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During the 2012 RHIC run, uranium, gold, and copper ions from EBIS were accelerated in Booster and AGS and delivered to RHIC for the physics program. In this note the nominal parameters for uranium are given. The parameters for gold and copper are given in a separate note.

In order to achieve the desired number of ions per bunch in RHIC, it was necessary to merge bunches in both Booster and AGS. This is described in Sections 11 and 12 of Reference [1].

1 Mass

A uranium 238 ion with charge eQ has $N = 146$ neutrons, $Z = 92$ protons, and $(Z - Q)$ electrons. Here Q is an integer and e is the positive elementary charge. The mass number is

$$A = N + Z = 238. \tag{1}$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$mc^2 = am_uc^2 - Qm_ec^2 + E_Q \tag{2}$$

where [2, 3]

$$a = 238.0507882(20) \tag{3}$$

is the relative atomic mass of the neutral uranium atom,

$$m_uc^2 = 931.494061(21)\ \text{MeV} \tag{4}$$

is the mass energy equivalent of the atomic mass constant, and

$$m_e c^2 = 0.510998928(11) \text{ MeV} \tag{5}$$

is the electron mass energy equivalent. The binding energy E_Q is the energy required to remove Q electrons from the neutral uranium atom. This amounts to [4, 5] 0.501 MeV for the helium-like uranium ion ($Q = 90$) and 0.762 MeV for the fully stripped ion. For $Q = 38$, 39, 40, 41 and 42 we have $E_Q = 25.1$, 26.4, 27.8, 29.1 and 30.6 KeV, respectively.

2 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference C with revolution frequency f. The radius of the orbit is defined to be $R = C/(2\pi)$. The velocity of the ion is then

$$v = 2\pi R f. \tag{6}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma, \quad W = mc^2(\gamma - 1) \tag{7}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2}. \tag{8}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kp/Q \tag{9}$$

where $k = 10^9/299792458$ and p is the momentum in units of GeV/c. The angular frequency is

$$\omega = 2\pi f. \tag{10}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{11}$$

where γ_t is the transition gamma.

3 RF Parameters

1. The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{12}$$

where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

2. The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S\,|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s|^{1/2} \tag{13}$$

where ϕ_s is the synchronous phase.

3. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{14}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

4. The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{15}$$

where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{16}$$

5. The area of a bucket is

$$A_{\mathrm{bk}} = \alpha(\phi_s) A_S \tag{17}$$

where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi|^{1/2}\, d\phi. \tag{18}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The integral $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [6]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{19}$$

6. The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s} \left\{ \frac{-h\eta_s eQV_g \cos\phi_s}{2\pi E_s} \right\}^{1/2} \tag{20}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g \cos\phi_s$, while measurement of dB/dt gives a value for $V_g \sin\phi_s$. These two can be used to obtain V_g and ϕ_s.

7. Let ϕ_l and ϕ_r be the phases at the left and right boundries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{21}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{22}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\} \tag{23}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\}. \tag{24}$$

If $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{25}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{26}$$

8. The half-height of a bunch matched to a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \,|\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s|^{1/2}\,. \tag{27}$$

9. The area of a bunch matched to a bucket is

$$A_{\rm b} = F(\phi_s, \Delta\phi)\, A_S \tag{28}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8}\int_{\phi_l}^{\phi_r} |\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s|^{1/2}\, d\phi. \tag{29}$$

The integral $F(\phi_s, \Delta\phi)$ must be evaluated numerically. If $\Delta\phi$ is small we have

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64}(\Delta\phi)^2\, |\cos\phi_s|^{1/2}\,. \tag{30}$$

4 Ring Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \delta C$	m
C_E	$C_a/4$	$4(C_r + \delta C)/19$	$C_r + \delta C$	m
ρ	13.8656	85.378351	242.7806	m
$\gamma_{\rm tr}$	4.832	8.5	22.89	

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{31}$$

meters. δC is the shift (if any) of the RHIC orbit circumference from the design value C_r. Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [7, 8]. The radius of curvature ρ in the Booster and AGS main dipoles is given in [7, 8, 9]. The RHIC ring parameters are taken from Ref. [10] and from MAD runs by Steve Tepikian.

The crossing angle at Store in the four non-experimental Intersection Regions (IR10, IR12, IR2 and IR4) is 2 milliradians. This gives an orbit circumference less than C_r. The shift δC has been calculated by Steve Tepikian and is given in the next section.

5 Initial Conditions and Assumptions

1. The revolution frequency in Booster of uranium ions from EBIS is 96.100 kHz at injection.

2. The revolution frequency of the U39+ ion at Booster extraction is $f = 653.5$ KHz [11]. The corresponding magnetic rigidity is $B\rho = 9.2874872541$ Tm. The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is $B\rho = 9.5$ Tm [12]. We have successfully extracted Au31+ ions (originating from Tandem) at $B\rho = 9.43$ Tm for several years.

