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An overview of particle flux models for 2000 GeV scaling

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ABSTRACT

Three formulations of particle yields in the collision of protons with target nucleons are considered, CKP, Trilling, Hagedorn-Ranft.

Cross-sections from each can be expressed as:

<u>CKP</u>: $\sim f_1(P)\ g_1(P_T)$

<u>Trilling</u>: $\sim f_2(P)\ g_1(P_T)$ + low energy term

<u>Hagedorn-Ranft:</u> $\sim f_3(\lambda)\ \{L(\lambda)\ f_i(\epsilon'_1 T(\lambda)\}$: $\lambda \sim \frac{\gamma-1}{\gamma_o-1}$

$g_1(P_T)$ is well established

$f_i(\epsilon'_1\ T(\lambda)\ )$ is based on Hagedorn's thermodynamics and has $g_1(P_T)$ built into it.

Crucial comparison between the models can be made on the basis of $f_{1,2,3}$, which must be fitted to the experimental data.

CKP: exponential

$$f_1(p) \sim p^2 \ell^{-\frac{p}{BPo^{3/4}}}$$

Trilling: Gaussian

$$f_2(p) \sim p^2 \ell^{-\frac{p^2}{Cp_o^2}}$$

Hagedorn-Ranft: exponential

$$f_3(\lambda) \sim (1-\lambda)\ell^{-a\lambda}$$

It is recommended that CKP, with its limits understood, is the most soundly based formulation for extrapolation to 2000 GeV.

AN OVERVIEW OF PARTICLE FLUX MODELS FOR 2000 GeV SCALING

Introduction

Three formulations of secondary particle yield in high energy collisions are widely used for estimating particle yields at proposed very high energy accelerator installations, CKP[1], Trilling[2], Hagedorn-Ranft[3].

It seems useful to attempt to make some comparison of the assumptions underlying the various formulations in order to get an estimate of the confidence that might be placed in these extrapolations.

The bases of the CKP formulation, derived from accelerator and cosmic ray data up to about 10^5 GeV primary energy, are six observations. Since these are relevant to any model they are listed below.

a. Pion energy distribution in the laboratory: a roughly exponential tail at least up to $E_\pi = 2/3\ E_o$ for $E_o > 20$ GeV.

b. Transverse momentum distribution:

$$g\,(P_T)\,d\,P_T \approx \frac{P_T}{P_o^2}\,\ell^{-P_T/P_o}\,d\,P_T$$

where $P_o = \langle P_T \rangle / 2$

and $\langle P_T \rangle = \langle P_T(m) \rangle = 0.30$ GeV/c for pions [4]

$\langle P_T \rangle = \langle P_T(E_o) \rangle$, though not strongly dependent.

c. Pion multiplicity: increases slowly with the primary energy.

$n_\pi = a\,E_o^m$ with $m \approx \frac{1}{4}$, $a = 2.7$

d. Pion inelasticity:

$$K_\pi = \frac{\Sigma E_\pi}{E_o} \approx 0.3 - 0.5 \text{ for } 25 \text{ GeV} < E_o < 10^5 \text{ GeV}$$

e. K/π ratio; kaon transverse momentum distribution:

The K/π ratio is fairly constant at 10% for $E_o > 25$ GeV for

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1. G. Cocconi, L. Koester, D. Perkins, UCRL 10022(1961) p. 167
2. G. Trilling, UCRL 16830 (1966), p. 25
3. R. Hagedorn, J. Ranft, CERN Preprint TH.851(1967)
   R. Hagedorn, CERN Preprint TH.853
   J. Ranft, KHEL/R 165 (1968)
4. G. Cocconi, CERN/NP Int. Report 68-17(1968)

e. (cont.)

all secondary energies E.

The kaon transverse momentum dependence is about the same as that for pions.

f. Proton c.m. spectrum, inelasticity, transverse momentum distribution:

In the c.m. system the proton energy spectrum is peaked towards high values.

Proton inelasticity, $K_p \approx 0.5$ over a wide range of E_o.

Proton transverse momentum distribution is similar to that for pions.

The present state of these data should be reviewed.

The Models:

1. The CKP approaches: (It seems to be generally overlooked that there are two different ones.)

a. Laboratory system: guess at pion (proton) energy spectrum in laboratory. Combine this with the transverse momentum distribution to obtain the pion (proton) flux in analytical form.

