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# BNL MAD Program Notes: Field Tracking Results in AGS Magnets

J. Niederer

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Collider Accelerator Department

**Brookhaven National Laboratory**

**U.S. Department of Energy**

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Accelerator Division
Alternating Gradient Synchrotron Department
BROOKHAVEN NATIONAL LABORATORY
Upton, New York 11973


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Technical Note

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## Field Tracking Results in AGS Magnets

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### 1. Summary

Some results of computing charged particle trajectories in a magnetic field described by a vertical field map at a central plane are presented. This is early experience with a tracking section added to the BNL MAD Program, using an AGS 90 inch magnet mapped at a field corresponding to about 2 gev/c protons. Results are given from several schemes for interpolating field values from the grid of measured field values at steps along a trajectory.

### 2. Introduction

The field tracking routine computes a reference orbit that is presumed to be symmetric in the beam axis z direction with respect to a mirror plane perpendicular to the orbit which splits the magnet into an entrance half and an exit half. The routine tries to find that orbit by adjusting an entrance offset X0 such that the entrance and exit angles are identical, and equal to half of a prescribed total bending angle. If the field is expressed as a set of measurements over the first half of the magnet, which are then reflected to obtain the field in the second half, one measure of the fit is that the computed exit offset Xf should be the same as the entrance offset X0. Any difference presumably means that the calculation has some inherent asymmetry, basically an error, of the same magnitude to be expected in tracking calculations. When the same field measurements are used for both parts of the trajectory, such residuals in displacements are computational issues that are largely independent of the accuracy of the field data.

### 3. Field Interpolation Methods

A feature of interest is the relative accuracy of various ways of interpolating field points along the trajectory from among the much coarser grid of measured points. Generally an integration step along the path of the order of 10% of the measurement grid size is a reasonable compromise between computing speed and accuracy. At some level smaller step sizes accumulate too much round off error, and larger steps also begin to give results which change with step size. In the case of AGS magnets, an integration step of .1 inch matches the predominant spacing in z measurements of 1 inch. At the magnet ends, where the field changes rapidly, the grid spacing is 1/4 inch in z. For these strong gradient magnets, the measured horizontal grid spacing (X) is .1 inch. The dominant source of error would appear to be the precision with which the coordinates of the grid positions was maintained during the measurements.

During the code development, several methods of interpolation were introduced as options to help in evaluating results. Following a predecessor method, we began with a linear interpolation of the field B(x,y) at each step coordinate (x,y) from a rectangle formed by the four nearest field points. This computation is fast and simple. A field value is interpolated linearly in x between each of the two pairs of points having the same z: B(x,z1) and B(x,z2). The desired value B(x,z) is then interpolated in z along the line joining B(x,z1) and B(x,z2). Results for the strong gradient AGS magnet were rather sensitive to step size, which prompted a closer look at more capable interpolation schemes.

This simple four point scheme was followed by one that performed a quadratic interpolation in the same spirit using nine field points also arranged in a rectangular grid. This second grid included the four points of the first one, plus an adjacent row of x measurements, and an adjacent row of z measurements. The nine point grid places the particle in the -dx, -dz quadrant of a four quadrant grid formed of nine nearby field values, in effect sampling the field ahead of the step coordinates. Field values of B(x,z1), B(x,z2), and B(x,z3) are found from each of the three trios of points at a same z. B(x,z) is interpolated quadratically from the line joined by formed the first three values in z. This enhancement is still fast, uses all nine points, and yields an expression:

B(x,z) = B0 + C1*x + C2*z + C3*x^2 + C4*x*z + C5*z^2 + C7*x^2*z + C8*x*z^2 + C12*x^2*z^2

However, its results differ from the four point scheme in the third significant figure, and it is perhaps a little suspect as to whether it gets the most reliable answer from among the nine grid points.

Accordingly, a set of more complicated weighted least squares fits were introduced in hopes of obtaining some convergence of results among the methods. While substantially slower, and occasionally quirky, least squares methods are a kind of familiar standard, and allow both weighting and exclusion of individual values within each local grid. Fits to x,z polynomials in orders of 1 through 5 are available.

