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Abstract. In accelerators, due to quadrupole roll errors and solenoid fields, the polarized proton acceleration often encounters coupling spin resonances at the horizontal betatron tune. In the Brookhaven AGS, the coupling effect comes from the solenoid partial snake which is used to overcome imperfection resonances. The coupling spin resonance strength is proportional to the amount of coupling as well as the strength of the corresponding intrinsic spin resonance. Generally in accelerators, the betatron tunes in the horizontal and vertical planes are separated to reduce the coupling strength and therefore the depolarization due to the coupling spin resonance. However, the coupling resonance can still cause substantial beam polarization loss if its corresponding intrinsic spin resonance is very strong. A new method of using an horizontal rf dipole to induce a full spin flip crossing both the intrinsic and its coupling spin resonances is studied in the Brookhaven's AGS. In this method, both the horizontal and vertical unperturbed betatron tunes are set to be equal, and the coupling resonance is as strong as the intrinsic resonance. Numerical simulations show that a full spin flip can be induced after crossing the two resonances by using a horizontal rf dipole to induce a large vertical coherent oscillation.

I INTRODUCTION

In an accelerator, particles undergo betatron oscillations in both horizontal and vertical planes while they circulate around the machine. In a perfect machine, both oscillations are independent of each other. However, this independence can be broken if there is any dipole roll error, quadrupole misalignment or solenoid field. In this case, the horizontal motion then gets coupled to the vertical oscillation. Unlike the uncoupled case, the frequency spectrum of the betatron oscillation in either of the two transverse plane then consists of two components ν_1 and ν_2 given by [1,2]

$$\nu_1 = \frac{1}{2}(\nu_x + \nu_z) + \frac{1}{2}\sqrt{(\nu_x - \nu_z)^2 + \Delta Q_{min}^2} \rightarrow \nu_x;\ \text{without coupleing} \tag{1}$$

$$\nu_2 = \frac{1}{2}(\nu_x + \nu_z) - \frac{1}{2}\sqrt{(\nu_x - \nu_z)^2 + \Delta Q_{min}^2} \rightarrow \nu_z;\ \text{without coupling} \tag{2}$$

[1] The work was performed under the auspices of the US Department of Energy

where ν_x and ν_z are the unperturbed horizontal and vertical tune. ΔQ_{min} is the minimum tune split between the two eigen tunes when $\nu_x = \nu_z$. It is given by

$$\Delta Q_{min} = \frac{1}{2\pi} \oint \sqrt{\beta_x \beta_z} A_{xz} e^{i(\nu_x \phi_x - \nu_z \phi_z - (\nu_x - \nu_z - l)\frac{s}{R}))} ds, \tag{3}$$

where $\beta_{x,z}$ are the betatron amplitude functions for the horizontal and vertical planes and A_{xz} is proportional to the strength of the coupling elements [3]. For a solenoid magnet,

$$A_{xz} = \frac{B_{//}}{2B\rho}[(\frac{\alpha_x}{\beta_x} - \frac{\alpha_x}{\beta_x}) + i(\frac{1}{\beta_x} + \frac{1}{\beta_x})], \tag{4}$$

where $B_{//}$ is the solenoid magnetic field strength and $B\rho$ is the momentum per charge. With weak coupling,

$$\nu_1 \simeq \nu_x \tag{5}$$

$$\nu_2 \simeq \nu_z. \tag{6}$$

In the Brookhaven AGS, the main coupling source comes from the solenoid partial snake which is used to overcome the imperfection spin resonances in the AGS [4]. The snake is 2.4384 m long and located in the $I10$ straight section where $\beta_x =$ 12.2 m, $\beta_z = 18.2$ m, $\alpha_x = 1.050$ and $\alpha_z = -1.470$. For a 5% snake, $\frac{B_{//}}{B\rho} =$ 0.023 m^{-1}, and the minimum tune split ΔQ_{min} from a 5% partial snake is

$$\Delta Q_{min} = 0.01435. \tag{7}$$

In a perfect planar circular machine, the spin vector of a particle precesses around the guiding magnetic field by $G\gamma$ times in one orbital revolution [5]. Here, $G =$ 1.7928474 is the abnormal gyro-magnetic g factor for the proton and γ is the Lorentz factor. The vertical betatron oscillation causes the spin motion of the particle to be perturbed by the horizontally oriented focusing magnetic field. When the frequency of this perturbation coincides with the spin precession frequency, a spin resonance occurs. In the absence of the coupling effect, there is only one spin resonance, namely the intrinsic spin resonance, at $G\gamma = kP \pm \nu_z$ [5]. Here ν_z is the vertical betatron tune, k is an integer and P is the super-periodicity of the machine.