Probability of one pion in $(E_1\ p_T)$

$$\mathcal{P}(E,p_T) = f(E)\ g\ (P_T) = \left(\frac{1}{T} e^{-E/T}\right) \left(\frac{p_T}{p_o^2} e^{-p_T/p_o}\right) dp_T\ dE$$

where T = mean pion energy

$$n_\pi T = K_\pi E_o$$

$$T \propto E_o^{3/4}$$

This leads to

$$\boxed{\frac{d^2 N_\pi (E,\theta)}{dE\ d\Omega} = \frac{n_\pi E^2}{2\pi\ p_o^2\ C^2\ T} e^{-E\left(\frac{1}{T} + \frac{\theta}{p_o C}\right)}}$$

The questionable assumption here is the form of f (E)

$$f(E) = \frac{1}{T} e^{-E/T}.$$

The formula does not hold with old BNL data for $E > {}^2/_3 E_o$, but this is expected to be less important for large values of E_o so extrapolation upwards to very high energies will be fairly reliable.

b. <u>C.M. distributions</u>: Spectra were calculated and weighted in the C.M. on the assumptions of constant transverse momentum, and exponential longitudinal momentum distribution. The weighted distribution was point-by-point transformed to the laboratory system. The results were in agreement with the laboratory system formula of a) above. Aside from normalizations attention in evaluating this approach should be paid to the assumption of an exponential form for the longitudinal momentum distribution.

2. <u>The Trilling Formula</u>

Trilling, like CKP, part a) above, seeks a laboratory system formula more consistent with the then available CERN data at 0^o. He assumes two mechanisms for pion production, an isotropic boil-off of pions in the c.m. contributing to the low energy pion spectrum, and the decay of nucleon isobars contributing to the high-energy component. For the high energy part the longitudinal momentum distribution in the laboratory is assumed to be Gaussian, in contrast to the exponential form of CKP. The low-energy boil-off contributions are set up in the c.m. and transformed by an approximate Lorentz transformation to a laboratory system form. From this, leaving normalization aside, Trilling obtains

$$\frac{d^2 n}{dp\, d\Omega} = \underbrace{A p^2 e^{\left\{-2p/\sqrt{p_B}\right.} \; e^{\left.-bp\sqrt{p_B}\,\theta^2\right\}}}_{\text{low energy}} + \underbrace{\left\{B \frac{p^2}{p_B} e^{-c\left(\frac{p}{p_B}\right)^2} e^{-dp\theta}\right\}}_{\text{isobar contribution}} \frac{\text{pions/interacting part.}}{\text{Sr/BeV/c}}$$

The parameters are evaluated mainly from the Dekkers data at 18.5 and 23.1 GeV/c. Yields for various target materials are evaluated by using the measured cross-sections per nucleus. Approximate scaling factors are used to get π^-, $K^{\pm}$, etc. yields. The low energy term is not safely extrapolated beyond the order of a hundred GeV. The comparison with CKP then reduces, aside from normalizations, to the question of whether the laboratory distribution of longitudinal momentum is better fitted by a Gaussian than by the CKP exponential. Reference 4, while explicitly treating of the transverse momentum distribution, also has some relevance here. A more trenchant and detailed criticism by Cocconi,[5] points out that the Trilling formula fails to maintain the constant transverse momentum dependence, which has become increasingly well established over the past few years.

With modification in the normalization suggested by the newer data, and with the additional information that p_T is a function of the mass of the produced particle, CKP would still seem to be preferable to Trilling. In using the formula it should be noted that the advertised range of validity is $< {}^2/_3\, E_o$, where E_o is the energy of the primary proton.

3. Hagedorn-Ranft

Hagedorn and Ranft[3] have proposed a much more elaborate formulation of particle production based on a statistical thermodynamics of strong interaction processes into which known production and decay vertices are explicitly introduced along with appropriate conservation laws. Hagedorn-Ranft evaluates the yield in the c.m., unlike CKP and Trilling. The basic relation is:

$$W_i\,(p) = \int_{-1}^{1} F(\gamma_o,\lambda)\,\left\{L\,(\beta)\; f_i\,(\varepsilon',\lambda)\right\}\,d\lambda$$

where $\lambda \sim \dfrac{\gamma-1}{\gamma_o-1}$, the energy in the c.m.

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[5] G. Cocconi, UCRL 16830, vol. III, p. 17 (1966)

f_i (ϵ', λ) is the production vertex, where particles are produced isotropically according to statistical thermodynamics of strong interactions (fireball).

$L(\beta)$ is the Lorentz transformation from the "fireball" to the interaction c.m.

$F(\gamma_o, \lambda)$ is a "universal" weighting function which weights the longitudinal momentum distribution in the c.m. The universality of this is determined by fitting experimental distributions. It has the form

$$F(\gamma_o, \lambda) \approx F(\lambda) \sim (1-\lambda)\, e^{-a\lambda}, \text{ an exponential.}$$

Leaving all the thermodynamics concepts aside, Hagedorn-Ranft seems to reduce basically to the CKP form in choosing on the basis of experimental data, an exponential form for the c.m. momentum distribution.

Conclusion

On balance it would seem that the most firmly based approach to extrapolation of particle yields would be to update the CKP constants on the basis of newer data, determine the error bars on the CKP curves and then use the CKP relations within the limits of validity indicated by the authors.

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