N = 1: B1 = B0 + C1*x + C2*z

N = 2: B2 = B1 + C3*x^2 + C4*x*z + C5*z^2

N = 3: B3 = B2 + O(x^3, z^3, etc)

N = 4: B4 = B3 + O(x^4, z^4, etc)

N = 5: B5 = B4 + O(x^5, z^5, etc)

In practice, the fitted AGS results are not strongly dependent upon weighting, most likely because the field data are consistent along the various grid traversals. Results here are given for equally weighted field points.

**4. Symmetry in Field Calculations.**

The particular AGS field measurements considered here cover the region from well outside the fringe field to the center of the magnet along the beam axis. For these half maps to be used for tracking, the program must either track through the given field and its mirror image, or the field data must be massaged to represent the full magnet. Without laboring the technical points here, there are symmetry considerations which lead to different reference orbit results between full and half map integrations if they are not taken into account in the programs. In odd order local field square grids, such as the four point(N = 1) and sixteen point (N = 3), the stepped z coordinate can be chosen to lie in a center cell(pivot) of the surrounding grid, so there are equal numbers of rows of points on all sides. So for odd ordered fit computations, the inherent symmetry should give the same answer for the full and half magnet data sets, within generally very small round off tolerances. In even order grids, more rows of values lie on one side of the pivot cell than on the other, causing different grid points to be used in the second half by the full and the half magnet integrations. For even ordered fits, the program must intervene to obtain equivalent computations, with or without symmetry, as directed.

A third measure consists of repeating these reference orbit calibrations with smoothed input field data. (Smoothing performed by R. Thern)

**5. Integration**

The field integration is a modified predictor corrector method that has the z step size for its only external parameter. Ideally results should be stable over a range of step sizes, so the z step parameter can be chosen to optimize the computing rate within some safety margin. In practice, the range of step sizes that give consistent, symmetric fits may be rather limited, and should be established through testing. A step size that is an exact fraction of the measured grid, such as .1 inch, can give fits that seem to have a deceptive precision, such as found in Tables 1 and 2. Also, the proper step size may differ somewhat among interpolation methods. More realistic results are found in Table 5, which compares step size effects for two of the interpolation schemes.

The integration method appears to be very satisfactory. It gives a properly symmetric reference orbit at the magnet center, with all odd field derivatives effectively zero to ten or more decimal places, for data and parameters which are supposed to give a properly symmetric orbit. At each step the field is computed once for the prediction step and once for the corrector step. These two values seldom differ within the first ten decimal places. At one time, the second calculation was inadvertently omitted without noticeably changing the result.

## 6. Results

Tables showing test results are attached at the end of this note. They result from applying the tracking calculation to data taken from AGS magnet B74 that extend to 45 inches into the magnet, treating it as a 90 inch magnet, with a prescribed bend angle. These tables present exit / entrance offset data compared among the interpolation methods, and sets of tracking results for initial offsets of the order of a millimeter, and of a centimeter. Tables 1 through 4 are computed with a .1 inch integration step, which happens to lead to displacement agreements which are much better than the precision warranted by the data and parameters. Quantities which are supposed to be the same among the methods, such as the Bdl integral, do appear to cluster properly.

The full and half map schemes agree reasonably well among the nine point interpolation cases, with the full map case showing a slight asymmetry as expected. The four and nine point interpolation results differ appreciably for all four cases, falling into two distinct groups. The program interpolation schemes give quite similar results with both the original and the smoothed field maps. The faster four and nine point interpolation schemes give practically identical results to the presumably more rigorous least squares calculations. The cubic fits of the least squares method, hopefully an even better use of the grid data, give a slightly different result than the symmetric quadratic schemes. The third, fourth and fifth order least squares fits give about the same results, differing among themselves by a only few parts per million.

The consistency among these results suggests that it is worth paying attention to maintaining inherent symmetry in the field data interpolations. The tracking results of Tables 3 and 4 give an idea of the spread among the least squares grids explored. Differences appear at the 10^-7 meter level. Thus it appears marginally worthwhile going to the higher order interpolations.

