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Maximum intensity pion beams

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MAXIMUM INTENSITY PION BEAMS

The question of how many pions can be transported in a secondary beam should be answered in order to decide whether it is worth while to upgrade any existing beams or build a new high intensity beam. Specifically, is it worth while to build high field, superconducting quadrupoles in order to increase the yield? To answer this question, it is probably sufficient to study the acceptance of a quadrupole doublet under a point to parallel constraint in each plane, since this condition is not radically different from that in the first two quads of any secondary beam front end. Fox[1] has calculated data and curves for maximizing the acceptance of such doublets for quads of equal lengths and apertures. In the present work, the apertures and lengths were allowed to differ in order to capitalize on any possible gains in flux.

It is interesting to consider, first, the acceptance of a doublet composed of ideally thin quadrupoles ($s = 0$, $f_y = -f_x$) adjusted for the point to parallel condition in both planes. Since the angular acceptance of any quadrupole is importantly affected by the bending moment near the pole tips, let us characterize the thin lenses by their pole tip radii $\underline{a}$ and by their moments

$$M = e\int B_{\text{pole tip}}\, ds \text{ (in GeV/c)}$$

(M is positive for both lenses)

Let subscripts denote values for the first and second lenses. If the radii a_1 and a_2 also are the aperture radii, and the accepted solid angle is defined as

$$\Omega = \pi \text{ (Hor. half angle accepted)} \times \text{(Ver. half angle accepted)}$$

we find

$$\frac{1}{\pi}\left(\frac{a_1}{a_2}\right)\left(\frac{p}{M_1}\right)^2 \Omega = \left(\frac{a_1}{a_2}\,\frac{M_2}{M_1}\right)\left\{\frac{1-\left(\frac{a_1}{a_2}\,\frac{M_2}{M_1}\right)}{1+\sqrt{1-\left(\frac{a_1}{a_2}\,\frac{M_2}{M_1}\right)}}\right\} \tag{1}$$

From this, one concludes:

$$\frac{\partial\Omega}{\partial M_1} > 0 \rightarrow \Omega \text{ increases monotonically with } M_1 \tag{2}$$

$$\frac{\partial\Omega}{\partial(a_1/a_2)} < 0 \rightarrow \Omega \text{ decreases monotonically with } (a_1/a_2) \tag{3}$$

$$\frac{\partial\Omega}{\partial\left(\frac{a_1 M_2}{a_2 M_1}\right)} = 0 \text{ at } \left(\frac{a_1}{a_2}\,\frac{M_2}{M_1}\right) = 5/9 \tag{4}$$

Equation (2) shows that, to maximize Ω, one should set M_1 = maximum attainable value. From (3), we see that (a_2/a_1) should be as large as possible, and from (4), that $\left(\frac{a_1}{a_2}\,\frac{M_2}{M_1}\right) = 5/9$. At this point:

$$\Omega_{max} = \frac{4\pi}{27}\left(\frac{a_2}{a_1}\right)\left(\frac{M_1}{p}\right)^2 \tag{5}$$

Equation (5) shows that, for these lenses, the maximum solid angle is proportional to $\left(\frac{a_2}{a_1}\,M_1^{\,2}\right)$. The upper curve in Fig. 1 illustrates the variation of Ω with the parameter $\left(\frac{a_1}{a_2}\,\frac{M_2}{M_1}\right)$. However, real quadrupoles used for a doublet fall short of this ideal acceptance for large values of Ω, as the quadrupole focal lengths become comparable to their physical lengths. The family of lower curves in Fig. 1 was calculated for a doublet composed of three 8Q48's and one 12Q60.

$$S_1 = 156''$$
$$S_2 = 66''$$
$$\frac{a_2}{a_1} = 3/2$$

The excluded, nonphysical regions are where one or both of the drift lengths (ℓ_1 or ℓ_2, which are adjusted to produce the point to parallel condition) become negative.

