BNL-99343-2013-TECH

C-A/AP/192;BNL-99343-2013-IR

Modeling Injection Trajectories on the Midplane of the C5 Dipole in Booster

C. J. Gardner

February 2005

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

C-A/AP/#192
February 2005

Modeling Injection Trajectories on the Midplane of the C5 Dipole in Booster

C.J. Gardner

Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973

Modeling Injection Trajectories on the Midplane of the C5 Dipole in Booster

C.J. Gardner

February 2, 2005

1 The Booster Dipole Lamination

A scale drawing of the Booster dipole lamination is shown in **Figure 1**. Here the pole width is 254 mm. The distance from the pole to the backleg is 121 mm. The width of the backleg is 133 mm. The vertical gap between the poles is 82.55 mm. The shims on the poles are 19 mm wide and the vertical gap between them is 77.75 mm. The height of the lamination is 596.55 mm and the height of the H-shaped opening is 330.55 mm. The brown rectangles indicate magnet coils. G10 insulating strips are indicated by the thin green rectangles. The thin pink rectangles indicate trim windings.

2 Top View of the Dipole

A top view of the Booster dipole is shown in **Figure 2**. Here the beam direction is from left to right. The horizontal and vertical axes give the x and y coordinates on the magnet midplane. The units are meters. The radial and azimuthal coordinates are defined by

$$x = r\sin\theta, \quad y = r\cos\theta. \tag{1}$$

The line $x = 0$ bisects the magnet. The magnetic length is

$$L = 2.42 \text{ m} \tag{2}$$

and the bend angle is

$$\theta_B = 2\pi/36. \tag{3}$$

The dotted black curve is the nominal trajectory of circulating beam on the midplane; it is a circular arc with radius-of-curvature

$$\rho_0 = L/\theta_B = 13.8656 \text{ m}. \tag{4}$$

The center of curvature is the point $x = 0$, $y = 0$. As noted by Bleser [1] the centerline of the dipole is actually displaced radially outward by 0.18 mm with respect to the nominal trajectory. The practical consequences of this are negligible and we shall assume that the centerline and nominal trajectory coincide. The red lines and curves show the perimeter of the magnet iron. The left and right edges of the magnet iron lie on the lines

$$\theta = \mp\theta_2/2 \tag{5}$$

where $\theta_2 = 9.5763$ degrees as given by Bleser [1]. The gray curves are the projections of the walls of the vacuum chamber onto the midplane; these are 82.55 mm from the magnet centerline. The black curves are the projections of the pole and backleg boundaries. Those closest to the centerline mark the pole boundaries; they are 127 mm from the centerline. The outer black curves mark the backleg boundaries; these are 248 mm from the centerline.

3 Magnetic Field on the Midplane

Using the dimensions of the lamination, Wuzheng Meng has calculated the field on the dipole midplane. This was done using the Opera code with the assumption that the field is independent of azimuthal coordinate θ. A plot of the field as a function of radius is shown in **Figure 3**. Here the blue curve gives the field in units of the field B_c at the center of the magnet. The horizontal axis gives the radial distance from the projection of the pole edge on the midplane. The units are mm. The black lines at 0 and 121 mark the pole and backleg edges. The dotted line at -127 mm marks the center of the magnet; the red line at -44.45 mm marks the vacuum chamber wall. Numerical integration of the equations of motion using this field gives the trajectory of the H-minus beam on the midplane of the C5 dipole.

Following is a list of the field data obtained by Meng. The first number in each pair is the radial distance from the magnet centerline in cm; the second number is the midplane field in Gauss.

Magnetic Field on Midplane of Booster Dipole

0.0	1550.32809468678				
0.1	1550.32816031908	3.1	1550.35072817815	6.1	1550.57861174407
0.2	1550.32822595138	3.2	1550.35300673234	6.2	1550.59321058839
0.3	1550.32829158368	3.3	1550.35528528653	6.3	1550.60780943272
0.4	1550.32837187513	3.4	1550.35786153424	6.4	1550.62240827704
0.5	1550.32850314418	3.5	1550.36107154500	6.5	1550.63387149466
0.6	1550.32863441322	3.6	1550.36428155576	6.6	1550.64481852892
0.7	1550.32876568227	3.7	1550.36749156652	6.7	1550.65576556318
0.8	1550.32893649864	3.8	1550.37070157728	6.8	1550.66671259745
0.9	1550.32919026212	3.9	1550.37491285293	6.9	1550.67765963171
1.0	1550.32944402560	4.0	1550.37993564101	7.0	1550.68440038160
1.1	1550.32969778907	4.1	1550.38495842908	7.1	1550.68327556832
1.2	1550.33001257114	4.2	1550.38998121716	7.2	1550.68215075504
1.3	1550.33047072633	4.3	1550.39500400523	7.3	1550.68102594175
1.4	1550.33092888151	4.4	1550.40134445368	7.4	1550.67990112847
1.5	1550.33138703670	4.5	1550.40835992284	7.5	1550.67877631519
1.6	1550.33187868078	4.6	1550.41537539201	7.6	1550.62770798804
1.7	1550.33255950149	4.7	1550.42239086117	7.7	1550.55667344091
1.8	1550.33324032219	4.8	1550.42940633034	7.8	1550.48563889377
1.9	1550.33392114290	4.9	1550.43806376965	7.9	1550.41460434663
2.0	1550.33460196361	5.0	1550.44761966103	8.0	1550.34356979950
2.1	1550.33559121098	5.1	1550.45717555242	8.1	1550.27253525236
2.2	1550.33662313995	5.2	1550.46673144380	8.2	1550.07280072485
2.3	1550.33765506892	5.3	1550.47628733519	8.3	1549.86721841522
2.4	1550.33868699790	5.4	1550.48724719141	8.4	1549.66163610558
2.5	1550.33997207344	5.5	1550.49956897096	8.5	1549.45605379595
2.6	1550.34152849548	5.6	1550.51189075051	8.6	1549.25047148631
2.7	1550.34308491751	5.7	1550.52421253006	8.7	1549.02405205726
2.8	1550.34464133954	5.8	1550.53653430961	8.8	1548.54008471874
2.9	1550.34619776158	5.9	1550.54941405542	8.9	1548.05611738022
3.0	1550.34844962396	6.0	1550.56401289975	9.0	1547.57215004169

Note that a proton with 200 MeV kinetic energy has magnetic rigidity $B\rho = 2.149636$ Tm. In order for it to follow a circular arc with nominal radius-of-curvature $\rho = \rho_0 = 13.8656$ m, the magnetic field must be 1550.34 Gauss.