The methods seem to be far too responsive to decreasing grid size, for which a likely cause is that a different fit is applied as a particle crosses a measurement cell boundary. The smaller the step, the larger the effect is if the locally fitted field polynomials do not join entirely smoothly at the cell boundaries. There is a certain artificial dependence on step size in the magnet edge regions, where the measured grid is somewhat finer than elsewhere. This kind of problem is affecting tracking results by a part or so in ten thousand, which may be at the edge of reasonable accuracy for this class of technique. In these calculations, one often has to look at the seventh or higher digits for signs of mistakes, in one case to sixteenth digits, so the checkout is rather demanding. The debugging traces print the contributions of each polynomial term to the locally computed field value, and none of these show grossly irregular behavior. One remaining sign of a problem is a single fifth order full magnet data case for which one grid point out of 75 fails to match its mirror calculation. Ostensibly due to roundoff, its grid points differ in the fourteenth digit, which somehow propagates in the fitting process to produce a difference in the fourth digit of the interpolated field. Similarly slight differences in other cells do not cause this kind of mischief.

## 7. Timing

Relative rates for one pass through one magnet for the various methods are:

| | |
|---|---|
| Simple 4 Point, Linear: | 16.4 / second |
| Simple 9 Point, Quadratic: | 16.0 |
| Least Square 4 Point: | 13.9 |

| | |
|---|---|
| Least Square 9 Point: | 10.6 |
| Least Square 16 Point: | 8.4 |
| Least Square 25 Point: | 2.6 |
| Least Square 36 Point: | 1.3 |

These numbers apply to a pass with about 1200 integration steps ( .1 inch), and about 150 grid changes involving a new field polynomial. They were obtained on an Iris 4 with debugging level code.

**8. Remarks**

These notes present a set of tests for matching the tracking calculations to a particular class of magnet. Similar tests should be carried out for any other magnets to be tracked this way in a lattice, to tune the parameters and to look for surprises. Most of the result data was obtained by using the detailed debugging trace features of the field map tracking codes, described in the *Fields* manual pages.

**Documentation Files**

Manual sources are in Unix troff format.
**Host:** **rapt.ags.bnl.gov**

This Report: /usr/disc2/jn/Docum+/Fields.res

Fields Manual: /usr/disc2/jn/Docum+/Fields.man

**Table 1. Reference Orbit Centering Results. AGS 90 Inch Magnet**

Half Map, Four Points
Xf = X0 = .007015783392

Full Map, Four Points
Xf = X0 = .007015783392

Smoothed Half Map, Four Points
Xf = X0 = .007016424934

Half Map, Nine Points, Symmetric
Xf = X0 = .007038264016

Full Map, Nine Points, Not Symmetric
Xf = .007035335173
X0 = .007035738082

Smoothed Half Map, Nine Points, Symmetric
Xf = X0 = .007038974688

**Table 2. Fits To Reference Orbit. AGS 90 Inch Magnet**

**Least Squares Interpolation Method, By Order N = 1, 5**

| N | Xf Meters | X0 Meters | PXf = PX0 Radians | Path = B * dl Ts - Meter |
|---|---|---|---|---|
| Measured Field Data | | | | |
| 1. | -.007015880723 | -.007015880723 | -0.01398251535 | 2.704886658 |
| 2. | -.007038238362 | -.007038238362 | -0.01398251535 | 2.704886566 |
| 3. | -.007035732196 | -.007035732196 | -0.01398251535 | 2.704886588 |
| 4. | -.007035626468 | -.007035626468 | -0.01398251535 | 2.704886594 |
| 5. | -.007035711802 | -.007035711802 | -0.01398251535 | 2.704886592 |
| Smoothed Field Data | | | | |
| 1. | -.007016503752 | -.007016503752 | -0.01398251535 | 2.704886658 |
| 2. | -.007039001159 | -.007039001158 | -0.01398251535 | 2.704886566 |
| 3. | -.007039001159 | -.007039001158 | -0.01398251535 | 2.704886566 |
| 4. | -.007036329633 | -.007036329633 | -0.01398251535 | 2.704886594 |
| 5. | -.007036398015 | -.007036398015 | -0.01398251535 | 2.704886592 |

