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The global betatron decoupling on the ramp is an important issue for the operation of the Relativistic Heavy Ion Collider (Rhic). In the polarized proton run, the betatron tunes are required to keep almost constant on the ramp to avoid spin resonance line crossing and the beam polarization loss. Also it is demanded for the horizontal and vertical orbit corrections separately on the ramp. Some possible correction schemes on the ramp, like three-ramp correction, the coupling amplitude modulation and the coupling phase modulation, have been found. Among the three schemes, the coupling phase modulation correction is the most promising one. However, this scheme has never been tested in a real machine. The on-line application program for the skew quadrupole modulations, including the coupling amplitude modulation and the coupling phase modulation, is being developed at Rhic. It will be ready for the next Rhic Run'05. Here we introduce the principles for these schemes, and the comparisons and comments are given. The Rhic skew quadrupoles' contributions to the global coupling and their groupings are also discussed.

1 Introduction

For a hadron machine like Rhic, the working points are constrained in a very narrow space. It is demanded to keep the two tunes close to the linear difference coupling resonance line in order to get good beam lifetime. Rhic presently has been equipped with applications for the global coupling correction at the injection and the store. However, the coupling correction on the ramp is still an open problem. Coupling correction on the ramp becomes more and more important in the past few years, especially during the polarized proton operations. In order to avoid the spin resonance line crossing and polarization loss, it is essential to keep the tunes almost flat on the ramp. The coupling situation changes on the ramp with the changes of the beam optics and the closed orbit. During the non-stop energy ramp, the beta squeezes, transition, re-bucketing, cogging and so on are included. Global coupling correction is much different to that at the injection and store. A fast, robust scheme and a set of reliable diagnostics instrumentations are needed to accomplish the task.

The skew quadrupole strength scan is the general way to decouple the machine globally for a static machine. It is used for the global coupling correction at the store at Rhic. However, it is not suitable for the coupling correction on the ramp because it needs to move the tunes to the difference coupling resonance line before scanning. The procedure of skew quadrupole or skew quadrupole family's strength scan is slow. And its large range scan sometime will make the beam unstable even cause the beam loss. In the most correction procedures at Rhic, this technique adopts the tune meter to obtain the two tunes, its bad resolution sometime makes it hard to find the minimum tune split during the skew quadrupole strength scan.

As a logical extension, T. Roser suggested the skew quadrupole modulation to replace the skew quadrupole scan for the global coupling correction on the ramp. The idea is to reduce the amplitude ratio of $1f$ peak to $2f$ peak in the fast Fourier transform (FFT) plots of $(Q_1 - Q_2 - p)^2$. The f is the skew quadrupole strength modulation frequency, Q_1, Q_2 are the two eigentunes. p is the integer part split for the uncoupled tunes $Q_{x,0}, Q_{y,0}$. For Rhic optics, $p = 28 - 29 = -1$. The idea is proved to be true in the following analytical solution.

However, the idea doesn't hint how to minimize the ratio of the $1f$ peak to the $2f$ peak. The ratio of the amplitudes of the peaks at $1f$ and $2f$ during the skew quadrupole modulation is only a proof or observable of the global coupling. The skew quadrupole modulation doesn't tell us how to reduce it. Based on the analytical solution, it is found that this technique can be used to measure the residual coupling's projection on the direction of the modulated skew quadrupole family introduced coupling. With at least two different family modulations, the residual coupling can be figured out based on the coupling directions of the two families. The directions of the coupling contributions from skew quadrupole families are obtained from the optics model. Then the correction can be carried out based on the residual coupling measurements. Some Dedicated beam experiments had been done at Rhic in its Run'04. Positive experiment results had been achieved at the injection and at the store. Since the skew quadrupole modulation measurement technique is faster than the traditional coupling correction, it is hopeful to be applied to the global coupling measurement on the ramp.

During the residual coupling projection measurement beam experiment on the ramp, we encountered some challenges from the phase lock loop (PLL) system. The tunes in the skew quadrupole modulations are obtained from the PLL system, which is much faster and has much higher resolution. We found the PLL sometimes lose locking. So besides the PLL system developments, we tried to find other data analysis techniques and new schemes to to shorten the PLL data taking time. The linear regression of the PLL data is one promising technique for the projection ratio processing. The data taking time then is greatly reduced to two or three skew quadrupole modulation periods, which is below 10 seconds.

Another great breakthrough is the coupling phase modulation correction scheme. This idea is invented by Y. Luo. The idea is to modulate two orthogonal skew quadrupole families simultaneously. The modulations are out-of-phase. It introduces a rotating coupling into the machine. Then the right direction of the residual coupling is easy to be found. The correction strengths are the modulating skew quadrupoles' strengths at the time point of the minimum tune split's square multiplied by a positive factor. The scheme has many advantages over other schemes. It doesn't care the detailed PLL data, no FFT or fitting needed. The correction strength factor is only obtained from the maximum and minimum tune splits. The data processing is very simple and straight. And the PLL tune data taking time is much shorter than the coupling amplitude modulation. We define the skew quadrupole modulation which introduces the amplitude modulated coupling in the machine as the coupling amplitude modulation, and the skew quadrupole modulation which introduces the phase modulated coupling as the coupling phase modulation.

D. Trbojecvic also put forth an idea to do the global coupling correction on the ramp. This idea suggests three ramps with different skew quadrupole strength settings at dedicated time stone one the ramp. From the three tune splits from the three ramp, the residual coupling, including the amplitude and the phase, can be solved out based on the optics model. The correction is carried out according the measurement. This scheme is much expensive and low efficient. It is the last choice for the coupling correction on the ramp.

In the following, we will first review the skew quadrupole families' global coupling contributions and their groupings for the Rhic rings. The basis of the skew quadrupole modulation and its theory are investigated. The three global coupling correction schemes are given one by one, followed by their comparisons. One thing should be mentioned here is that the following schemes dedicated for the global coupling corrections on the ramp also work at the injection and store.

2 Rhic skew quadrupole families

2.1 Six fold Rhic structure

For each Rhic ring, the Blue ring or the Yellow ring, there are 48 skew quadrupoles for the coupling corrections. All of them are located between the interaction region and the arc in the six sextants of the Blue or Yellow rings. They are divided into 3 families according to their coupling contributions' directions. Each family has 16 individual skew quadrupole magnets. Fig. 1 shows the skew quadrupole locations and their power supplies in the Blue ring.