Magnetic Field on Midplane of Booster Dipole

9.1	1547.08818270317	12.1	1409.02901917878	15.1	925.931627436054
9.2	1546.60421536465	12.2	1397.20012266527	15.2	908.552676229051
9.3	1546.06496402176	12.3	1385.37122615176	15.3	891.756861059250
9.4	1544.78367915136	12.4	1373.54232963826	15.4	875.680716012216
9.5	1543.50239428096	12.5	1361.18650442872	15.5	859.604570965183
9.6	1542.22110941056	12.6	1346.50007702568	15.6	843.528425918149
9.7	1540.93982454016	12.7	1331.81364962265	15.7	827.452280871115
9.8	1539.65853966976	12.8	1317.12722221961	15.8	811.376135824081
9.9	1538.37725479937	12.9	1302.44079481657	15.9	795.299990777048
10.0	1535.84895174343	13.0	1287.75436741353	16.0	779.554826592807
10.1	1533.24105178204	13.1	1273.06794001050	16.1	765.102430182768
10.2	1530.63315182064	13.2	1257.35441810100	16.2	750.650033772729
10.3	1528.02525185925	13.3	1240.31193161505	16.3	736.197637362691
10.4	1525.41735189785	13.4	1223.26944512909	16.4	721.745240952652
10.5	1522.80945193646	13.5	1206.22695864314	16.5	707.292844542612
10.6	1518.63779148480	13.6	1189.18447215718	16.6	692.840448132574
10.7	1513.75313684671	13.7	1172.14198567123	16.7	678.388051722535
10.8	1508.86848220861	13.8	1155.09949918528	16.8	665.418872404800
10.9	1503.98382757052	13.9	1137.59438019233	16.9	652.740128720984
11.0	1499.09917293242	14.0	1119.73111395477	17.0	640.061385037168
11.1	1494.21451829432	14.1	1101.86784771721	17.1	627.382641353352
11.2	1488.52758034307	14.2	1084.00458147965	17.2	614.703897669536
11.3	1481.05366214793	14.3	1066.14131524209	17.3	602.025153985720
11.4	1473.57974395278	14.4	1048.27804900453	17.4	589.346410301904
11.5	1466.10582575763	14.5	1030.41478276698	17.5	577.127415357433
11.6	1458.63190756248	14.6	1012.82638347107	17.6	565.781904687268
11.7	1451.15798936733	14.7	995.447432264067	17.7	554.436394017103
11.8	1443.68407117218	14.8	978.068481057064	17.8	543.090883346938
11.9	1432.68681220579	14.9	960.689529850061	17.9	531.745372676773
12.0	1420.85791569228	15.0	943.310578643057	18.0	520.399862006608

Magnetic Field on Midplane of Booster Dipole

18.1	509.054351336444	20.1	321.023568964196	22.1	171.435711669852
18.2	497.708840666279	20.2	312.868437718537	22.2	164.498603596021
18.3	487.334395320800	20.3	304.713306472877	22.3	157.803943701659
18.4	477.319840413637	20.4	296.558175227218	22.4	151.171404419460
18.5	467.305285506473	20.5	288.403043981558	22.5	144.538865137261
18.6	457.290730599310	20.6	280.333956719487	22.6	137.906325855062
18.7	447.276175692147	20.7	272.789337109045	22.7	131.273786572862
18.8	437.261620784984	20.8	265.244717498604	22.8	124.641247290663
18.9	427.247065877821	20.9	257.700097888161	22.9	118.008708008464
19.0	417.232510970657	21.0	250.155478277719	23.0	111.376168726265
19.1	408.242840757854	21.1	242.610858667277	23.1	104.854869498242
19.2	399.262961160166	21.2	235.066239056835	23.2	98.4763899492302
19.3	390.283081562478	21.3	227.521619446394	23.3	92.0979104002185
19.4	381.303201964790	21.4	219.995468186664	23.4	85.7194308512066
19.5	372.323322367102	21.5	213.058360112834	23.5	79.3409513021949
19.6	363.343442769414	21.6	206.121252039004	23.6	72.9624717531829
19.7	354.363563171726	21.7	199.184143965173	23.7	66.5839922041710
19.8	345.488962701174	21.8	192.247035891343	23.8	60.2055126551592
19.9	337.333831455515	21.9	185.309927817512	23.9	53.8270331061473
20.0	329.178700209855	22.0	178.372819743682	24.0	47.5693325627023
				24.1	41.3172123640681
				24.2	35.0650921654340
				24.3	28.8129719668001
				24.4	22.5608517681659
				24.5	16.3087315695320
				24.6	10.0566113708978
				24.7	3.80449117226369

4 LTB Centerline in the C5 Dipole

The position of the LTB (Linac To Booster) centerline in the C5 dipole is given by Bleser [1] and is shown in **Figure 4**. Here the beam direction is from left to right. The dotted violet line is the centerline; it's angle with respect to the x axis is $\theta_4 = 11.88573$ degrees. The equation of the line is

$$y = mx + b \tag{6}$$

where

$$m = -\tan\theta_4, \quad b = 13.939951 \text{ m}. \tag{7}$$

The x and y coordinates of the entrance to the dipole along the centerline are defined to be

$$x_I = -\{\rho_0 + W_L/2\}\sin(\theta_B/2), \quad y_I = mx_I + b \tag{8}$$

where $W_L = 0.762$ m is the width of the dipole lamination.

Note that the LTB centerline intersects the inside wall of the vacuum chamber 10.7 inches upstream of the downstream end of the C5 magnet iron. Other possible neutral beam trajectories in the C5 dipole are shown in **Figure 5**. Here again the dotted violet line is the centerline. At the upstream end of the magnet, the solid violet lines are displaced 20 mm from the dotted line. The lower (upper) line has angle −2 (+2) mrad with respect to the dotted line. These lines intersect the inside wall of the vacuum chamber 17.1 and 3.6 inches upstream of the downstream end of the magnet iron. This gives some idea of where neutral hydrogen atoms coming down the LTB line may hit the C5 vacuum chamber.

