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Shorter separators for beam 3

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SHORTER SEPARATORS FOR BEAM 3

This note investigates the possibility of saving dc separator units in Beam 3 by shortening the separators from triplets to doublets. In this connection the question of the optimum separator gap is re-examined in the light of what has been learned about the performance of Beam 3 and about the aberrations of the beam optics.

It is concluded that one may take out, if necessary, one of the three separators in each stage. To maintain the same top beam energy, however, one must then reduce the separator gap to 2" exploiting an expected gain in maximum electric field of approximately 1.4. For a given separation factor, the demands on machine intensity will increase over most of the energy range.

In the following, fluxes of negative K-mesons are compared in some detail for three separator configurations assuming no changes in the present beam optics:

(a) 15 meter (triplet) separator, 4" gap, 50 kV/cm (present).

(b) 10 meter (doublet) separator, 4" gap, 50 kV/cm.

(c) 10 meter (doublet) separator, 2" gap, 70 kV/cm.

Beam 3 now provides K^- mesons between ~ 2.9 GeV/c and 5.5 GeV/c. The K^- flux is separation limited at the higher momentum and limited by the separator aperture and beam optics at the lower momentum. After shortening the separator one can expect to operate with equal ease but generally less flux, between 3.0 GeV/c and 4.7 GeV/c, if the 4" gap is maintained. With a 2" gap one may then operate between 3.3 GeV/c and 5.4 GeV/c.

The remainder of this note is concerned with the detailed arguments leading to these conclusions.

(A) The Maximum K^- Flux

The 9^o production yields for π^--mesons (30 GeV primary proton energy) and K^-/π^- ratios measured by Baker et al.[1] on thin internal Be targets are used to calculate[2] the yield of thick internal tungsten targets at 7^o production angle in Beam 3. The maximum beam acceptance is 24 mrad horizontally and 4 mrad vertically using a 4" gap. To match flux predictions with reality a downward correction factor of 4 is applied to the data. This is attributable to mass slit losses (factor ~2), to lower targeting energy (25 GeV/c, not 30 GeV/c), to uncertain target efficiencies and the ever present beam alignment losses (together: factor ~2).

Figure 1 shows the maximum K^- flux available (4" gap) at the chamber, normalized to 10^{11} protons per pulse (ppp) used on the target, and to a 1% momentum bite. The beam accepts a 1.5% momentum bite. Measurements between 4 and 5.5 GeV/c support the calculation within ± 20%. For a 2" gap, due to the limitations of the beam optics, the flux would be limited to one half these values.

(B) Separation Factor

The different separator configurations are compared on the basis of a common figure of merit, the separation factor η. It is defined, at the output of the separator stage, as the spatial separation between images of wanted and unwanted particles measured in units of the image size.

The spatial separation, at the focal point of the separator output lens (focal length f_o), between K-mesons and π-mesons is given by:

$$S = \alpha_{k\pi} \cdot f_o$$

where $$\alpha_{k\pi} = \frac{e\,E\,L}{2(pc)^3}\,[m_k^2 - m_\pi^2]\,c^4 \text{ (radians)}$$

is the angular separation introduced by the electric field E, acting over the length L. Here e is the electronic charge, p is the momentum, c is the velocity of light, and m_k and m_π are the masses of K-mesons and π-mesons, respectively

We are using, at the input of the separator, a lens of adjustable focal length f , and the beam is vertically "parallel" in the electric field region. The image of the target of height t is located at the focal point of the output lens, and has, in the absence of aberrations, the size:

$$i = t \cdot \left(\frac{f_o}{f}\right)$$

The image aberrations i.e. the width of an image of an infinitesimally thin target can be written as:

$$\varepsilon = f\,f_o \cdot k(f) \cdot \varphi \quad + \quad 0(\varphi^2)$$

where φ is the vertical angular spread of the beam at the target. We ignore, in subsequent calculations, the higher order terms $0(\varphi^2)$. The function k(f) is discussed further below.

The image width, including aberrations, may be defined in various ways. Because of its "bell shape" the full width of the image at half maximum is sufficiently well approximated by adding i and ε in quadrature. This definition of image size is adequate for purpose of this note, but one should not be mislead into believing that the image tails show Gaussian behavior.

The separation factor η is a measure of the separation between unwanted and wanted particle images in units of the image size. Noting that the beam size a in the separator is, approximately, given by:

$$a = \varphi \cdot f$$

we obtain
$$\eta = \frac{\alpha_{k\pi}}{[(\frac{\varphi t}{a})^2 + (k(f)\ a)^2]^{\frac{1}{2}}} \qquad (1)$$

(C) The Aberration [k(f).a]

The factorization of the aberration term displays the fact that image aberrations scale proportional to the angular divergence at the source (first order terms are predominant and the effect of target height on beam size is ignored). The factor k(f), which is, to first order, independent of a, is a function of the lens excitations used to adjust the focal length f for a desired image size i.