In order to illustrate the gain in solid angle given by long quads and by higher pole tip fields, and to produce results directly comprehensible in terms of a high energy beam, the acceptance curves of Fig. 2 were calculated for two different pole tip fields, (12 kG as in our present room temperature quads, and 24 kG as might be realized by superconducting quads) and for four different momenta (10, 15, 20, and 25 GeV/c). For each field and momentum, the length S_2 of the second quad was varied to find the maximum solid angle at a given value of S_1, always under the point to parallel constraint. The aperture radii used were $a_1 = 4''$ and $a_2 = 6''$. The solid angle maxima so found are plotted as a function of the length S_1 of the first quad. The curves terminate approximately where the physical limits $\ell_1 \leq 0$ or $\ell_2 \leq 0$ are reached. Clearly, at any momentum, there is a first quad length beyond which it hardly pays to go. One also sees that as the limiting acceptance is approached, doubling the pole tip field will only double Ω instead of quadrupling it as at small values of Ω.

The benefits of increasing S_1 and/or B_{pole} are even more limited if one considers the flux actually captured. Figure 3 contains the results of integrating the π^- yield (as given by the Sanford-Wang formula[2]) over the acceptance regions of the same doublet configurations as in Fig. 2. This was done for a central production angle of 0 degrees, the best angle for high momenta. The momentum bite was taken as ± 1%. Here, the flattening out of the intensity as a function of S_1 is even more pronounced than that of Ω and doubling the field gives less than twice the flux in the region of the limiting intensity. All this is due, of course, to the rapid fall off of the pion production with angle and would be similar for other secondaries.

In a real, practical beam, one would position the quads with the values of ℓ_1 and ℓ_2 that satisfy the point to parallel constraint for $(B_{pole})_{max}$ at some p_{max}. Then, of course, the quads could be tuned downward to lower momenta as desired. Figure 4 shows the π^- fluxes obtainable (again for $\Delta p/p = \pm 1\%$) as a function of p for room temperature doublets optimized at 10, 15, 20, and

25 GeV/c. Real AGS quads were employed, with real spacings between. The solid angles and fluxes obtained at p_{max} are also indicated by on Figs. 2 and 3 at the value of total gradient length of the quads composing the first element of the doublet.

Figure 5 has similar curves constructed for hypothetical superconducting quads with B_{pole} = 24 kG. The solid angles and fluxes obtained at p_{max} are also shown on Figs. 2 and 3 by . Figures 6 and 7 are similar flux curves for π^+ production. For all of these flux curves, remember that they are in terms of <u>interacting</u> protons.

The last five figures indicate that one would normally stick to room temperature magnets as the acceptance limiting elements in the high radiation field near the target, since the higher fields produced by superconducting quads, even up to 36 kG or so, will allow flux gains of only a factor of two or less. I would judge that only if it is important to gain every particle possible <u>and</u> simultaneously to decrease power costs to the absolute minimum (by going superconducting in <u>every</u> important magnet in the beam line), would the difficulties of putting relatively delicate magnets near the target be attempted. Such a situation is not likely.

The quadrupole apertures used here are those of existing magnets. A flux gain of perhaps 1.5 could be achieved by doubling the apertures. However, this would require that the aperture of the momentum defining dipole (which would follow the doublet in any high performance beam) be substantially greater than in the standard AGS dipoles; (i.e. a gap 10" high by 24" wide would be needed, not including the sagitta). This marginal gain must again be balanced against the extra cost which would be incurred.

<u>References</u>

1. J. D. Fox, "Maximizing Solid Angle Acceptance in High Energy Beams", BNL Summer Study on AGS Utilization, 1970, BNL 16000.

2. J. R. Sanford and C. L. Wang, "Empirical Formulas for Particle Production in P-Be Collision Between 10 and 35 GeV/c", AGS Internal Reports JRS/CLW-1 and JRS/CLW-2, 1967.

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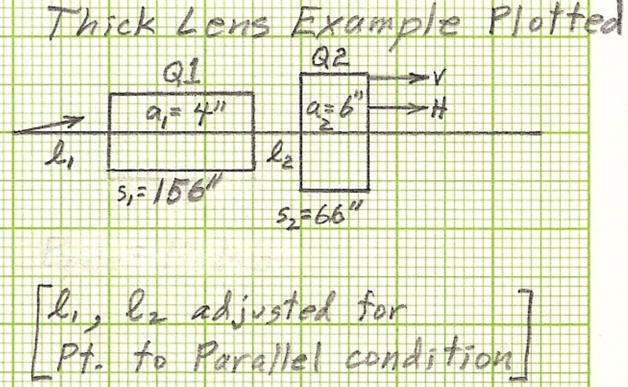
SOLID ANGLE VS POLE TIP BENDING MOMENT