2.2 Coupling coefficient

The coupling contribution from the skew quadrupoles in one ring is :

$$C^- = \frac{1}{2\pi}\oint \sqrt{\beta_x\beta_y}k_s e^{i(\Psi_x-\Psi_y-\Delta\frac{2\pi s}{L})}dl, \tag{1}$$

$$\Delta = Q_{x,0} - Q_{y,0} - p, \tag{2}$$

where $Q_{x,0}$, $Q_{y,0}$ are the uncoupled tunes, $k_s dl$ is the integrated skew quadrupole strength, β_x andβ_y , Ψ_x and Ψ_y are the beta functions, betatron phases at the skew quadrupoles. $p$is the integer split of the uncoupled tunes. For Rhic, $p = 28 - 29 = -1$. Δ is the distance to the linear coupling resonance line in the $Q_x - Q_y$ plot. So it is the decimal split of the two uncoupled tunes. Fig. 2 shows the coupling contribution directions from all positively powered skew quadrupoles for the Blue ring at the injection.

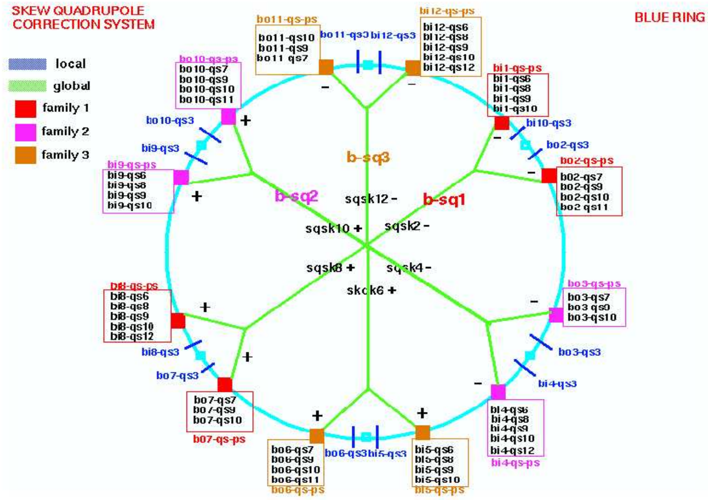

Figure 1: The skew quadrupole corrector locations in the Rhic Blue ring.

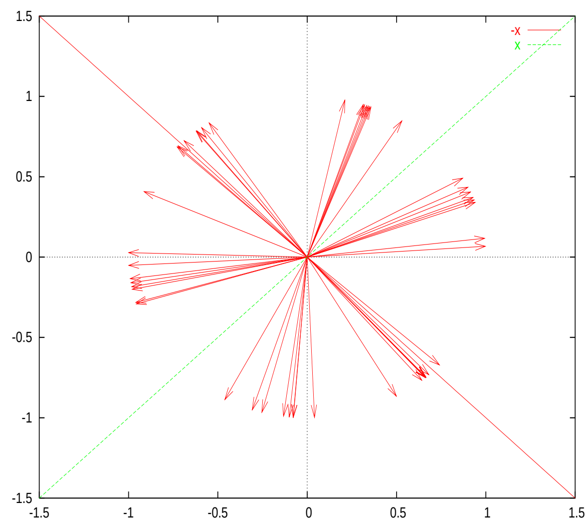

Figure 2: The coupling contribution directions of all positively powered skew quadrupoles for the Blue ring at the injection.

The decimal parts of the two tunes in the horizontal and vertical planes are very close for most circular accelerators. For example, for Rhic Δ is about 0.01. The maximum coupling angle difference coming from $\Delta\frac{2\pi s}{L}$ in the Eq.(1) is small. If $\Delta = 0.01$, the maximum $\Delta\frac{2\pi s}{L}$ is about $360° \times 0.01 = 3.6°$. In the following discussion, we ignore this part. The coupling is then approximated by:

$$C^- \simeq \sum \frac{1}{2\pi}\sqrt{\beta_x\beta_y}k_s e^{i(\Psi_x-\Psi_y)}dl. \tag{3}$$

The coupling contribution's direction from one individual skew quadrupole is then decided by its betatron phase difference $(\Psi_x - \Psi_y)$. This is the criterion for the skew quadrupole grouping.

2.3 Skew quadrupoles close to one IR

Since the small β^* are used for the high colliding luminosity in the colliders, the betatron phase advances in the horizontal and vertical planes across the interaction region are about 180° different. The beta functions at the symmetric points range from 10 m at the injection to 1 m at the IP6 and IP8 at the store. We also ignore the phase advances between the skew quadrupoles on each side of one interaction region, then the betatron phase differences $(\Psi_x - \Psi_y)$ between the left and right sides of one IR are the same since

$(\Psi_x + 180^\circ) - (\Psi_y + 180^\circ) = (\Psi_x - \Psi_y)$. According to Eq.(3), the contributions from the skew quadrupoles on each sides of one IR are almost same if their strengths are same. Then they are sorted into one same skew quadrupole family.

2.4 Three skew quadrupole families

There are 6 interaction regions in each ring of Rhic. The optics of the six arcs are almost same. Then the betatron phase advances in each sextant are almost the same. Since the tune integer part split of the uncoupled tunes for Rhic Blue and Yellow ring are 1 unit, so the total $(\Psi_x - \Psi_y)$ of the whole ring is 360°. The $(\Psi_x - \Psi_y)$s between two adjacent IRs are $360^\circ/6 = 60^\circ$. So it is recommended to divide the six skew quadrupole sub-families into three families. The skew quadrupoles located close to IR8 and IR2 are grouped into family F1, the skew quadrupoles located close to IR10 and IR4 are grouped into family F2, the skew quadrupoles located close to IR12 and IR6 are grouped as family F3. In each family, the skew quadrupoles from different IR are oppositely powered in order to get the same directions of their coupling contributions. The default positive polarizations of the skew quadrupoles are shown in Fig. 1. The three families' coupling contribution directions are shown in Fig. 3. It is found that the directions of the three families are not 120° apart. It is due to the conventional definition of the polarizations of the three skew quadrupole families.

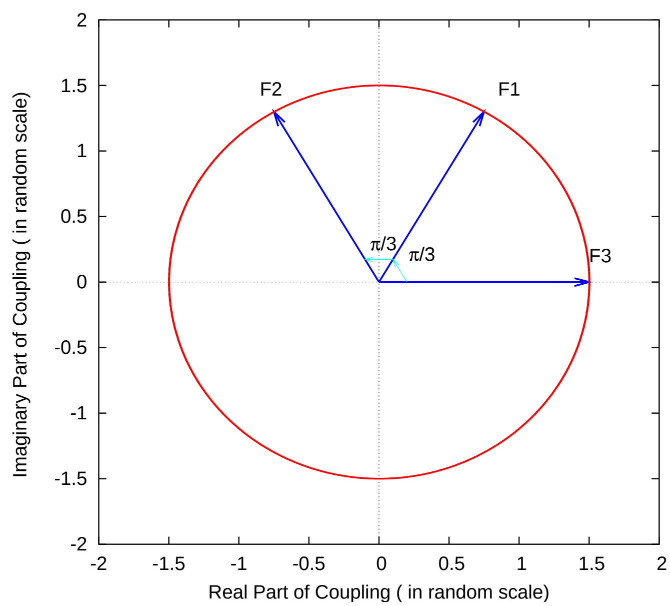

Figure 3: The coupling angles from the three skew quadrupole families with the grouping in Fig. 1

2.5 Summary

Through the above discussion, we draw the following conclusion to the coupling contribution directions of the three skew quadrupole families. The directions of the three skew quadrupole families shown in Fig. 3 is valid when the following conditions are met:

- the betatron phase advances in every sextant are same, or almost same.
- the Q_x, Q_y integer part split are 1.
- the Q_x, Q_y decimal part split is small, say ≤ 0.015.