In a coupled machine, in addition to the intrinsic spin resonance at $G\gamma = kP \pm \nu_2$ ($G\gamma = kP \pm \nu_z$ without coupling) [5], the vertical betatron oscillation also drives a coupling spin resonances at $G\gamma = kP \pm \nu_1$. The strength of the coupling resonance is proportional to the amount of the coupling and its strength ϵ_{ν_x} is given by

$$\epsilon_{\nu_x} \propto C_x \sqrt{\varepsilon_u} \epsilon_{\nu_z} \tag{8}$$

where ϵ_{ν_z} is the strength of the adjacent intrinsic spin resonance and C_x is the coupling coefficient. For a fully coupled machine, $\nu_x = \nu_z$ and $C_x = 1$. For a decoupled machine, $C_x = 0$. ε_u is the beam emittance in the eigen direction [6] and equals the horizontal beam emittance if $C_x = 0$.

In the AGS, there are four strong intrinsic spin resonances at $0+\nu_z$, $12+\nu_z$ and $36 \pm \nu_z$ [5]. Traditionally in the AGS, the beam polarization loss at the coupling resonances is minimized by separating the horizontal and vertical set points. The coupling resonances around these four strong intrinsic spin resonances can produce about 35% polarization losses with the normal AGS polarized proton setting [7,8]. In order to achieve 70% polarization in the AGS, one needs to minimize the polarization loss at the coupling resonances. Because they are adjacent to the intrinsic resonances, it is very difficult to use the vertical rf dipole [9] to obtain two full spin flip at both the intrinsic and the coupling resonances.

II USE A HORIZONTAL RF DIPOLE TO CROSS THE INTRINSIC AND COUPLING RESONANCES

Analogous to the method of using a vertical rf dipole at the intrinsic spin resonance, one should also expect to obtain a full spin flip by inducing a strong artificial resonance if the intrinsic and its coupling spin resonances are fully overlapped. Because of the coupling effect, the two spin resonance can never be brought closer than the minimum tune split ΔQ_{min}. However, the ΔQ_{min} in general is small and a full spin flip still should be achievable if the induced resonance is strong enough. In a fully coupled machine, the unperturbed tunes are equal and the intrinsic and the coupling resonances are equally strong and located on either side of the unperturbed betatron tune at a distance of half of ΔQ_{min}.

Unlike using a vertical rf dipole to obtain a vertical coherence in an uncoupled machine [11], the vertical coherence is excited by a horizontal rf dipole instead in a fully coupled machine. To understand this, let's first transform the beam motion from the normal geometric coordinates (x, z, s) to the (u, v, s) coordinate system in which the betatron motions along the two eigen directions are fully decoupled [6], i.e

$$\begin{pmatrix} x \\ x' \\ z \\ z' \end{pmatrix} = R \begin{pmatrix} u \\ u' \\ v \\ v' \end{pmatrix} . \tag{9}$$

where R is the transformation matrix between (x, z, s) and (u, v, s). In a fully coupled machine with weak coupling coming from a solenoid magnet, namely the minimum tune split $\Delta Q_{min} << 1$, the R matrix is given by (See the Appendix)

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -a(\alpha_x+\alpha_z) & -a(\beta_x+\beta_z) \\ 0 & 1 & a(\gamma_x+\gamma_z) & a(\alpha_x+\alpha_z) \\ -a(\alpha_x+\alpha_z) & -a(\beta_x+\beta_z) & 1 & 0 \\ a(\gamma_x+\gamma_z) & a(\alpha_x+\alpha_z) & 0 & 1 \end{pmatrix} , \tag{10}$$

where $a = \frac{1}{\sqrt{\beta_x \gamma_z + \beta_z \gamma_x + 2(1-\alpha_x \alpha_z)}}$ and $\beta_{x,z}$ are the horizontal and vertical betatron functions at where the solenoid magnet is. $\gamma_{x,z} = \frac{1+\alpha_{x,z}^2}{\beta_{x,z}}$ and $\alpha_{x,z} = -\frac{1}{2}\beta'_{x,z}$ are the corresponding twiss parameters. Here $'$ is the derivative on the longitudinal coordinate s. With the weak coupling force, one also has [10]

$$\beta_u \simeq \beta_x; \quad \beta_v \simeq \beta_z \tag{11}$$
$$\alpha_u \simeq \alpha_x; \quad \alpha_v \simeq \alpha_z \tag{12}$$

where $\beta_{u,v}$ and $\alpha_{u,v}$ are the twiss parameters in the two eigen directions.