**Table 3. Tracking Results for AGS 90 Inch Magnet.**

**Compare Least Square Method Interpolations by Order N**

| | X | PX | Y | PY | DS |
|---|---|---|---|---|---|
| INITIAL PARTICLE POSITIONS | | | | | |
| 1 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 2 | 0.00141566 | -0.00000233 | 0.00084929 | 0.00002793 | 0.00000000 |
| 3 | 0.00005770 | 0.00005704 | -0.00027523 | 0.00008619 | 0.00000000 |
| 4 | -0.00141566 | 0.00000233 | -0.00084929 | -0.00002793 | 0.00000000 |
| 5 | -0.00005770 | -0.00005704 | 0.00027523 | -0.00008619 | 0.00000000 |
| FINAL PARTICLE 1 POSITIONS | | | | | |
| By Order: | | | | | |
| 1 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 2 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 3 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 4 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 5 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| FINAL PARTICLE 2 POSITIONS | | | | | |
| 1 | 0.00173851 | 0.00016944 | 0.00075681 | -0.00007226 | -0.00000162 |
| 2 | 0.00173861 | 0.00016948 | 0.00075690 | -0.00007222 | -0.00000162 |
| 3 | 0.00173852 | 0.00016944 | 0.00075692 | -0.00007221 | -0.00000162 |
| 4 | 0.00173859 | 0.00016947 | 0.00075691 | -0.00007221 | -0.00000162 |
| 5 | 0.00173852 | 0.00016944 | 0.00075692 | -0.00007221 | -0.00000162 |
| FINAL PARTICLE 3 POSITIONS | | | | | |
| 1 | 0.00030649 | 0.00007637 | 0.00007939 | 0.00009928 | -0.00000017 |
| 2 | 0.00030653 | 0.00007639 | 0.00007938 | 0.00009927 | -0.00000017 |
| 3 | 0.00030651 | 0.00007639 | 0.00007936 | 0.00009927 | -0.00000017 |
| 4 | 0.00030652 | 0.00007639 | 0.00007937 | 0.00009927 | -0.00000017 |
| 5 | 0.00030651 | 0.00007638 | 0.00007936 | 0.00009927 | -0.00000017 |
| FINAL PARTICLE 4 POSITIONS | | | | | |
| 1 | -0.00173813 | -0.00016923 | -0.00075721 | 0.00007206 | 0.00000165 |
| 2 | -0.00173809 | -0.00016920 | -0.00075736 | 0.00007198 | 0.00000165 |
| 3 | -0.00173805 | -0.00016919 | -0.00075732 | 0.00007200 | 0.00000165 |
| 4 | -0.00173806 | -0.00016919 | -0.00075735 | 0.00007199 | 0.00000165 |
| 5 | -0.00173806 | -0.00016919 | -0.00075732 | 0.00007200 | 0.00000165 |
| FINAL PARTICLE 5 POSITIONS | | | | | |
| 1 | -0.00030649 | -0.00007637 | -0.00007938 | -0.00009928 | 0.00000020 |
| 2 | -0.00030652 | -0.00007639 | -0.00007937 | -0.00009927 | 0.00000020 |
| 3 | -0.00030652 | -0.00007638 | -0.00007936 | -0.00009927 | 0.00000020 |
| 4 | -0.00030652 | -0.00007639 | -0.00007936 | -0.00009927 | 0.00000020 |
| 5 | -0.00030652 | -0.00007639 | -0.00007936 | -0.00009927 | 0.00000020 |

