# Measuring the chromatic nonlinearity parameter -alpha 1- in RHIC 

M. Blaskiewicz

July 2000

# Collider Accelerator Department <br> Brookhaven National Laboratory 

## U.S. Department of Energy <br> USDOE Office of Science (SC)

[^0]
## DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

## Measuring ${ }^{\mathbf{H}}{ }_{1}$ in RHIC

 M. Blaskiewicz, et al

Collider-Accelerator Department Brookhaven National Laboratory

Upton, NY 11973

# Measuring $\alpha_{1}$ in RHIC 

M. Blaskiewicz, J.M. Brennan, P. Cameron, A. Drees,<br>J. Kewish, T. Roser, K. Smith, C. Tang CAD Brookhaven National Laboratory


#### Abstract

The chromatic nonlinearity parameter, $\alpha_{1}$, has a strong impact on longitudinal dynamics in the vicinity of transition [1]. Measurements of the synchrotron frequency as a function of radius are used to constrain the value of $\alpha_{1}$.


The lattice parameters $\alpha_{0}$ and $\alpha_{1}$ relate the change in closed orbit path length $C$ with the reference value for the center of the beam pipe $C_{0}$ and the fractional momentum difference between the closed orbit momentum $p$ and the reference value for the center of the beam pipe $p_{0}$ via:

$$
\begin{equation*}
\frac{C}{C_{0}}=1+\alpha_{0} \delta\left(1+\alpha_{1} \delta\right)+O\left(\delta^{3}\right) \tag{1}
\end{equation*}
$$

where $\delta=\left(p-p_{0}\right) / p_{0}$ is the fractional momentum difference of the closed orbit and reference orbit momenta. To get the revolution frequency one needs the change in velocity ( $\beta c$ ) between the closed and reference orbits.
Define $u=p / m c$ then :

$$
\begin{align*}
\frac{1}{\beta} & =\left(1+u^{-2}\right)^{1 / 2}  \tag{2}\\
\frac{d}{d u} \frac{1}{\beta} & =-u^{-3}\left(1+u^{-2}\right)^{-1 / 2}  \tag{3}\\
\frac{d^{2}}{d u^{2}} \frac{1}{\beta} & =3 u^{-4}\left(1+u^{-2}\right)^{-1 / 2}-u^{-6}\left(1+u^{-2}\right)^{-3 / 2} \tag{4}
\end{align*}
$$

Since $\delta=\left(u-u_{0}\right) / u_{0}$

$$
\begin{equation*}
\frac{1}{\beta}=\frac{1}{\beta_{0}}\left(1-\frac{\delta}{\gamma_{0}^{2}}+\frac{\delta^{2}}{2}\left[\frac{3}{\gamma_{0}^{2}}-\frac{1}{\gamma_{0}^{4}}\right]\right)+O\left(\delta^{3}\right) \tag{5}
\end{equation*}
$$

The revolution period is $T=C / \beta c$ so

$$
\begin{equation*}
\frac{T}{T_{0}}=1+\left(\alpha_{0}-\frac{1}{\gamma_{0}^{2}}\right) \delta+\delta^{2}\left(\alpha_{0} \alpha_{1}-\frac{\alpha_{0}}{\gamma_{0}^{2}}+\frac{3}{2 \gamma_{0}^{2}}-\frac{1}{2 \gamma_{0}^{4}}\right)+O\left(\delta^{3}\right) \tag{6}
\end{equation*}
$$

The data are synchrotron frequency versus radius. These were obtained with the 2 GHz Schottky cavity and the values at the peaks in the synchrotron spectrum correspond to small amplitude synchrotron oscillations. Therefore, a linear expansion of the equations of motion about the stable fixed point will suffice. Define

$$
\begin{equation*}
\hat{\delta}=\frac{p-p_{s}}{p_{0}}=\frac{p-p_{0}}{p_{0}}-\frac{p_{s}-p_{0}}{p_{0}}=\delta-\delta_{s} \tag{7}
\end{equation*}
$$

where $p_{s}$ is the synchronous momentum, and let $\tau=T-T_{s}$ be the difference in revolution period between a particle and the synchronous particle. The experiment was done at constant magnetic field below transition so:

$$
\begin{align*}
\frac{d \hat{\delta}}{d n} & =\frac{q V}{\gamma_{0} \beta_{0} \beta_{s} m c^{2}}\left(\omega_{r f} \tau\right)  \tag{8}\\
\frac{d \tau}{d n} & =T_{0} \hat{\delta}\left(\frac{d}{d \delta} \frac{T}{T_{0}}\right)_{\delta=\delta_{s}} \tag{9}
\end{align*}
$$