5 H-minus Trajectory by Numerical Integration

Let

$$Z_1 = x, \quad Z_2 = \frac{dx}{ds} = x', \quad Z_3 = y, \quad Z_4 = \frac{dy}{ds} = y' \tag{9}$$

where s is the distance along the H-minus trajectory and the primes denote differentiation with respect to s. Then the equations of motion on the midplane are [2]

$$Z_1' = Z_2, \quad Z_2' = -KZ_4, \quad Z_3' = Z_4, \quad Z_4' = KZ_2 \tag{10}$$

where

$$Z_2^2 + Z_4^2 = 1 \tag{11}$$

and

$$K = \frac{e}{cp}B(Z_1, Z_3) = \frac{e}{cp}B(x, y) = \frac{1}{\rho_c}\{B(x, y)/B_c\}. \tag{12}$$

Here e is the proton charge, p is the particle momentum, $B(x, y)$ is the field on the midplane, B_c is the field at the center of the magnet and

$$\rho_c = \frac{cp}{eB_c}. \tag{13}$$

The ratio $B(x, y)/B_c$ is given by Meng's data. These equations can be integrated numerically to obtain the particle trajectory. Employing the fourth-order Runge-Kutta method [3, 4] we obtain the trajectory shown in **Figure 6**. Here the H-minus beam enters the C5 dipole along the LTB centerline. The field in the channel through the backleg is assumed to be zero, so the trajectory follows the LTB centerline until it enters the region between the pole and backleg. It then follows the curved path shown in blue in the figure. Here we have assumed that p and B_c are such that

$$\rho_c = \rho_0 = L/\theta_B. \tag{14}$$

The blue curve ends at the magnet exit defined by the line

$$\theta = \theta_B/2 = \pi/36. \tag{15}$$

Below this line the field is assumed to be zero. At the magnet exit we find

$$x = 1.2060 \text{ m}, \quad y = 13.7844 \text{ m} \tag{16}$$

and

$$x' = 0.99594, \quad y' = -0.090024. \tag{17}$$

6 Transport to the Injection Foil

At the C5 dipole exit we transform to coordinates appropriate for the C6 straight section. The centerline of the straight is perpendicular to the line $\theta = \theta_B/2$ and the transformation is

$$x_s = Cx - Sy, \quad y_s = Sx + Cy - \rho_0 \tag{18}$$

where

$$C = \cos(\theta_B/2), \quad S = \sin(\theta_B/2). \tag{19}$$

Using (16) we then have at the magnet exit

$$x_s = 0, \quad y_s = -28.48 \text{ mm}. \tag{20}$$

We also have

$$\frac{dx_s}{dx} = C - S\,\frac{dy}{dx} = C - S(y'/x') \tag{21}$$

$$\frac{dy_s}{dx} = S + C\,\frac{dy}{dx} = S + C(y'/x') \tag{22}$$

and

$$\frac{dy_s}{dx_s} = \frac{Sx' + Cy'}{Cx' - Sy'} = -2.8796 \text{ mrad.} \tag{23}$$

The drift length from the exit of the C5 dipole to the H-minus injection foil in the C6 straight is

$$L_F = 0.2726 \text{ m} \tag{24}$$

as given by Bleser [1]. Tracking the H-minus trajectory through the drift gives position and angle (with respect to the C6 centerline)

$$X_F = y_s + L_F \frac{dy_s}{dx_s} = -29.3 \text{ mm}, \quad X'_F = \frac{dy_s}{dx_s} = -2.88 \text{ mrad} \tag{25}$$

at the injection foil.

Now let X_0 and X'_0 be the position and angle of an H-minus particle with respect to the LTB centerline at the entrance to the C5 dipole. Tracking the particle through the C5 dipole as in the previous section and then tracking through the drift to the foil we obtain the positions and angles listed in **Table 1**.

Table 1: Position and Angle at Injection Foil obtained by numerical integration with various values of X_0 and X'_0. Units are mm and mrad.

X_0	X'_0	X_F	X'_F
10	0	-33.5	0.20
0	5	-40.6	-6.51
0	0	-29.3	-2.88
0	-5	-18.0	0.74
-10	0	-25.0	-5.95

7 Dispersion at Injection Foil

The calculations of the previous two sections were carried out with $\rho_c/\rho_0 = 1$. If we carry them out with ρ_c/ρ_0 equal to various values close to 1 then we can obtain the dispersion of the H-minus beam at the injection foil. We have

$$\rho_c = \frac{cp}{eB_c}, \quad \rho_0 = \frac{cp_0}{eB_c} = L/\theta_B \tag{26}$$

and

$$\rho_c/\rho_0 - 1 = p/p_0 - 1 = \frac{p - p_0}{p_0} = \frac{\Delta p}{p_0}. \tag{27}$$

Table 2 lists the positions X_F and angles X'_F obtained at the injection foil for various values of $\Delta p/p_0$.

Table 2: Position and Angle at Injection Foil obtained by numerical integration with various values of $\Delta p/p_0$.

ρ_c/ρ_0	$\Delta p/p_0$	X_F(mm)	X'_F(mrad)
1.020	0.020	−31.8	−5.16
1.015	0.015	−31.1	−4.60
1.010	0.010	−30.5	−4.03
1.005	0.005	−29.9	−3.45
1.000	0.000	−29.3	−2.88
0.995	−0.005	−28.6	−2.28
0.990	−0.010	−28.0	−1.69
0.985	−0.015	−27.3	−1.09
0.980	−0.020	−26.7	−0.48

Here we see that for $\Delta p/p_0 = 0.01$ we have

$$\Delta X_F = -1.27 \text{ mm}, \quad \Delta X'_F = -1.17 \text{ mrad}. \tag{28}$$

This gives the dispersion at the foil due to transport of H-minus beam through the C5 dipole.

8 Hard Edge Approximation

Consider the region on the midplane with radial and azimuthal coordinates such that

$$\rho_0 - H \le r \le \rho_0 + H, \quad -\theta_B/2 \le \theta \le \theta_B/2 \tag{29}$$

where

$$\rho_0 = L/\theta_B, \quad H = W_P/2 + d, \quad \theta_B = 2\pi/36. \tag{30}$$

Here $W_P = 254$ mm is the pole width and the pole edges are at radii $r = \rho_0 \pm W_P/2$. The parameter d gives the radial distance from the pole edges to the limits $r = \rho_0 \pm H$. As an approximation we assume that the field everywhere in this region is equal to the field B_c at the center of the magnet. The radius-of-curvature for particle motion in the region is

$$\rho_c = \frac{cp}{eB_c} \tag{31}$$

where p is the particle momentum. Outside the region the field is assumed to be zero. The radii $r = \rho_0 \pm H$ define the hard edges of the field. Under these assumptions the incoming H-minus beam follows the LTB centerline until it encounters the hard edge of the field at radial coordinate $r = \rho_0 + H$. At this point the beam begins to follow a circular arc with radius-of-curvature ρ_c. This continues until the beam reaches the magnet exit defined by the line

$$\theta = \theta_B/2 = \pi/36. \tag{32}$$

Below this line the field is zero and the beam follows a straight line to the injection foil.