FOR THIN LENS DOUBLET

AND TYPICAL THICK LENS DOUBLET

$$M = e\int B_{Pole}\, ds \ (\text{in GeV/c})$$



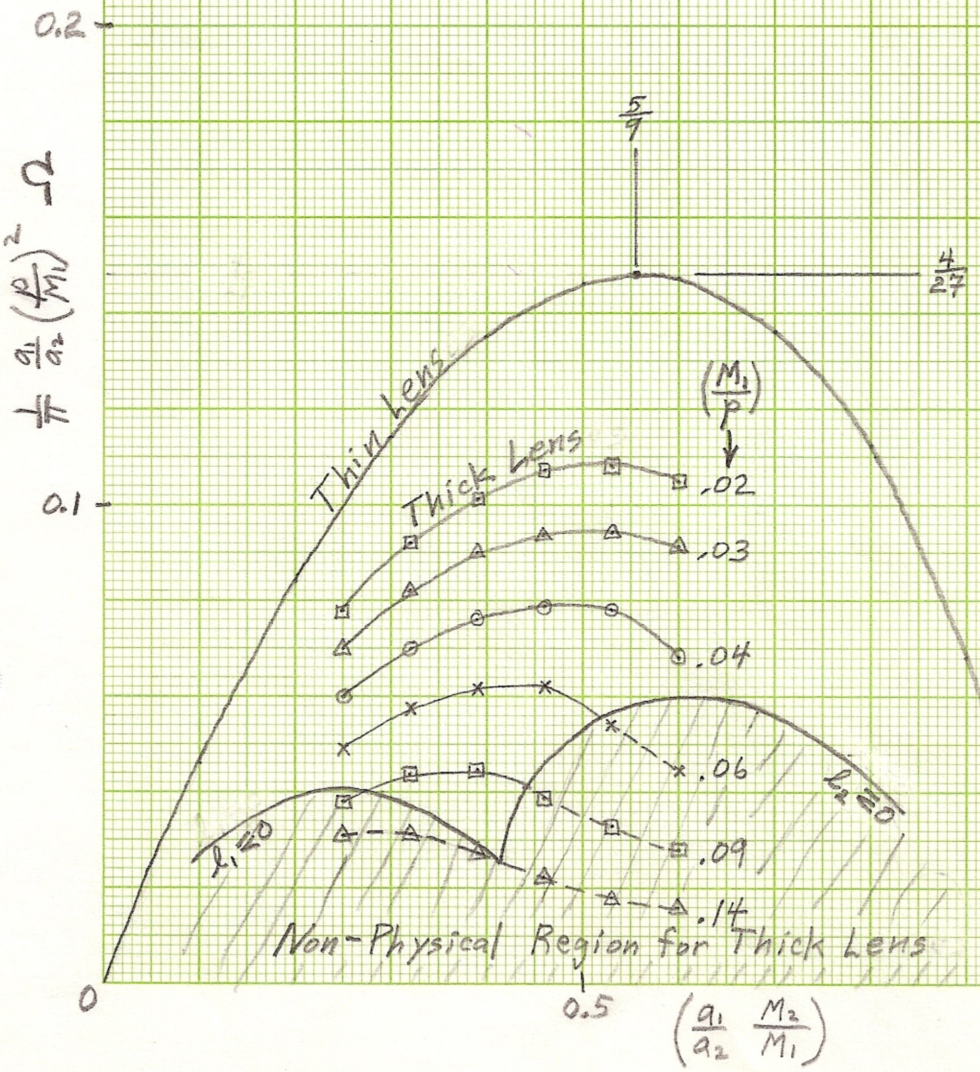


FIGURE 1

MAXIMUM SOLID ANGLE VS Q1 LENGTH

