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Longitudinal Stochastic Cooling Update, FY05

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Abstract

The bunched beam stochastic cooling system for RHIC is different from typical coasting beam stochastic cooling systems in two significant ways. The RHIC system is designed to keep beam within the RF bucket, as opposed to cooling it to a small energy spread. Also, the fact that the beam is bunched is used to minimize the kicker power requirements. These considerations allow for a viable cooling system at moderate cost. This note summarizes the theory and implementation of cooling in RHIC and shows what we may expect during FY05.

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I. INTRODUCTION

The value of stochastic cooling in RHIC has been appreciated for some time [1, 2] but the implementation of a broad band kicker system with beam energies of 100 GeV/nucleon was daunting. Progress in this area has been made and a longitudinal cooling system for RHIC is under construction [3, 4].

The first test of cooling in RHIC is scheduled for FY05 and it is appropriate to summarize our understanding of how the system will work. In section 1 the physics of cooling is reviewed. In section 2 the particulars of the RHIC cooling system are discussed and Fokker-Planck simulations of the cooling system are presented.

II. COOLING PHYSICS

Before discussing the details of the cooling system it is appropriate to review the underlying physics [1, 2, 5, 6]. The RHIC system will operate in the $4-8$ GHz frequency range and the Schottky signals in this range appear smooth with little evidence of individual synchrotron sidebands. Additionally, the prominent synchrotron sidebands occur at low order close to the revolution line and will be suppressed by the cooling filter. For such a case it has been confirmed that the stochastic cooling rates for bunched beams are the same as those for coasting beams with the same peak current [2]. Therefore, consider a coasting beam with N particles. Let θ denote azimuth with respect to the accelerator, T_0 be the synchronous revolution period, $\omega_0 = 2\pi/T_0$, and t be time as measured in the laboratory frame. We use longitudinal coordinate θ and its time derivative $\omega = d\theta/dt$ as dynamical varibles. The phase space distribution function for the beam is decomposed as

$$\begin{aligned} \Psi(\theta,\omega,t) &\equiv \sum_{k=1}^{N} \delta[\theta-\theta_k(t)]\delta[\omega-\omega_k(t)] && (1)\\ &= \Psi_0(\omega,t)+\Psi_1(\omega,\theta,t)+\Psi_s(\omega,\theta,t), && (2) \end{aligned}$$

Equation 1 is the Klimontovich distribution for the beam [7] which is just writing down the trajectories of the individual particles. In equation (2) $\Psi_0(\omega,t)$ is the average coarse grained phase space distribution, which evolves over the cooling time scale. The fine grained Schottky distribution corresponding to Ψ_0 is Ψ_s, and Ψ_1 is a coarse grained perturbation. The maximum stable system gain is limited by the requirement that Ψ_1 be well behaved.

Also, the effect of Ψ_1 on the observed Schottky signal is refered to as signal suppression. This is the primary tool for adjusting the gain of the system.

Over short time scales we neglect the time dependence of Ψ_0. The linearized Vlasov equation for Ψ_1 is,

$$\frac{\partial \Psi_1}{\partial t} + \omega \frac{\partial \Psi_1}{\partial \theta} - \frac{q\eta\omega_0^2}{\beta^2 E_T} V_K(t)\delta_p(\theta - \theta_K)\frac{d\Psi_0}{d\omega} = 0, \tag{3}$$

where the stochastic cooling kicker is located at azimuth θ_K, the *total* voltage across the kicker is $V_K(t)$, $E_T = \gamma mc^2$ is the total energy of a synchronous particle, q is the charge per particle, $\delta_p(\theta)$ is the periodic delta function, η is the frequency slip factor, and $\beta = v/c$.

The kicker voltage depends on the current at the pickup, located at θ_P. Consider the response of a filter cooling system at a single frequency $\tilde{\omega}$ so that $V_K(t) = \hat{V}\exp(-i\tilde{\omega}t)$. Then

$$\hat{V} = -Z_T(\tilde{\omega})[I_s(\tilde{\omega}, \theta_P) + I_1(\tilde{\omega}, \theta_P)], \tag{4}$$

where $Z_T(\omega)$ is the transfer impedance of the cooling system, $I_1(\tilde{\omega}, \theta_P)$ is the perturbation current at the pickup, and $I_s(\tilde{\omega}, \theta_P)$ is the Schottky current at the pickup. The minus sign is chosen so that a positive resistance results in energy loss. The plan is to take the Schottky current as the driving term for a beam transfer function and calculate the resulting $\hat{V}$ self-consistently including the effect of Ψ_1. This is conceptually the same as calculating the effect of high intensity on the stability of an RF system.

To proceed consider the identity

$$\delta_p(\theta) = \sum_{k=-\infty}^{\infty} \exp(ik\theta)/2\pi,$$

and set

$$\Psi_1(\omega, \theta) = \sum_{k=-\infty}^{\infty} f_k(\omega)\exp(ik[\theta - \theta_K] - i\tilde{\omega}t + \epsilon t). \tag{5}$$

The positive constant ϵ has been included to insure a causal response [8] and will be set to zero at the end. It is worth noting that we are not worried about exponential damping due to the feedback system, so the difficult parts of Landau's original work are not needed.