With a horizontal rf dipole $\Delta B_y L = \Delta B_{ym} L cos\nu_m\theta$, the horizontal excitation is $\delta x' cos\nu_m\theta = \frac{\Delta B_{ym} L}{B\rho} cos\nu_m\theta$ where $B\rho$ is the magnetic rigidity. The corresponding excitation in the (u, v) coordinates are

$$\delta u = 0 \tag{13}$$
$$\delta u' = \frac{1}{\sqrt{2}}\delta x' \tag{14}$$
$$\delta v = \frac{a}{\sqrt{2}}(\beta_x + \beta_z)\delta x' \tag{15}$$
$$\delta v' = -\frac{a}{\sqrt{2}}(\alpha_x + \alpha_z)\delta x'. \tag{16}$$

Similar to the case of using a vertical rf dipole in an uncoupled machine [11], the coherent betatron motion is a fixed point in the frame which rotates with the rf dipole modulation frequency as shown in Fig. 1. In every modulation period, the

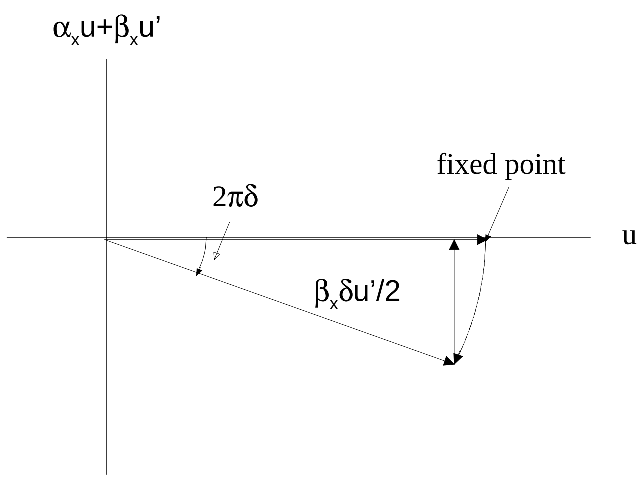

FIGURE 1. Derivation of the coherent betatron motion in the (u, u') plane in the rotating frame. In each modulation period, the phase-space vector rotates through an angle of $2\pi\delta$ and is then given an effective kick by the rf dipole field by an average amount of $\frac{\beta_u \delta u'}{2}$

particle's phase-space vector rotates through an angle of $2\pi\delta$ in the rotating frame where $\delta = |\nu_m - \nu_{1,2}|$. It then gets deflected by the rf dipole. The effective deflection is $\frac{1}{2}\beta_u \delta u^{'}$ due to the fact that the deflection given by the rf dipole is oscillating at the same frequency. Hence, the fixed point in the $(u, u^{'})$ plane is at

$$f_u = b\frac{a}{\sqrt{2}}\beta_x \delta x^{'} \tag{17}$$

$$f_{u'} = -\frac{\alpha_x}{\beta_x} b\frac{a}{\sqrt{2}}\beta_x \delta x^{'} \tag{18}$$

where $b = \frac{1}{4\pi\delta}$. The fixed point in the $(v, v^{'})$ plane can be calculated using the same method. Since $\delta v \neq 0$, the deflection given by the rf dipole in the $(v, v^{'})$ plane is not along the axis of $\alpha_v v + \beta_v v^{'}$. The fixed point in the $(v, v^{'})$ plane is given by

$$f_v = -b\frac{a}{\sqrt{2}}(\alpha_z\beta_x - \beta_z\alpha_x)\delta x^{'} \tag{19}$$

$$f_{v'} = b\frac{a}{\sqrt{2}}(\beta_x\gamma_z - \alpha_x\alpha_z + 1). \tag{20}$$