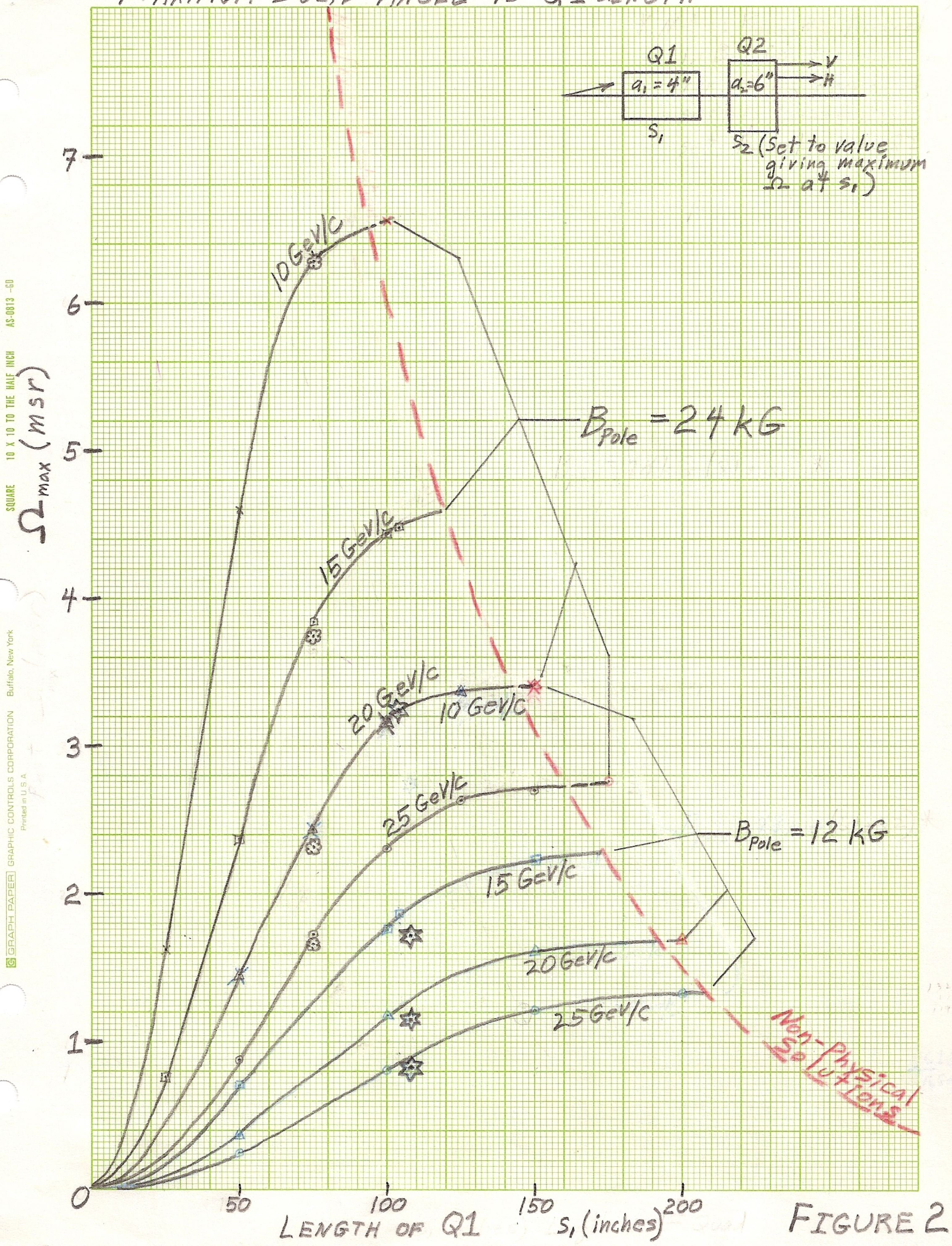


FIGURE 2

MAXIMUM π^- ACCEPTANCE vs