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MESB Design Notes: I. Tolerances on the uniformity of β Bd in D1 and D2

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March 1973

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U.S. Department of Energy

USDOE Office of Science (SC)

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BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

EP&S DIVISION TECHNICAL NOTE

No. 59

C.T. Murphy

March 28, 1973

MESB Design Notes: I. Tolerances on the Uniformity
of $\int B d\ell$ in D1 and D2.

ABSTRACT- It is found that $\int B d\ell$ in D1 must be uniform within 0.2% over a 10-inch region up to 6.6 Gev/c (15.4 KG) and within 0.2% over an 8" region from 6.6 to 9.3 Gev/c (21.8 KG), in order to hold aberrations in the y coordinate at the mass slit to a tolerable level. D2 is less critical since there is a nearby sextupole in the beam to use for correction, if an extra power supply can be found for it.

~~MESB Notes~~

~~Aberations at mass slit due to $D_1 + D_2$~~

Theory

If ~~the~~ $\int B dl$ of a bending magnet has a quadratic variation with x (horizontal coordinate):

$$\int B dl = A_0 \left(1 + \frac{\epsilon x^2}{x_0^2} \right) \quad (1)$$

where ϵ is the fractional change in $\int B dl$ at $x = x_0$, then the fringe focusing in the vertical plane has an x -dependence given by:

$$\frac{1}{f} = \frac{\alpha^2}{L} \left(1 + \frac{2L\epsilon x}{\alpha x_0^2} \right)$$

where $\alpha =$ bend angle (radians), $L =$ length. We care only about the variation about the central value:

$$\Delta \frac{1}{f} = \alpha \epsilon \frac{2x}{x_0^2} \quad (2)$$

(For reference, I note that such an aberration can be simulated by a sextupole magnet in various beam programs. In particular, Fox's program ~~from~~ BEAM has the sextupole focal lengths in the form

$$\frac{1}{f} = \frac{G}{1313 P} \quad 2x \quad (\text{units: KG/inch, inches, GeV/c})$$

where G is the sextupole "gradient". Since $\alpha = \int B' dl / 1313 P$,

(1) M.L. Good, ANL-6611 (1962); C.T. Murphy, U. of Mich. Buffle Chamber Note 42/65 (1965)

where B is the bending ~~mag~~ magnet field,

$$\epsilon = \frac{\epsilon}{x_0^2} \int B dl \quad (3)$$

~~Eq. (3)~~ gives a relationship between a measured, ϵ (value of ϵ) and the input "gradient" value which simulates the effect.)

The variation in f , Eq. (2), induces a variation in $y'_b = dy/dz$ at the bending magnet:

$$\Delta y'_b = y_b \Delta \left(\frac{1}{f} \right)$$

which may be related to the effective target ~~to~~ broadening, Δy_t by

$$\Delta y_t = b \Delta y'_b \quad (4)$$

where b is the "upper right hand corner" element of the transfer matrix, T_{tb} , from the target to the bending magnet:

$$T_{tb} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Let $y'_t =$ angle of a ray at the target. Then $y_b = b y'_t$.
Eq. (4) can then be written in a number of forms:

$$\Delta y_t = b^2 y'_t \Delta \left(\frac{1}{f} \right) = \frac{y_b}{y'_t} \Delta \left(\frac{1}{f} \right) = b y_b \Delta \left(\frac{1}{f} \right) \quad (5)$$

This may be related to the broadening at ^{the mass slit} (the first focus), Δy_m , and the slope at the mass slit, y'_m , by multiplying by the magnification (and note $y'_m = y'_t / m_v$):

(3)

$$\Delta y_m = b_{m_v}^2 \frac{y'_m}{y_m} \frac{\alpha \epsilon 2x_b}{x_0^2} \quad (6)$$

Summarizing the notation, Δy_m is the change of position of a ray at the mass slit which has slope y'_m and which passed through the bending magnet at x_b ; b_{m_v} is the transfer matrix element defined in Eq. (4); m_v is the magnification, y_z/y_m ; α is the bend angle (radians); x_0 is the position in the bending magnet where B differs by the fraction ϵ from its central value.

Data and examples

Table I shows input data for Eq. (6) for 5 modes of MESB. The value of x_b max includes half of the total sagitta (1.4") of the beam in D1 and D2. The bend angle, α , is $8^\circ = 0.14$ rad in all cases. The $K-\pi$ separation listed for each mode is the separation at the nominal momentum limit of that mode, i.e., the momentum at which the $K-\pi$ separation is twice the geometrical image width (except mode 4W, where the ratio is 1.5). (These momenta are 4, 6, and 6.6 GeV/c for modes 1, 3, and 4, respectively.)