Transform the fixed points in the (u, v) coordinate system back to the normal geometric (x, z) coordinates, the fixed points are

$$f_x/b = \frac{1}{\sqrt{2}}f_u - \frac{a}{\sqrt{2}}(\alpha_x + \alpha_z)f_v - \frac{a}{\sqrt{2}}(\beta_x + \beta_z)f_{v'} \tag{21}$$

$$f_{x'}/b = \frac{1}{\sqrt{2}}f_{u'} + \frac{a}{\sqrt{2}}(\gamma_x + \gamma_z)f_v + \frac{a}{\sqrt{2}}(\alpha_x + \alpha_z)f_{v'} \tag{22}$$

$$f_z/b = -\frac{a}{\sqrt{2}}(\alpha_x + \alpha_z)f_u - \frac{a}{\sqrt{2}}(\beta_x + \beta_z)f_{u'} + \frac{1}{\sqrt{2}}f_v \tag{23}$$

$$f_{z'}/b = \frac{a}{\sqrt{2}}(\gamma_x + \gamma_z)f_u + \frac{a}{\sqrt{2}}(\alpha_x + \alpha_z)f_{u'} + \frac{1}{\sqrt{2}}f_{v'} \tag{24}$$

$$\tag{25}$$

By substituting Eq. **??** to Eq. 25, one then obtains

$$f_x/b = 0.0 \tag{26}$$

$$f_{x'}/b = 0.0 \tag{27}$$

$$f_z/b = a(\beta_z\alpha_x - \beta_x\alpha_z)\delta x^{'} \tag{28}$$

$$f_{z'}/b = a(\beta_x\gamma z - \alpha_x\alpha_z + 1)\delta x^{'} \tag{29}$$

$$. \tag{30}$$

This demonstrates that in a fully coupled machine, a vertical coherence can be excited by applying a horizontal rf dipole. The amplitude of the vertical coherence is $\frac{BL}{4\pi B\rho\delta}\sqrt{\beta_x\beta_z}$.

Fig. 2 shows spin numerical spin tracking results at $G\gamma = 36 + \nu_z$. The dotted line shows the result with the nominal AGS tune setting ($\nu_x = 8.8, \nu_z = 8.7$) and no correction scheme for the intrinsic spin resonance. In this case, the depolarization at the coupling resonance is obvious. The solid line is the result of using a horizontal rf dipole with the horizontal and vertical betatron tunes set at 8.7. Due to the coupling from the solenoid partial snake, the two betatron tunes are split by 0.0144. The horizontal rf dipole tune was set to be 0.3. With a horizontal RF dipole amplitude 28.0 G-m, a full spin flip was achieved.

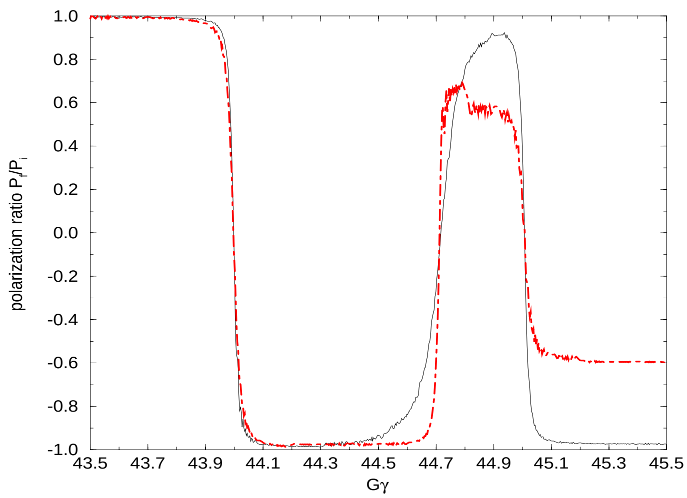

FIGURE 2. This figure shows the calculated polarization ratio P_f/P_i as a function of energy. The solid line is for the case of a fully coupled machine and a horizontal rf dipole was used to obtain an adiabatic vertical coherence. The dotted line is the result of a weakly coupled machine with the two betatron tunes set 0.1 apart. No correction scheme was used at the $G\gamma = 36 + \nu_z$ intrinsic spin resonance. For both cases, the horizontal and vertical emittance are 20π mm-mrad and 10π mm-mrad respectively.

III EXPERIMENTAL RESULTS

The method of using a horizontal rf dipole to excite a vertical coherence to cross the coupling spin resonance was tested in the AGS during the 2000 RHIC polarized proton commissioning run. The polarized H^- beam was pre-accelerated in the 200 MeV LINAC and then stripped and injected into the Booster. It was then injected into the AGS at $G\gamma = 4.7$ and accelerated up to $G\gamma = 46.5$. In the AGS, the nominal tune setting is $\nu_x = 8.8$ and $\nu_z = 8.7$.