Taking $\rho_c = \rho_0$ and $d = 37.7$ mm we obtain the trajectory shown in **Figure 7**. Here the dotted blue curves show the hard edges of the field. The drift length along the LTB centerline from the magnet entrance to the hard edge at $r = \rho_0 + H$ is $\lambda = 858.251$ mm. At the intersection of the centerline with the hard edge, the trajectory has angle $\psi = 76.4731$ degrees with respect to the line perpendicular to the hard edge as shown in **Figure 8**. This results in horizontal focusing (not defocusing!) of the H-minus beam. The trajectory follows the circular arc shown in blue in Figure 7. The turning angle of the arc is $\phi = 6.72156$ degrees. Tracking the trajectory through the drift between the magnet exit and the injection foil gives position and angle

$$X_F = -30.2 \text{ mm}, \quad X'_F = -2.87 \text{ mrad} \tag{33}$$

at the foil. Comparing these values with those given in (25), we see that the hard edge approximation with $d = 37.7$ mm gives essentially the same trajectory as that obtained by numerical integration of the equations of motion using the field calculated by Meng. **Table 3** lists the parameters λ, ψ, ϕ, X_F, and X'_F obtained for various values of the hard edge parameter d.

Figure 9 shows the trajectory of a proton which enters the C5 dipole along the LTB centerline. The proton travels along the centerline until it encounters the hard edge of the field. At this point it begins to follow a circular arc with radius-of-curvature ρ_c. Taking $\rho_c = \rho_0$ and hard edge parameter $d = 37.7$ mm we obtain the magneta curve shown in the figure. This curve intersects the inside wall of the vacuum chamber 21.0 inches upstream of the downstream end of the C5 magnet iron. This gives some idea of where protons coming down the LTB line may hit the C5 vacuum chamber.

Table 3: Parameters obtained using Hard Edge Approximation.

d (mm)	λ (mm)	ψ	ϕ	X_F (mm)	X'_F (mrad)
30	891.328	76.6045	6.58586	−34.6554	−5.23372
31	887.014	76.5874	6.60356	−34.0758	−4.92483
32	882.706	76.5703	6.62123	−33.4956	−4.61634
33	878.403	76.5532	6.63889	−32.9148	−4.30824
34	874.105	76.5361	6.65652	−32.3334	−4.00052
35	869.813	76.5190	6.67413	−31.7514	−3.69318
36	865.526	76.5020	6.69171	−31.1688	−3.38623
37	861.245	76.4850	6.70928	−30.5856	−3.07965
37.7	858.251	76.4731	6.72156	−30.1771	−2.86528
38	856.968	76.4680	6.72682	−30.0019	−2.77346
39	852.697	76.4511	6.74434	−29.4176	−2.46764
40	848.431	76.4341	6.76185	−28.8327	−2.16220
41	844.171	76.4172	6.77932	−28.2472	−1.85713
42	839.915	76.4003	6.79678	−27.6612	−1.55244
43	835.665	76.3835	6.81422	−27.0746	−1.24811
44	831.420	76.3666	6.83163	−26.4874	−0.94416
45	827.180	76.3498	6.84903	−25.8997	−0.64057

9 Linear Matrix for Transport of H-minus Beam through the C5 Dipole

The hard edge approximation allows us to write down a linear matrix for the transport of H-minus beam through the C5 dipole. In the horizontal plane the transfer matrix is [5]

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = \mathbf{BQL} \tag{34}$$

where

$$\mathbf{B} = \begin{pmatrix} C & \rho S & D \\ -S/\rho & C & D' \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \cos\phi, \quad S = \sin\phi \tag{35}$$

$$D = \rho(1 - \cos\phi), \quad D' = \sin\phi \tag{36}$$

and

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 0 \\ -T/\rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T = \tan\psi. \tag{37}$$

Here we take hard edge parameter $d = 37.7$ as before. This gives $\phi = 6.72156$ degrees, $\psi = 76.4731$ degrees, and $\lambda = 0.858251$ m. The radius-of-curvature is $\rho = \rho_0 = 13.8656$ m. Note that the effect of the angle ψ at the hard edge of the field is to focus the beam in the horizontal plane.

Carrying out the matrix multiplications we have

$$M_{11} = C - TS, \quad M_{12} = \rho S + \lambda(C - TS), \quad M_{13} = D \tag{38}$$

$$M_{21} = -(CT + S)/\rho, \quad M_{22} = C - \lambda(CT + S)/\rho, \quad M_{23} = D' \tag{39}$$

and

$$M_{31} = 0, \quad M_{32} = 0, \quad M_{33} = 1. \tag{40}$$

Putting in numbers we have

$$C = 0.993127, \quad S = 0.117044, \quad T = 4.15670 \tag{41}$$

and

$$D = 0.095303 \text{ m}, \quad D' = 0.117044. \tag{42}$$

In the vertical plane the transfer matrix is

$$\mathbf{N} = \mathbf{A}\mathbf{Q}^{-1}\mathbf{L} \tag{43}$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & \rho\phi & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{44}$$

Here the effect of the angle ψ at the hard edge of the field is to defocus the beam.

10 Transport to Foil using Linear Matrix

The linear matrix for the transport of H-minus beam from the C5 dipole entrance to the injection foil is

$$\begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} = \begin{pmatrix} 1 & L_F & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \tag{45}$$

where

$$L_F = 0.2726 \text{ m} \tag{46}$$

is the drift length from the dipole exit to the injection foil. The elements M_{ij} are given by (38–40). Carrying out the matrix multiplication we have

$$T_{11} = M_{11} + L_F M_{21}, \quad T_{12} = M_{12} + L_F M_{22}, \quad T_{13} = D + L_F D' \tag{47}$$

$$T_{21} = M_{21}, \quad T_{22} = M_{22}, \quad T_{23} = D' \tag{48}$$

$$T_{31} = 0, \quad T_{32} = 0, \quad T_{33} = 1. \tag{49}$$

Let X_0 and X'_0 be the position and angle of an H-minus particle with respect to the LTB centerline at the C5 dipole entrance. Let Δp be the deviation of the particle's momentum from the nominal momentum p_0 given by (26). Then the position and angle (with respect to the C6 straight section centerline) at the foil are

$$X_F = X_{F0} - T_{11}X_0 - T_{12}X'_0 - T_{13}\Delta p/p_0 \tag{50}$$

$$X'_F = X'_{F0} - T_{21}X_0 - T_{22}X'_0 - T_{23}\Delta p/p_0 \tag{51}$$

where X_{F0} and X'_{F0} are the nominal position and angle given by (33). **Table 4** lists the values of X_F and X'_F obtained from (50) and (51) for various values of X_0 and X'_0 with $\Delta p/p_0 = 0$. These numbers are in good agreement with those obtained by numerical integration and listed in **Table 1**.