Substituting equation (5) into equation (3) yields the set of equations

$$(\epsilon - i\tilde{\omega})f_n(\omega) + i\omega n f_n = \frac{\hat{V}\eta q\omega_0^2}{2\pi\beta^2 E_0}\frac{d\Psi_0(\omega)}{d\omega}, \tag{6}$$

which gives

$$f_n(\omega) = \frac{1}{\epsilon + i(n\omega - \tilde{\omega})} \frac{\hat{V}\eta q \omega_0^2}{2\pi\beta^2 E_0} \frac{d\Psi_0(\omega)}{d\omega}. \tag{7}$$

Integrating (7) over ω and normalizing appropriately yields the perturbation current for each harmonic.

$$I_n = \frac{\hat{V}\eta q \omega_0^2}{2\pi\beta^2 E_0} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} \frac{d\omega \bar{I}}{\epsilon + i(n\omega - \tilde{\omega})} \frac{d\hat{\Psi}(\omega)}{d\omega}, \tag{8}$$

where $\bar{I}$ is the average beam current, the full frequency spread of the beam is $\pm\Delta\omega$, and the normalized frequency distribution $\hat{\Psi}(\omega)$ satisfies $\int d\omega \hat{\Psi}(\omega) = 1$. The perturbation current at the pickup is then given by

$$I_1(\tilde{\omega}, \theta_P) = \sum_{m=-\infty}^{\infty} I_m e^{im(\theta_P - \theta_K)}. \tag{9}$$

Now set $I_1(\tilde{\omega}, \theta_P) = Y_b \hat{V} B(\tilde{\omega})$ where

$$B(\tilde{\omega}) = \omega_0^2 \sum_{k=-\infty}^{\infty} \lim_{\epsilon\to 0^+} \int d\omega \frac{e^{ik(\theta_P - \theta_K)}}{\epsilon + i(k\omega - \tilde{\omega})} \frac{d\hat{\Psi}(\omega)}{d\omega}, \tag{10}$$

is the beam transfer function with

$$Y_b = \frac{\bar{I} q \eta}{2\pi\beta^2 E_0}. \tag{11}$$

A closed form for the infinite sum is given in Appendix A. With these definitions equation (4) becomes

$$\hat{V} = -Z_T(\tilde{\omega})[I_s(\tilde{\omega}, \theta_P) + \hat{V} Y_b B(\tilde{\omega})],$$

yielding

$$\hat{V} = \frac{-Z_T(\tilde{\omega}) I_s(\tilde{\omega}, \theta_P)}{1 + Z_T(\tilde{\omega}) Y_b B(\tilde{\omega})} \tag{12}$$

$$\equiv -Z_D(\tilde{\omega}) I_s(\tilde{\omega}, \theta_P), \tag{13}$$

where we have defined the "dressed" impedance, $Z_D(\omega)$. Equation (12) relates the total kicker voltage to the Schottky current, including the coherent effects of the beam. Since the circuitry between the pickup and the kicker is linear the supression by $1 + Z_T(\tilde{\omega}) Y_b B(\tilde{\omega})$ is present in the signal at the pickup, upstream of any filtering. Also, since the derivation nowhere relied on V_s being a Schottky signal, the same suppression will be operative in beam transfer function measurements.

To understand how including the coherent response limits the system gain consider the denominator on the right side of equation (12). If there is a value of $\tilde{\omega}$ where the denominator is very small then a small I_s can yield a large $\hat{V}$. When the gain is just right so that $1 + Z_T(\tilde{\omega})Y_b B(\tilde{\omega}) = 0$, one can have an arbitrary $\hat{V}$ for no drive at all. This is the stability limit. If the gain is increased one gets exponentially growing $\hat{V}$ for no drive. To check whether a given gain is stable one makes a parametric plot of $Z_T(\tilde{\omega})Y_b B(\tilde{\omega})$ on the complex plane, as a function of $\tilde{\omega}$. If this curve does not encircle -1, the system is stable.

To proceed we must construct the equations of motion for Ψ_0[9]. To do this consider a time interval Δt that is large compared to the revolution period but small compared to the cooling time. Let $T(\omega, \Omega)d\Omega$ be the probability that a particle with revolution frequency ω at time t has a revolution frequency in the interval $[\omega+\Omega, \omega+\Omega+d\Omega]$ at $t+\Delta t$. Since particles are conserved $\int T(\omega, \Omega)d\Omega = 1$. Also, since Δt contains a large number of independent kicks we may use the central limit theorem and put

$$T(\omega, \Omega) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\Omega - \bar{\Omega})^2/2\sigma^2}. \tag{14}$$

The particle motion during Δt is the combination of a smooth force and a random walk. Hence $\bar{\Omega} = \tilde{F}(\omega, t)\Delta t + o(\Delta t)$ and $\sigma^2 = 2\tilde{D}(\omega, t)\Delta t + o(\Delta t)$. [14]

Applying the conditional probability to the phase space distribution gives,

$$\Psi_0(\omega, t + \Delta t) - \Psi_0(\omega, t) = \int d\Omega \left[T(\omega - \Omega, \Omega)\Psi_0(\omega - \Omega, t) - T(\omega, \Omega)\Psi_0(\omega, t)\right]. \tag{15}$$

Assuming Δt is short compared to the time interval over which Ψ_0 changes appreciably and expanding the integrand in a Taylor series in Ω yields

$$\Delta t \frac{\partial \Psi_0}{\partial t} = \frac{\partial}{\partial \omega}\left[-\Psi_0(\omega, t)\int d\Omega T(\omega, \Omega)\Omega + \frac{\partial}{\partial \omega}\left\{\Psi_0(\omega, t)\int d\Omega T(\omega, \Omega)\Omega^2/2\right\}\right] + o(\Delta t) \tag{16}$$

$$= \frac{\partial}{\partial \omega}\left[-\bar{\Omega}\Psi_0(\omega, t) + \frac{\partial}{\partial \omega}\left\{\frac{\sigma^2}{2}\Psi_0(\omega, t)\right\}\right] + o(\Delta t). \tag{17}$$

Dividing out Δt and taking $\Delta t \to 0$ yields

$$\frac{\partial \Psi_0}{\partial t} = \frac{\partial}{\partial \omega}\left[-\tilde{F}(\omega, t)\Psi_0(\omega, t) + \frac{\partial}{\partial \omega}\left\{\tilde{D}(\omega, t)\Psi_0(\omega, t)\right\}\right]. \tag{18}$$

To calculate $\tilde{D}$ and $\tilde{F}$ consider the dressed wake potential

$$W_D(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z_D(\omega) e^{-i\omega t} d\omega, \tag{19}$$

where the dressed impedance is defined in equation (12) and the integral is well defined for stable cooling. Using the dressed wake potential the kicker voltage as a function of time is given by

$$V_K(t) = \int_0^{\infty} I_s(\theta_P, t - \tau) W_D(\tau) d\tau, \tag{20}$$

where the Schottky current is due to N uncorrelated particles.

