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RHIC Resistive Wall Coupled Bunch Instability

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Abstract

The RHIC resistive wall coupled bunch instability is reviewed for gold and proton beams at the injection, top energy, and storage. The gold beam around the transition is also discussed. The strongest resistive wall coupled bunch instabilities are at the injection, for both gold and proton beams. With proper set-up of the machine chromaticity, however, most of these instabilities can be eliminated, or significantly reduced.

1 Introduction

One of the primary concerns of the RHIC transverse coupled bunch instability is the resistive wall impedance caused instability [1,2]. In general, the resistive wall may also cause a single bunch instability. This type of instability will not happen at the RHIC, because the fractional tune is 0.2, which is smaller than the half integer. On the other hand, the coupled bunch mode instability has chance to develop under certain conditions, such as an unfavorable chromaticity.

The transverse coupled bunch mode is determined by $(nM + \nu + n_c)f_0$, where f_0 is the revolution frequency, n is an integer, M is the bunch number, ν is the tune, and n_c is the coupled bunch mode number. At the negative frequency, the real part of the resistive wall impedance is negative. Coupled with the beam, the negative real impedance can excite beam instabilities. Since the wall impedance is proportional to $1/\sqrt{\omega}$, the coupled bunch mode sampling the closest negative frequency to zero will be dominant. Taking the

RHIC horizontal tune $\nu_x = 28.2$, and let $n = -1$, then the coupled bunch mode $n_c = 31$ gives rise to the sampling frequency $(-60 + 28.2 + 31) f_0 = -62.56 \text{ kHz}$ for 60 bunches and $f_0 = 78.2 \text{ kHz}$. The real part of the wall impedance at the frequency of -62.56 kHz is $-6.55 \text{ M}\Omega/m$.

In this note, the resistive wall coupled bunch instability for 60 bunches of gold and proton beams at the injection, top energy, and storage will be reviewed. One of the most critical parameters, the chromaticity, is considered in a reasonable range, and both azimuthal mode $m = 0$ and $m = 1$ are included. The situation of the gold beam at the transition, with the gamma jump, is also discussed. The fastest possible growth time is at the proton beam injection, which is 16.1 ms . With the usual chromaticity at the proton beam injection, which is slightly positive, this instability can be eliminated. The coupled bunch instability of the gold beam around the transition, with the gamma jump, poses no serious problem. On the other hand, the relatively weak instability of $m = 1$ mode at the high energy, with the positive chromaticity, may need to be observed.

2 Resistive wall Impedance

Assuming a smooth cylindrical vacuum chamber, the longitudinal and transverse resistive wall impedances are,

$$Z_\ell(\omega) = (\text{sgn}(\omega) + j) \frac{\beta Z_0 \delta_s}{2b} \frac{\omega}{\omega_0} \quad (1)$$

and

$$Z_T(\omega) = (\text{sgn}(\omega) + j) \frac{R Z_0 \delta_s}{b^3} \quad (2)$$

where Z_0 is the impedance in free space, $377 \text{ }\Omega$, b is the radius of the vacuum chamber, R is the machine radius, and $\omega_0 = 2\pi f_0$. The skin depth at the frequency ω is defined as,

$$\delta_s = \sqrt{\frac{2\rho}{\mu_0|\omega|}} \quad (3)$$

where ρ is the resistivity of the vacuum chamber, for stainless steel we take $\rho = 1 \times 10^{-6} \text{ }\Omega m$ for warm, $\rho = 0.5 \times 10^{-6} \text{ }\Omega m$ for cold regions, and $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ is the permeability of free space.

For RHIC, $R = 610.175 \text{ m}$, and the beam pipe diameter is 6.91 cm for cold region of $2,955 \text{ m}$, 12.28 cm for warm region of 879 m . At the revolution frequency 78.2 kHz , we get the average $\delta_s = 1.27 \text{ mm}$, $\delta_s = 1.80 \text{ mm}$, for the cold and warm region, respectively. The longitudinal and transverse resistive wall impedances at the revolution frequency are

$$Z_\ell(\omega_0) = 6.61(1 + j)\Omega \quad (4)$$

and

$$Z_T(\omega_0) = 5.86(1 + j)M\Omega/m \quad (5)$$

3 Coupled Bunch Instability

The following equation is used to calculate the coupled bunch instability,

$$\omega - \omega_\beta = \frac{j e M I_0 \bar{\beta}_\perp}{2 R^2 m_0 \gamma \omega_0} \sum_{n=-\infty}^{\infty} Z_T(n) h_m(n') \quad (6)$$

where the beam current I_0 is defined for the number of charges per bunch, N_{bh} ,

$$I_0 = N_{bh} e f_0 \quad (7)$$

Also $Z_T(n)$ is the transverse impedance at the sampling harmonic of $n f_0$. In the beam power spectrum $h_m(n')$, n' represents the chromatic effect in frequency domain.

