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Longitudinal Stochastic Cooling Power Requirements: A Coasting-Beam Approach

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Abstract

A practical obstacle for stochastic cooling in RHIC is the large amount of power needed for the cooling system. This paper discusses the cooling power needed for the longitudinal cooling process. Based on the coasting-beam Fokker-Planck equation, we analytically derived the optimum cooling rate and cooling power for a beam of uniform distribution and a cooling system of linear gain function. The results indicate that the usual back-of-envelope formula over-estimated the cooling power by a factor of the mixing factor M. On the other hand, the scaling laws derived from the coasting-beam Fokker-Planck approach agree with those derived from the bunched-beam Fokker-Planck approach if the peak beam intensity is used as the effective coasting-beam intensity. A longitudinal stochastic cooling system of 4 – 8 GHz bandwidth in RHIC can effectively counteract intrabeam scattering, preventing the beam from escaping the RF bucket becoming debunched around the ring.

I. INTRODUCTION

Stochastic cooling [1,2] has long been recognized as a viable approach to counteract the emittance growth and beam loss caused by intrabeam scattering in RHIC [3,4]. Theoretically, with a transverse cooling system of frequency bandwidth from 4 to 8 GHz, the (normalized 95%) emittance of a gold beam of 10^9 particles per bunch can be preserved at 30 $\pi\mu$m. With a longitudinal cooling system of the same frequency bandwidth, the beam escaping from the RF bucket debunched around the ring can be eliminated [5].

A possible technical obstacle is the existence of very strong coherent components at GHz frequency range that would saturate the electronics of the cooling system and swamp the true stochastic information. Due to this problem, attempts at implementing bunched-beam stochastic cooling at the Tevatron and the SPS were unsuccessful. On the other hand, cooling of the heavy ion beam in RHIC has the advantage that the signal-to-noise ratio is high due to the high charge state, and that longitudinally the beam occupies a large fraction of the RF bucket. Recent measurements of Schottky signals of the gold beam indicate that stochastic cooling in RHIC would not be impeded by anomalous coherent components in the Schottky signals [6,7].

Practically, the obstacle for stochastic cooling in RHIC is the large amount of power needed for the cooling system [3]. Early study using the Fokker-Planck approach indicated that the power needed is proportional to the energy spread of the beam to the forth power [4]. With a total kicker coupling-resistance of 6.4 kΩ, the power needed for longitudinal cooling at beam storage is several kilo Watts at a frequency range from 4 to 8 GHz. However, a comparison between the Fokker-Planck calculation [4] and the estimate given in Ref. [3] indicates a difference in the scaling behavior of the cooling power when the mixing factor [2] is larger than unity. The purpose of this note is to present the analytical derivation of the cooling power scaling law and to discuss applications in RHIC.

II. FOKKER-PLANCK APPROACH

A. Formalism

Define the quantity W as

$$W \equiv \frac{\Delta E}{\omega_s},$$

where ω_s is the revolution frequency and ΔE is the deviation in energy. Assume that the evolution of the beam distribution is slow during a synchrotron-oscillation period. The evolution of the longitudinal density function $\Psi(W)$ can be described by the Fokker-Planck equation [8,9,2]

$$\frac{\partial \Psi}{\partial t} = -\frac{\partial}{\partial W}(F\Psi) + \frac{1}{2}\frac{\partial}{\partial W}\left[(D_{sh} + D_t)\frac{\partial \Psi}{\partial W}\right]. \tag{1}$$

The boundary condition of Eq. 1 is that Ψ vanishes at the energy aperture W_{max} and the flux vanishes at $W = 0$,

$$\begin{cases} -F\Psi + \frac{1}{2}(D_{sh} + D_t)\frac{\partial \Psi}{\partial W} = 0 & W = 0 \\ \Psi = 0 & W = W_{max} \end{cases} \tag{2}$$

The drifting coefficient F corresponds to the coherent component of the cooling force, and the diffusion coefficients D_{sh} and D_t correspond to Schottky noise and system thermal noise,

$$F(W) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\langle \int_0^{\Delta t} dt\ U_W(t)\rangle \tag{3}$$

$$D_{sh}(W) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\langle \int_0^{\Delta t} dt \int_0^{\Delta t} dt'\ U_W(t)U_W(t')\rangle \tag{4}$$

Thermal noise is usually small for cooling of heavy-ion beams and thus neglected in this discussion. The energy kick U_W applied on a test particle i by the cooling system contains both the deterministic (coherent) part resulting from its own signal and a fluctuating (incoherent) part resulting from the signal generated by all other particles.