$$I_s(\theta, t) = \sum_{k=1}^{N} \frac{q\omega_0}{2\pi} \sum_{m=-\infty}^{\infty} e^{im[\theta - \theta_k(t)]} \tag{21}$$

Assume that ω_k varies slowly compared to the decay time of $W_D(t)$ so that $\theta_k(t - \tau) \approx \theta_k(t) - \tau\omega_k(t)$. This gives

$$V_K(t) = -\sum_{k=1}^{N} \sum_{m=-\infty}^{\infty} \frac{q\omega_k(t)}{2\pi} e^{im[\theta_P - \theta_k(t)]} Z_D[m\omega_k(t)], \tag{22}$$

The voltage is localized in θ so the particle energy changes in discrete steps. Assuming no overlap of the Schottky bands and dropping the fast terms in the force yields the single particle equations of motion $d\theta_j/dt = \dot{\theta}_j = \omega_j(t)$ and

$$\begin{aligned}
\dot{\omega}_j &= \tilde{C} \sum_{k=1}^{N} \sum_{m=-\infty}^{\infty} e^{im[\theta_P - \theta_K + \theta_j(t) - \theta_k(t)]} \omega_k(t) Z_D[m\omega_k(t)], \\
&= \tilde{C} \sum_{m=-\infty}^{\infty} e^{im[\theta_P - \theta_K]} \omega_j(t) Z_D[m\omega_j(t)] \\
&+ \tilde{C} \sum_{k \neq j} \sum_{m=-\infty}^{\infty} e^{im[\theta_P - \theta_K + \theta_j(t) - \theta_k(t)]} \omega_k(t) Z_D[m\omega_k(t)] && (23) \\
&= F(\omega_j) + \sum_{k \neq j} G[\theta_j - \theta_k, \omega_k] && (24)
\end{aligned}$$

where $\tilde{C} = \eta q^2 / (T_0^2 \beta^2 E_T)$ and the force has been decomposed into a self-force and the force due to the other particles. Since $\tilde{D} = O(\tilde{C}^2)$ we need first order perturbation theory to calculate $\tilde{D}$. For $\tilde{F}$ the leading term is first order in $\tilde{C}$ but a second order term is also present. To start the expansion set $\theta_{j0}(t) = \omega_{j0}t + \psi_j$ then

$$\omega_{j1}(t) = F(\omega_{j0})t + \sum_{k\neq j} \int_0^t dt_1 G[\theta_{j0}(t_1) - \theta_{k0}(t_1), \omega_{k0}], \tag{25}$$

$$\theta_{j1}(t) = \int_0^t dt_1 \omega_{j1}(t_1), \tag{26}$$

where $\theta_m(t) \approx \psi_m + \omega_{m0}t + \theta_{m1}(t)$. Next expand all terms to second order in $\tilde{C}$ and get the total force on particle j,

$$\begin{aligned} \dot{\omega}_{j,all} &= F(\omega_{j0} + \omega_{j1}(t)) + \sum_{k\neq j} G[\theta_{j0}(t) + \theta_{j1}(t) - \theta_{k0}(t) - \theta_{k1}(t), \omega_{k0} + \omega_{k1}(t)]. & (27) \\ &\approx F[\omega_{j0} + \omega_{j1}(t)] + \sum_{k\neq j} G[\theta_{j0}(t) - \theta_{k0}(t), \omega_{k0}] + \omega_{k1}(t) G_\omega[\theta_{j0}(t) - \theta_{k0}(t), \omega_{k0}] \\ &+ \sum_{k\neq j} [\theta_{j1}(t) - \theta_{k1}(t)] G_\theta[\theta_{j0}(t) - \theta_{k0}(t), \omega_{k0}], & (28) \end{aligned}$$

where the subscripts on G denote partial derivatives. Upon averaging, the terms proportional to $\theta_{k1}(t)$ and $\omega_{k1}(t)$ in (28) describe the effect of particle j acting on the each of the other particles in the beam and having these particles subsequently back react on particle j. This effect was included completely when the impedance was dressed, so the terms proportional to $\theta_{k1}(t)$ and $\omega_{k1}(t)$ will be ignored. The term proportional to θ_{j1} describes the effect of the random voltage from the other particles causing the phase of particle j to change, altering the cooling rate. This term will be kept. To second order the net force on particle j is then

$$\dot{\omega}_j = F[\omega_{j0} + \omega_{j1}(t)] + \sum_{k\neq j} G[\theta_{j0}(t) - \theta_{k0}(t), \omega_{k0}] + \theta_{j1}(t) G_\theta[\theta_{j0}(t) - \theta_{k0}(t), \omega_{k0}]. \tag{29}$$

The Fokker-Planck coefficients are

$$\tilde{F}(\omega_j) = \frac{1}{\Delta t} \left\langle \int_0^{\Delta t} \dot{\omega}_j(t) dt \right\rangle_{\omega_{k0}, \psi_k, \psi_j} \tag{30}$$

$$\tilde{D}(\omega_j) = \frac{1}{2\Delta t} \left\langle \left| \int_0^{\Delta t} \dot{\omega}_j(t) dt \right|^2 \right\rangle_{\omega_{k0}, \psi_k, \psi_j} \tag{31}$$

where the angular brackets denote averages over all initial phases and over the energies of the particles other than particle j. The time interval Δt is short compared with the cooling time but long compared to the coherence time of any statistical fluctuations. A detailed calculation is given in Appendix B.

$$\tilde{D}(\omega) = \pi \tilde{C}^2 \sum_{m=-\infty}^{\infty} \omega^2 \left| Z_D(m\omega) \right|^2 \frac{\Psi_0(\omega)}{|m|} \tag{32}$$

$$\tilde{F} - \frac{\partial \tilde{D}}{\partial \omega} = \tilde{C} \sum_{m=-\infty}^{\infty} e^{im[\theta_P - \theta_K]} \omega Z_D[m\omega] = F(\omega). \tag{33}$$