Q1 LENGTH $\left(\frac{\Delta p}{p} = \pm 1\%\right)$

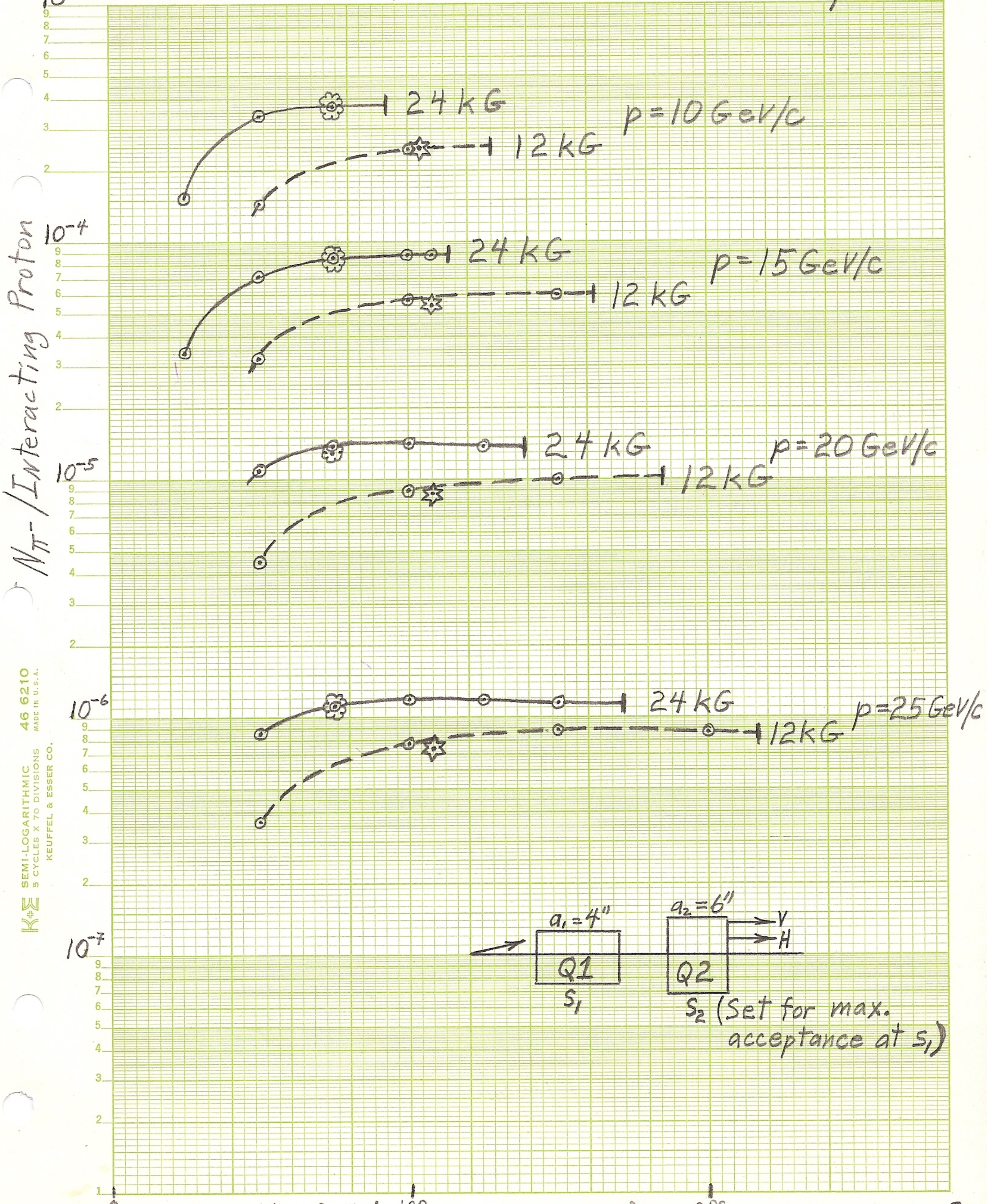