$$U_{W,i} = \frac{Ze}{\omega_s}\sum_{n=-\infty}^{\infty} V_k(t)\ \delta\left(t - \frac{2\pi n}{\omega_i} - \frac{\phi_i}{\omega_i} - \frac{\theta_k}{\omega_i}\right), \tag{5}$$

where ϕ_i is the phase of the test particle, θ_k is the azimuthal location of the kicker in the ring, the subscript k indicates the kicker, and

$$V_k(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega) I_{pu} \exp(i\omega t) d\omega \tag{6}$$

where $G(\omega)$ is the gain function, and the frequency component of the beam current is given by

$$I_{pu}(\omega) = \int_{-\infty}^{\infty} I_{pu} \exp(-i\omega t)\ dt, \tag{7}$$

$$I_{pu}(t) = Zc \sum_{m=-\infty}^{\infty} \sum_{j=1}^{N} \delta\left(t - \frac{2\pi m}{\omega_j} - \frac{\phi_j}{\omega_j} - \frac{\theta_{pu}}{\omega_j}\right), \tag{8}$$

where the subscript pu indicates the signal detector pick-up. Use the relation on the gain function imposed by the causality condition

$$G(-\omega) = G^*(\omega) \tag{9}$$

and mathematical relations

$$\sum_{m=-\infty}^{\infty} \exp -i\frac{2\pi m\omega}{\omega_i} = \omega_i \sum_{m=-\infty}^{\infty} \delta\left(\omega - m\omega_i\right) \tag{10}$$

$$\lim_{\Delta t \to} \frac{1}{\Delta t}\int_0^{\Delta t} dt \exp(at) = \begin{cases} 1 & a = 0 \\ 0 & a \neq 0 \end{cases} \tag{11}$$

The drifting coefficient becomes

$$F = \frac{z^2 e^2 \omega_i}{4\pi^2} \sum_{m=-\infty}^{\infty} G(m\omega_i) \exp\left[-im\left(\theta_{pu} - \theta_k\right)\right] \tag{12}$$

where the factor $\exp\left[-im\left(\theta_{pu} - \theta_k\right)\right]$ is the effect from the delay between the pick-up and the kicker, and the summation is over the effective frequency range of the cooling system. The diffusion coefficient becomes

$$D_{sh} = \frac{z^4 e^4 \omega_i^2}{8\pi^3} \sum_{j=1}^{N} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{|G(m\omega_j)|^2}{|m|} \rho(\omega_j)\Big|_{\omega_j = \frac{n}{m}\omega_i} \tag{13}$$

where the summation over j indicates the contribution from all the particles, and the double summation over m and n considers the case of frequency overlapping.

The average power required for cooling is

$$\bar{P} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_0^{\Delta t} dt \, \frac{\langle V_k^2(t) \rangle}{n_k R_k} \equiv \frac{\langle V_{cool} \rangle^2}{n_k R_k} \tag{14}$$

where $\langle V_{cool} \rangle$ is defined as the mean cooling voltage, n_k is the number of kicker pairs, and R_k is the coupling resistance of each kicker pair.

III. SCALING LAWS FOR COOLING RATE AND POWER

The cooling rate of the beam energy spread

$$\langle W \rangle = 2 \int_0^{W_{max}} W \; \Psi(W) \; dW$$

can be obtained from the Foller-Planck equation as

$$\frac{\partial \langle W \rangle}{\partial t} = 2 \int_0^{W_{max}} dW \; \left(F + \frac{1}{2} \frac{\partial D}{\partial W} \right) \Psi(W) \tag{15}$$

To obtain a scaling of the cooling rate, we assume a linear gain function "notched" at multiples of the revolution frequency

$$G(m\omega) = gmW, \quad \Delta\omega = \omega - m\omega_s = -\frac{\eta \omega_s^2}{E_s \gamma^2} W \tag{16}$$

where $m\omega_s$ is the nearest multiple of the revolution frequency to ω, $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$ is the slip factor, E_s is the beam energy, γ is the Lorentz factor, and γ_t is the γ value of the transition energy. Denote the effective frequency range of cooling from $n_1\omega_s$ to $n_2\omega_s$, the drifting and diffusion coefficients becomes

$$F \approx \frac{Z^2 e^2 \omega_s}{2\pi^2} \; \bar{n} \; \Delta n \exp\left[i\bar{n}(\theta_k - \theta_{pu})\right] \; g \; W \tag{17}$$

$$D \approx \frac{Z^4 e^4 \omega_s^2}{4\pi^3} \; \bar{n} \; \Delta n \; g^2 \; W^2 \; \rho(\omega_i) \tag{18}$$

