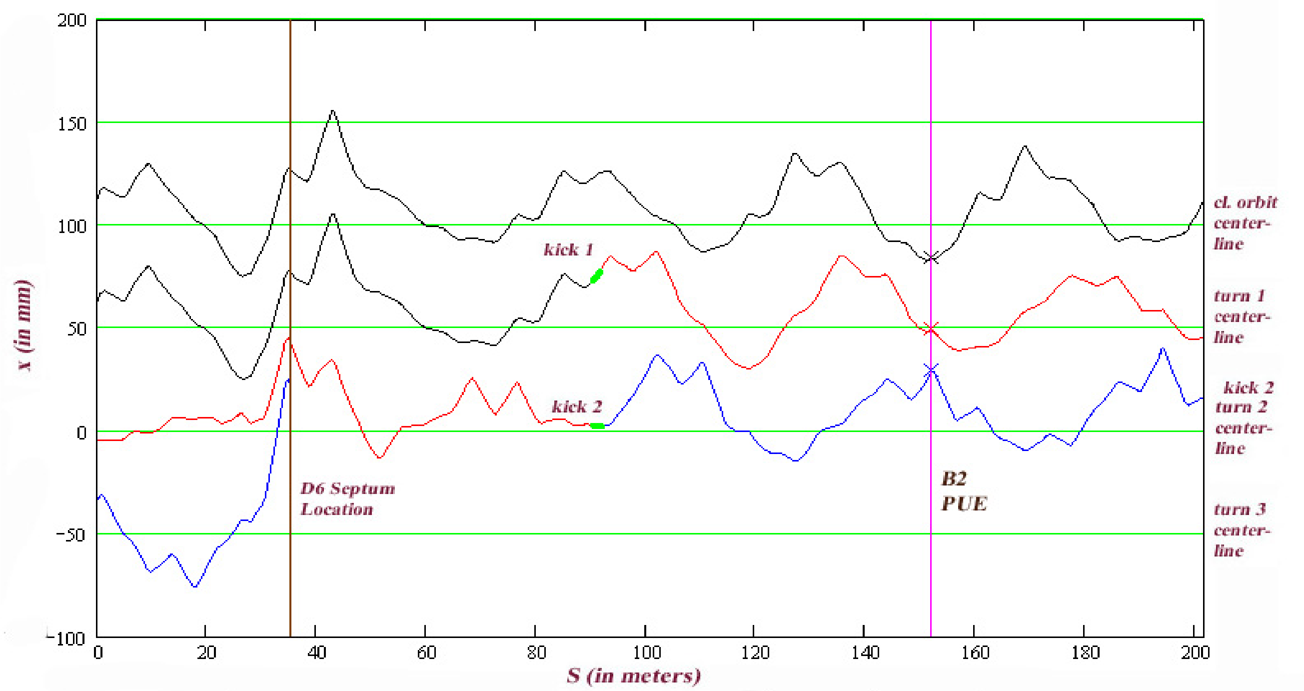

Figure 22: Model of kicks at F3 superimposed onto closed orbit using the modified BPM fit with D6. Spacing between grid lines is 50 mm. The centerline for each trace is marked on the right. The top trace is the closed orbit, the 2nd trace is the trajectory on the turn with the first kick, the 3rd trace is the trajectory on the turn with the 2nd kick, and the bottom trace is the next turn, where the beam exits at D6. Q=4.77 for this case, and the F3 kick is 3.58 mrad. Also shown is the location of the B2 PUE.

preferred over one another except in the sense that one gives a solution which is consistent with the observations. Second, the strength of the F3 kicker is somewhat uncertain. The value of 3.58 mrad that was arrived at is consistent with magnetic measurements, which give an expected value of 3.85 mrad, and the fact that the cap bank had likely discharged some of its charge before the F3 discharged. However, it is likely that other significantly different 'solutions', involving different kicker strengths, different fits to the dipole error field induced component of the BPM data, and a different tune could also exist.

The tune used for the component of the orbit associated with the bump, as well as the response from the kicks at F3 was chosen because it resulted in the smallest rms error between the measured and predicted orbits for both the raw and difference orbit cases. It was also chosen because the predicted behavior at B2 was close to what was observed.

There is some reason to believe that a scenario using the D3 septum to bend the beam might be preferable since the D3 septum is very thin (0.76 mm thick), and all the F3 kicker has to be able to do is displace the beam by twice its width (~13 mm) there. It's kick could still contribute to the displacement at D6 because, given its apparent strength at this rigidity, it need not be completely in phase with D3 to accomplish this.

Although, for a given F3 kick the total displacement at D6 is greater in the two kick case than in the one kick case, it does not seem that that is the most important consideration. What is obviously critical is the amount of displacement on the turn where the beam passes across the septum. For two kicks of 3.58 mrad, the maximum displacement is 31.8 mm when the tune is 4.80. But, for one kick, a maximum displacement of 31.8 mm also occurs, but for a tune of 4.54. In other words, a one turn kick with a bump at D6 that brings the beam very close to the septum, and possibly some other modifications to the orbit, should be able to accomplish the same thing. More kick is also probably available from the kicker, since the stop charge was not set up optimally.