For the case in which $X_0 = 0$, $X'_0 = 0$, and $\Delta p/p_0 = 0.01$ we obtain

$$\Delta X_F = X_F - X_{F0} = -1.27 \text{ mm} \tag{52}$$

and

$$\Delta X'_F = X'_F - X'_{F0} = -1.17 \text{ mrad} \tag{53}$$

which give the dispersion at the foil due to transport of H-minus beam through the C5 dipole. These numbers are in good agreement with those obtained by numerical integration and listed in (28).

Table 4: Position and Angle at Injection Foil obtained using linear transfer matrix with various values of X_0 and X_0'. Units are mm and mrad.

X_0	X_0'	X_F	X_F'
10	0	-34.4	0.20
0	5	-41.5	-6.52
0	0	-30.2	-2.87
0	-5	-18.9	0.79
-10	0	-25.9	-5.93

11 Acknowledgement

I would like to thank Wuzheng Meng for his calculations of the field on the midplane of the Booster dipole.

References

[1] E. Bleser, "Geometry of the Booster Injection Region", Booster Technical Note 216, January 6, 1993

[2] C.J. Gardner, "Notes on Orbit Equations in the AGS", C-A/AP/Note 164, September, 2004, p. 3

[3] W.H. Press, et al, "Numerical Recipes (Fortran Version)", Cambridge University Press, 1989, pp. 550–554.

[4] N. Gershenfeld, "The Nature of Mathematical Modeling", Cambridge University Press, 1999, pp. 67–72.

[5] D.C. Carey, "The Optics of Charged Particle Beams, Harwood Academic Publishers, New York, 1987.

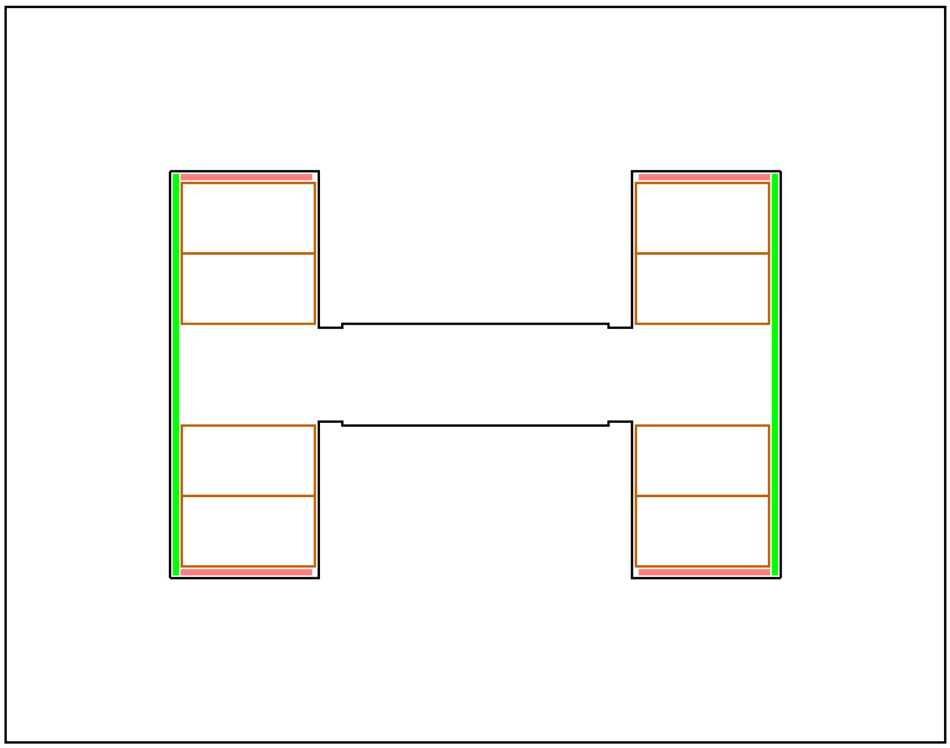

Figure 1: Scale drawing of Booster dipole lamination. The pole width is 254 mm. The distance from the pole to the backleg is 121 mm. The width of the backleg is 133 mm. The vertical gap between the poles is 82.55 mm. The shims on the poles are 19 mm wide and the vertical gap between them is 77.75 mm. The height of the lamination is 596.55 mm and the height of the H-shaped opening is 330.55 mm. The brown rectangles indicate magnet coils. G10 insulating strips are indicated by the thin green rectangles. The thin pink rectangles indicate trim windings.