Values of $(\Delta y_m)_{\max}$ are listed under the assumption that $\epsilon = 0.2\%$ at $x_0 = 5''$, an apparently arbitrary case. In fact, this represents case is the maximum tolerable non-uniformity^{in D1}, as will be

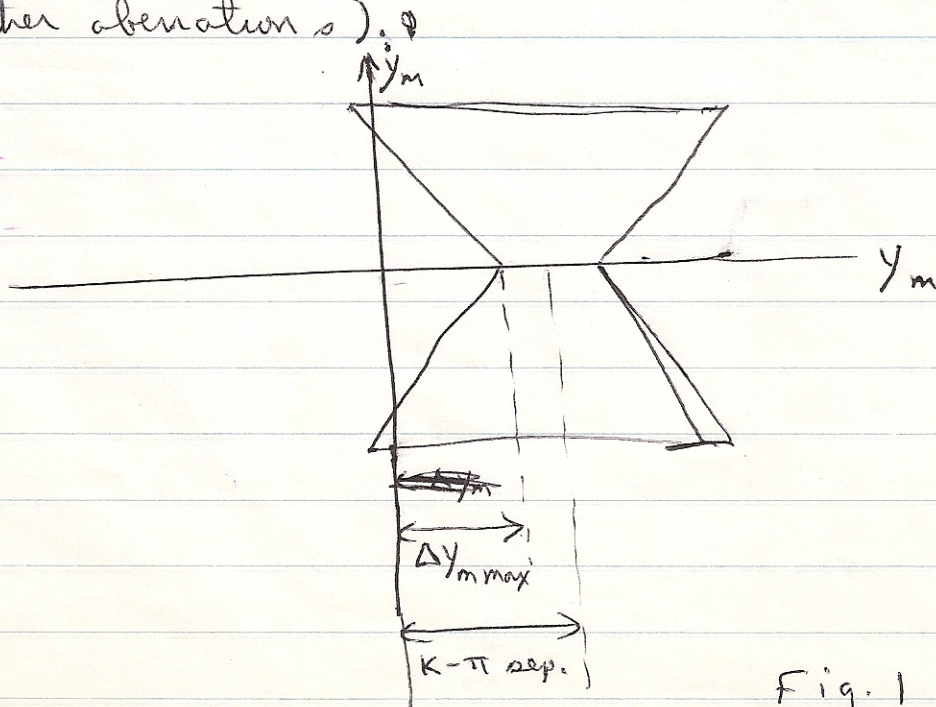
(4)

shown in the next section. The ratio of $(\Delta y_m)_{\max}$ to the $K-\pi$ separation is also listed and is 0.88 for mode ~~4W~~ ^{4W} and slightly less for other modes, ~~Thus mode 4W is the mode most sensitive to~~ considering only D1.

The same ratio is, ^{usually} considerably smaller for D2. ~~Further~~ Furthermore, D2 has a nearby sextupole magnet which can be used to correct the aberration. Therefore D2 is of no concern, unless it has aberrations ~~which are very large in~~ which are of higher order than sextupole and are very large.

A criterion for maximum allowable non-uniformity in D1

As can be seen from Eq. (b), the non-uniformity of D1 will produce a phase space plot, ^{for pions} at the mass slit which has the qualitative appearance of Fig. 1 (ignoring all other aberrations).



(+ this fig. is in the text.)

Fig. 1

(5)

~~This ~~shape~~ shape is reminiscent of ~~the shape~~ that of the chromatic aberration~~

This shape is qualitatively the same as that ~~is~~ shown in Fig. 15 of EP+S 72-1, a shape which results from the unremovable chromatic aberrations.

The mass slit has been shaped to as to roughly cover this shape (see separate report).

Thus a reasonable criterion for the maximum acceptable non-uniformity in DI ~~is~~ seems to be that the aberration be no worse than ~~than~~ the chromatic aberration. Comparing Δy_{\max} (line 8, Table I) with Δy_{\max} , chromatic (line 4) we see that the criterion is met for mode 3 and slightly exceeded for modes 1 and 4. To be more exact, I display in Figs. 2 and 3 the actual phase space plots for modes 1E and 4W. Fig. (a) shows the shape derived from the Monte Carlo in BEAM, including chromatic aberration but assumed no aberration from DI. In the Monte Carlo, a 4" target was assumed; the momentum bites were ~~to~~ 3% for mode 1 and 1½% for mode 4. The effective shape of the mass slit is shown as a dotted line (assuming that the mass slit is 40" long and has a slope of 0.6 mrad). Fig. (b) shows the phase space derived from Eq. (6) with $\epsilon = 0.2\%$ at $x_0 = 5"$, i.e., resulting from DI aberrations alone. The mass slit is supposed to

(6)

intercept any pion to the right of the dotted line. We see that Mode 1E is predicted to have some leak through. However, these hand-calculated phase spaces cannot be taken literally, since chromatic effects and clipping by collimators has been ignored. Hence Monte Carlos were run with a sextupole added at the end of D1, with G given by Eq. (3) ($\epsilon = 0.2\%$, $x_0 = 5''$). The resulting phase space plots (computer print out) are shown in Fig. 4; both chromatic and D1 aberrations are now correctly included. The number of ~~particles~~ π 's which leak thru the mass slit is $\approx 1\%$ or less.