3. The set revolution frequency of the U90+ ion at AGS injection is $f = 162.000$ KHz. This gives an energy loss of 2.079 MeV per nucleon in the BTA stripper.

4. The magnetic rigidity of the U92+ ion at RHIC injection is taken to be $B\rho = 81.1137824$ Tm.

5. The circumference at RHIC injection is C_r.

6. The circumference at Store is 1.400 mm **less than** C_r.

7. The magnetic rigidity of the U92+ ion at RHIC Store is the same as that of a Au79+ ion with energy $E = 100$ GeV per nucleon.

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be measured; this is done for computational convenience.

6 Inflector Voltage

At Booster injection, the voltage V_I required for particles with mass m, velocity $c\beta$, and charge Q to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma. \tag{32}$$

Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the calculated values of β and γ at Booster injection, we obtain $V_I = 57.262$ kV for the U39+ ion from EBIS.

Because of an unresolved calibration error, the actual setpoint for the inflector voltage needs to be

$$V_I(\text{setpoint}) = 1.034\, V_I \tag{33}$$

which gives

$$V_I(\text{setpoint}) = 59.209\,\text{kV}. \tag{34}$$

7 Booster Injection Field

The nominal magnetic field in the Booster dipoles at injection is

$$B = (B\rho)/\rho \tag{35}$$

where $B\rho$ is given by (9) and ρ is the nominal radius of curvature. Using the calculated values of $B\rho$ we obtain $B = 886.5$ Gauss for the U39+ ion from EBIS.

The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field given by the Gauss Clock. For U39+ ions from EBIS the measured field at injection is 883.1 Gauss [13].

8 AGS Injection Field

Similarly, the nominal magnetic field in the AGS dipoles at injection is $B = 466.4$ Gauss for the U90+ ion.

9 BTA Stripper

The stripper used to strip uranium ions in the BTA (Booster-To-AGS) transfer line consists of a 4.38 mg/cm^2 nickel foil followed by a 9.0 mg/cm^2 aluminum foil.

10 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$B = (B\rho)/\rho \tag{36}$$

where $B\rho$ is the magnetic rigidity of the beam and $\rho = 18.625$ m [15] is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$NI = gB/\mu_0 \tag{37}$$

where $N = 1$ is the number of conductor turns; $g = 0.0467$ m [15] is the magnet gap; and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

For U90+ ions at injection, the magnetic rigidity is $B\rho = 3.98224861797$ Tm. This gives $B = 0.21381$ T and $I = 7946$ A.

For Au77+ ions at injection, the magnetic rigidity is $B\rho = 3.88434088$ Tm. This gives $B = 0.208555$ T and $I = 7750$ A.

For Cu29+ ions at injection, the magnetic rigidity is $B\rho = 3.3405536$ Tm. This gives $B = 0.179359$ T and $I = 6665$ A.

For comparison, the magnetic rigidity of polarized protons at AGS injection is $B\rho = 7.205178$ Tm. This gives $B = 0.3869$ T and $I = 14380$ A.

11 AGS Injection Kicker Current

The current required in the A5 kicker is [14, 15]

$$I = \frac{B\rho}{K} \sin\phi \tag{38}$$

where

$$K = 1.8718 \times 10^{-5} \text{ Tm/A} \tag{39}$$

and

$$\phi = 3.35 \text{ milliradians} \tag{40}$$

is the desired kick angle. Using the calculated values of $B\rho$ at AGS injection we obtain a current of 712.7 A for U90+ ions. The maximum available current is 1100 A.