For the RHIC, the average β_\perp function at IR is about 70 m , and at the arc it is about 30 m . The IR occupies about $1,000 \text{ m}$, and the arcs about $2,834 \text{ m}$, we thus take the average β_\perp function of the ring as $\bar{\beta}_\perp = 40 \text{ m}$.

For a Gaussian distribution, the power spectrum of the first orthogonal polynomial of the $m = 0$ mode is

$$h_0(n) = \frac{1}{2\pi} e^{-(n\omega_0\sigma_\tau)^2} \quad (8)$$

where σ_τ is the rms bunch length in second. For $m = 1$ mode, the first order approximation of Bessel function yields,

$$h_1(n) = \frac{(n\omega_0\sigma_\tau)^2}{4\pi} e^{-(n\omega_0\sigma_\tau)^2} \quad (9)$$

The derivation of the Gaussian beam power spectra is shown in the Appendix.

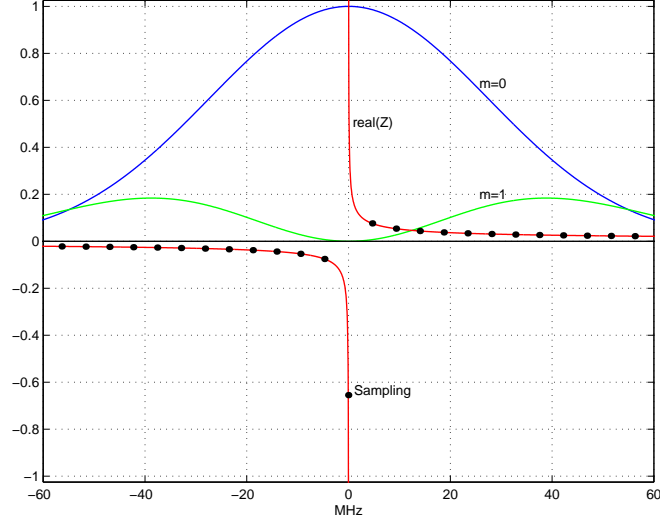


Figure 1: Coupled bunch instability with the resistive wall impedance.

In Fig.1, the mechanism of coupled bunch instability is illustrated, where the beam power spectra of $m = 0$ and $m = 1$, the resistive wall impedance, and the sampling of the coupled bunch mode $n_c = 31$ are shown. The chromaticity in Fig.1 is zero.

In the calculation, the beam power spectrum is shifted by the chromatic frequency f_ξ , represented by

$$n' = n - \frac{f_\xi}{f_0} \quad (10)$$

where

$$f_\xi = \frac{\xi\nu}{\eta} f_0 \quad (11)$$

and the chromaticity is normalized, i.e.,

$$\xi = \frac{\Delta\nu/\nu}{\Delta p/p} \quad (12)$$

where p and Δp are the beam momentum, and momentum spread, respectively.

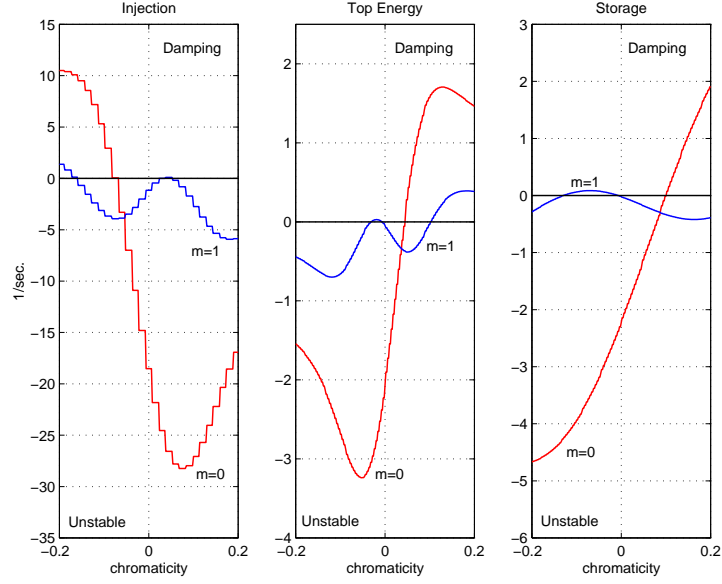


Figure 2: Gold beam coupled bunch instability with the resistive wall impedance.