Details: Momentum dependence and the sagitta question

The criterion that $\epsilon = 0.2\%$ at $\pm 5''$ is dictated by modes 1 and 4. Mode 1 is supposed to ~~work up to 6 GeV (\bar{p} 's)~~ to separate \bar{p} 's up to 6 GeV/c; Mode 4 is supposed to make a cleaner K beam at 6 GeV/c (with a loss of flux), or perhaps can be used to extend the momentum limit for K^- to 6.6 GeV/c. (15.4 RF).

Above 6.6 GeV/c, neither mode 1 nor mode 4 will be used to separate masses. Mode 3 will be adequate to separate π^+ from p up to the momentum limit of 9.3 GeV/c. If anybody tries to separate \bar{p} from π^- at 9 GeV/c, the loss of flux caused by switching to mode 4 is intolerable.

Thus we can demand that above 6.6 GeV/c, the D1 non-uniformity must satisfy mode 3, which has a looser tolerance (see Table I). For equal chromatic and D1 aberrations, one can allow $\epsilon \leq 0.32\%$ at ~~$x_0 = 5''$~~
 $x_0 = 5''$; since only $\pm 4''$ are used in mode 3, this criterion is better stated as $\epsilon \leq 0.20\%$ at $x_0 = \pm 4''$.

~~If these criteria cannot be met without the addition of pole face windings under this condition~~
 The ~~situation~~ phase space plots for mode 3W are shown in Figs. 5(a) ~~and~~ (Monte Carlo, no D1 aberration), and 5(b) (hand calculation of aberrated D1) and 6 (Monte Carlo including chromatic and D1 aberrations).

The maximum x -values assumed included and extd 0.7" in all modes, corresponding to the half sagitta of the path of the particle through D1. If a curved SBdL coil is used, or if a model can be used to calculate SBdL along curved paths, the tolerances are loosened slightly: at 6.6 GeV/c, $\epsilon \leq .18\%$ at $x = \pm 4.5''$; at 9.3 GeV/c, $\epsilon \leq .18\%$ at $x_0 = \pm 3.5''$.

If these tolerances cannot be achieved without the addition of pole face windings, it would of course be desirable to design the windings to achieve ϵ values about one-half as large as those stated above

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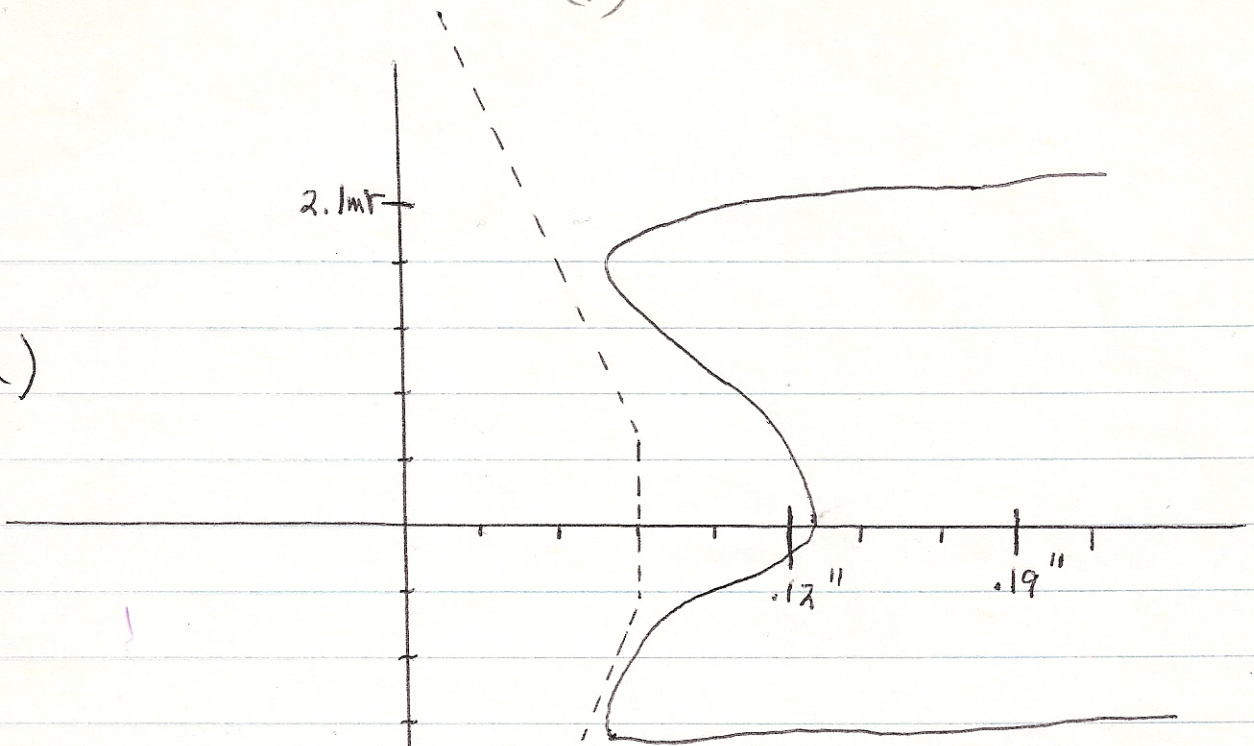
Table I - Aberrations from non-uniformities in SBdl for D1 & D2