A combination of beam tests and ray trace investigations provide the following empirical function k(f) for Beam 3, with a full 1.5% momentum bite and sextupole correctors on:

$$k(f) = [6.5 + \frac{17.5}{(f/1000'')^3}] \times 10^{-6} \text{ radians/inch} \qquad (2)$$

As one might expect, it approaches a constant for long focal lengths (high energies). Experience with the beam indicates that this function gives a fairly good prediction for the full width at half maximum of an image of a very thin target and that the aberration decreases somewhat for smaller momentum bites*). The size of the f^{-3} term is not well known, and the formula (2) has not yet been adequately verified by experiment below $f \lesssim 1500''$.

(D) Optimum Flux for a Desired Separation Factor

The flux is proportional to the vertical angular divergence φ at the target. The beam size a in the separator should be chosen carefully balancing [φt/a] against the aberration [k(f) a] to optimize the flux at a desired separation factor. A very simple calculation shows that the largest beam size does not always provide the optimum flux.

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*Because of horizontal chromatic aberrations and, for low focal length, nonlinear dispersion effects at the focus on the sextupole, the sextupole is unable to provide the ideal correction of vertical chromatic aberrations.

Using the definition of the separation factor η, formula (1), the vertical angular divergence φ is, implicitly, given by:

$$\varphi t = \{(\frac{\alpha_{k\pi}}{\eta}a)^2 - (k(f)a^2)^2\}^{\frac{1}{2}} \text{ (with } f = \frac{a}{\varphi}) \tag{3}$$

φ is limited to 4 mrad for a 4" gap and 2 mrad for a 2" gap, in Beam 3.

In the absence of aberrations ($k(f)=0$) it is indeed best to use the largest possible gap and beam size, even if the maximum electric field scales as the inverse square root of the gap. This is established doctrine[2].

In the presence of aberrations, one can conveniently optimize the flux if $k(f)$ is a constant, k. This is the case for larger f (high energies) and the optimum beam size a_o, for a certain choice of field, is then obtained by differentiating (3):

$$a_o = \frac{\alpha_{k\pi}}{\sqrt{2\, k\, \eta}} \qquad (k(f) = k, \text{ constant}) \tag{4}$$

independent of the focal length f. In Beam 3, a_o is smaller than the present 4" gap for the highest energies and larger than this gap at the lower energies. The flux is "separator limited" at high energies, "gap limited" at lower energies.

In the presence of the f^{-3} term of the aberration, formula (4) gives only a first approximation to the optimum. Suffice it to assert here that, for Beam 3, the influence of this term on the optimum flux becomes important only at lower energies where a_o is much larger than 4". The flux is gap limited in these cases.

The optimum flux for a desired separation factor η is calculated from formula (3), using a = gap, when the beam is gap limited, and $a = a_o$ when the beam is separator limited.

Before proceeding to detailed flux calculations, it is well to anticipate qualitatively what to expect. At "<u>very high energies</u>" the separation angle $\alpha_{k\pi}$ is so small that, aside from a need to choose a small basic image size <u>i</u> (large f), <u>there is a great stake in minimizing the aberrations</u>. This can be done by choosing a focal length <u>f</u> so as to decrease the beam size <u>a</u>. An optimum is reached when the angular spread due to target width, $[\varphi t/a]$ (see formula (1)), equals the aberration term [ka]. This gives the optimum beam size a_o ($a_o \leq$ gap) computed above, at high energies. The flux may be called "separator limited" and it scales as

$$\varphi_o \sim \frac{1}{k} \frac{(EL)^2}{\eta^2} \quad \text{(at high energies)}. \qquad (5)$$

Thus a decisive advantage is gained by narrowing the gap to increase the field as long as $a_o \lesssim$ gap. An increase of the field by $\sqrt{2}$ achieved by narrowing the gap may gain a factor of 2 in flux at high energies.

At lower energies, on the other hand, $\alpha_{k\pi}$ is so large that the optimum beam size a_o exceeds all reasonable gap sizes. It is best to fill the gap completely. If the aberration [k(f) a] remains relatively small, and if the maximum electric field scales as the inverse square root of the gap, the old fashioned argument becomes more true: the flux, at a certain separation factor, scales as

$$\varphi \sim \sqrt{gap} \quad \text{(at low energies)} \qquad (6)a$$

subject to the limitations of other apertures in the beam and the detailed behavior of k(f). Whatever the aberrations, if one stays within the limitations of the optics for the larger gap (f > 1000") the flux can scale at best as

$$\varphi \sim gap \quad \text{(at low energies)} \quad . \qquad (6)b$$

The extent to which the above extremes represent realistic operating conditions for any beam, depends on the size of the aberrations, and the desired flux and separation factor. For Beam 3, the high energy extreme (5) is reached at reasonable flux and purity levels. At low energies, due to the details of the behavior of the aberration k(f) the flux lies somewhere between the extremes 6(a) and 6(b).