where $\Delta n = n_2 - n_1$, $\bar{n} = \dfrac{n_1 + n_2}{2}$, and the Schottky bands are assumed to be non-overlapping. For a uniform density distribution

$$\rho(W) = \begin{cases} \dfrac{N}{2\Delta\omega_s} & |W| < W_0 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

where

$$\Delta\omega_s = -\eta\omega_s(\Delta\hat{p}/p)_0 = -\frac{\eta\omega_s^2}{E_s\gamma^2}W_0$$

the maximum cooling rate that corresponds to the minimum cooling time τ_{min} is obtained as

$$\tau_{min}^{-1} = \frac{1}{\langle W\rangle}\frac{\partial\langle W\rangle}{\partial t}|_{max} \approx -\frac{\Delta n\omega_s}{2\pi NM} \tag{20}$$

where the "bad mixing" factor is neglected, and the mixing factor M is given by

$$M = \frac{1}{\bar{n}|\eta|(\Delta\hat{p}/p)_0} = \frac{1}{\sqrt{3}\bar{n}|\eta|\sigma_p}, \quad \text{for} \quad M > 1 \tag{21}$$

where σ_p is the rms spread in momentum $(\Delta p/p)$. The optimum gain function to achieve the maximum cooling rate is given by

$$g_{opt} \approx -\frac{2\pi|\eta|(\Delta\hat{p}/p)_0}{Z^2e^2N} \tag{22}$$

The average power needed for cooling is

$$\bar{P} \approx \frac{2}{f_s n_k R_k}\frac{1}{\tau_{min}M}\left(\frac{\beta^2 E_s\sigma_p}{Ze}\right)^2 \tag{23}$$

where $f_s = \omega_s/2\pi$ is the revolution frequency.

IV. NUMERICAL EXAMPLE: LONGITUDINAL COOLING IN RHIC

Coasting-beam estimate of the cooling rate and cooling power is the following. Consider longitudinal stochastic cooling of a gold beam ($Z = 79$, $A = 197$) at RHIC storage ($\gamma = 108$) with $h = 2520$ RF system of 6 MV voltage, and a bunch of intensity $N_0 = 10^9$. The revolution frequency is $\omega_s/2\pi = 78$ kHz. The slip factor is $\eta = 1.9 \times 10^{-3}$. The central harmonic corresponding to a $4 - 8$ GHz system is $\bar{n} = 7.7 \times 10^4$. During a 10-hour store under intrabeam scattering and longitudinal stochastic cooling [10], the bunch length σ_l varies from about 0.11 m to 0.19 m (σ_ϕ varies from about 27° to about 45°), and the bunching factor B varies from 0.19 to 0.31. With cooling, the reduction in beam intensity is small during the store, and the effective number of particle in the ring $N = hN_0/B$ varies from about 1.33×10^{13} to about 0.81×10^{13}. The rms momentum spread σ_p varies from about 0.42×10^{-3} to about 0.71×10^{-3}, and the mixing factor M varies from 9.4 to 5.6 (Eq. 21). According to Eq. 20, the optimum cooling time varies from 8.7 to 3.2 hours. The mean cooling voltage $\langle V_{cool} \rangle$ varies from about 1.0 kV to about 3.6 kV (Eq. 14). Assume $n_k = 128$ pairs of kickers each at $R_k = 50\ \Omega$ coupling resistance. The average cooling power varies from 0.15 kW to 2.0 kW (Eq. 23). The peak power is proportional to the 4th power of the momentum spread σ_p.

V. DISCUSSIONS AND SUMMARY

Based on the coasting-beam Fokker-Planck equation, we analytically derived the optimum cooling rate and cooling power for a beam of uniform distribution and a cooling system of a linear gain function. The results indicate that the usual back-of-envelope formula [3] over-estimated the cooling power by a factor of the mixing factor M. On the other hand, the scaling laws derived from the coasting-beam Fokker-Planck approach agree with those derived from the bunched-beam Fokker-Planck approach [4] if the peak beam intensity is used as the effective coasting-beam intensity. Although we have ignored signal suppression for the entire discussion, the conclusion holds.

A longitudinal stochastic cooling system of $4 - 8$ GHz bandwidth in RHIC can effectively counteract intrabeam scattering, preventing the gold beam from escaping the RF bucket becoming debunched around the ring. Combining with a transverse stochastic cooling system of the same frequency bandwidth to contain the transverse emittance, we expect a significant increase in the average luminosity.

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