	Mode				
	1W (MPS)	1E	3W	3E	4W
m_v	2.13	2.41	0.62	0.61	1.00
$y'_m \text{ max}$	1.8mr	1.6	1.9	1.9	1.2
K- π separation	0.17"	0.19"	0.049"	0.049"	0.060"
Δy_{max} , chromatic	0.09"	0.71	0.040	0.042	0.040
<u>At D1:</u>					
b	340"	325	645	620	645
χ_{max} { includes $\frac{1}{2}$ sagitta }	5.3"	5.2	4.2	4.0	4.1
$(L_{im} \chi_0^2 / \epsilon)_{\text{max}}$	1400 in ³	1440	320	305	540
$\Delta y_m \text{ max}^*$	0.11"	0.12	0.025	0.024	0.044
$\Delta y_m \text{ max} / K-\pi \text{ sep.}$	0.65	0.64	0.51	0.49	0.73
<u>At D2:</u>					
b	200"	163"	520	645	372
χ_{max}	4.7"	3.7	3.3	1.8	2.3
$(\Delta y_m \chi_0^2 / \epsilon)_{\text{max}}$	1500 in ³	260	175	190	105
$\Delta y_m \text{ max}^*$	0.12"	0.02"	0.014	.008	.0085
$\Delta y_m \text{ max} / K-\pi \text{ sep.}$	0.70	.10	.29	.16	.14

* if $\epsilon = \frac{1}{5}\%$ at $\chi_0 = .5"$

(9)

(a)



(b)

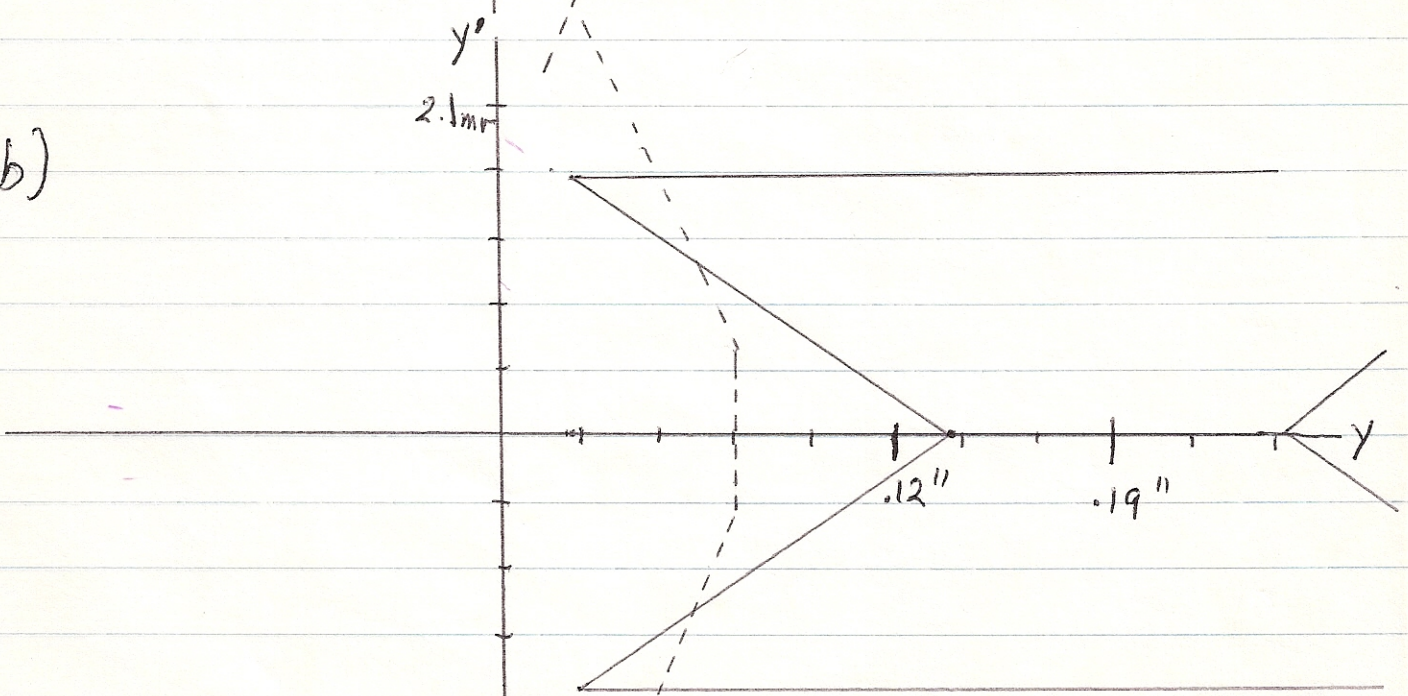
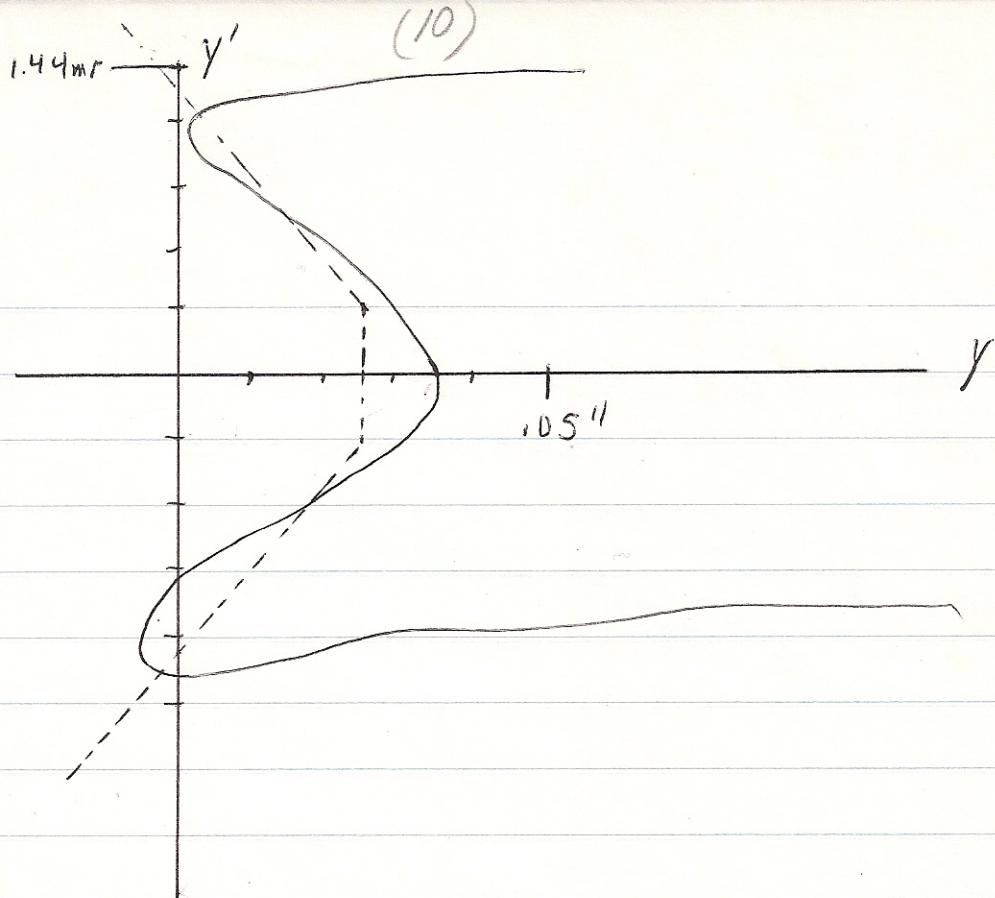