3.1 Gold beam

The gold beam parameters are shown in Table 1, where h is the harmonic number, and σ_ℓ is the bunch length in meter.

	Injection	Top Energy	Storage	Unit
γ	10.52	108.4	108.4	
η	-71	18	18	10^{-4}
N_{bh}	1	1	1	10^{11}
I_0	1	1	1	mA
h	360	360	2520	
σ_τ	4.1	1.59	0.46	ns
σ_ℓ	1.23	0.48	0.14	m

Table 1

In Fig.2, the gold beam resistive wall coupled bunch instability is shown for the injection, top energy, and the storage. Positive region represents

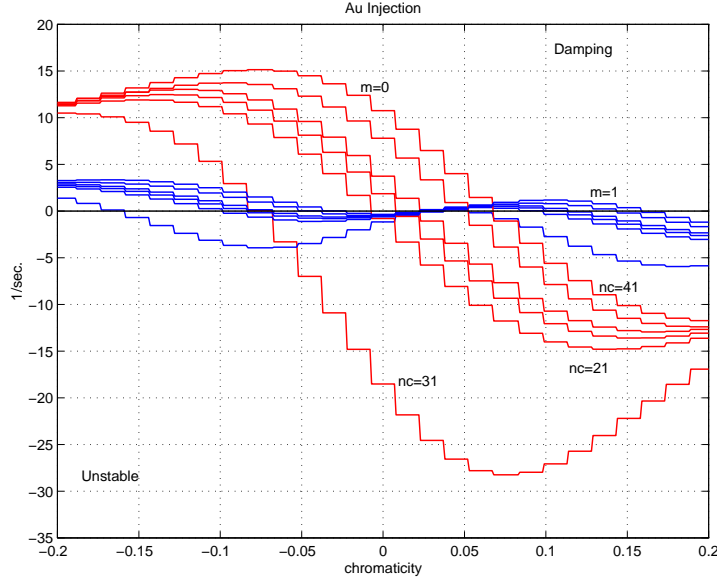


Figure 3: Coupled bunch modes at the Gold beam injection. The modes between 21 and 41 are 11, 1, and 51.

damping, and negative is antidamping, all in the unit of $1/sec$. We may observe that

- The fastest growth time of $35.4 ms$ occurs at the injection, with the chromaticity of $\xi \approx 0.074$. Together with other considerations, such as the broadband impedance induced instabilities, the chromaticity at the injection has to be slightly negative, therefore, this instability is not expected to happen.
- For negative chromaticity, the $m = 1$ mode is not stable, with the fastest growth rate of $250 ms$ at $\xi \approx -0.08$. This instability, however, might be Landau damped.
- At the top energy and storage, the positive chromaticity in general satisfies the $m = 0$ mode stability. However, the $m = 1$ mode stability condition might be violated at the positive chromaticity, but with slow growth rate.

For 60 bunches in the ring, there are total 60 coupled bunch modes. The mode $n_c = 31$ is the dominant one in terms of the growth rate. In Fig.3, this