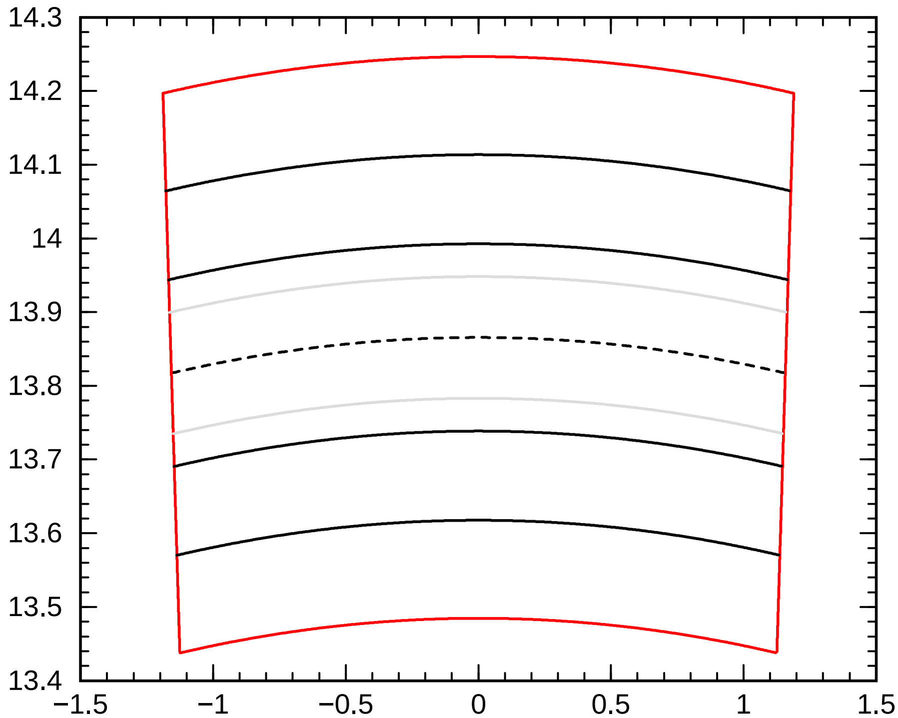

Figure 2: Top view of Booster Dipole. Beam direction is from left to right. The horizontal and vertical axes give the x and y coordinates on the magnet midplane. The units are meters. The radial and azimuthal coordinates are defined by $x = r \sin\theta$ and $y = r\cos\theta$. The line $x = 0$ bisects the magnet. The magnetic length is $L = 2.42$ m and the bend angle is $\theta_B = 2\pi/36$. The dotted black curve is the nominal trajectory of circulating beam on the midplane; it is a circular arc with radius-of-curvature $\rho = L/\theta_B = 13.8656$ m. The center of curvature is the point $x = 0$, $y = 0$. As noted by Bleser [1] the centerline of the dipole is actually displaced radially outward by 0.18 mm with respect to the nominal trajectory. The practical consequences of this are negligible and we assume that the centerline and nominal trajectory coincide. The red lines and curves show the perimeter of the magnet iron. The left and right edges of the magnet iron lie on the lines $\theta = \mp\theta_2/2$, where $\theta_2 = 9.5763$ degrees as given by Bleser [1]. The gray curves are the projections of the walls of the vacuum chamber onto the midplane; these are 82.55 mm from the magnet centerline. The black curves are the projections of the pole and backleg boundaries. Those closest to the centerline mark the pole boundaries; they are 127 mm from the centerline. The outer black curves mark the backleg boundaries; these are 248 mm from the centerline.

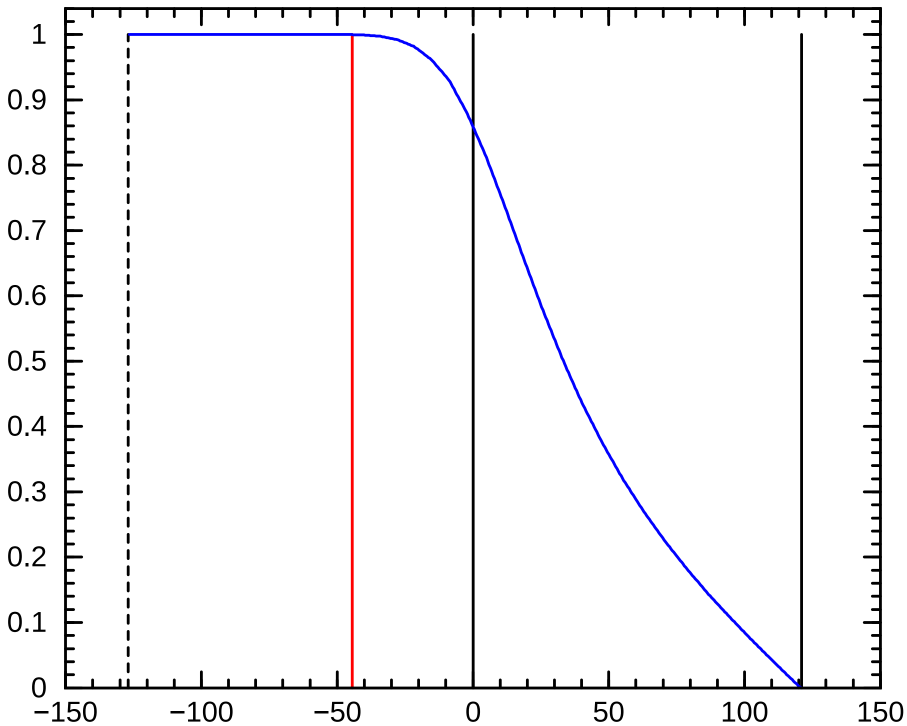

Figure 3: Magnetic field on the midplane obtained by Wuzheng Meng using the Opera code. The blue curve gives the field in units of the field B_c at the center of the magnet. The horizontal axis gives the radial distance from the projection of the pole edge on the midplane. The units are mm. The black lines at 0 and 121 mark the pole and backleg edges. The dotted line at -127 mm marks the center of the magnet; the red line at -44.45 mm marks the vacuum chamber wall. Numerical integration of the equations of motion using this field gives the trajectory of the H-minus beam on the midplane of the C5 dipole.

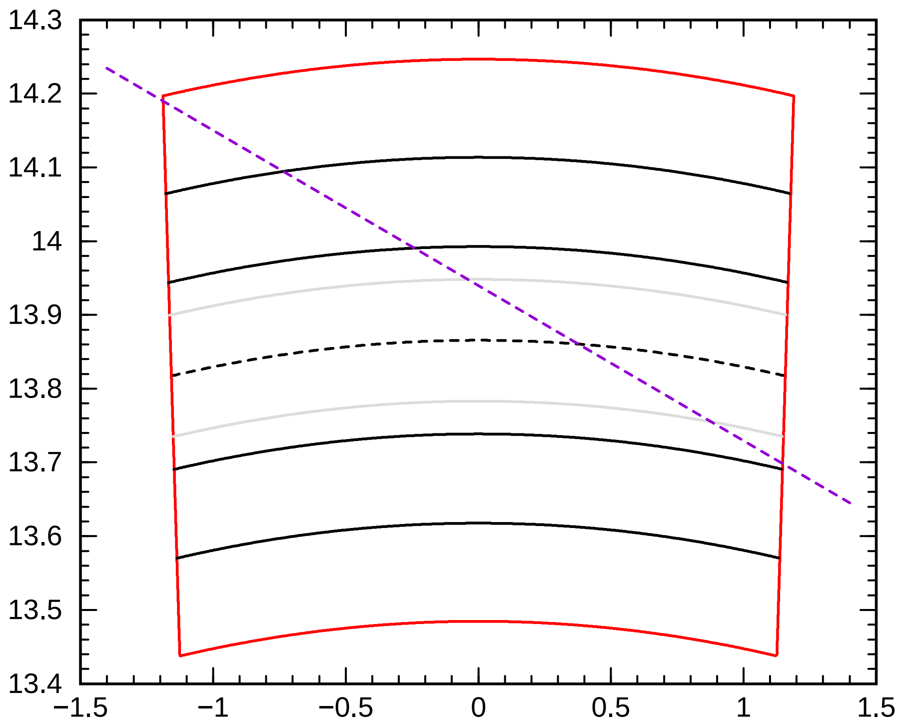

Figure 4: Position of the LTB centerline in the C5 Dipole as given by Bleser [1]. Here the beam direction is from left to right. The dotted violet line is the centerline; it's angle with respect to the x axis is $\theta_4 = 11.88573$ degrees. The equation of the line is $y = mx + b$ where $m = -\tan\theta_4$ and $b = 13.939951$ m. The x and y coordinates of the entrance to the dipole along the centerline are defined to be $x_I = -\{\rho_0 + W_L/2\}\sin(\theta_B/2)$, $y_I = mx_I + b$ where $W_L = 0.762$ m is the width of the dipole lamination. The centerline intersects the inside wall of the vacuum chamber 10.7 inches upstream of the downstream end of the C5 magnet iron.