Fig. 2 - Mode I E phase spaces, and mass slit projection at the mass slit.

(10)

(a)



(b)

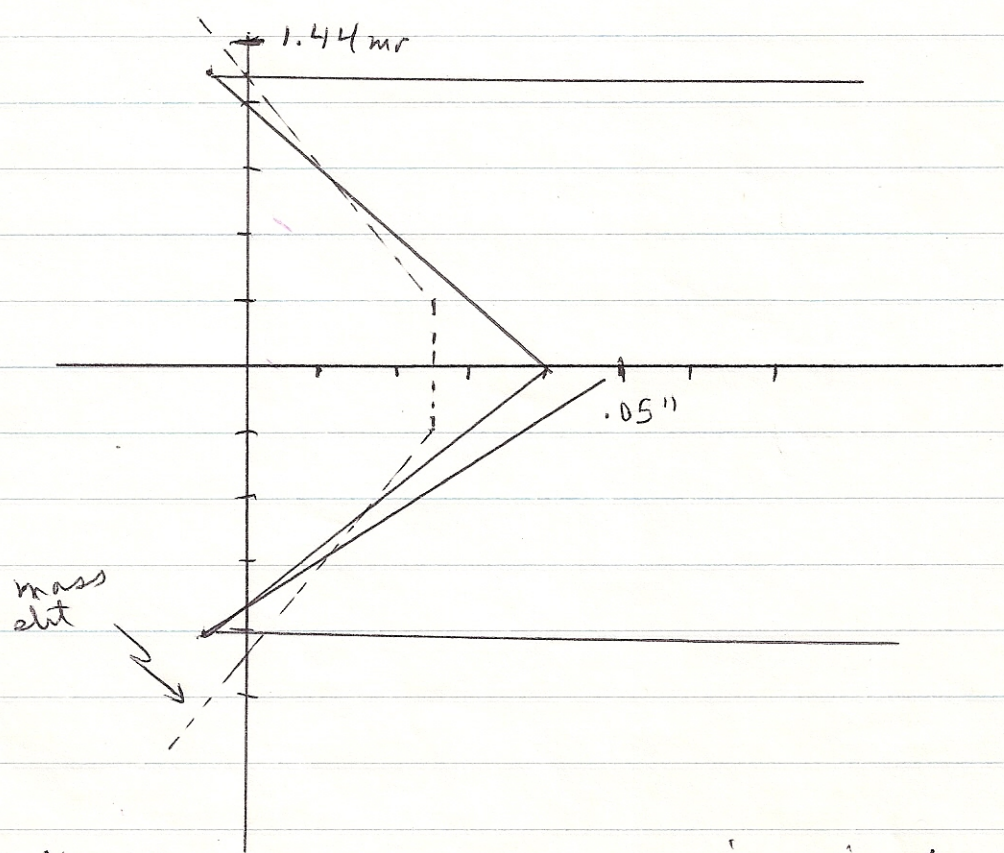


Fig. 3 - Mode 4 W phase spaces and mass slit projections

S	JPRINT	XMAX	DXMAX	YMAX	DYMAX	K1	K2	K3	K4	X1	DX1	Y1													
MASS	MASS	4,00000	,10000	,09975	,00144	3	5	3	4	,001	,02000	,020													
0	0	0	0	0	0	0	1	1	0	3	1	8													
0	0	0	0	0	0	1	1	1	2	2	2	31													
0	0	0	0	0	0	0	0	0	4	2	5	56													
0	0	0	0	0	0	0	0	0	2	9	7	87													
0	0	0	0	0	0	0	0	0	3	2	15	130													
0	0	0	0	0	0	0	0	0	1	2	4	141													
0	0	0	0	0	0	0	1	3	0	7	18	193													
0	0	0	0	0	0	0	1	0	4	7	18	209													
0	0	0	0	0	0	0	0	1	2	8	17	219													
0	0	0	0	0	0	0	1	0	3	7	19	244													
0	0	0	0	0	0	0	1	1	4	4	13	219													
0	0	0	0	0	0	0	1	0	2	5	15	208													
0	0	0	0	0	0	0	0	1	0	2	5	211													
0	0	0	0	0	0	0	0	1	1	2	10	154													
0	0	0	0	0	0	0	0	0	3	2	13	155													
0	0	0	0	0	0	0	0	2	2	3	6	121													
0	0	0	0	0	0	0	0	1	1	3	2	82													
0	0	0	0	0	0	0	0	0	1	1	4	64													
0	0	0	0	0	0	0	0	0	1	0	1	28													
0	0	0	0	0	0	0	0	0	0	1	3	6													
0	0	0	0	0	0	3	6	18	40	73	199	,611 =													
0	0	0	0	0	0	0	0	0	0	0	0	0													
0	0	0	0	0	0	0	0	1	1	2	5	41													
0	0	0	0	0	0	1	1	4	4	9	20	148													
0	0	0	0	0	0	0	0	3	6	12	8	154													
0	0	0	0	0	0	0	1	3	4	7	13	137													
0	0	0	0	0	0	0	0	0	4	9	22	186													
0	0	0	0	0	0	0	0	0	1	5	16	155													
0	0	0	0	0	0	0	0	0	0	13	30	172													
0	0	0	0	0	0	0	0	0	0	10	29	172													
0	0	0	0	0	0	0	0	0	0	4	49	181													
0	0	0	0	0	0	0	0	0	0	3	41	171													
0	0	0	0	0	0	0	0	0	0	6	28	164													
0	0	0	0	0	0	0	0	0	0	12	21	191													
0	0	0	0	0	0	0	0	1	5	10	26	162													
0	0	0	0	0	0	0	0	1	4	7	19	162													
0	0	0	0	0	0	0	0	1	4	6	16	158													
0	0	0	0	0	0	0	1	4	8	6	14	153													
0	0	0	0	0	0	1	3	2	8	10	10	58													
0	0	0	0	0	0	1	0	0	0	0	0	1													
0	0	0	0	0	0	0	0	0	0	0	0	0													
0	0	0	0	0	0	3	6	18	40	73	199	,507 =													
= ,048 (CENTRAL RAY Y)/WAIST=												1,253	YMAX, YMIN=	,085	,036	YMIN/GEOM, IN									
MISSION TO MASS=												,540	NTHIT=	2345	KILLK=	1291	NK=	1364	KOUT/KIN=	,514	NPI=	6700			
TUM DISTR												6	28	64	82	121	155	154	211	208	219	244	219	209	193
S IN BEAM																									
RT	Q1	Q2	D1	SEXD	DRF1	Q3	ENTR	SEP1	WALL	STOP	SEP														
	74	83	0	0	0	0	37	0	0	0	122														
		SEXT	PLIM	DR64	DRF3	D2	DD2	Q6	DRF4	MAS2	MAS														
	0	0	0	0	0	0	0	0	0	0	89														
B E A M												MPS	4**	4**	DATE= 03/										

Fig. 4 (b)