This is the usual result [5, 6] and the density obeys a damped diffusion equation

$$\frac{\partial \Psi_0}{\partial t} = \frac{\partial}{\partial \omega}\left[-F(\omega)\Psi_0(\omega,t) + \tilde{D}(\omega,t)\frac{\partial \Psi_0}{\partial \omega}\right]. \tag{34}$$

III. THE RHIC SYSTEM

Intrabeam scattering rates for gold beams in RHIC are of order one hour [1, 3, 4], setting the scale for the cooling system. Optical fibers will transmit the signal from the pickup to the kicker. To keep cost down the cables are inside the tunnel and we are working on the yellow (counter-clockwise) ring first. The pickup is in the 12 o'clock straight section and the kicker is in the 4 o'clock straight section. At storage energy the frequency slip is dominated by dispersion in the arcs and the pickup and kicker are in straight sections. Therefore, the efffective delay will be very close to 0.66 turns or $T_d = 8.5\mu$s.

Substituting $\exp(-im\omega_0 T_d)$ for $\exp(im[\theta_P - \theta_K])$ in equation (33) and writing $Z_T(m\omega) = Z_t(m\omega)\exp(im\omega T_d)$ yields

$$F(\omega) = \tilde{C} \sum_{m=-\infty}^{\infty} e^{imT_d(\omega - \omega_0)} \omega \frac{Z_t(m\omega)}{1 + Z_T(m\omega) Y_b B(m\omega)}. \tag{35}$$

The standard solution for $Z_t(\omega)$ is to use a one turn delay notch filter. For an input signal $V_{in}\exp(-i\omega t)$ one gets $V_{out} = [1 - \exp(i\omega T_0)]V_{in}$. For two such filters in series $V_{out} = [1 - \exp(i\omega T_0)]^2 V_{in}$. To judge the relative effectiveness of various schemes consider the dimensionless variable $x = m(\omega - \omega_0)T_0$. Figure 1 shows plots of the various filters. From the plots it is clear that the two filters in series have cooling out to a larger value of x than the single filter. This translates into a larger allowable frequency range for the two filters in series. Another property of the two filters in series is that both the cooling and diffusion terms, and their derivatives, vanish at $x = 0$. With the two filters in series the cooling system is "blind" to particles near the center of the bucket. It is worth noting that the imaginary parts of $[1 - \exp(ix)]^n \exp(ixT_d/T_0)$ are plotted in Figure 1. The cooling force in (35) is real so these filters require broad band phase shifters.

Along with the low level system one must create the necessary voltage. Previous estimates [11] gave rms values between 1 and 4kV, depending on beam parameters. Taking

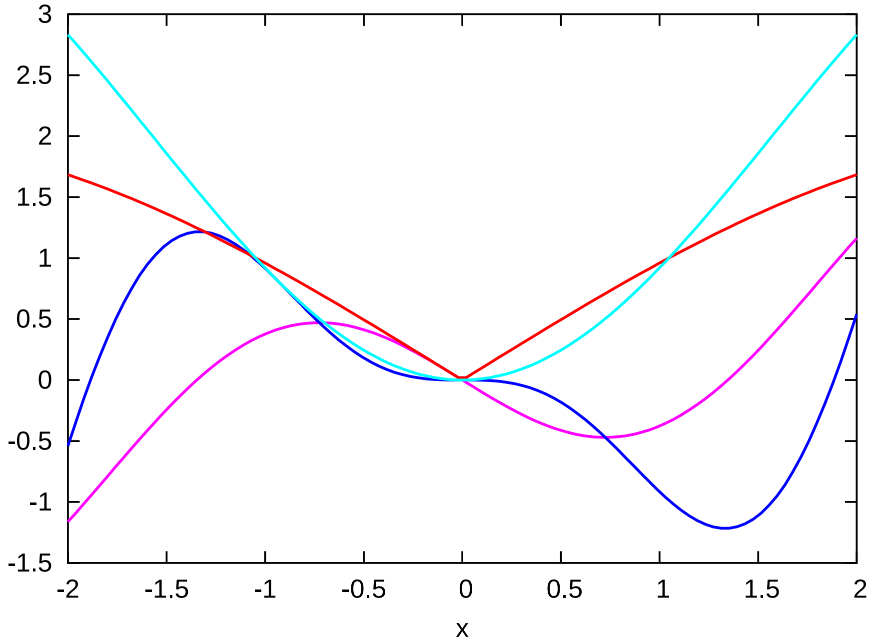

FIG. 1: Plots of various filter schemes: lavendar, $Im\{\exp(ixT_d/T_0)[1-\exp(ix)]\}$; dark blue, $Im\{\exp(ixT_d/T_0)[1-\exp(ix)]^2\}$; red, $|\exp(ixT_d/T_0)[1-\exp(ix)]|$; light blue, $|\exp(ixT_d/T_0)[1-\exp(ix)]^2|$. The delay is set to $T_d/T_0 = 0.66$, as in RHIC.

a 1kΩ transfer impedance gives a kicker power of 16kW. In principle one could scale up the broad band kickers used at FNAL or CERN to supply the necessary voltage but this would involve 10 or more broad band kickers and traveling wave tube amplifiers. Such a system is quite complicated and an alternate solution is desirable. The key to the problem involved reinterpreting earlier work [10]. In Boussard's original paper the fact that the beam is bunched allowed for a large reduction in data transmission rates. With modern fiber optic links this is no longer an issue, but his notion of using a Fourier series to decompose the signal from the bunch naturally leads to using a Fourier series to represent the kicker voltage. The concept is illustrated in Figure 2. For bunches of length τ_b a cooling voltage covering the band from $f_- = n_-/\tau_b$ to $f_+ = n_+/\tau_b$ can be represented as

$$V(t) = \sum_{k=n_-}^{n_+} a_n(t)\sin(2\pi kt/\tau_b) + b_n(t)\cos(2\pi kt/\tau_b), \tag{36}$$

where $a_n(t)$ and $b_n(t)$ have appropriate values as each bunch passes and vary smoothly between bunches. Each term in the sum is generated using a narrow band cavity. With a bunch length of $\tau_b = 5\text{ns} = 1/200\text{MHz}$ we need 21 cavities to span the 4-8GHz band. The broad band phase shifter is replaced by a $\lambda/4$ length of cable for each cavity. With