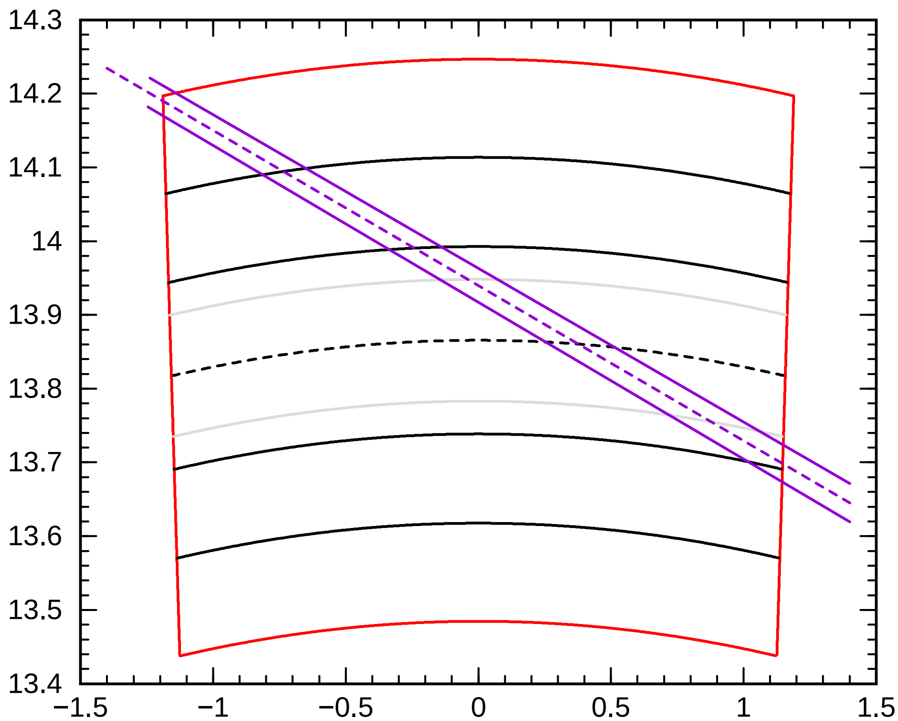

Figure 5: Neutral Beam Trajectories on the midplane. Here again the dotted violet line is the LTB centerline; it intersects the inside wall of the vacuum chamber 10.7 inches upstream of the downstream end of the C5 magnet iron. At the upstream end of the magnet, the solid violet lines are displaced 20 mm from the dotted line. The lower (upper) line has angle -2 ($+2$) mrad with respect to the dotted line. These lines intersect the inside wall of the vacuum chamber 17.1 and 3.6 inches upstream of the downstream end of the magnet iron. This gives some idea of where neutral hydrogen atoms coming down the LTB line may hit the C5 vacuum chamber.

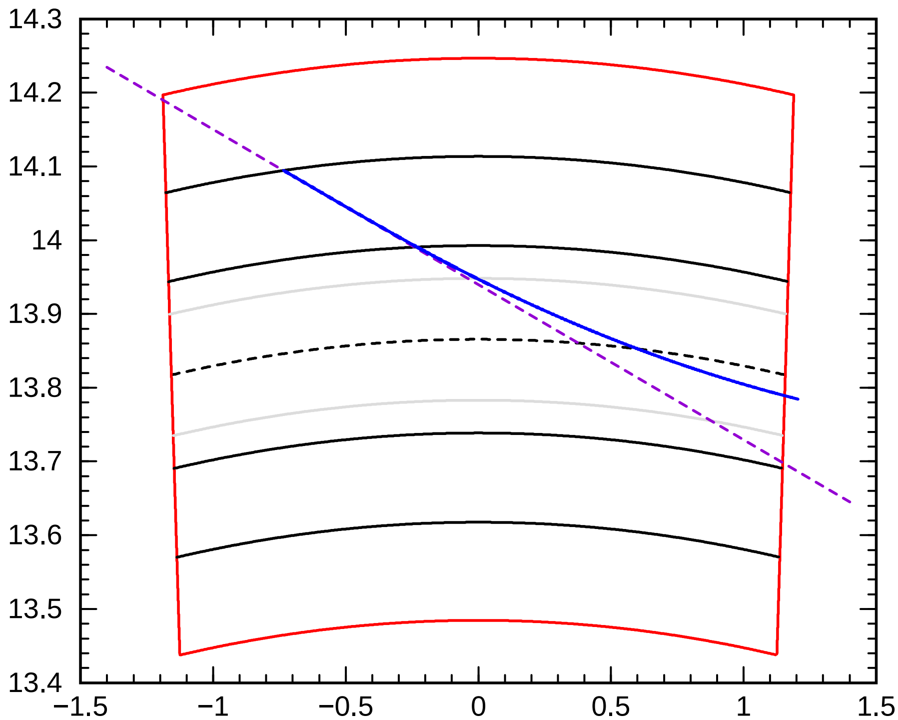

Figure 6: H-minus Trajectory on C5 Dipole Midplane. Here, as before, the dotted violet line is the LTB centerline. The blue curve is the H-minus trajectory obtained by numerical integration of the equations of motion using the field calculated by Meng. The field is assumed to be zero in the channel that passes through the backleg and is assumed to be independent of the azimuthal position on the midplane. The incoming H-minus beam travels along the LTB centerline until it enters the region between the pole and backleg where the field is nonzero. The blue curve ends at the magnet exit defined by the line $\theta = \theta_B/2 = \pi/36$. Below this line the field is assumed to be zero. The drift length from the exit of the magnet to the H-minus injection foil in the C6 straight section is 0.2726 m as given by Bleser [1]. Tracking the trajectory through the drift gives position -29.3 mm and angle -2.88 mrad (with respect to the C6 centerline) at the foil.