**Table 4. More Tracking Results for AGS 90" Magnet.**

**Compare Least Square Method Interpolations by Order N**

| | X | PX | Y | PY | DS |
|---|---|---|---|---|---|
| INITIAL PARTICLE POSITIONS | | | | | |
| Particle: | | | | | |
| 2 | 0.00447670 | -0.00000735 | 0.00465176 | 0.00015298 | 0.00000000 |
| 3 | 0.00018247 | 0.00018038 | -0.00150749 | 0.00047207 | 0.00000000 |
| 4 | -0.00447670 | 0.00000735 | -0.00465176 | -0.00015298 | 0.00000000 |
| 5 | -0.00018247 | -0.00018038 | 0.00150749 | -0.00047207 | 0.00000000 |
| FINAL PARTICLE 2 POSITIONS | | | | | |
| By Order: | | | | | |
| 1 | 0.00549813 | 0.00053602 | 0.00414530 | -0.00039579 | -0.00000506 |
| 2 | 0.00549804 | 0.00053598 | 0.00414583 | -0.00039556 | -0.00000506 |
| 3 | 0.00549811 | 0.00053601 | 0.00414599 | -0.00039546 | -0.00000506 |
| 4 | 0.00549808 | 0.00053600 | 0.00414595 | -0.00039549 | -0.00000506 |
| 5 | 0.00549811 | 0.00053601 | 0.00414604 | -0.00039544 | -0.00000506 |
| FINAL PARTICLE 3 POSITIONS | | | | | |
| 1 | 0.00096923 | 0.00024153 | 0.00043488 | 0.00054378 | -0.00000026 |
| 2 | 0.00096931 | 0.00024158 | 0.00043484 | 0.00054377 | -0.00000026 |
| 3 | 0.00096928 | 0.00024156 | 0.00043475 | 0.00054375 | -0.00000026 |
| 4 | 0.00096929 | 0.00024157 | 0.00043478 | 0.00054376 | -0.00000026 |
| 5 | 0.00096927 | 0.00024155 | 0.00043474 | 0.00054375 | -0.00000026 |
| FINAL PARTICLE 4 POSITIONS | | | | | |
| 1 | -0.00549619 | -0.00053499 | -0.00414815 | 0.00039421 | 0.00000529 |
| 2 | -0.00549610 | -0.00053496 | -0.00414845 | 0.00039405 | 0.00000529 |
| 3 | -0.00549610 | -0.00053496 | -0.00414882 | 0.00039389 | 0.00000529 |
| 4 | -0.00549610 | -0.00053496 | -0.00414862 | 0.00039399 | 0.00000529 |
| 5 | -0.00549608 | -0.00053495 | -0.00414881 | 0.00039391 | 0.00000529 |
| FINAL PARTICLE 5 POSITIONS | | | | | |
| 1 | -0.00096923 | -0.00024152 | -0.00043472 | -0.00054374 | 0.00000093 |
| 2 | -0.00096933 | -0.00024157 | -0.00043465 | -0.00054372 | 0.00000093 |
| 3 | -0.00096929 | -0.00024154 | -0.00043460 | -0.00054370 | 0.00000093 |
| 4 | -0.00096931 | -0.00024155 | -0.00043463 | -0.00054371 | 0.00000093 |
| 5 | -0.00096930 | -0.00024155 | -0.00043460 | -0.00054371 | 0.00000093 |

**Table 5. Fits As Function of Integration Z Step for AGS 90 Inch Magnet**

| N | Xf Meters | X0 Meters | PXf = PX0 Radians | Path = B * dl Ts - Meter |
|---|---|---|---|---|
| N = 2 Simple Fits | | | | |
| .010 | -.007038343840 | -.007038314527 | -.01398251535 | 2.704886338 |
| .025 | -.007038386617 | -.007038313314 | -.01398251535 | 2.704886348 |
| .050 | -.007038310189 | -.007038310189 | -.01398251535 | 2.704886398 |
| .010 | -.007038264016 | -.007038264016 | -.01398251535 | 2.704886588 |
| .025 | -.007041495525 | -.007038168521 | -.01398251535 | 2.704887147 |
| .050 | -.007037122974 | -.007037130357 | -.01398251535 | 2.704895059 |
| N = 3 Least Square Fits | | | | |
| .010 | -.007035764769 | -.007035735455 | -.01398251535 | 2.704886338 |
| .025 | -.007035808741 | -.007035735433 | -.01398251535 | 2.704886348 |
| .050 | -.007035739583 | -.007035739583 | -.01398251535 | 2.704886398 |
| .100 | -.007035732196 | -.007035732196 | -.01398251535 | 2.704886588 |
| .250 | -.007039062760 | -.007035735905 | -.01398251535 | 2.704887147 |
| .500 | -.007035722429 | -.007035723569 | -.01398251535 | 2.704895059 |