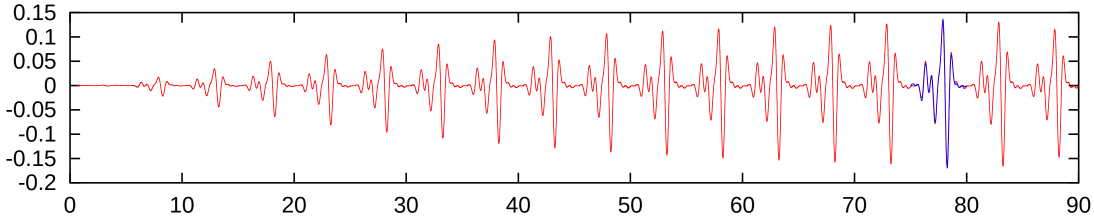

FIG. 2: Illustration of the narrow band kicker concept. The blue trace is the voltage one would have using a broad band system, and the bunch is present only during the time it is non-zero. The red trace is a set of narrow band cavities with frequencies $1/\tau_b, 2/\tau_b \ldots$.

21 cavities incoherently adding to produce 4kV the voltage per cavity is less than 1kV. The cavities have $R/Q \approx 100\Omega$. With 100ns between bunches a 10MHz bandwidth gives 3 e-folding times for the voltage to settle between bunches. With 40W amplifiers the 8 GHz cavity can generate an rms voltage of 1.8kV, which should be adequate.

To take advantage of the narrow cavity bandwidth the drive signal for each cavity is piecewise sinusiodal. The raw signal from the pickup, $S_0(t)$ is lagged and added producing

$$
\begin{aligned}
S_1(t) &= \sum_{k=0}^{M-1} S_0(t - k\tau_b) \\
S_1(\omega) &= S_0(\omega) \sum_{k=0}^{M-1} e^{ik\omega\tau_b} \\
&= S_0(\omega) \frac{\sin(M\omega\tau_b/2)}{\sin(\omega\tau_b/2)} e^{i\omega\tau_b(M-1)/2} \qquad (37)
\end{aligned}
$$

where $M = 16$ for the present configuration and both time and frequency domains are shown. Signals vary in time as $\exp(-i\omega t)$. The filter $S_1(\omega)/S_0(\omega)$ is composed of lines with 3dB full width $= 0.885/\sqrt{M^2-1}\tau_b = 11$MHz, separated by $1/\tau_b = 200$MHz. Next, $S_1(t)$ is processed through the one turn delay notch filters giving.

$$
\begin{aligned}
S_2(t) &= S_1(t - T_d) - 2S_1(t - T_d - T_0) + S_1(t - T_d - 2T_0) \\
S_2(\omega) &= S_1(\omega) e^{i\omega T_d} \left[1 - e^{i\omega T_0}\right]^2 \qquad (38)
\end{aligned}
$$

where the time is now referenced to the kicker via the delay T_d, T_0 is the revolution period, and we have taken two cascaded one turn delay notch filters. Next this signal is split and put through bandpass filters of full width 100 MHz centered on multiples of 200 MHz. Each

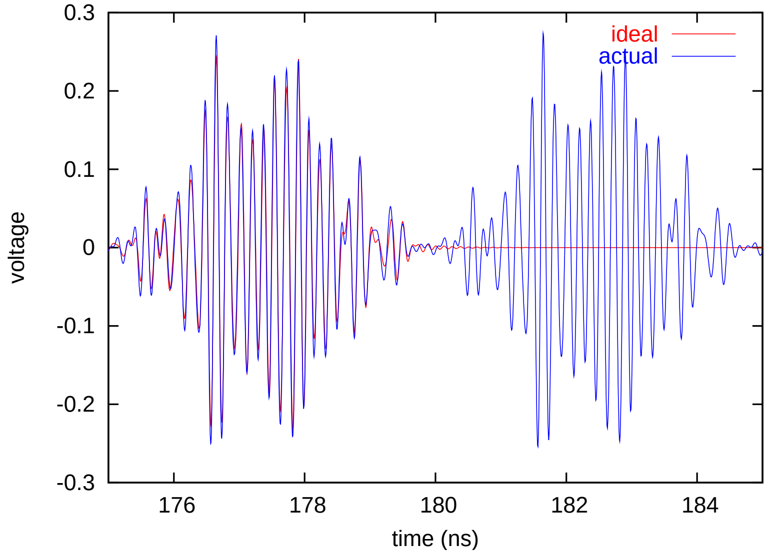

FIG. 3: Realistic simulation of the narrow band kicker system. The red trace is the ideal voltage for a broad band system, and the bunch is present only during this time it is non-zero. The blue trace is the voltage from 21 narrow band cavities. Three bunches preceeded the one shown. There is very little cross talk.

of the signals is nearly piecewise sinusiodal and each goes to its kicker. Figure 3 shows simulation results for the actual system, including realistic errors.