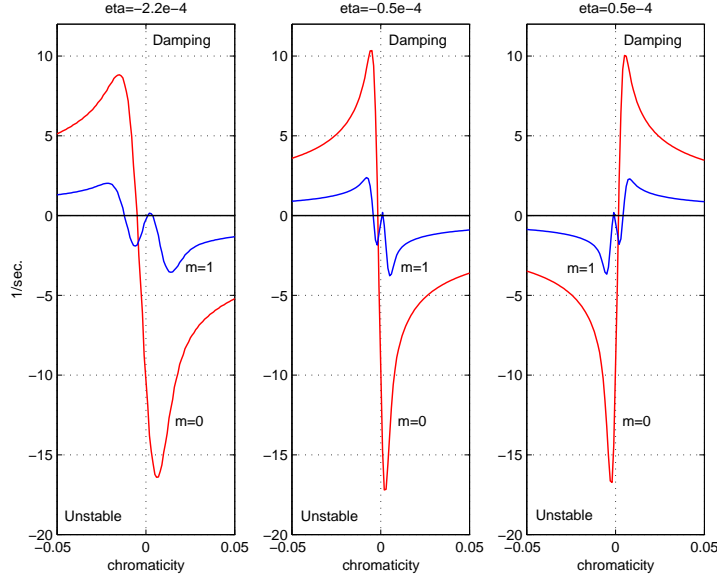


Figure 4: Gold beam coupled bunch instability around the transition.

mode is compared with other mode, such as $n_c = 41, 51$, to 1, 11, and 21.

In Fig.4, the coupled bunch instability around the transition is shown, assuming the gamma jump of ± 0.4 unit in ± 100 ms. Three conditions with $\eta = -2.2 \times 10^{-4}$, and $\eta = \pm 0.5 \times 10^{-4}$ are considered. Under this gamma jump scenario, the acceleration period with either $\eta = 0.5 \times 10^{-4}$ and $\eta = -0.5 \times 10^{-4}$ will last for about 200 ms, therefore, the fastest growth rate of about 60 ms might be tolerable. By swiftly changing the chromaticity, even this instability can be avoided.

3.2 Proton beam

In Table 2, the relevant proton beam parameters are shown.

In Fig.5, the resistive wall coupled bunch instability with mode $n_c = 31$ is shown. We note,

- The fastest growth time is again at the injection, with the slightly negative chromaticity, it is 16.1 ms. Setting a slightly positive chromaticity, this instability will not happen.
- At the top energy and the storage, the mode $m = 1$ needs again to be

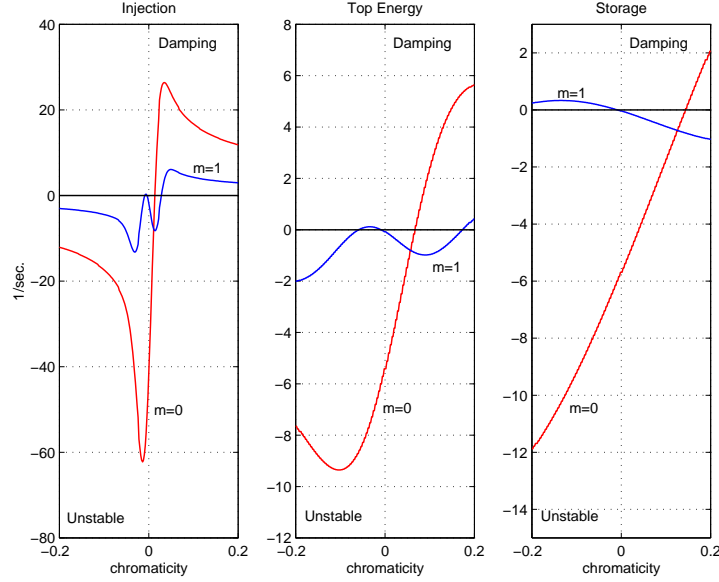


Figure 5: Proton beam coupled bunch instability with the resistive wall impedance.

watched, if positive chromaticity is chosen to stabilize the $m = 0$ mode.

	Injection	Top Energy	Storage	Unit
γ	31.2	268.3	268.3	
η	9	19	19	10^{-4}
N_{bh}	2	2	2	10^{11}
I_0	2.5	2.5	2.5	mA
h	360	360	2520	
σ_τ	2.95	0.94	0.27	ns
σ_ℓ	0.89	0.28	0.08	m

Table 2

4 Summary

The largest growth rate of the mode $m = 0$ is shown in Table 3. In general, these unstable conditions can be avoided, simply using the rule of negative chromaticity below the transition, and positive above.

		Growth rate	η	ξ
<i>Au</i>	Injection	28.2	-71	0.074
	Below transition	17.0	-0.5	0.002
	Above transition	17.0	0.5	-0.001
	Top energy	3.2	18	-0.048
	Storage	4.7	18	-0.224
<i>p</i>	Injection	62.1	9	-0.012
	Top energy	9.3	19	-0.096
	Storage	14.5	19	-0.44
Unit		1/sec .	10^{-4}	

Table 3. Largest growth rate for Gold and proton beams

The most critical resistive wall coupled bunch instability is at the injection, for both gold and proton beams. In Table 4, the growth and damping rate with respect to the chromaticity is shown. With a setup of $\xi = -0.2$ at the gold beam injection, and $\xi = 0.1$ at the proton beam injection, both $m = 0$ and $m = 1$ modes will be stable.