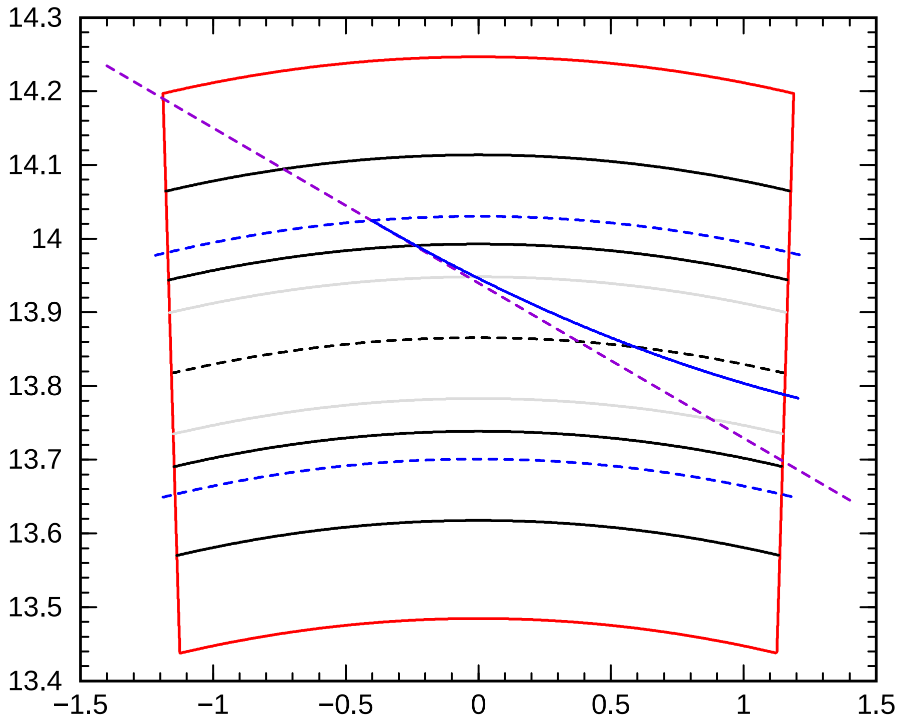

Figure 7: H-minus Trajectory on C5 Dipole Midplane. Here the hard edge approximation has been used to calculate the beam trajectory. The hard edges of the field are shown by the dotted blue curves. These are $d = 37.7$ mm from the pole edges. The solid blue curve is the H-minus trajectory. The resulting position and angle of the H-minus beam at the injection foil are -30.2 mm and -2.87 mrad, in good agreement with the position and angle obtained by numerical integration.

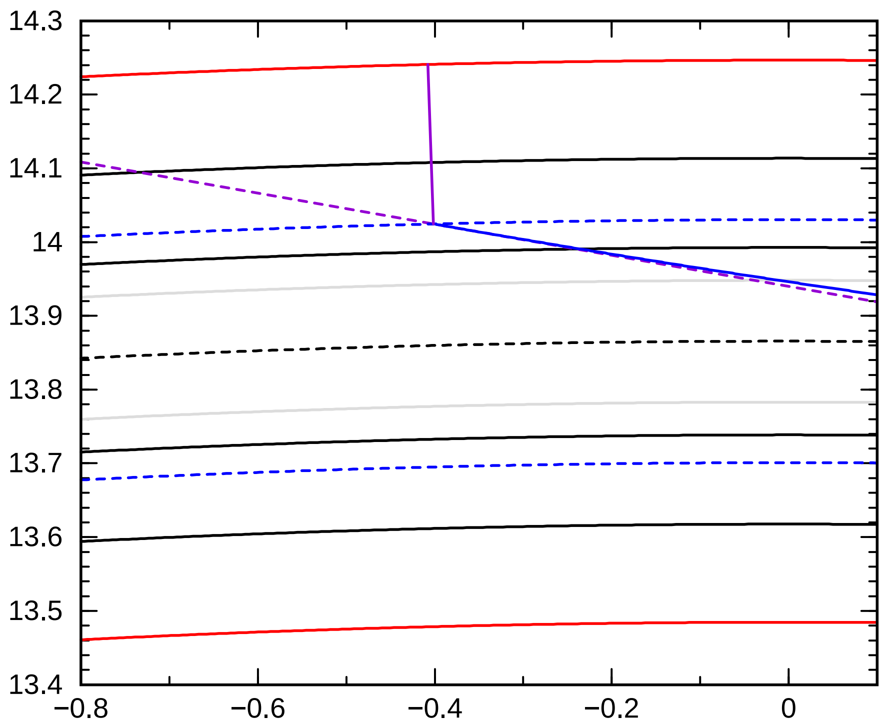

Figure 8: Expanded view of **Figure 7**. Here the solid violet line is perpendicular to the hard edge (dotted blue curve) of the field at the intersection of the centerline (dotted violet line) with the hard edge. The angle between the centerline and the solid violet line is $\psi = 76.4731$ degrees. This results in horizontal focusing of the H-minus beam. The focal length is $f = \rho / \tan\psi = 3.34$ m.

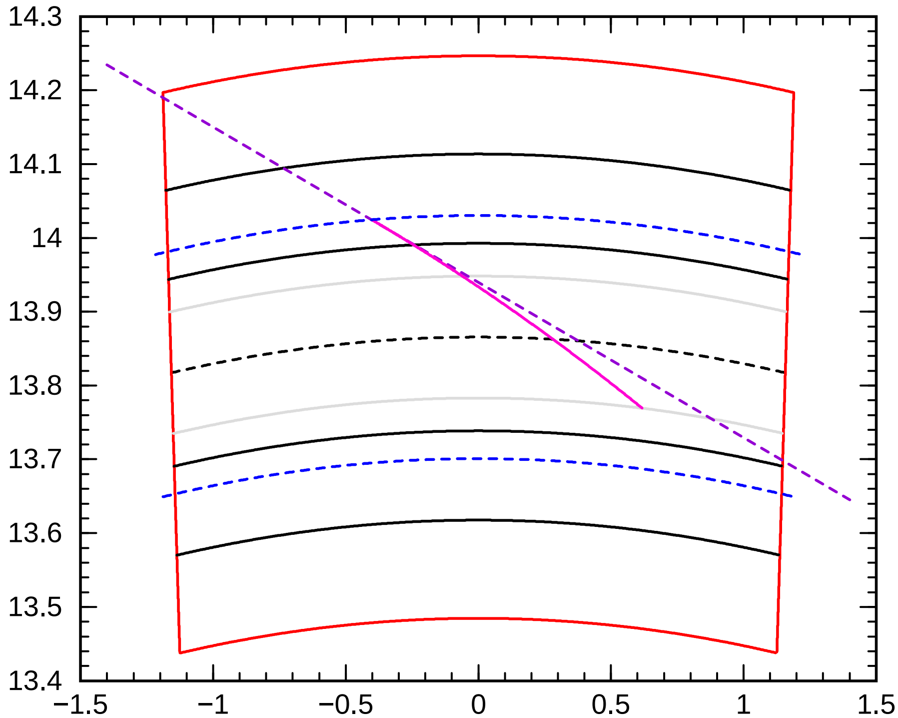

Figure 9: Proton Trajectory on C5 Dipole Midplane. Dotted violet line is the LTB center line. The solid magenta line is the proton trajectory obtained using the hard edge approximation with d = 37.7 mm. The dotted blue curves show the hard edges of the field. The proton beam hits the inside wall of the vacuum chamber 21.0 inches upstream of the downstream end of the C5 magnet iron.