During the FY05 copper run we plan on having the 7-8 GHz band of the stochastic cooling system operational in the yellow ring. A computer code was written to evaluate the dressed impedance and numerically integrate equation (34). The distribution $F_0(\omega, t)$ was assumed symmetric about ω_0, and the inital distribution was taken to be that for the center of the RF bucket. The boundary condition at the edge of the RF bucket was $\Psi_0 = 0$. Figure 4 shows the expected Schottky suppression at 7 GHz for a typical copper bunch and optimal gain with 4MV on the storage cavities. Figure 5 is for 6 MV. Cooling calculations were done with and without intrabeam scattering (IBS). For IBS calculations the first step was to calculate IBS growth rates using a simple model [13].

$$\frac{1}{\tau_{||}} = \frac{1}{\sigma_p^2}\frac{d\sigma_p^2}{dt} = \frac{r_i^2 c N_b \Lambda}{8\beta^3\gamma^3\epsilon_x^{3/2} < \beta_\perp^{1/2} > \sqrt{\pi/2}\sigma_p^2} \tag{39}$$

$$\frac{1}{\tau_\perp} = \frac{1}{\epsilon_x}\frac{d\epsilon_x}{dt} = \frac{\sigma_p^2}{\epsilon_x}\left\langle \frac{D_x^2 + (D_x'\beta_x + \alpha_x D_x)^2}{\beta_x} \right\rangle \frac{1}{\tau_{||}} \tag{40}$$

In equations (39) and (40), σ_p is the rms value of $(p - p_0)/p_0$, ϵ_x is the rms un-normalized

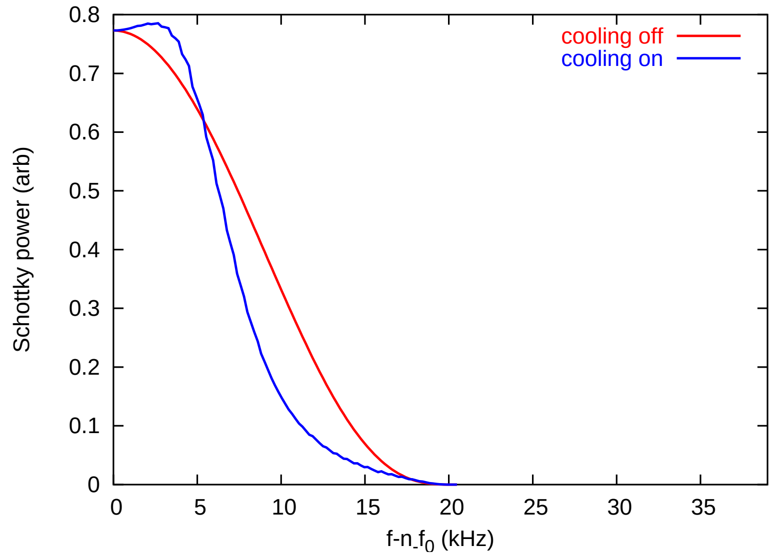

FIG. 4: Schottky signal suppression upstream of the notch filters at 7 GHz for 2×10^9 copper ions with 4MV of RF and near optimal gain. The spectrum on the upper half of the revolution line closest to 7 GHz is shown.

horizontal emittance, r_i is the classical radius of a copper nucleus, N_b is the number of ions per bunch, $\Lambda \approx 18.5$ is the Coulomb logarithm, $<\beta_\perp^{1/2}>= 5.5\mathrm{m}^{1/2}$ is the average of the root of the transverse beta function, and

$$\left\langle \frac{D_x^2 + (D_x'\beta_x + \alpha_x D_x)^2}{\beta_x} \right\rangle \approx 0.045\mathrm{m}$$

is the average dispersion squared over beta. The transverse distributions were assumed to be fully coupled and Gaussion. The rms momentum spread was calculated using Ψ_0,

$$\sigma_\omega^2(t) = \frac{\displaystyle\int (\omega-\omega_0)^2 \Psi_0(\omega,t)d\omega}{\displaystyle\int \Psi_0(\omega,t)d\omega}.$$

The diffusion coefficient was modified to

$$\tilde{D}(\omega) \rightarrow \tilde{D}(\omega) + \frac{\sigma_\omega^2(t)}{2\tau_{||}(t)},$$

so that the evolution of the rms agreed with the simple model. Simulations for copper with and without IBS and 4 MV on the storage cavities are shown in Figure 6. Figure 7 is for 6 MV.

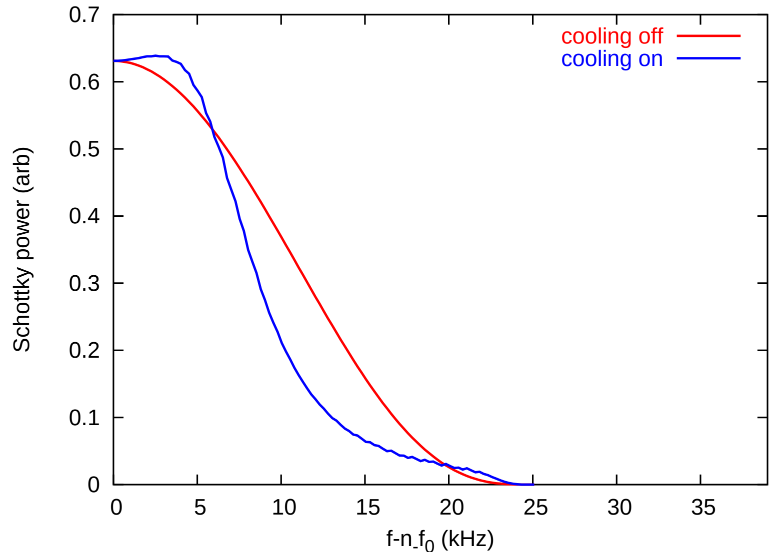

FIG. 5: Schottky signal suppression upstream of the notch filters at 7 GHz for 2×10^9 copper ions with 6MV of RF and near optimal gain. The spectrum on the upper half of the revolution line closest to 7 GHz is shown.