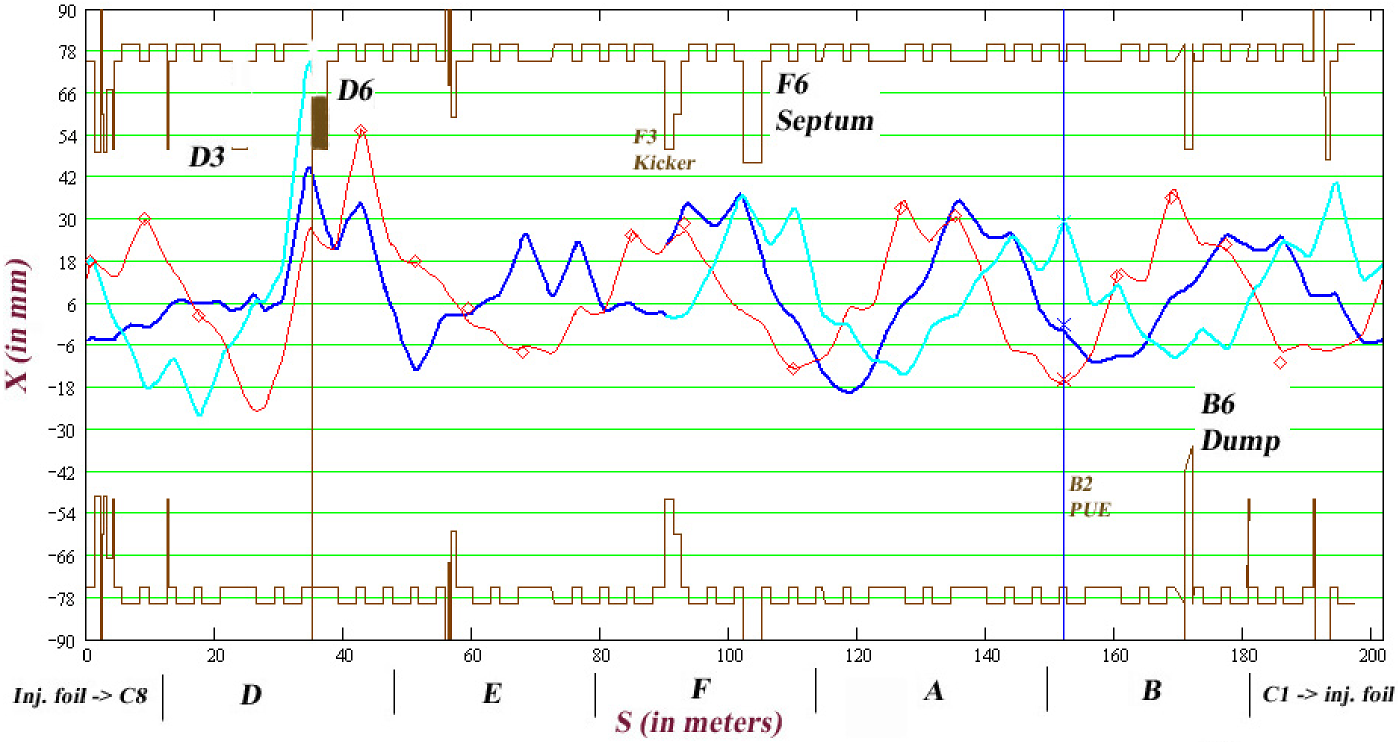

Figure 23: Two Kick FEB extraction to R line. The red trace is the modeled closed orbit, the blue trace is a modeled trajectory after the first kick, and the cyan trace is a modeled trajectory after the 2nd kick. Diamond shaped points are the BPM orbit measurement shown in figure 13, and X shaped points are the positions at B2 consistent with figure 7 (see text). The tune is 4.77 and the F3 kick is 3.58 mrad. The brown vertical line marks the entrance to the D6 septum, the blue vertical line the position of the B2 PUE.

In the case where one turn extraction was accomplished, the tune was around 4.40. This tune would give a one turn displacement of 25.8 mm. In this setup, there were losses around D6 before the kicker fired, and the beam was not kicked far enough to clear the septum (figure 11). Since the displacement at D6 after one kick with Q=4.54 is the same as the displacement after the second kick with Q=4.80, the question arises as to whether it is easy to create a bump at Q=4.54 that is large enough at D6 to move the closed orbit there as close to the septum as can be accomplished by the first kick in the two kick scenario, while still not scraping somewhere else. For a tune of 4.54, a 40.0 mm bump at D6 can be created, though it appears difficult to create one larger than that, and that bump causes scraping at F6.[28] A more standard bump, where the orbit is moved to the outside at D3 and D6, together with the use of the D3 septum might resolve this potential problem with a one-kick scenario.

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[28] The following magnet currents would give a bump at D6 of 40.0 mm for an orbit centered on R_0 and with no error field induced harmonics for a tune of 4.54: C7=-600A, D1=600A, D4=600A, D7=450A, E1=600A. The residuals for this bump are about $\pm$8 mm.