The conclusion holds at the injection, store , and on the ramp. The coupling angle differences among the three families are very important, it is the basic assumption for the following residual coupling measurements and corrections for the three-ramp correction and the coupling amplitude modulation correction schemes.

In the operation, each skew quadrupole in one family has the same integrated strength absolute value. The relation between the coupling amplitude and the integrated strength for each family can be estimated from the optics model according to Eq (1):

$$C^- \simeq (\sum_i \frac{1}{2\pi}\sqrt{\beta_{i,x}\beta_{i,y}})|_{\text{ one family}} \times (k_s dl) \tag{4}$$

$$|C^-| \simeq 60 \times |k_{1s} dl| \tag{5}$$

The scaling factor is calculated out at the injection. For the specific lattice, the scaling factor will shift a little. It's better to re-calculate it after the optics model changed.

3 Skew quadrupole modulation

Skew quadrupole modulation is to modulate the currents, therefore the integrated strengths of the skew quadrupoles. In Rhic, we modulation one skew quadruple family's strength each time. And the sinuous modulations are chosen so that the currents of the power supplies start modulation from zero currents and end modulation with zero currents.

3.1 Analytical solution

There are several approaches to tackle the linear coupling. Since we are interested in the two global tunes' response to the skew quadrupole modulation, perturbation theory [1, 2] based on the Hamiltonian mechanism is more straight and simple, and therefore is adopted here. The two eigentunes Q_1, Q_2 are given by:

$$\left\{ \begin{array}{lcl} Q_1 & = & Q_{x,0} - \frac{\Delta}{2} + \frac{1}{2}\sqrt{\Delta^2 + (C^-)^2}, \\ Q_2 & = & Q_{y,0} + \frac{\Delta}{2} - \frac{1}{2}\sqrt{\Delta^2 + (C^-)^2}. \end{array} \right. \tag{6}$$

the tune separation's square is obtained:

$$(Q_1 - Q_2)^2 = \Delta^2 + (C^-)^2. \tag{7}$$

Please pay attention to that the coupling coefficient C^- is a complex number normally.

In order to distinguish the different sources of coupling coefficient, in the following we designate C^-_{tot} as the total coupling coefficient or coupling, C^-_{res} the residual coupling, C^-_{mod} the introduced coupling by the skew quadrupole modulation.

Assuming the modulation amplitude of the coupling is $C^-_{mod,amp}$, the introduced and total coupling are given by:

$$C^-_{mod} = C^-_{mod,amp} \sin(2\pi f t) \tag{8}$$

$$C^-_{tot} = C^-_{res} + C^-_{mod} \tag{9}$$

We introduce the slow varying approximation, that is, the skew quadrupole modulation frequency is slow enough so that the eigentunes at one time are only determined by the modulation strengths at that time. Of course, the modulation amplitude should be small so that the above perturbation theory still holds. Substituting Eq. (8) into Eq. (22), we obtain:

$$\begin{array}{rcl} (Q_1 - Q_2)^2 & = & \Delta^2 + |C^2_{res}|^2 + \frac{1}{2}|C^2_{mod,amp}|^2 \\ & & +2|C^-_{res}||C^-_{mod,amp}|\cos(\varphi)\sin(2\pi f t) \\ & & -\frac{1}{2}|C^-_{mod,amp}|^2 \cos(4\pi f t) \end{array} \tag{10}$$

where the φ is the angle difference between C^-_{res} and $C_{mod,amp}$. $|C^-_{res}|\cos(\varphi)$ is the projection of the residual coupling onto the modulation coupling.

Now it is clear from Eq. (10) that the $2f$ item is only related to the skew quadruple modulation amplitude, the $1f$ item is related to the dot multiplication of the modulation coupling and the residual coupling. In the frequency domain of $(Q_1 - Q_2)^2$ during skew quadruploe modulation, we will see two peaks located at $1f$ and $2f$ if the machine is originally coupled, or only the $2f$ peak if the machine originally well decoupled. So the $1f$ peak is one reflection, or observable of the global residual coupling in one accelerator.

3.2 Projection ratio

A straight application of Eq. (10) is the projection ratio of the residual coupling onto the modulation coupling direction. It is defined as:

$$\kappa = \frac{|C^-_{res}|\cos(\varphi)}{|C^-_{mod,amp}|}. \tag{11}$$

κ is a dimensionless quantity, however it has its sign which is decided by the difference angle φ between the residual and modulation couplings. From FFT of $(Q_1 - Q_2)^2$, we could get $1f$ and $2f$ peaks' amplitudes, say A_{1f}, A_{2f}, then

$$|\kappa| = (\frac{A_{1f}}{A_{2f}})/4. \tag{12}$$

κ's sign is the same to that of $\sum_{i=1}^{N}(Q_1 - Q_2)^2_i \times i_{mod,courrent}$, where $i_{mod,courrent} = I_0 \sin(2\pi ft)$, is the skew quadrupole modulation power supply current, I_0 is normally positive.

3.3 Linear regression

In order to shorten the PLL data taking time on the ramp, linear regression for Eq. (10) is used. The fitting function is assumed as:

$$\begin{aligned} f(t_i) \quad = \quad & A + B_1\sin(2\pi ft_i) + B_2\cos(2\pi ft_i) \\ & + C_1\sin(4\pi ft_i) + C_2\cos(4\pi ft_i) \\ & + Et_i + Ft_i^2. \end{aligned} \tag{13}$$