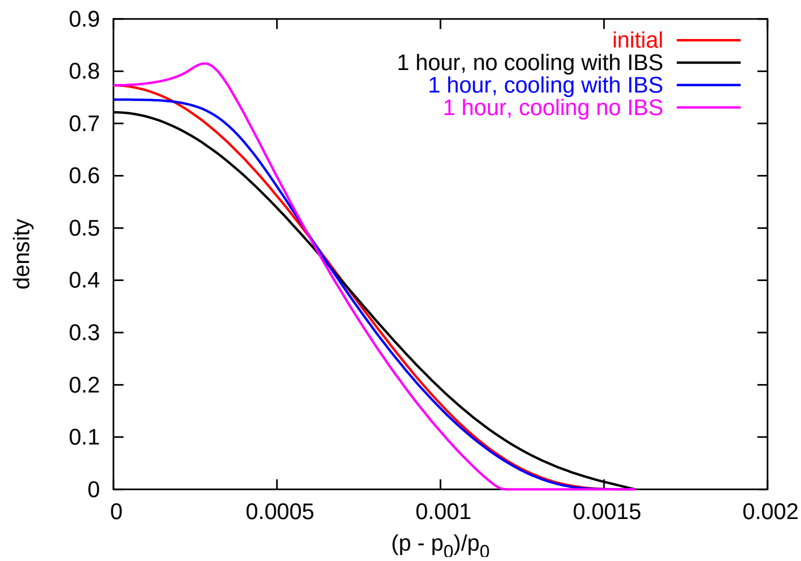

FIG. 6: Cooling with 7-8 GHz system for 2×10^9 copper ions with 4MV of RF and near optimal gain.

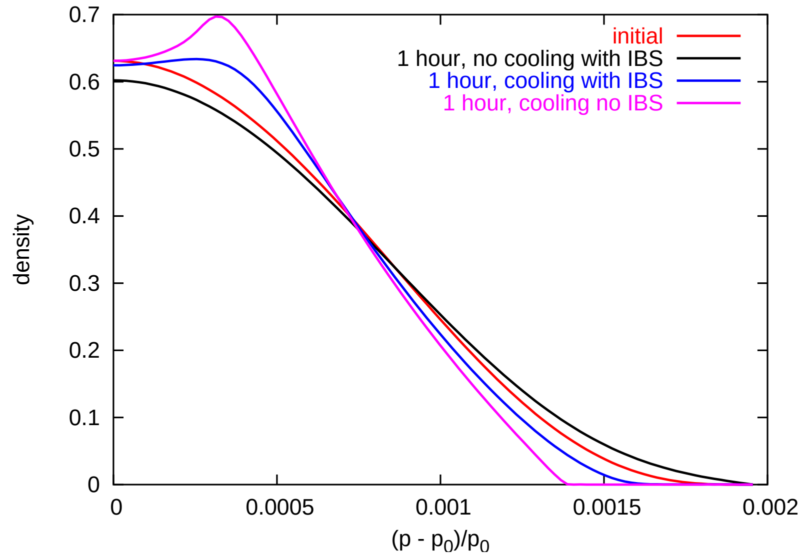

FIG. 7: Cooling with 7-8 GHz system for 2×10^9 copper ions with 6MV of RF and near optimal gain.

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[14] Remember that $o(x)$ means $o(x)/x \to 0$ as $x \to 0$.

APPENDIX A: EVALUATION OF THE INFINITE SUM IN EQUATION (10)

The infinite sum is given by $G(\theta_P - \theta_K)$ with

$$G(\theta) = \sum_{k=-\infty}^{\infty} \frac{e^{ik\theta}}{\lambda + ik\omega}, \tag{A1}$$

where $\lambda = \epsilon - i\tilde{\omega}$. The trick is to find a differential equation for $G(\theta)$ that has a closed form solution. Consider

$$\frac{dG}{d\theta} = \sum_{k=-\infty}^{\infty} \frac{ike^{ik\theta}}{\lambda + ik\omega} \tag{A2}$$

$$= \frac{1}{\omega} \sum_{k=-\infty}^{\infty} \frac{(ik\omega + \lambda - \lambda)e^{ik\theta}}{\lambda + ik\omega} \tag{A3}$$

$$= \frac{1}{\omega} \sum_{k=-\infty}^{\infty} e^{ik\theta} - \frac{\lambda}{\omega} G(\theta) \tag{A4}$$

$$= \frac{2\pi\delta_p(\theta)}{\omega} - \frac{\lambda}{\omega} G(\theta). \tag{A5}$$

For $0 < \theta < 2\pi$ we have $G(\theta) = A\exp(-\lambda\theta/\omega)$, where A is a constant. From equation (A1) we see that $G(\theta) = G(\theta + 2\pi)$, so for small positive x, $G(2\pi + x) = A\exp(-\lambda x/\omega)$. Next we integrate both sides of equation (A5) from $2\pi - x$ to $2\pi + x$ and then take the limit as $x \to 0$. This gives

$$A\left(1 - e^{-2\pi\lambda/\omega}\right) = 2\pi/\omega. \tag{A6}$$

Substituting this expression for A gives

$$G(\theta) = \frac{2\pi}{\omega} \frac{e^{-\lambda\theta/\omega}}{1 - e^{-2\pi\lambda/\omega}}, \tag{A7}$$

where this expression is good for $0 < \theta < 2\pi$ and $G(\theta) = G(\theta + 2\pi)$.

APPENDIX B: FOKKER-PLANCK COEFFICIENTS

Going back to explicit expressions, and neglecting the terms that will disappear upon averaging, equation (29) becomes

$$\dot{\omega}_j = F(\omega_{j0}) + \tilde{C} \sum_{k \neq j} \sum_{m} \omega_{k0} Z_D^*(m\omega_{k0})[1 - im\theta_{j1}] e^{-im(\theta_{j0} - \theta_{k0} + \theta_P - \theta_K)}, \tag{B1}$$

where $\theta_{j0} = \theta_{j0}(t)$ etc. Since $\dot{\omega}$ is a real number we were free to complex conjugate the terms in the sum and Z_D^* is the complex conjugate of Z_D. For $\tilde{F}$ the average of the first term yields $F(\omega_j)$. For the second term , since $< \int \sum X >= \int \sum < X >$ for arbitrary X, we may consider

$$f(\omega_j, m, t) = \left\langle \omega_{k0} Z_D^*(m\omega_{k0})[1 - im\theta_{j1}(t)] e^{-im(\theta_{j0}(t) - \theta_{k0}(t) + \theta_P - \theta_K)} \right\rangle_{\omega_{k0}, \psi_k, \psi_j},$$

with

$$\langle \dot{\omega}_j(t) \rangle_{\omega_{k0}, \psi_k, \psi_j} = F(\omega_{j0}) + N\tilde{C} \sum_{m=-\infty}^{\infty} f(\omega_j, m, t) \tag{B2}$$