where $n$ is the turn number. The synchrotron frequency is given by

$$
\begin{equation*}
f_{s}^{2}=\frac{q V}{\gamma_{0} \beta_{0} m c^{2}} \frac{\omega_{r f}^{3}}{4 \pi^{2} h^{2}} \frac{T_{0}}{\beta_{s}}\left(\frac{d}{d \delta} \frac{T}{T_{0}}\right)_{\delta=\delta_{s}} \tag{10}
\end{equation*}
$$

where $h=360$ is the harmonic number. In equation (10) only $\omega_{r f}, \beta_{s}$ and the derivative term vary with radial steering and they are tightly related since

$$
\begin{equation*}
\frac{\omega_{r f}}{\omega_{r f, 0}}=\frac{T_{0}}{T_{s}} \tag{11}
\end{equation*}
$$

Taking the logarithm of equation (10), differentiating with respect to $\delta_{s}$, and evaluating at $\delta_{s}=0$ yields

$$
\begin{equation*}
\frac{2}{f_{s}} \frac{d f_{s}}{d \delta_{s}}=2 \frac{\alpha_{1}+\frac{3}{2 \gamma_{0}^{2} \alpha_{0}}+O\left(\alpha_{0}\right)}{1-\frac{1}{\gamma_{0}^{2} \alpha_{0}}}-3\left(\alpha_{0}-1 / \gamma_{0}^{2}\right)-\frac{1}{\gamma_{0}^{2}} \tag{12}
\end{equation*}
$$

In equation (12) the $O\left(\alpha_{0}\right)$ appearing in the numerator of the first term on the right are found in equation (6) and are neglected since they produce a very small correction. Also the second and third terms on the right of equation (12) are very small near transition and will be neglected.


Figure 1: Measured synchrotron frequency versus radial steering setpoint. The first measurement was at $x=0$ and the second at $x=1 \mathrm{~mm}$. An ideal $\chi^{2}$ requires $1 \sigma$ errors of 0.046 Hz . The resolution bandwidth of the spectrum analyzer was 1 Hz and measurements were made using 11 synchrotron lines $\left(10 f_{s}\right)$ yielding an expected error $\sim \sqrt{2} /(10 \sqrt{12})=0.04 \mathrm{~Hz}$, where the $\sqrt{12}$ comes from a boxcar distribution and the $\sqrt{2}$ for the two independent measurements at the ends of the "picket fence".

The data for the yellow ring and a least squares fit are shown in Figure 1. The reference value for the energy was $\gamma_{0}=20$ and both the horizontal and vertical chromaticities were $\lesssim 1$. Assume a bare value of $\gamma_{t} \equiv 1 / \sqrt{\alpha_{0}}=22.76$. Assuming the frequency steering is accurate at this value of gamma

$$
\begin{equation*}
\frac{d f_{s}}{d \delta}=R_{0} \alpha_{0} \frac{d f_{s}}{d x} \tag{13}
\end{equation*}
$$

where $R_{0}=610.2 \mathrm{~m}$ is the reference radius and $x$ is the horizontal position. From the fit to the data

$$
\begin{equation*}
\frac{1}{f_{s}} \frac{d f_{s}}{d \delta_{s}}=-2.68 \pm 0.23 \tag{14}
\end{equation*}
$$

Now since

$$
1-\frac{\gamma_{t}^{2}}{\gamma_{0}^{2}}=-0.295, \quad \frac{3 \gamma_{t}^{2}}{2 \gamma_{0}^{2}}=1.94
$$

one obtains the final result

$$
\alpha_{1}=-1.15 \pm 0.10
$$

As an additional result one may obtain the values of $\gamma_{s}$ and $\delta_{s}$ for which the synchrotron frequency vanishes. To leading order these are related via:

$$
\begin{equation*}
\frac{\gamma_{s}-\gamma_{t}}{\gamma_{t}}=-\delta_{s}\left(\alpha_{1}+1 / 2\right) \tag{15}
\end{equation*}
$$

where $\gamma_{t} \equiv 1 / \sqrt{\alpha_{0}}$ as above.

## References

[1] Handbook of Physics and Accelerator Engineering, Eds. Chao \& Tigner, World Scientific, (1998), p 92. (beware of definitions!)


[^0]:    Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