The linear and quadratic terms are included considering the possible tune shifts on the ramp. We use the linear regression technique to minimize the error function χ^2:

$$\chi^2 = \sum_{i=1}^{N}[(Q_1 - Q_2)^2 - f(t_i)]^2. \tag{14}$$

To minimize χ^2 in Eq.(14) is equivalent to solve the following linear matrix equation:

$$\mathbf{M}\cdot\mathbf{X} = \mathbf{V}, \tag{15}$$

where

$$\mathbf{X} = \begin{pmatrix} A \\ B1 \\ B2 \\ C1 \\ C2 \\ E \\ F \end{pmatrix}, \quad \mathbf{V} = \sum_{\mathbf{i}} \begin{pmatrix} \Delta Q_i^2 \\ \Delta Q_i^2\sin(2\pi ft_i) \\ \Delta Q_i^2\sin(2\pi ft_i) \\ \Delta Q_i^2\sin(4\pi ft_i) \\ \Delta Q_i^2\cos(4\pi ft_i) \\ \Delta Q_i^2 t_i \\ \Delta Q_i^2 t_i^2 \end{pmatrix}, \tag{16}$$

$$\mathbf{M} \quad = \quad \sum_i \Delta Q_i^2 \cdot \begin{pmatrix} 1 & \sin(2\pi ft_i) & \cos(2\pi ft_i) & \sin(4\pi ft_i) & \cos(4\pi ft_i) & t_i & t_i^2 \\ \sin(2\pi ft_i) & \sin(2\pi ft_i)\sin(2\pi ft_i) & \sin(2\pi ft_i)\cos(2\pi ft_i) & \sin(2\pi ft_i)\sin(4\pi ft_i) & \sin(2\pi ft_i)\cos(4\pi ft_i) & \sin(2\pi ft_i)t_i & \sin(2\pi ft_i)t_i^2 \\ \cos(2\pi ft_i) & \cos(2\pi ft_i)\sin(2\pi ft_i) & \cos(2\pi ft_i)\cos(2\pi ft_i) & \cos(2\pi ft_i)\sin(4\pi ft_i) & \cos(2\pi ft_i)\cos(4\pi ft_i) & \cos(2\pi ft_i)t_i & \cos(2\pi ft_i)t_i^2 \\ \sin(4\pi ft_i) & \sin(4\pi ft_i)\sin(4\pi ft_i) & \sin(4\pi ft_i)\cos(2\pi ft_i) & \sin(4\pi ft_i)\sin(4\pi ft_i) & \sin(4\pi ft_i)\cos(4\pi ft_i) & \sin(4\pi ft_i)t_i & \sin(4\pi ft_i)t_i^2 \\ \cos(4\pi ft_i) & \cos(4\pi ft_i)\sin(4\pi ft_i) & \cos(4\pi ft_i)\cos(2\pi ft_i) & \cos(4\pi ft_i)\sin(4\pi ft_i) & \cos(4\pi ft_i)\cos(4\pi ft_i) & \cos(4\pi ft_i)t_i & \cos(4\pi ft_i)t_i^2 \\ t_i & t_i\sin(2\pi ft_i) & t_i\cos(2\pi ft_i) & t_i\sin(4\pi ft_i) & t_i\cos(4\pi ft_i) & t_i t_i & t_i t_i^2 \\ t_i^2 & t_i^2\sin(2\pi ft_i) & t_i^2\cos(2\pi ft_i) & t_i^2\sin(4\pi ft_i) & t_i^2\cos(4\pi ft_i) & t_i^2 t_i & t_i^2 t_i^2 \end{pmatrix}. \tag{17}$$

Then the solution to $\mathbf{X}$ is :

$$\mathbf{X} = \mathbf{M}^{-} \cdot \mathbf{V} \quad . \tag{18}$$

The projection ratio is:

$$|\kappa| = \sqrt{B_1^2 + B_2^2} / \sqrt{C_1^2 + C_2^2} \ . \tag{19}$$

Using the linear regression, the modulation measurement time can be reduced below 10 s and it opens the possibility of the continuous measurement of coupling on the whole ramp. However, FFT data analysis is still needed sometimes, especially under situations where the PLL data quality is not good enough. FFT is more robust than fitting, and reveals more physics.

3.4 Simulation

Simulation is important for the skew quadrupole modulation study. The coupling measurement and correction simulations have been performed based on the Rhic optics model. There are two ways to perform the skew quadrupole modulation simulations. The simple one is to use the smooth accelerator model. The complicate one is to use the element-by-element tracking.

A simulation program has been developed with the smooth accelerator model. The uncoupled motion is simply represented by

$$\left\{ \begin{array}{lcl} x^{''} + (\frac{Q_{x,0}}{R})^2 x & = & 0 \\ y^{''} + (\frac{Q_{y,0}}{R})^2 y & = & 0 \end{array} \right. \tag{20}$$

where R is the average radius of the circular accelerator. Three skew quadrupoles presenting the three families are distributed along the ring. The 4×4 uncoupled transfer matrix between the skew quadrupoles are adopted.

In the model, each quadrupole could have constant, modulated strengths, or both. The constant strengths of the three skew quadrupoles present the residual coupling. The modulating strengths are used for the residual coupling measurements.

In the code, the uncoupled tunes are the same as those of the Rhic, that is, $(Q_{x,0}, Q_{y,0}) = (28.22, 29.23)$. The average radius $R = L/2\pi$, L is the Rhic ring circumference 3833.84 m. For every tracking job, one particle with the initial horizontal and vertical displacement coordinates is launched. The successive particle x and y coordinates are recorded at the observation point. One set of the two eigentunes are obtained from 4096 turns' $(x + y)$. The projection ratio is from the FFT of 1024 sets of $(Q_1 - Q_2)^2$.

Fig. 4 shows the two tunes and Fig. 5 is the FFT of the $(Q_1 - Q_2)^2$ from simulation when the third skew quadrupole strength modulates with integrated strength amplitude $((k_s dl)_3)_{amp,mod} = 0.0005 \text{ m}^{-1}$, modulation frequency 0.5 Hz, under condition that the first and the second skew quadruple strengths are constant at $(k_s dl)_{1,2} = 0.0005 \text{ m}^{-1}$.

Beside the simplified simulation with the smooth accelerator model, the element-by-element tracking simulations have been performed with the SAD [3] code. The Rhic optics model is used. The interaction region nonlinear field errors are also included in the model. The results from element-by-element tracking can be used for comparison with the on-line measurement.

The smooth accelerator model and the element-by-element tracking both verify the principle of the skew coupling modulation technique. The projection ratios are close to the predication under the simulation condition where the known coupling sources are intentionally introduced into the model. The element-by-element tracking simulation requires a lot of computer calculation time. So most of the simulations are performed with the smooth accelerator model.

By now all the coupling measurement and correction simulations are taken in a static model. Energy ramp is not taken into account. Further element-by-element tracking on the ramp with the skew quadrupole modulation is in progress. However, if the machine situation changes not too fast, the conclusion from the static machine simulation is still valid for the ramp.