Mode 4 W

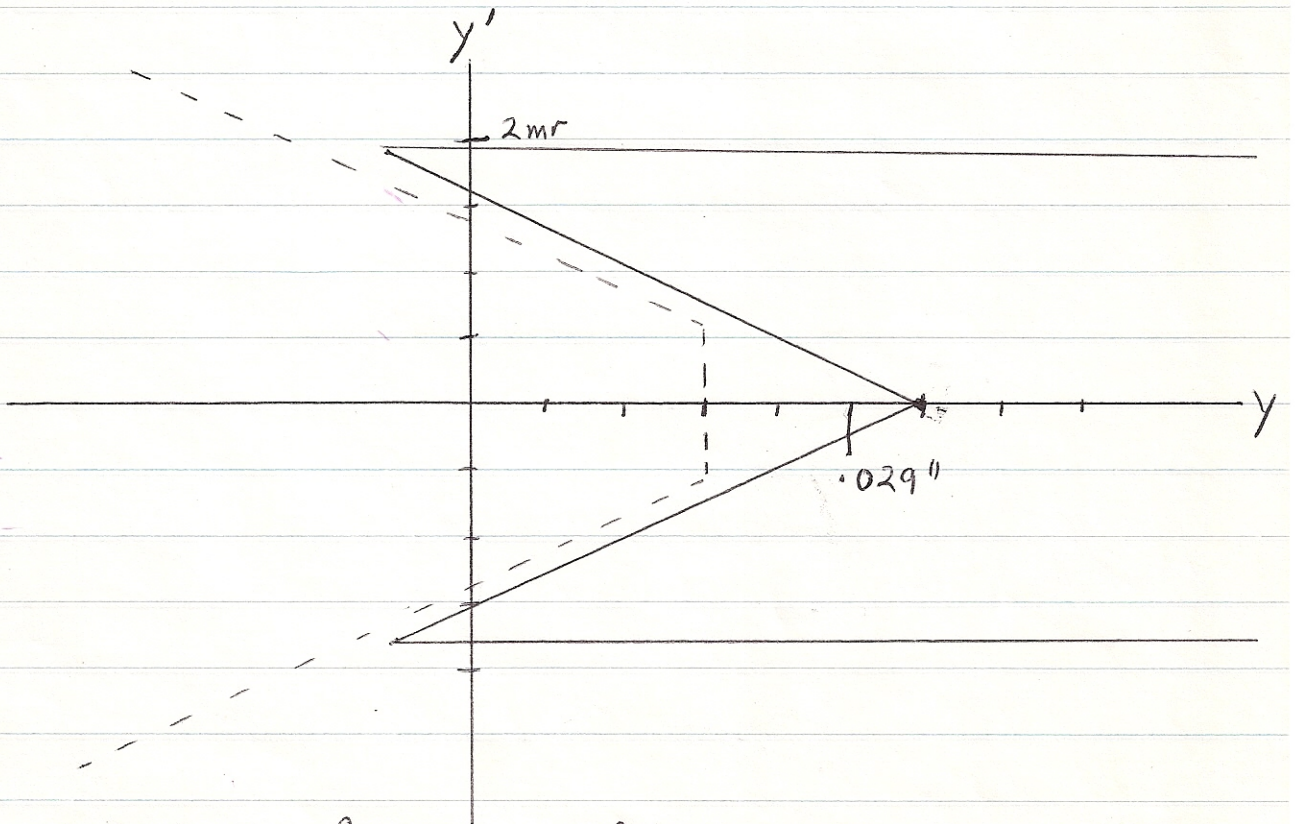
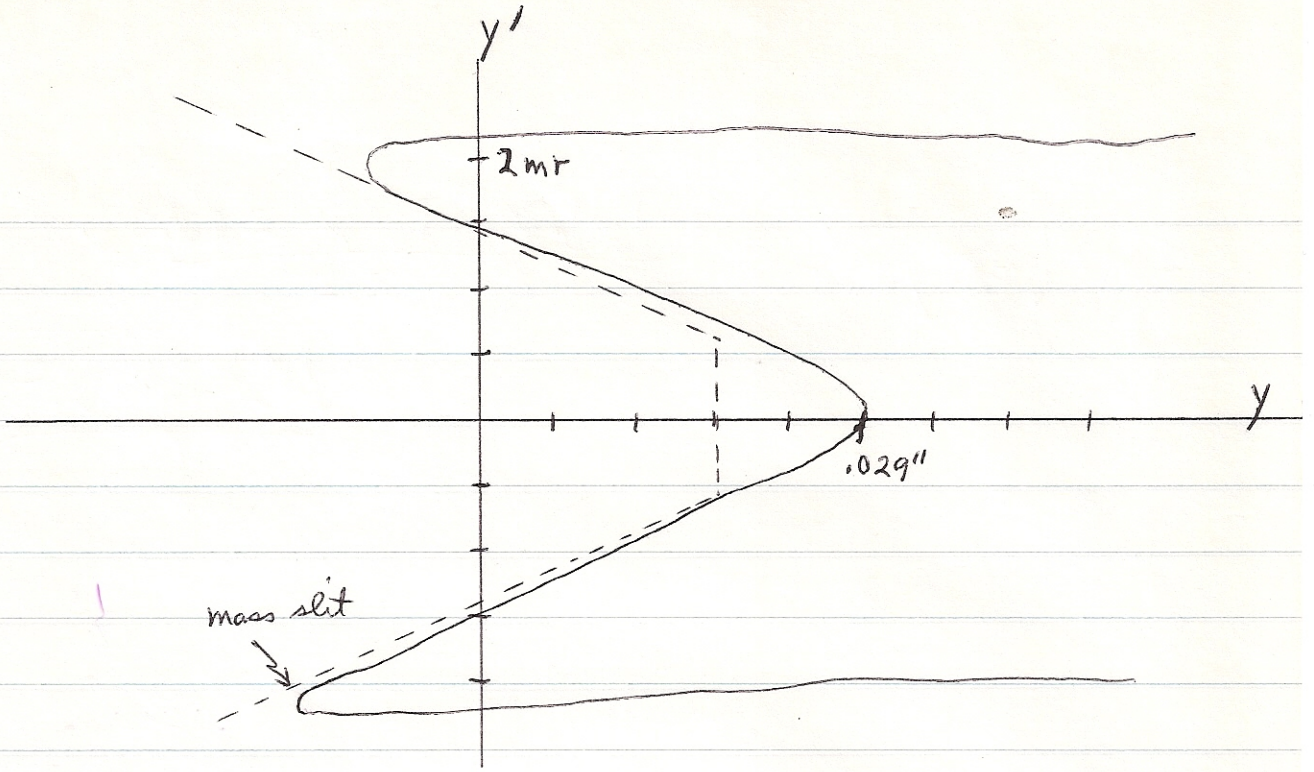