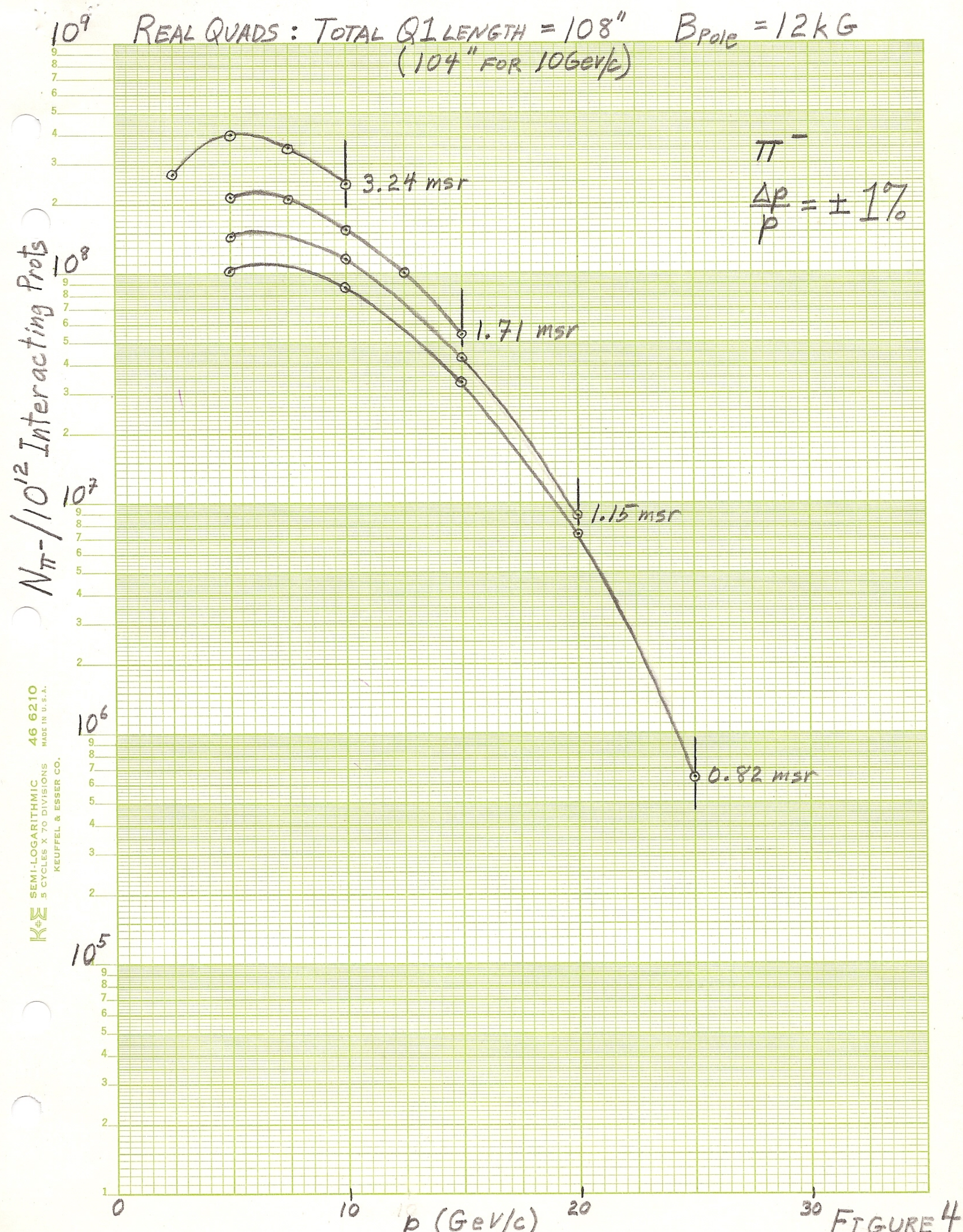
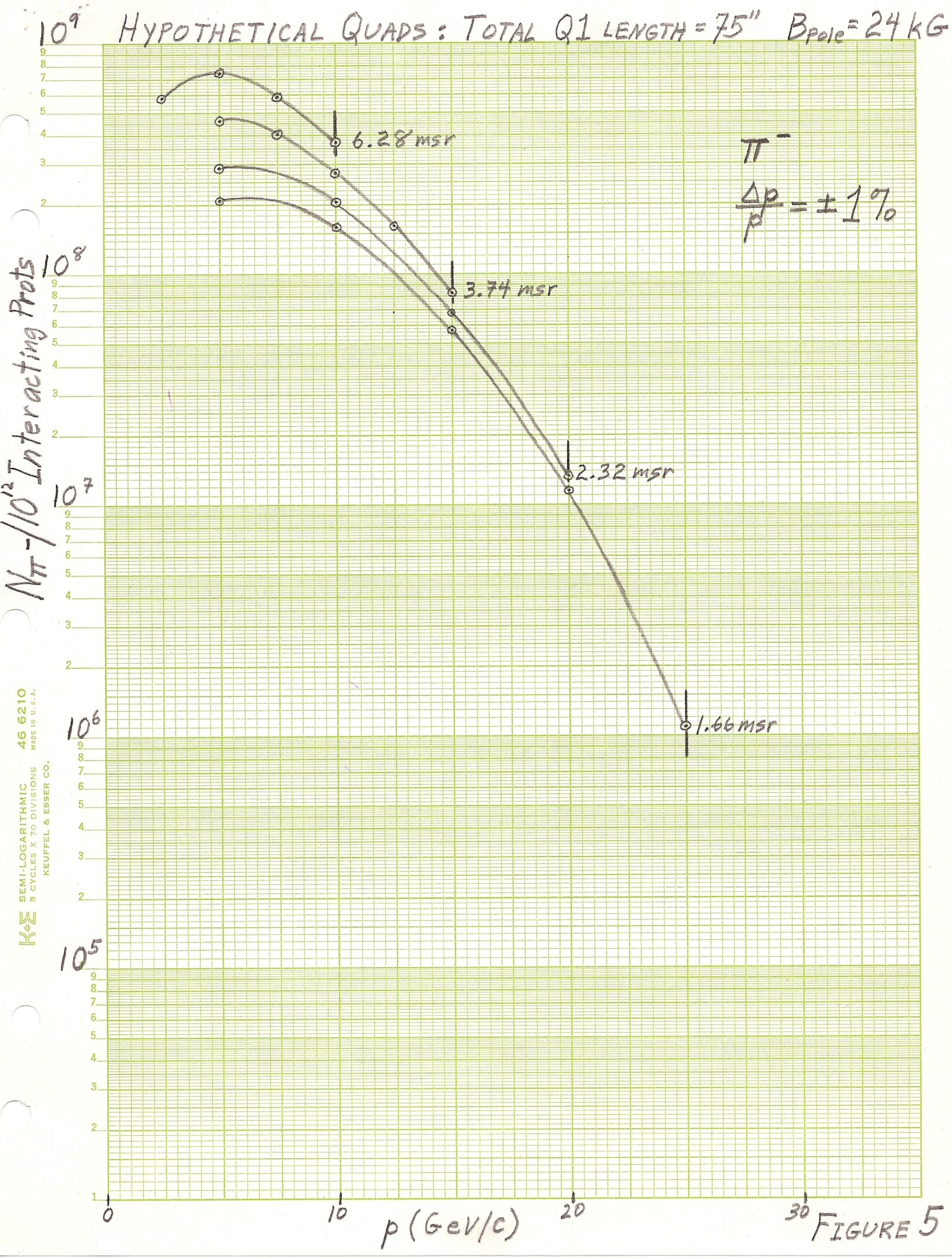