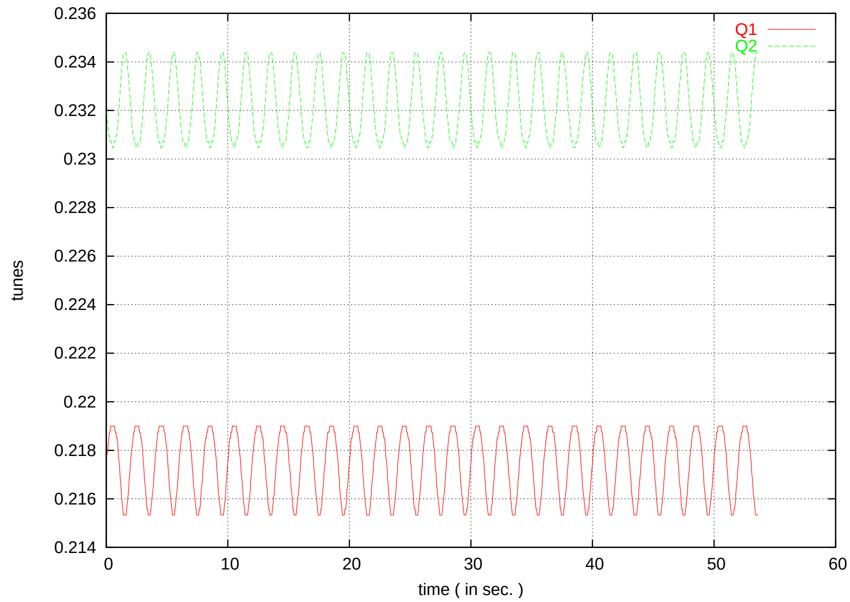

Figure 4: An example of tunes from simulation during skew quadruploe modulation. Every set of tunes (Q_1,Q_2) are achieved from FFT of 4096 turns' $(x + y)$ data.

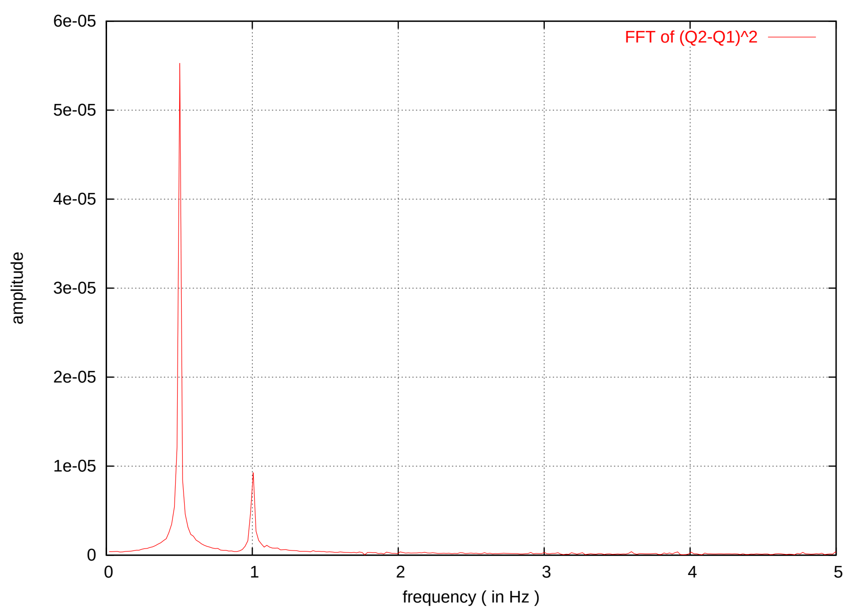

Figure 5: An example of tunes from simulation during skew quadruploe modulation. Every set of tunes (Q_1,Q_2) are achieved from FFT of 4096 turns' $(x + y)$ data.

4 Three-ramp correction

D. Trbojevic suggested using three ramps to measure and correct the residual coupling at one specific ramp time stone. This idea is quit simple and straight. With three ramps with the different three skew quadrupole families' strengths settings at the specific ramp stone, the residual coupling can be solved out from the three tune splits.

For every ramp at one ramp time stone, the total introduced coupling from the three families' strengths are calculated according to Eq.(5). Their relative coupling contribution directions are shown in Fig. **??**.

$$C^-_{ind} = C^-_{F1} + C^-_{F2} + C^-_{F3}, \tag{21}$$

where $C^-_{F1}, C^-_{F2}, C^-_{F3}$ are the coupling contributions from the three skew quadrupole families. Then the tune splits at that time stone for the three ramps are obtained:

$$\begin{cases} \Delta Q_1 = \Delta^2 + (C_{res,r} + C_{ind-1,r})^2 + (C_{res,i} + C_{ind-1,i})^2 \\ \Delta Q_2 = \Delta^2 + (C_{res,r} + C_{ind-2,r})^2 + (C_{res,i} + C_{ind-2,i})^2 \\ \Delta Q_3 = \Delta^2 + (C_{res,r} + C_{ind-3,r})^2 + (C_{res,i} + C_{ind-3,i})^2 \end{cases} , \tag{22}$$

where $C_{res,r}, C_{res,i}$ are the real and imaginary parts of the residual coupling. $C^-_{ind-j,r}$, $C_{ind-j,i}$ are the real and the imaginary parts of the coupling from the three skew quadrupole families at the time stone. The subscribe j in $C^-_{ind-j,r}$ means the three separate ramps.

In the above equations, everything is real. There are three unknowns: Δ^2, $C_{res,r}$, $C_{res,i}$. So with three ramps, they can be solved out. Then the correction to the residual coupling is easily obtained. By the way, since there are three families of skew quadrupoles, the corrections also have different combination choices of the three families.

This method was proved effective in the real operational coupling corrections in the Rhic Run'04. The shortcoming of this method is clear. It needs three ramps, which is much expensive. Another solution to this is to automatically change the skew quadrupole families' settings close to the dedicated ramp time stone. So one ramp is sufficient to figure out the residual coupling. And the coupling measurements at different time stones can be sandwiched into one ramp, which will reduce the price of the correction.

This method has tight connections to the lattice. We should have the knowledges of the directions of the three skew quadrupole families, and the scaling factor between the coupling coefficient and the integrated strength. Both of them only can be obtained from the optics model.

This method also works at injection and store. Maybe it is faster than the skew quadrupole scan.

The resolution of the tune measurements are still of concern. Up to now, this method uses the tune meter data. Tunes are calculated from the 1024 turns' BPM reading. Its resolution is not good enough. If the PLL data are used, the effectiveness will be improved.

This method is expensive so that it is the last choice for the coupling correction on ramp.

5 Coupling amplitude modulation

Since the skew quadrupole modulation can determinate the projections of the residual coupling onto the modulated coupling directions, it can be used for the residual coupling measurements and therefore corrections. At least two skew quadrupole modulations of two different skew quadrupole families are needed to determinate the residual coupling. The directions of the coupling contributions from the the modulated skew quadrupole families are needed.

It is wise to choose two orthogonal modulation directions to get two orthogonal projections. Then the residual coupling and its correction are easy to be figured out:

$$\begin{cases} (k_s dl)_{corr_1} & = & -\kappa_1 \times (k_s dl)_{amp,mod_1} \\ (k_s dl)_{corr_2} & = & -\kappa_2 \times (k_s dl)_{amp,mod_2} \end{cases} \tag{23}$$

Each Rhic ring has three correction skew quadrupole families, F1,F2 and F3. There are several combinations of them to construct the two orthogonal modulations. For example, according to Fig. 3 , we modulate F1 and F2 simultaneously with the same frequency and the same modulation amplitude. The introduced coupling is then normal to that from F3 modulation. It is shown in Fig. 6.