If one emphasises beam operation at higher energies one will favor a smaller gap with its higher electric field. If, on the other hand, one emphasises lower beam energies one should favor a larger gap.

(E) Options for Beam 3

Fluxes for various separator configurations can now be compared using a reasonable value of the separation factor η. One must not strain the quantitative precision of the flux predictions. Uncertain quantities enter into this calculation, one does not know precisely how to compute the image size and there is some operational latitude. One must stress, instead, the qualitative aspects of the comparison on the basis of a common figure of merit noting that the calculations are normalized to the operational results for the presently installed separator (see comments below).

It is evident from operating experience that the images for K's and π's must be separated by at least 2.5 image widths (full width at half maximum) and η=2.5 is chosen, accordingly, for the comparison. The following quantities and assumptions enter into the calculations:

Target width:	t=40 mils (as at present)
Electric field:	E=50 kV/cm for a 4" gap
	E=70 kV/cm for a 2" gap
Beam layout:	Unmodified
Optics:	Unmodified (1000" $\leq$f<10000")
Separator Length:	L = 15m (triplets), L=10m (doublets), each stage.

K^- fluxes, and focal lengths f, are displayed in Fig. 1.

Assume now that one has 3.3 x 10^{11} protons per pulse on the target, and a 1.5% momentum bite, and requires an average of 12K^- tracks per picture, or more, with a separation factor of η=2.5 or better in each stage. Then the following K^- momenta can be provided by the separator configurations considered (see horizontal line in Fig. 1):

(a) L=15m, 4" gap, E=50 kV/cm:	2.9 - 5.5 GeV/c K^- (at present)
(b) L=10m, 4" gap, E=50 kV/cm:	3.0 - 4.7 GeV/c K^-
(c) L=10m, 2" gap, E=70 kV/cm:	3.3 - 5.4 GeV/c K^-

The computations bear out the qualitative conclusions above:

The present configuration (a) is superior to the other two. If separators have to be economized (options (b) and (c)) a 2" gap provides a wider and higher momentum range. Economizing separators costs flux.

(F) Comments

Beam 3 now has problems which make it difficult to run K^- beams at the limits of the momentum bands indicated. Ray traces do not properly predict the amount of the aberrations, yet their size is rather important to the arguments. It is well, therefore, to stress again the qualitative aspects of the comparison and to point out its strengths and weaknesses.

The calculations match the present situation quite well. The empirical adjustments necessary to achieve this match are basically sound. One of these, a scale factor on the target yield and the system transmission, is well understood [Section (A)]. The other is a scale factor, applied to the chromatic aberrations predicted by the ray traces, to match presently achievable image sizes. The functional form of the aberration k(f) is not surprising: For focal lengths f long compared to the lens spacings, all aberrations approach a constant, characteristic of the lens arrangement, since the typical trajectories hardly change with large f. The f^{-3} term (whose size is, unfortunately, somewhat uncertain) is a result of the relatively violent ray encursions which occur when one wishes to achieve short focal lengths in a complex lens system with relatively large lens spacings. The chromatic aberration predicted by the ray trace programs is merely one example of the generally reasonable behavior of k(f), and applying some empirical scale factor to allow for other aberrations is sufficiently accurate for the purpose of our arguments.

The arguments in favor of a 2" gap become stronger if the aberrations are larger than indicated, if larger separation factors are required, or if higher order aberrations, $O(\varphi^2)$, are significant.

An important formal objection to the above optimization, is that it was done for the maximum possible momentum bite whereas it is really possible to improve the aberrations by choosing a smaller bite. Smaller aberrations would imply a larger optimum beam size a_o. A calculation to optimize the flux (φ times momentum bite) for a variable beam size and a variable bite, however, is too speculative to be of any value. In any event, the larger electric field achievable with a 2" gap remains a decisive advantage at the highest energies.

One may also object that the definition of the separation factor should have emphasized the behavior of image tails (rather than image widths of half maximum) i.e., one should have used linear addition of ideal image widths and aberrations (rather than addition in quadrature). Such a calculation, unfortunately, is hard to verify experimentally since a measurement of widths of images at their base is very inaccurate. At any rate, the calculation has been made and its predictions agree in all important details with the results presented here. The argument in favor of a smaller gap for high energy operation is strengthened.