FIGURE 3



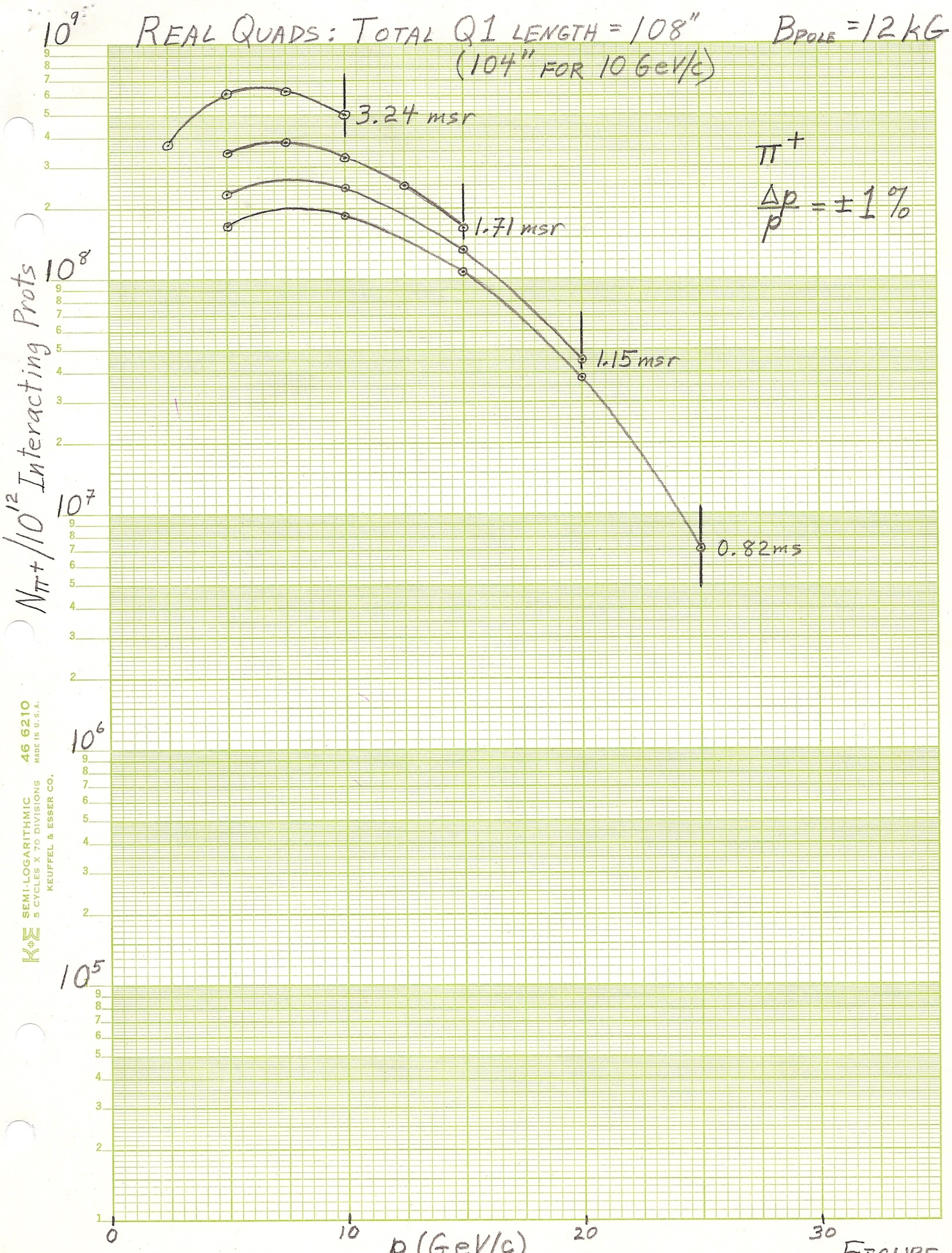
REAL QUADS : TOTAL Q1 LENGTH = 108" (104" FOR 10GeV/c)
B_{Pole} = 12 kG
π^-
$\frac{\Delta p}{p} = \pm 1\%$
3.24 msr
1.71 msr
1.15 msr
0.82 msr
$N_{\pi^-}/10^{12}$ Interacting Protons
10^9
10^8
10^7
10^6
10^5
0
10
20
30
p (GeV/c)
SEMI-LOGARITHMIC
5 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO.
46 6210
MADE IN U.S.A.

FIGURE 4



HYPOTHETICAL QUADS: TOTAL Q1 LENGTH = 75" B_{pole} = 24 kG
π^-
$\frac{\Delta p}{p} = \pm 1\%$
6.28 msr
3.74 msr
2.32 msr
1.66 msr
$N_{\pi^-}/10^{12}$ Interacting Protons
10^9
10^8
10^7
10^6
10^5
0
10
20
30
p (GeV/c)
K+E SEMI-LOGARITHMIC 5 CYCLES X 70 DIVISIONS KEUFFEL & ESSER CO.
46 6210 MADE IN U.S.A.

FIGURE 5



REAL QUADS: TOTAL Q1 LENGTH = 108"
(104" FOR 10 GeV/c)
B_{POLE} = 12 kG
π^+
$\frac{\Delta p}{p} = \pm 1\%$
3.24 msr
1.71 msr
1.15 msr
0.82 ms
$N_{\pi^+}/10^{12}$ Interacting Prots
p (GeV/c)

FIGURE 6


Hypothetical Quads: Total Q1 length = 75" B_{pole} = 24 kG
π^+
$\frac{\Delta p}{p} = \pm 1\%$
$N_{\pi^+}/10^{12}$ Interacting Prots
10^{10}
10^9
10^8
10^7
10^6
6.28msr
3.74msr
2.32msr
1.66msr
0
10
20
30
p (GeV/c)
K+E SEMI-LOGARITHMIC 46 6210
5 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

FIGURE 7