This method is fast. For the Rhic PLL tune measurement system, 177 sets of tunes are obtained each second. If we use FFT technique to achieve the projections, for one measurement it will take at least 13 seconds to get 2048 sets of tunes. So two separate modulations to determinate the residual coupling need 26 seconds. If adopting the linear regression technique, the PLL data taking time will be reduced to two or three modulation periods for each projection measurement. To avoid the PLL lose locking, we choose not too fast modulation frequency. The default modulation frequency is 0.5Hz. So two periods modulation time is 4 seconds. The two separate projection measurements need 8 seconds. The two separate modulations are arranged close to the dedicated time stone on the ramp.

The residual coupling correction can be carried out in two modes: feed-forth and feed-back. By now we use feed-forth mode for the global coupling correction since it is not frequently demanded to do the global coupling correction.

To determine the residual coupling, the angle informations of the three families' coupling contributions are needed. For the orthogonal modulations, the correction strengths are from the projection ratios, the scaling

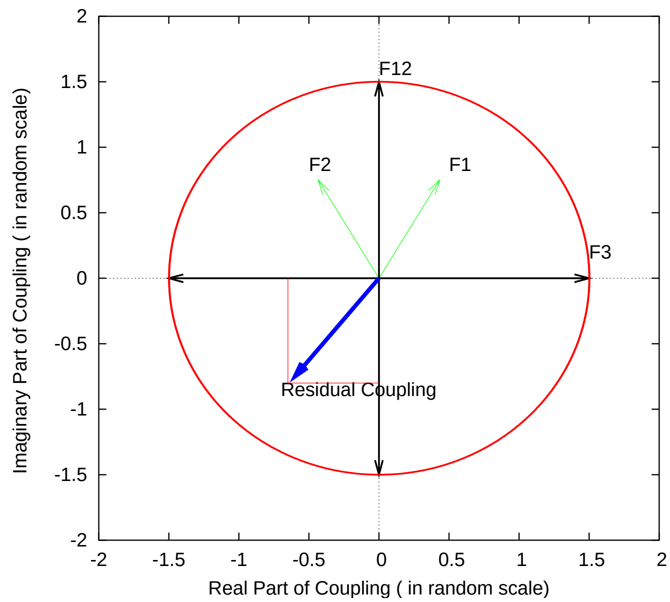

Figure 6: Orthogonal modulations in the coupling amplitude modulation measurement and correction.

factor between the coupling and the integrated strength for the skew quadrupole family is not needed. If two modulation families are used, even they are not strictly orthogonal, the residual coupling still could be corrected by iterations.

6 Coupling phase modulation

6.1 Principle

To further shorten the time period occupied by the PLL data taking, one promising correction scheme was put forth. The above discussed coupling amplitude modulation introduces an amplitude modulated coupling in the machine, the direction of coupling doesn't change. Here we introduce the coupling phase modulation. It will introduce a rotating coupling with constant amplitude into the machine.

To construct the rotating coupling, we should have two orthogonal modulations simultaneously. Their coupling modulation amplitude and the modulation frequency are also the same. However, the two orthogonal modulations are out-of-phase, it means that one modulation is co-sinuous function modulation, the other one is the sinuous function modulation, as shown in Fig. 7.

The couplings introduced from the coupling phase modulation is:

$$\left\{ \begin{array}{lcl} C_1^- & = & C^-_{mod,amp} \cdot \cos(2\pi f t) \\ C_2^- & = & C^-_{mod,amp} \cdot \sin(2\pi f t) \cdot i, \end{array} \right. \tag{24}$$

where $C^-_{mod,amp}$ is the amplitude of the two orthogonal modulations, which is a positive number. f is the modulation frequency. The imaginary number unit "i" in the second equation comes from the assumption that their coupling phases are out-of-phase. Then the total introduced coupling is:

$$C^-_{mod,ind} = C_1^- + C_2^- = C^-_{mod,amp} \cdot e^{i2\pi f t}. \tag{25}$$

So it is clear that the total introduced coupling is rotated, the rotation frequency is the same as the modulation frequency. The couping phase linearly increased with the time like $2\pi f t$. For one cycle modulation, the coupling phase change $2\pi = 360°$.

Now we discuss the advantage of the coupling phase modulation. In the Fig. **??**, when the introduced coupling rotates, it will scan 360° per modulation period. When the introduced coupling overlaps onto the residual coupling, or their coupling directions are the same, the total coupling in the machine is the biggest. Then we will observe the biggest eigen tune split. However, when the introduced coupling rotates to the opposite direction of the residual coupling, the total coupling in the machine is the smallest, then we observe

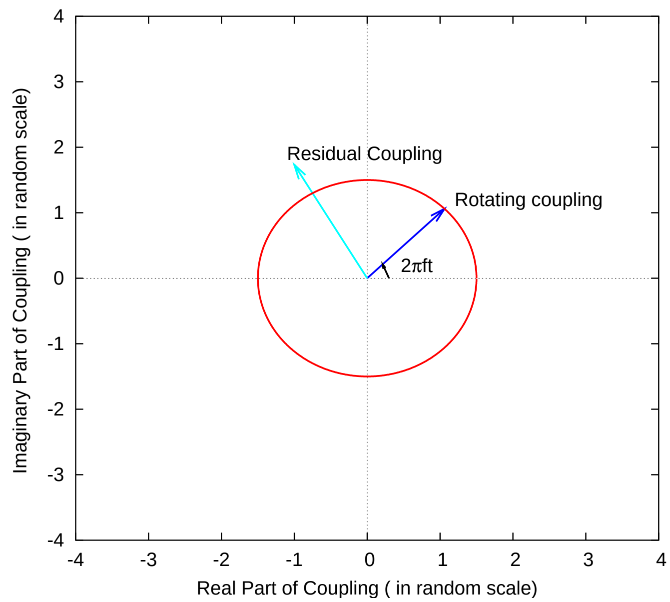

Figure 7: Skew quadrupoles' Currents in the operational coupling phase modulation

the smallest eigen tune split. The tune split's square in the coupling phase modulation is given according to Eq.(7):

$$
\begin{aligned}
(Q_1 - Q_2 - p)^2 &= \Delta^2 + |C^-_{res} + C^-_{mod,amp}|^2 \\
&= \Delta^2 + |C^-_{res,amp} \cdot e^{i\phi_{res}} + C^-_{mod,amp} \cdot e^{i2\pi f t}|^2 \\
&= \Delta^2 + |C^-_{res}|^2 + |C^-_{mod,amp}|^2 + 2|C^-_{res}||C^-_{mod,amp}| \cos{(2\pi f t - \phi_{res})}.
\end{aligned} \tag{26}
$$

From Eq.(26), it is clear that there are only one peaks located at the modulation frequency in the FFT plot of $(Q_1 - Q_2 - p)^2$, which is different from the coupling amplitude modulation. For the coupling amplitude modulation, there are two peaks which are located at the $1f$ and $2f$ frequencies, respectively.