A decision in favor of a 2" gap for the shortened separator provides the best insurance for success of the beam at high energies. It need be reviewed only if ways are found to reduce the aberration by more than a factor of 2, at the present beam width of a=4". It must be emphasized, however, that this decision depends critically on the postulated improvement in electric field. For most separators the maximum field scales as the inverse square root of the gap (or more favorably), but this has not been verified for the Beam 3 separator. Narrowing the gap from 4" to 2" without being able to improve the maximum field will cause a major embarassment.

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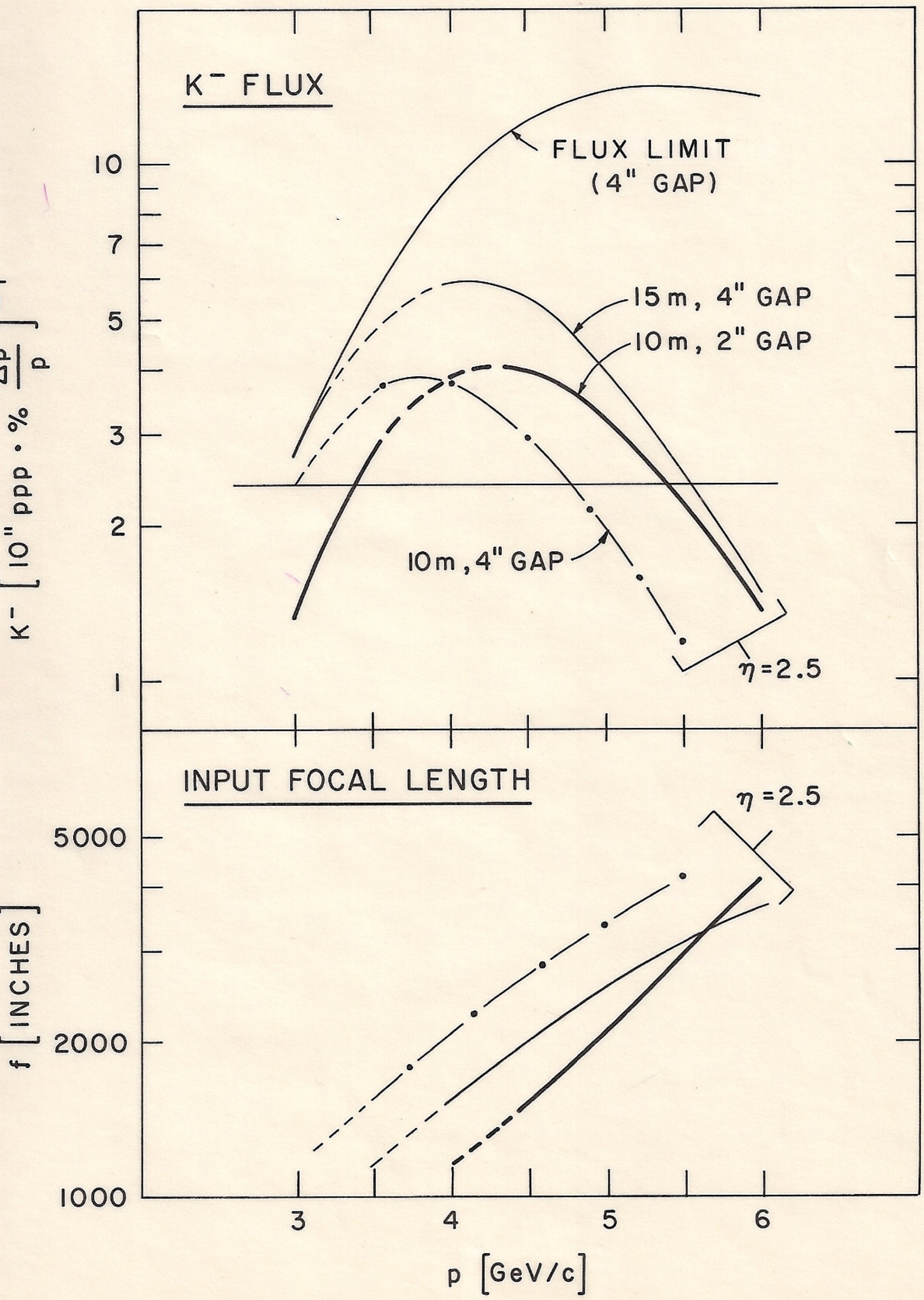
K⁻ FLUX
FLUX LIMIT
(4" GAP)
15 m, 4" GAP
10 m, 2" GAP
10 m, 4" GAP
η = 2.5
10
7
5
3
2
1
K⁻ [10¹¹ ppp · % Δp/p]⁻¹
INPUT FOCAL LENGTH
η = 2.5
5000
2000
1000
f [INCHES]
3
4
5
6
p [GeV/c]