During the experiment, the AGS skew quadrupoles were all set to 17 A. Due to a hardware limit, the partial snake strength at $G\gamma = 36 + \nu_z$ is actually only about 3.5% instead of 5%. The combined effect of the skew quadrupoles and the weaker snake gave a smaller minimum tune spilt ΔQ_{min} of 0.007. The horizontal rf dipole was set in the middle of the two betatron tunes ν_1 and ν_2. The turn by turn beam position monitor data confirmed that a vertical coherence was excited without horizontal response as shown in the left part of Fig. 3. The horizontal response was not zero once the rf dipole tune deviated from the average of the two eigen tunes as shown on the right of Fig. 3.

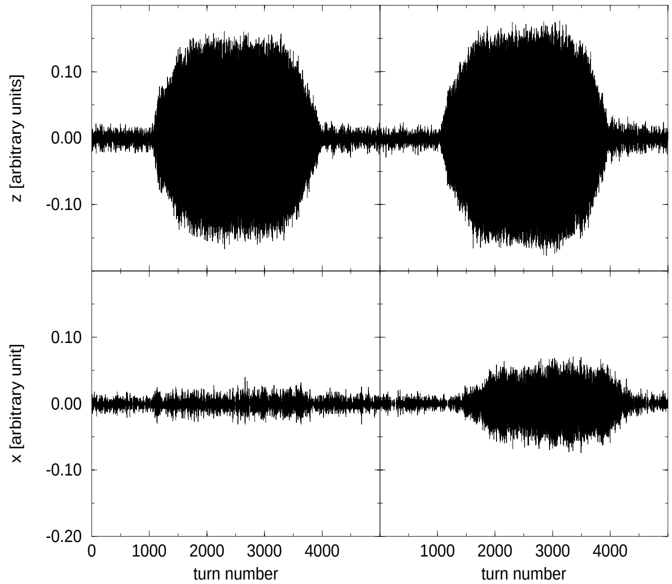

FIGURE 3. The top and bottom plots on the left are vertical and horizontal turn-by-turn beam position data when the horizontal rf dipole modulation tune $\nu_m = \frac{1}{2}(\nu_1 + \nu_2)$. As shown, no horizontal coherence was excited. The two plots on the right correspond to the case where the horizontal modulation tune $\nu_m \neq \frac{1}{2}(\nu_1 + \nu_2)$ and the horizontal coherence was no longer zero.

Table 1 shows the comparison of the measured beam asymmetries of using vertical rf dipole, no correction and using horizontal rf dipole at $G\gamma = 36 + \nu_z$. Comparing the measured asymmetry when using the horizontal rf dipole with the case of no correction, it is clear that the horizontal rf dipole did help to recover the beam polarization. However, the excited coherence was not optimized and about 70%

TABLE 1. measured asymmetry

	measured asymmetry (x10^{-3})	condition
1	1.50$\pm$ 0.04	with vertical rf dipole
2	1.25$\pm$ 0.1	with horizontal rf dipole
3	0.067$\pm$ 0.063	no correction

beam emittance growth was observed. Because of limitations of the AGS sextupole power supplies, we could not achieve small chromaticities in both planes and obtain a fully adiabatic excitation. This is the most likely reason that the horizontal rf dipole did not recover 100% beam polarization as expected.

IV CONCLUSION

It has been demonstrated in the AGS that in a fully coupled machine, a vertical coherence can be excited by an horizontal rf dipole. Although beam polarization was improved, we think the residual polarization loss was due to the not fully adiabatic beam motion.

V ACKNOWLEDGEMENT

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VI APPENDIX

In an accelerator with a solenoid magnet as the only source of coupling, the one turn matrix T is

$$T = \begin{pmatrix} M & n \\ m & N \end{pmatrix} = \begin{pmatrix} p & q \\ -q & p \end{pmatrix} \begin{pmatrix} M_1 & 0 \\ 0 & N_1 \end{pmatrix}. \tag{31}$$

where M_1 and N_1 are the matrices starting from the beginning of the solenoid to the end of the solenoid. M and N are the horizontal and vertical matrices.