Also from Eq.(26), we obtain the maximum $(Q_1 - Q_2 - p)^2$ when the residual and the rotating coupling have the same coupling phase, and the minimum of $(Q_1 - Q_2 - p)^2$ when the residual and the rotating coupling have the opposite coupling phases. Since the rotating coupling's amplitude is assumed here to be constant and the residual coupling's amplitude also constant. We designate the residual coupling's amplitude is k times of the rotating coupling's amplitude. Then according to Eq.(26), the maximum and minimum tune split's squares are obtained:

$$
\left\{ \begin{aligned}
\Delta Q^2_{min} &= \Delta^2 + (k-1)^2 \cdot |C^-_{mod,amp}|^2 \\
\Delta Q^2_{max} &= \Delta^2 + (k+1)^2 \cdot |C^-_{mod,amp}|^2
\end{aligned} \right. . \tag{27}
$$

Together with ΔQ^2_0 without any introduced coupling,

$$
\Delta Q^2_0 = \Delta^2 + k^2 \cdot |C^-_{mod,amp}|^2 \tag{28}
$$

The factor k is easily obtained:

$$
k = (\frac{\Delta Q^2_{max} - \Delta Q^2_0}{\Delta Q^2_{max} - \Delta Q^2_{min}} - \frac{1}{2})^{-1}. \tag{29}
$$

The factor k has very significant meaning for the coupling correction. As discussed above, the minimum tune split is achieved when the rotating coupling has opposite direction to the residual coupling, so the correction coupling should also in the direction of the rotating coupling at that time point. Since we know the modulation strengths (or currents) for the skew quadrupoles involved in the rotating coupling, the correction strengths are therefore the strengths of those skew quadrupoles at the minimum tune split's square multiplied by a positive factor. This factor is the ratio of the residual coupling amplitude to that of the rotation coupling, that is, k in Eq.(29).

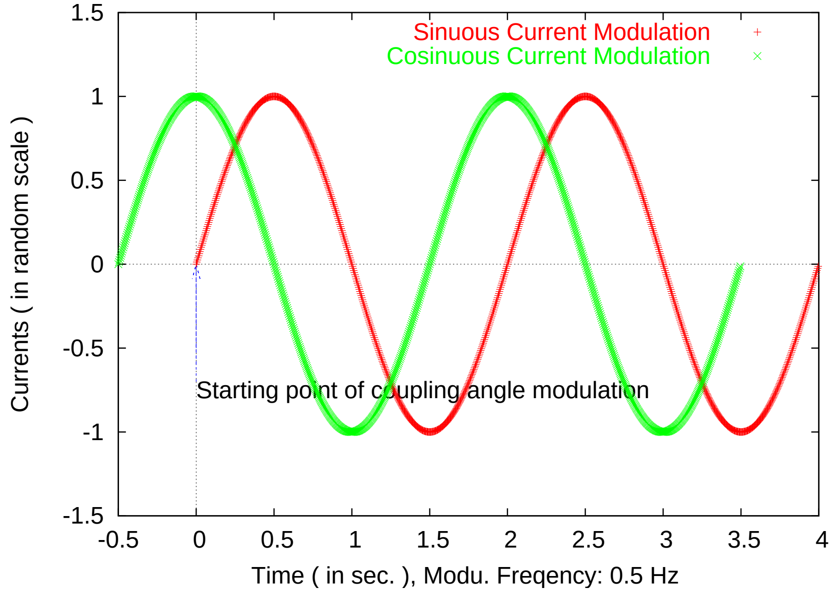

Figure 8: Skew quadrupoles' Currents in the operational coupling phase modulation

6.2 Comments

The advantage of the coupling phase modulation is very clear. It doesn't need FFT or FIT to the PLL tune data, only the ΔQ_0^2 under no introduced coupling situation, ΔQ_{max}^2 and ΔQ_{min}^2 during the coupling phase modulation are needed. The correction strengths are also easily obtained. They are just the modulation strengths at the minimum tune split square time point multiplied by the positive factor k. We doesn't need to figure out the residual coupling first. The correction strengths are directly found in the modulation procedure.

The time period occupied by the coupling phase modulation is much shorter. In principle, one modulation period is enough to find the correction strengths. Since the power supplies' currents need to rise from zero current, the co-sinuous function modulation is obtained operationally by starting a sinuous function modulation 1/4 period ahead the coupling phase modulation, as shown in Fig. 8. So the time occupied in the modulation is about 2 modulation periods.

We have encountered problems in the modulation frequency choice in the coupling strength modulation. Faster modulation will introduce more peaks in the FFT of $(Q_1 - Q_2 - p)^2$. And also the PLL system may not be able to track the tune's modulation. Too slow modulation frequency will lengthen the PLL data taking time, and the two peaks in FFT plot of $(Q_1 - Q_2 - p)^2$ locate in the very tight narrow frequency region, which makes it hard to distinguish the two peaks. However, here we have not this problem. The modulation frequency could be slow or fast, just up to the capability of the PLL system.

This correction scheme is also robust since only the ΔQ_0^2, ΔQ_{max}^2 and ΔQ_{min}^2 are needed. It doesn't care too much about the spikes in the detailed tune data from PLL. In order to swept off the large spikes, the co-sinuous fitting according Eq.(26) can be used to find the ΔQ_{max}^2 and ΔQ_{min}^2.

It is also easy to prove that the orthogonal modulations are not strictly demanded. The above solution to factor k is still valid even if the two modulation is not strictly out-of-phase, or the two modulation amplitudes are not exactly the same. There the rotating coupling traces out an ellipse instead the circle in Fig. 7. Considering the radial symmetry of the ellipse, the rotating coupling at the maximum and the minimum tune split are 180° different, the amplitudes are the same. So we still could use Eq. (29) to figure out the factor k. So the coupling phase modulation scheme has less connections to the optics.

6.3 Simulation example

Here we use the combination of skew quadrupole F1 and F2 to construct a F12, which is orthogonal to F3. In order to make the modulation amplitude of F12 and F3 are the same, F1 and F2 's modulation strengths are set to $1/\sqrt{3} = 0.577$ of F3.

We give an simulation correction example based on the smooth accelerator model. The residual coupling is introduced by setting the constant integrated strengths of F1,F2,F3 to 0.m^{-1}, -0.001 m^{-1}, -0.0015 m^{-1}. Please be reminded that there are only three skew quadrupoles in the tracking model to represent the three families. The integrated strengths given here is not the same to the individual skew quadrupole integrated strength in the real lattice model.