Fig. 5. Mode 3W phase space plots

JPRINT	XMAX	DXMAX	YMAX	DYMAX	K1	K2	K3	K4	X1	DX1	Y1							
MASS	MASS	4,00000	,10000	,06169	,00233	3	5	3	4	,001	,02000	,020						
0	0	0	0	1	0	0	1	0	1	0	1	0	2	0	2	8		
0	0	0	1	1	1	0	1	1	1	3	1	2	4	9	7	6	7	45
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0	0	0	0	0	0	2	0	4	0	2	5	8	7	21	20	15	13	97
0	0	0	0	0	0	0	1	1	2	6	5	9	23	25	27	21	28	148
0	0	0	1	0	1	0	0	2	3	5	9	13	16	30	33	34	24	171
0	0	0	1	0	2	2	3	1	2	9	10	12	24	38	34	35	26	199
0	0	0	0	0	0	1	4	2	4	6	12	13	18	41	30	37	31	199
0	0	0	0	0	0	2	1	1	4	6	8	11	20	28	32	27	33	173
0	0	0	0	1	0	0	2	4	4	6	6	12	20	19	47	29	36	186
0	0	0	0	0	0	1	1	3	4	3	6	8	10	33	34	31	32	166
0	0	0	0	1	0	0	1	1	0	2	7	11	14	23	36	29	34	159
0	0	0	0	0	0	1	0	1	2	4	11	10	24	27	39	39	28	186
0	0	0	0	0	0	0	1	4	0	5	6	10	13	26	34	34	32	165
0	0	0	0	0	0	0	0	2	3	4	3	11	11	25	28	39	35	161
0	0	0	0	0	0	2	1	2	2	1	6	9	14	23	22	33	19	134
0	0	0	0	0	0	0	0	1	1	0	5	5	5	21	24	24	32	118
0	0	0	0	0	0	0	0	1	0	2	1	6	11	9	18	18	16	82
0	0	0	0	0	0	0	0	0	0	1	1	0	9	13	10	15	8	57
0	0	0	0	0	0	0	0	1	0	1	0	1	3	1	2	0	4	13

0	0	0	3	3	5	11	19	38	34	69	103	158	257	420	488	486	451	,672 =	
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0	0	0	0	0	0	0	0	0	4	11	16	24	26	34	29	29	14	173	
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0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	3	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

1.6% leak thru

mass slit

2.33 mr

0.0617"

0 0 0 3 3 5 11 19 38 34 69 103 158 257 420 488 486 451 ,592 =

.040 (CENTRAL RAY Y)/WAIST= 1.332 YMAX, YMIN= ,073 ,033 YMIN/GEOM, I

MISSION TO MASS= ,690 NTHIT= 1590 KILLK= 700 NK= 2710 KOUT/KIN= ,795 NPI= 11300

UM DISTR 13 57 82 118 134 161 165 186 159 166 186 173 199 199

IN BEAM

T	Q1	Q2	D1	SEXD	DRF1	Q3	SEP1	WALL	SEP2	EXIT
73	87	0	0	0	0	0	1	34	639	132 79
1	PLIM	SEX2	DRF3	D2	DD2	Q6				MASS
9	452	0	0	0	0	10 13	0	0	0	865

MESB, MODE MPS 3 ** 3 ** CRAM IT AL IN 12 INCH TARGET DATE= 03

Fig. 6