Only the term proportional to θ_{j1} will survive averaging over ψ_k,

$$f(\omega_j, m, t) = -im \left\langle \omega_{k0} Z_D^*(m\omega_{k0}) \int_0^t dt_1 \omega_{j1}(t_1) e^{-im(\theta_{j0}(t) - \theta_{k0}(t) + \theta_P - \theta_K)} \right\rangle_{\omega_{k0}, \psi_k, \psi_j}.$$

Using equation(25) and the fact that the ψ_ks are independently distributed

$$f(\omega_j, m, t) = -im\tilde{C} \left\langle |\omega_{k0} Z_D(m\omega_{k0})|^2 \int_0^t dt_1 \int_0^{t_1} dt_2 e^{-im(\omega_{j0} - \omega_{k0})(t - t_2)} \right\rangle_{\omega_{k0}}.$$

Using the generic formula

$$\int_0^x dx_1 \int_0^{x_1} g(x_2) dx_2 = \int_0^x (x - x_1) g(x_1) dx_1$$

$$
\begin{aligned}
f(\omega_j, m, t) &= -im\tilde{C}\left\langle |\omega_{k0} Z_D(m\omega_{k0})|^2 \int_0^t dt_1 (t-t_1) e^{-im(\omega_{j0}-\omega_{k0})(t-t_1)} \right\rangle_{\omega_{k0}} \\
&= \tilde{C}\frac{\partial}{\partial\omega_{j0}}\left\langle |\omega_{k0} Z_D(m\omega_{k0})|^2 \int_0^t dt_1 e^{-im(\omega_{j0}-\omega_{k0})(t-t_1)} \right\rangle_{\omega_{k0}} \\
&= \tilde{C}\frac{\partial}{\partial\omega_{j0}}\left\langle |\omega_{k0} Z_D(m\omega_{k0})|^2 \int_0^t dt_1 e^{-im(\omega_{j0}-\omega_{k0})t_1} \right\rangle_{\omega_{k0}}
\end{aligned}
\tag{B3}
$$

Inserting (B3) into (B2) and noting that $f(\omega_j, m, t) = f^*(\omega_j, -m, t)$

$$
\langle \dot{\omega}_j(t) \rangle_{\omega_{k0},\psi_k,\psi_j} = F(\omega_{j0}) + \frac{N\tilde{C}^2}{2}\sum_{m=-\infty}^{\infty}\frac{\partial}{\partial\omega_{j0}}\left\langle |\omega_{k0} Z_D(m\omega_{k0})|^2 \int_{-t}^t dt_1 e^{-im(\omega_{j0}-\omega_{k0})t_1} \right\rangle_{\omega_{k0}}
\tag{B4}
$$

Equation (B3) gives the average force on particle j. Now we use the assumption that Δt is large compared to any statistical correlation time, which turns the integral over t_1 into a delta function. This gives the final result

$$
\tilde{F}(\omega_j) = F(\omega_j) + \pi\tilde{C}^2\frac{\partial}{\partial\omega_j}\sum_{m=-\infty}^{\infty}|\omega_j Z_D(m\omega_j)|^2\frac{\Psi_0(\omega_j)}{|m|}
\tag{B5}
$$

For the diffusion coefficient we only need the term

$$
\tilde{D}(\omega_j) = \frac{1}{2\Delta t}\left\langle |\omega_{j1}(\Delta t)|^2 \right\rangle_{\omega_{k0},\psi_k,\psi_j}.
\tag{B6}
$$

Since Δt is small compared to the cooling time the term $F(\omega_{j0})t$ in equation (25) will be neglected. This gives

$$
\tilde{D}(\omega_j) = \frac{1}{2\Delta t}\left\langle \left| \int_0^{\Delta t} dt\tilde{C}\sum_{k=1}^{N}\sum_{m=-\infty}^{\infty} e^{im[\theta_P - \theta_K + \theta_{j0}(t) - \theta_{k0}(t)]}\omega_{k0} Z_D[m\omega_{k0}] \right|^2 \right\rangle_{\omega_{k0},\psi_k,\psi_j}
\tag{B7}
$$

Writing out the double sum and averaging over ψ_k diagonalizes with respect to k and m

$$
\tilde{D}(\omega_j) = \frac{N\tilde{C}^2}{2\Delta t}\left\langle \sum_{m=-\infty}^{\infty}|\omega_{k0} Z_D[m\omega_{k0}]|^2 \int_0^{\Delta t} dt_1 \int_0^{\Delta t} dt_2 e^{im(\omega_{j0}-\omega_{k0})(t_1-t_2)} \right\rangle_{\omega_{k0}}
\tag{B8}
$$

Set $\tau_1 = t_1 - t_2$ and $\tau_2 = (t_1 + t_2)/2$ so that $dt_1 dt_2 = d\tau_1 d\tau_2$.

$$
\tilde{D}(\omega_j) = \frac{N\tilde{C}^2}{2\Delta t}\left\langle \sum_{m=-\infty}^{\infty}|\omega_{k0} Z_D[m\omega_{k0}]|^2 \int_0^{\Delta t} d\tau_2 \int_{-2\tau_2}^{2\tau_2} d\tau_1 e^{im(\omega_{j0}-\omega_{k0})\tau_1} \right\rangle_{\omega_{k0}}
\tag{B9}
$$

Now we use the assumption that δt is large compared with any correlation time so that the value of τ_2 may be considered large over almost the whole range of integration. This turns the second integral into $2\pi\delta[m(\omega_{j0} - \omega_{k0})]$ yielding

$$\tilde{D}(\omega_j) = \pi\tilde{C}^2 \sum_{m=-\infty}^{\infty} |\omega_{j0} Z_D[m\omega_{j0}]|^2 \frac{\Psi_0(\omega_{j0})}{|m|}, \tag{B10}$$

the desired result. It is worthwhile to note that a comparable result holds for bunched beams [12].