**Table 6. Tracking Results for AGS 90 Inch Magnet.**

**Compare N = 2 Simple Fit As Function of Integration Z Step**

| | X | PX | Y | PY | DS | |
|---|---|---|---|---|---|---|
| INITIAL PARTICLE POSITIONS. | | | | | | |
| 1 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 2 | 0.00447670 | -0.00000735 | 0.00465176 | 0.00015298 | 0.00000000 | 0.00000000 |
| 3 | 0.00018247 | 0.00018038 | -0.00150749 | 0.00047207 | 0.00000000 | 0.00000000 |
| 4 | -0.00447670 | 0.00000735 | -0.00465176 | -0.00015298 | 0.00000000 | 0.00000000 |
| 5 | -0.00018247 | -0.00018038 | 0.00150749 | -0.00047207 | 0.00000000 | 0.00000000 |
| FINAL PARTICLE 1 POSITIONS. | | | | | | |
| Step: | | | | | | |
| .010 | -0.00000003 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .025 | -0.00000007 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .050 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .100 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .250 | -0.00000333 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .500 | 0.00000001 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| FINAL PARTICLE 2 POSITIONS. | | | | | | |
| .010 | 0.00549802 | 0.00053598 | 0.00414589 | -0.00039554 | -0.00000506 | 0.00000000 |
| .025 | 0.00549798 | 0.00053598 | 0.00414588 | -0.00039554 | -0.00000506 | 0.00000000 |
| .050 | 0.00549805 | 0.00053598 | 0.00414589 | -0.00039554 | -0.00000506 | 0.00000000 |
| .100 | 0.00549806 | 0.00053599 | 0.00414590 | -0.00039553 | -0.00000506 | 0.00000000 |
| .250 | 0.00549485 | 0.00053598 | 0.00414579 | -0.00039554 | -0.00000506 | 0.00000000 |
| .500 | 0.00549830 | 0.00053610 | 0.00414605 | -0.00039546 | -0.00000506 | 0.00000000 |
| FINAL PARTICLE 3 POSITIONS. | | | | | | |
| .010 | 0.00096928 | 0.00024158 | 0.00043483 | 0.00054377 | -0.00000025 | 0.00000000 |
| .025 | 0.00096924 | 0.00024158 | 0.00043483 | 0.00054377 | -0.00000025 | 0.00000000 |
| .050 | 0.00096931 | 0.00024158 | 0.00043482 | 0.00054377 | -0.00000026 | 0.00000000 |
| .100 | 0.00096932 | 0.00024159 | 0.00043483 | 0.00054377 | -0.00000026 | 0.00000000 |
| .250 | 0.00096604 | 0.00024158 | 0.00043495 | 0.00054377 | -0.00000028 | 0.00000000 |
| .500 | 0.00096938 | 0.00024162 | 0.00043495 | 0.00054381 | -0.00000031 | 0.00000000 |
| FINAL PARTICLE 4 POSITIONS. | | | | | | |
| .010 | -0.00549613 | -0.00053497 | -0.00414845 | 0.00039407 | 0.00000529 | 0.00000000 |
| .025 | -0.00549617 | -0.00053497 | -0.00414845 | 0.00039407 | 0.00000529 | 0.00000000 |
| .050 | -0.00549610 | -0.00053497 | -0.00414844 | 0.00039408 | 0.00000529 | 0.00000000 |
| .100 | -0.00549610 | -0.00053497 | -0.00414845 | 0.00039407 | 0.00000529 | 0.00000000 |
| .250 | -0.00549955 | -0.00053497 | -0.00414837 | 0.00039406 | 0.00000529 | 0.00000000 |
| .500 | -0.00549608 | -0.00053497 | -0.00414810 | 0.00039425 | 0.00000528 | 0.00000000 |
| FINAL PARTICLE 5 POSITIONS. | | | | | | |
| .010 | -0.00096935 | -0.00024156 | -0.00043467 | -0.00054372 | 0.00000092 | 0.00000000 |
| .025 | -0.00096940 | -0.00024156 | -0.00043467 | -0.00054372 | 0.00000092 | 0.00000000 |
| .050 | -0.00096932 | -0.00024156 | -0.00043467 | -0.00054372 | 0.00000092 | 0.00000000 |
| .010 | -0.00096932 | -0.00024156 | -0.00043467 | -0.00054372 | 0.00000093 | 0.00000000 |
| .250 | -0.00097271 | -0.00024156 | -0.00043480 | -0.00054372 | 0.00000095 | 0.00000000 |
| .500 | -0.00096932 | -0.00024156 | -0.00043465 | -0.00054372 | 0.00000098 | 0.00000000 |