12 AGS Injection Kicker Short Pulse Waveforms

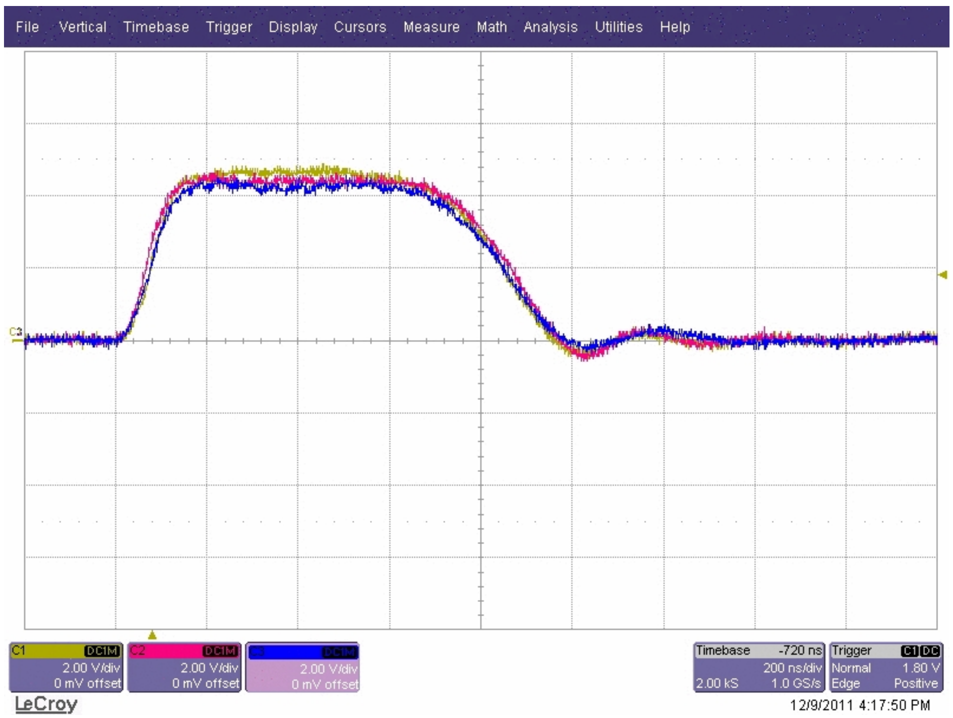

Figure 1: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 386 ns for U90+ ions. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to the bucket width minus the bunch width. The rise time is approximately 100 ns, which implies that the bunch width must be less than or equal to 286 ns. A single bunch with this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is some 1200 ns. With this kicker pulse one could in principle fill 14 of the 16 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This was not an issue during the run as only 8 of the buckets needed to be filled. The filling pattern was four adjacent filled buckets followed by four adjacent empty buckets.

13 AGS Injection Kicker Long Pulse Waveforms

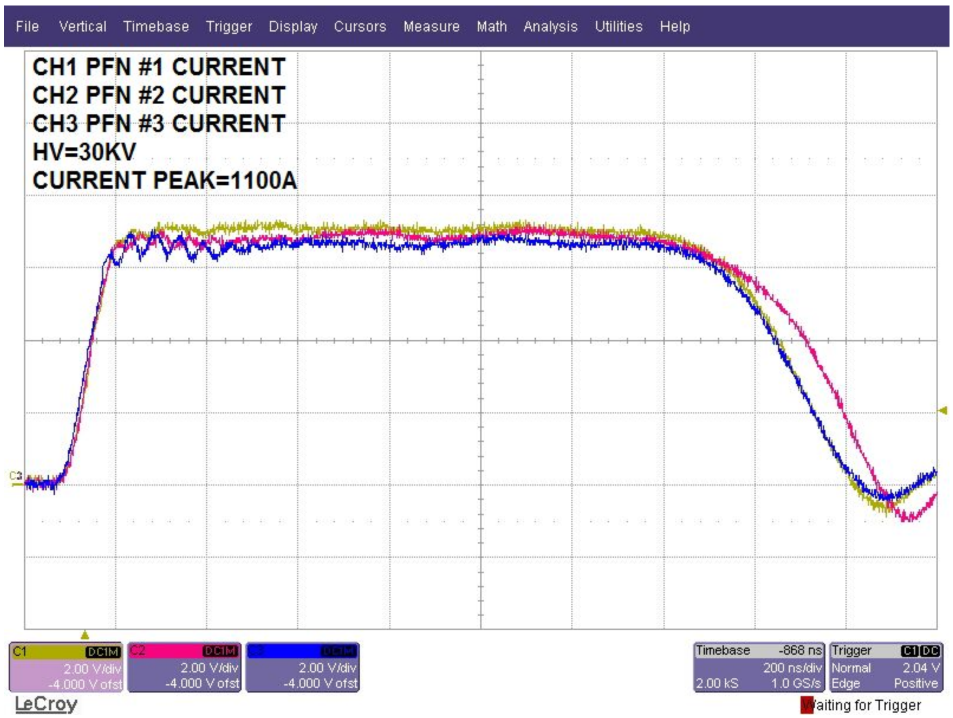

Figure 2: AGS injection kicker waveforms in the long pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan in Oct 2010. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 386 ns for U90+ ions. Here the flattop portion of the pulse is some 1300 ns long. The total pulse width is some 2000 ns. In principle this kicker pulse could be used to fill 8 buckets wtih a filling pattern of four adjacent filled buckets followed by four adjacent empty buckets.