		ξ	Growth $m = 0$	Damping $m = 0$	Growth $m = 1$	Damping $m = 1$
<i>Au</i> injection	0.2	16.9			5.8	
	0.1	27.1			2.7	
	0	18.5			1.2	
	-0.1			5.3	3.1	
	-0.2			10.5		1.4
<i>p</i> injection	0.2			11.9		3.0
	0.1			16.7		4.3
	0	47.0			0.7	
	-0.1	17.2			4.4	
	-0.2	12.1			3.0	
Unit			1/sec .	1/sec .	1/sec .	1/sec .

Table 4. Growth rate for Gold and proton beams at the injection.

5 Appendix: Gaussian Beam Power Spectra

Consider a normalized Gaussian distribution in phase space,

$$\psi_0(r) = \frac{2}{\pi r_\ell^2} e^{-2r^2/r_\ell^2} \quad (13)$$

where r_ℓ is half bunch length in radius, or twice the rms bunch length. The beam spectrum is defined as,

$$\lambda_m^{(k)}(n) = \int_0^\infty W_T(r) f_m^{(k)}(r) J_m(nr) r dr \quad (14)$$

where the transverse weight function is,

$$W_T(r) = \psi_0(r) \quad (15)$$

and the orthogonal polynomials are defined as,

$$f_m^{(k)}(r) = \left(\frac{\sqrt{2}r}{r_\ell} \right)^m \left(\frac{2\pi k!}{(m+k)!} \right)^{1/2} L_k^{(m)} \left(\frac{2r^2}{r_\ell^2} \right) \quad (16)$$

with $L_m^{(k)}(x)$ being the generalized Laguerre polynomial

$$L_m^{(k)}(x) = \sum_{i=0}^k (-1)^i \binom{m+k}{k-i} \frac{x^i}{i!} \quad (17)$$

For the beam spectrum of mode $m = 0$, we have the beam spectrum of the first orthogonal polynomial, $k = 0$, as,

$$\lambda_0(n) = 2\sqrt{\frac{2}{\pi}} \int_0^\infty e^{-2r^2/r_\ell^2} J_0(nr) \left(\frac{r}{r_\ell} \right) d \left(\frac{r}{r_\ell} \right) = \frac{1}{\sqrt{2\pi}} e^{-n^2 r_\ell^2 / 8} \quad (18)$$

where the following equation [3] is used,

$$\int_0^\infty e^{-a^2 x^2} x^{\nu+1} J_\nu(bx) dx = \frac{b^\nu}{(2a^2)^{\nu+1}} e^{-b^2/4a^2} \quad (19)$$

with $\nu = 0$, $a^2 = 2$, $x = r/r_\ell$, and $b = nr_\ell$.

For the beam spectrum of mode $m = 1$, we have,

$$\lambda_1(n) = \frac{4}{\sqrt{\pi}} \int_0^\infty e^{-2r^2/r_\ell^2} J_1(nr) \left(\frac{r}{r_\ell}\right)^2 d\left(\frac{r}{r_\ell}\right) = \frac{nr_\ell}{4\sqrt{\pi}} e^{-n^2 r_\ell^2/8} \quad (20)$$

Using $r_\ell = 2\omega_0\sigma_\tau$, the power spectra can be written as,

$$h_0(n) = \frac{1}{2\pi} e^{-n^2 r_\ell^2/4} = \frac{1}{2\pi} e^{-(n\omega_0\sigma_\tau)^2} \quad (21)$$

and

$$h_1(n) = \frac{n^2 r_\ell^2}{16\pi} e^{-n^2 r_\ell^2/4} = \frac{(n\omega_0\sigma_\tau)^2}{4\pi} e^{-(n\omega_0\sigma_\tau)^2} \quad (22)$$

References

- [1] Workshop on the RHIC Performance, BNL, March, 1988.
- [2] S. Peggs and W.W. MacKay, RHIC/AP/36, Sep. 1994.
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