We modulate F3 with integrated strength amplitude 0.0005 m^{-1}, modulate F1 and F2 with $0.0005/\sqrt{3}$ m^{-1}. The modulation frequency is 0.2 Hz. And F3's strength is modulated like co-sinuous function, F1 and F2 's strengths are modulated like sinuous function. It is shown in Fig. 9. Fig. 10 shows the tunes' modulation during the coupling phase modulation. Fig. 11 shows the $(Q_1 - Q_2 - p)^2$ during the coupling phase modulation. Fig. 12 shows the tunes in the whole procedure of coupling phase modulation correction. The minimum $(Q_1 - Q_2 - p)^2$ happens at the modulation phase 380°. The correction strengths are given by the modulation strengths at that time point multiplied by the factor $k = 5.0$. After correction, we see in Fig. 12 that the tunes restore to the design tunes.

7 Conclusion

The principles for the three candidate methods to the global decoupling on the ramp in the Rhic have been reviewed. Some simulation results are given. The grouping of the Rhic skew quadrupoles are also discussed. The coupling phase modulation correction is the most promising scheme for the coupling correction on the ramp. It will be tested in the coming Rhic Run'05. If it is successful, it will be the fastest scheme for the global decoupling among those ever found . All the schemes discussed in this note are also suitable for the global coupling correction at the injection, store and on the ramp.

8 Acknowledgments

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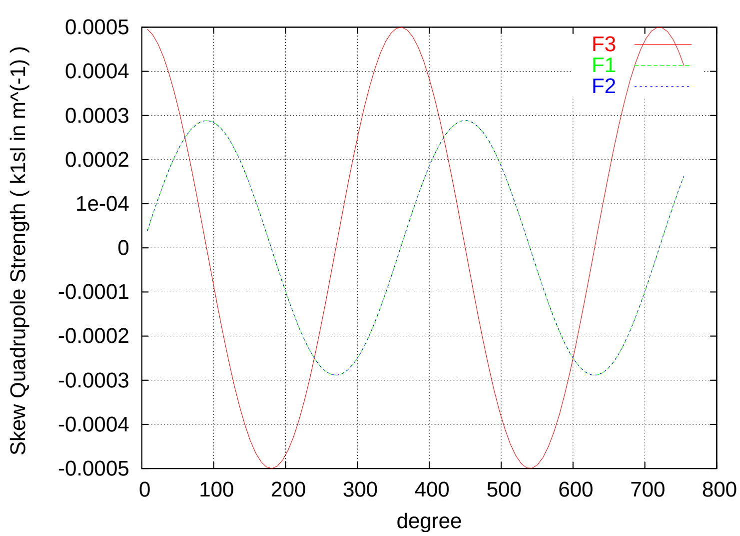

Figure 9: Skew quadrupoles' currents during the coupling phase modulation.

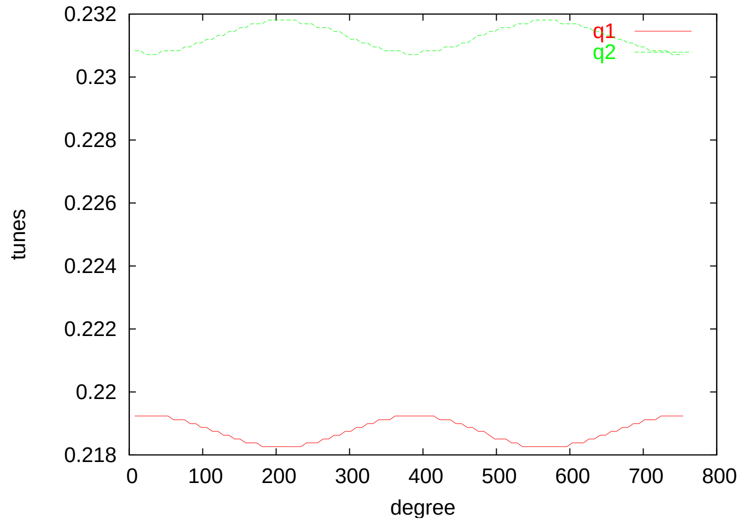

Figure 10: Tunes' modulation during the coupling phase modulation.

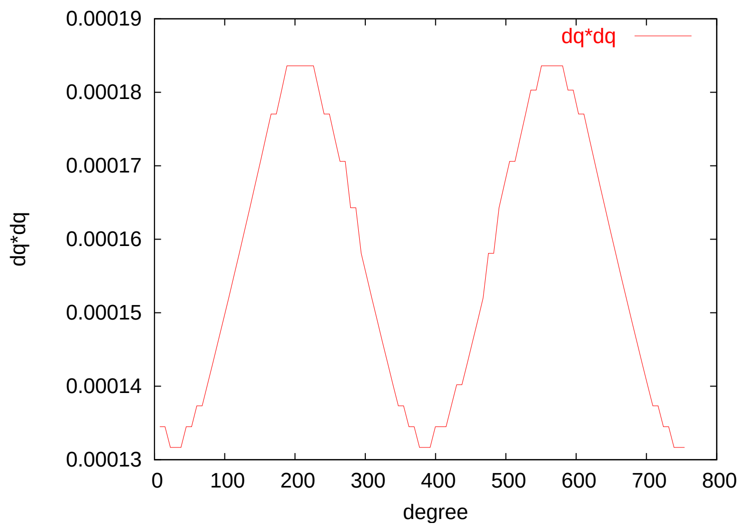

Figure 11: The $(Q_1 - Q_2 - p)^2$ during the coupling phase modulation.

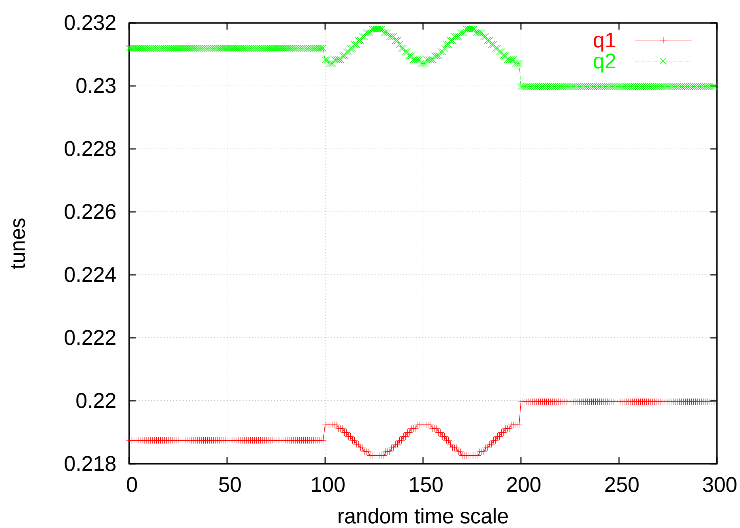

Figure 12: The tunes in the whole procedure of global decoupling in the simulation.