**Table 7. Tracking Results for AGS 90 Inch Magnet**

**Compare N = 3 Least Square Fit As Function of Integration Z Step**

| | X | PX | Y | PY | DS |
|---|---|---|---|---|---|
| INITIAL PARTICLE POSITIONS. | | | | | |
| 1 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 2 | 0.00447670 | -0.00000735 | 0.00465176 | 0.00015298 | 0.00000000 |
| 3 | 0.00018247 | 0.00018038 | -0.00150749 | 0.00047207 | 0.00000000 |
| 4 | -0.00447670 | 0.00000735 | -0.00465176 | -0.00015298 | 0.00000000 |
| 5 | -0.00018247 | -0.00018038 | 0.00150749 | -0.00047207 | 0.00000000 |
| | | | | | |
| FINAL PARTICLE 1 POSITIONS. | | | | | |
| Step: | | | | | |
| .010 | -0.00000003 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .025 | -0.00000007 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .050 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .100 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .250 | -0.00000333 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| .500 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| FINAL PARTICLE 2 POSITIONS. | | | | | |
| .010 | 0.00549808 | 0.00053601 | 0.00414599 | -0.00039547 | -0.00000506 |
| .025 | 0.00549804 | 0.00053601 | 0.00414598 | -0.00039547 | -0.00000506 |
| .050 | 0.00549811 | 0.00053601 | 0.00414599 | -0.00039547 | -0.00000506 |
| .100 | 0.00549811 | 0.00053601 | 0.00414599 | -0.00039546 | -0.00000506 |
| .250 | 0.00549491 | 0.00053601 | 0.00414590 | -0.00039546 | -0.00000506 |
| .500 | 0.00549810 | 0.00053601 | 0.00414598 | -0.00039546 | -0.00000506 |
| FINAL PARTICLE 3 POSITIONS. | | | | | |
| .010 | 0.00096925 | 0.00024156 | 0.00043475 | 0.00054375 | -0.00000025 |
| .025 | 0.00096921 | 0.00024156 | 0.00043475 | 0.00054375 | -0.00000025 |
| .050 | 0.00096928 | 0.00024156 | 0.00043475 | 0.00054375 | -0.00000026 |
| .100 | 0.00096928 | 0.00024156 | 0.00043475 | 0.00054375 | -0.00000026 |
| .250 | 0.00096599 | 0.00024155 | 0.00043487 | 0.00054375 | -0.00000028 |
| .500 | 0.00096928 | 0.00024156 | 0.00043475 | 0.00054375 | -0.00000031 |
| FINAL PARTICLE 4 POSITIONS. | | | | | |
| .010 | -0.00549613 | -0.00053496 | -0.00414881 | 0.00039390 | 0.00000529 |
| .025 | -0.00549617 | -0.00053496 | -0.00414881 | 0.00039390 | 0.00000529 |
| .050 | -0.00549610 | -0.00053496 | -0.00414881 | 0.00039389 | 0.00000529 |
| .100 | -0.00549610 | -0.00053496 | -0.00414882 | 0.00039389 | 0.00000529 |
| .250 | -0.00549956 | -0.00053496 | -0.00414875 | 0.00039388 | 0.00000529 |
| .500 | -0.00549617 | -0.00053500 | -0.00414888 | 0.00039386 | 0.00000528 |
| FINAL PARTICLE 5 POSITIONS. | | | | | |
| .010 | -0.00096932 | -0.00024154 | -0.00043460 | -0.00054370 | 0.00000092 |
| .025 | -0.00096937 | -0.00024154 | -0.00043460 | -0.00054370 | 0.00000092 |
| .050 | -0.00096929 | -0.00024154 | -0.00043460 | -0.00054370 | 0.00000092 |
| .100 | -0.00096929 | -0.00024154 | -0.00043460 | -0.00054370 | 0.00000093 |
| .250 | -0.00097268 | -0.00024154 | -0.00043473 | -0.00054370 | 0.00000095 |
| .500 | -0.00096932 | -0.00024156 | -0.00043458 | -0.00054370 | 0.00000098 |