$$M = \begin{pmatrix} cos\mu_x + \alpha_x sin\mu_x & \beta_x sin\mu_x \\ -\gamma_x sin\mu_x & cos\mu_x - \alpha_x sin\mu_x \end{pmatrix} \tag{32}$$

and

$$N = \begin{pmatrix} cos\mu_z + \alpha_z sin\mu_z & \beta_z sin\mu_z \\ -\gamma_z sin\mu_z & cos\mu_z - \alpha_z sin\mu_z \end{pmatrix} \tag{33}$$

where $alpha_{x,z}$, $\beta_{x,z}$ and $\gamma_{x,z}$ are the horizontal and vertical twiss parameters. $\mu_{x,z}$ are the horizontal and vertical phase advances. p and q are the components in the solenoid transfer matrix T_s.

$$T_s = \begin{pmatrix} p & q \\ -q & p \end{pmatrix} = \begin{pmatrix} C^2 & \frac{SC}{K} & SC & \frac{S^2}{K} \\ -KSC & C^2 & -kS^2 & SC \\ -SC & -\frac{S^2}{K} & C^2 & \frac{SC}{K} \\ kS^2 & -SC & -KSC & C^2 \end{pmatrix}, \tag{34}$$

where $K = \frac{B_{//}}{2B\rho}$, $C = cos(KL)$, $S = sin(KL)$ and L is the length of the solenoid. $B_{//}$ is the strength of the solenoid. From Eq. 31, one has

$$pM_1 = M; \tag{35}$$
$$pN_1 = N; \tag{36}$$
$$n = qN_1 = qp^{-1}N; \tag{37}$$
$$m = -qM_1 = qp^{-1}M; \tag{38}$$
$$\tag{39}$$

In a fully coupled machine where $\mu_x = \mu_z$, the transformation matrix R which diagnoses the one turn matrix T in Eq. 31 is given by [4]

$$R = \begin{pmatrix} \sqrt{1-|E|} & -sE\ s \\ -E & \sqrt{1-|E|} \end{pmatrix} \tag{40}$$

where

$$s = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{41}$$

and

$$E = -\frac{\sqrt{2}}{4sin\mu sin(\pi\Delta Q_{min})}(m - sn\ s). \tag{42}$$

where ΔQ_{min} is the minimum tune split given by [3]

$$\Delta Q_{min} = \frac{KL}{2\pi}\sqrt{\beta_x\beta_z}[(\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{beta_z}) + i(\frac{1}{\beta_x} + \frac{1}{\beta_z})], \tag{43}$$

where $KL = \frac{B_{//}L}{2B\rho}$. The size of the minimum tune split is

$$\Delta Q_{min} = \frac{KL}{2\pi}a = \frac{KL}{2\pi}\sqrt{\beta_x\gamma_z + \beta_z\gamma_x + 2(1-\alpha_x\alpha_z)}. \tag{44}$$

In a fully coupled machine with weak coupling strength, namely $\Delta Q_{min} \ll 1$ and $KL \ll 1$, we then have

$$sin(\pi\Delta Q_{min}) \simeq \pi\Delta Q_{min}, \tag{45}$$

and

$$\frac{sin(KL)}{cos(KL)} \simeq KL. \tag{46}$$

Substituting Eq. 45, Eq. 46, m in Eq. 39 and n in Eq. 38into Eq. 42, one then gets

$$E = -\frac{1}{\sqrt{2}} a \begin{pmatrix} -(\alpha_x + \alpha_z) & -(\beta_x + \beta_z) \\ (\gamma_x + \gamma_z) & (\alpha_x + \alpha_z) \end{pmatrix}. \tag{47}$$

Hence, the R matrix is

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -a(\alpha_x + \alpha_z) & -(\beta_x + \beta_z) \\ 0 & 1 & (\gamma_x + \gamma_z) & (\alpha_x + \alpha_z) \\ -a(\alpha_x + \alpha_z) & -(\beta_x + \beta_z) & 1 & 0 \\ (\gamma_x + \gamma_z) & (\alpha_x + \alpha_z) & 0 & 1 \end{pmatrix} \tag{48}$$

and its inverse matrix R^{-1} is

$$R^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & a(\alpha_x + \alpha_z) & (\beta_x + \beta_z) \\ 0 & 1 & -(\gamma_x + \gamma_z) & -(\alpha_x + \alpha_z) \\ a(\alpha_x + \alpha_z) & (\beta_x + \beta_z) & 1 & 0 \\ -(\gamma_x + \gamma_z) & -(\alpha_x + \alpha_z) & 0 & 1 \end{pmatrix} \tag{49}$$

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