14 U39+ Parameters in Booster

Parameter	Injection	Merge Porch	Extraction	Unit
Q	39	39	39	
mc^2	221.7229929	221.7229929	221.7229929	GeV
W/A	1.954927816	49.27863476	105.7255165	MeV
cp/A	60.38442907	306.9938830	456.2538936	MeV
E/A	0.9335641415	0.9808878485	1.037334730	GeV
$B\rho$	1.229183188	6.249156042	9.287487254	Tm
β	0.064681607	0.3129755185	0.4398328527	
$\gamma - 1$	0.0020984419	0.0528963	0.113486980297	
η	-0.953	-0.859	-0.764	
ϵ_H (95%)	12.0π	12.0π	12.0π	mm mrad
ϵ_V (95%)	5.64π	5.64π	5.64π	mm mrad
h	4	1	1	
hf	384.400	465.000	653.5	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during multi-turn injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parameter	Injection	Extraction	Extraction	Extraction	Unit
V_g	0.6	30	30	30	kV
A_S	6.313	420.5	420.5	420.5	eV s
dB/dt	0	**70**	**35**	**0**	G/ms
ϕ_s	0	40.753	19.050	0	degrees
F_s	0.3756	0.9815	1.096	1.128	kHz
A_{bk}	6.313	85.64	214.0	420.5	eV s
A_b	5.804	28.56	28.56	28.56	eV s
Δt	2239	315.4	293.9	289.1	ns
ΔE	1.861	58.3	62.2	63.1	MeV

Parameter	Injection	Extraction	Unit
No. of Bunches	4	1	
Bucket Spacing	2601.4568	1530.2219	ns
Ions/Bunch	1.21/4	0.588	10^9 [16]
Bunch Area	0.09755/4	0.12	eV s/A

15 Uranium Parameters in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	90	90	90	
mc^2	221.6974065	221.6974065	221.6974065	GeV
W/A	0.1036346103	6986262810	8.514536614	GeV
cp/A	0.4514551645	7.862779378	9.399997050	GeV
E/A	1.035136318	7.917764519	9.446038322	GeV
$B\rho$	3.982248618	69.35692573	82.91659551	Tm
β	0.4361311225	0.9930554715	0.9951258644	
γ	1.111255416	8.5000	10.14065593	
η	-0.796	0.0	0.00412	
ϵ_H (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
ϵ_V (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
h	16	12	12	
hf	2.592	4.426420719	4.43546341731	MHz
R	128.4526	128.4526	128.45798	m

Parameter	Injection	Injection	Extraction	Unit
h	16	4	12	
V_g	35.11	22	148.9	kV
A_S	42.26	267.6	5629	eV s
dB/dt	0	0	0	G/ms
ϕ_s	0	0	180	degrees
F_s	1.894	0.7496	0.0804	kHz
A_{bk}	42.26	267.6	5629	eV s
A_b	33.32	133.28	133.28	eV s
Δt	289.1	849.3	25	ns [17]
ΔE	79.45	103.6	3398	MeV

Parameter	Injection	Injection	Extraction	Unit
h	16	4	12	
Bucket Spacing	385.802	1543.210	225.456	ns
No. of Bunches	8	2	2	
Ions/Bunch	1.74/8	0.87	0.39	10^9 [16]
Bunch Area	0.14	0.56	0.56	eV s/A [18, 19]

16 Uranium Parameters in RHIC

Parameter	Injection	Transition	Store	Unit
Q	92	92	92	
mc^2	221.696645523	221.696645523	221.696645523	GeV
W/A	8.51450738740	20.3905023971	95.4628241434	GeV
cp/A	9.39996478450	21.3016439091	96.3898218198	GeV
E/A	9.44600589800	21.3220009077	96.3943226540	GeV
$B\rho$	81.1137824	183.815253399	831.763013151	Tm
β	0.995125864413	0.999045258528	0.999953308098	
γ	10.1406559329	22.8900	103.483066862	
η	-0.00782	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.0134531620	28.1237867061	197.045514282	MHz
δC	0	0	-1.400	mm

Parameter	Injection	Store	Unit
h	360	2520	
V_g	357.3	3000	kV
A_S	184.9	191.8	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.200	0.231	kHz
A_{bk}	184.9	191.8	eV s
A_b	133.28	133.28	eV s
A_b	0.56	0.56	eV s/A [20]
Δt	25	3.46	ns
ΔE	3627	26066	MeV

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[16] K.L. Zeno, Booster-AGS-EBIS-2013 elog, 11 May 2012, entry 23:06

[17] K.L. Zeno, Booster-AGS-EBIS-2013 elog, 11 May 2012, entry 17:54

[18] K.L. Zeno, Booster-AGS-EBIS-2013 elog, 8 May 2012, entry 21:28

[19] Here we assume that the 4-to-1 merge and subsequent acceleration conserve the effective emittance.

[20] Here we take the longitudinal emittance to be the same as that at AGS extraction. This gives a lower bound on the longitudinal emittance in RHIC.