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THE BUCKET AND TIPE BUNCH WITH A SECOND RF HARMONIC

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> Accelerator Division Technical Note

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SECOND RF HARMONIC

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THE BUCKET AND THE BUNCH WITH A 2.ND RF HARMONIC A.Luccio

In the presence of a second RF harmonic the voltage acting on a particle is

(1)
$$eV = eV_1 \sin(\omega_{RF}t + \varphi_1) + eV_2 \sin(2\omega_{RF}t + \varphi_2).$$

With the positions

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(2)
$$V_0 \equiv V_1, \quad \chi = \frac{V_2}{V_o}, \quad \phi = \omega_{RF}t + \varphi_1, \quad \Delta \varphi = \varphi_2 - 2\varphi_1,$$

the differential equation for the energy change during the acceleration becomes

(3)
$$\tau \frac{d}{dt} (\Delta E) = eV_o \left\{ \sin \phi - \sin \phi_s + \chi \left[\sin(2\phi + \Delta \phi) - \sin(2\phi_s + \Delta \phi) \right] \right\}$$

with ϕ_s the synchronous phase, to be calculated by solving the transcendental Equation

(4)
$$\frac{\Delta E_s}{eV_0} = \sin \phi_s + \chi \sin(2\phi_s + \Delta \varphi).$$

To find the bucket separatrix -the "bucket"-, let us start from the differential equation for the phase

(5)
$$\frac{d^2\phi}{dt^2} = \frac{2\pi}{\tau^2} \frac{h\eta}{\beta^2} \frac{eV_0}{E_s} f_b,$$

with f_b the function on the rhs of Eq. (3)

(6)
$$f_b = \sin \phi - \sin \phi_s + \chi [\sin(2\phi + \Delta \phi) - \sin(2\phi_s + \Delta \phi)],$$

and with the harmonic number h, the dispersion η , and the relativistic velocity β evaluated for the synchronous particle (we dropped the subscript "s").

An integral of motion is found by integration in the standard way¹

(7)
$$\int \frac{d^2\phi}{dt^2} \frac{d\phi}{dt} dt = \frac{2\pi}{\tau^2} \frac{h\eta}{\beta^2} \frac{eV_0}{E_s} \int f_b \frac{d\phi}{dt} dt.$$

The integration yields the following

(8)
$$\frac{K}{2} \left(\frac{\Delta E}{E_s}\right)^2 + eV_0 \tilde{b}(\phi, \phi_s) = H = const,$$

with

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(9)
$$K = 2\pi \frac{h\eta}{\beta^2} E_s,$$

and

(10)
$$\tilde{b}(\phi,\phi_s) = \cos\phi + \phi \sin\phi_s + \chi \left[\frac{1}{2}\cos(2\phi + \Delta\phi) + \phi \sin(2\phi_s + \Delta\phi)\right].$$

The constant H of Eq.(3) is the Hamiltonian of the system

(11)
$$H = \frac{1}{2}KW^2 + U,$$

and the bucket is found using the two fixed points ϕ_s and π - ϕ_s of the distribution²

(12)
$$H(\Delta E, \phi) = H(0, \pi - \phi_s).$$

The resulting bucket is

(13)
$$b(\phi,\phi_s) = \cos\phi + \phi\sin\phi_s + \cos\phi_s - (\pi - \phi_s)\sin\phi_s + \chi[\frac{1}{2}\cos(2\phi + \Delta\phi) + \phi\sin(2\phi_s + \Delta\phi) - \cos(2\phi_s - \Delta\phi) - (\pi - \phi_s)\sin(2\phi_s + \Delta\phi)]$$

¹ D.A.Edwards and M.J.Syphers An Introduction to the Physics of High Energy Accelerators Wiley, NY 1993, p.38

² J.M.Kats Synchronous Particle and Bucket Dynamics AGS/AD/89-1, BNL-52171, October 3, 1988

To accommodate the bunch in the bucket, we want to find the two extreme points of the separatrix, ϕ_1 and ϕ_2 ($\phi_2 < \phi_1$). With some manipulation of Eq. (13), a convenient form of the transcendental Equation to be solved to find these points, using as a parameter - the bunch length $\Delta \phi$ can be written as

(14)
$$\sin \psi = A - \frac{\chi}{4\sin(\frac{1}{2}\Delta\phi)}\sin(2\psi + \Delta\phi),$$

with the positions

结

(15)
$$\phi_1 = \psi + \frac{\Delta \phi}{2}, \quad \phi_2 = \phi_1 - \Delta \phi, \quad A = \frac{\frac{1}{2}\Delta \phi}{\sin(\frac{1}{2}\Delta \phi)} \left[\sin\phi_s + \chi \frac{1}{2}\Delta\phi \sin(2\phi_s + \Delta\phi)\right]$$

Eq.(4) for the synchronous phase and Eq.(14) for the bucket extreme points can be solved numerically. A very efficient and fast routine for this task is the Newton-Raphson iteration, based on the simple algorithm

(16)
$$x_{n+1} = x_n - \frac{f}{f'}$$
,

that solves the Equation $f \approx 0$ -f' is the derivative of f- A convenient starting point for the NR iteration, for Eq. (4) is the synchronous phase with no second harmonic

(17)
$$\phi_s^{(1)} = \arcsin \frac{\Delta E_s}{eV_0}$$

(actually a point slightly beyond ϕ_s). A starting point for solving Eq. (14) is

(18)
$$\phi_1^{(1)} = \frac{1}{2}\Delta\phi + \arg\left(\frac{\frac{1}{2}\Delta\phi}{\sin\frac{1}{2}\Delta\phi}\sin\phi_s\right).$$

Indeed, if $\chi = 0$, the exact solutions of Eqs. (4) and (14) are given by Eqs. (17) and (18), respectively.

Examples of how the synchronous phase is changing by adding a second harmonic to the RF field are shown in Figures 1 and 2. The stable phase is found as the (first positive) zero point of these curves. Figures 3, 4 and 5 show buckets. Parameters are:

 $\frac{eV_0}{\Delta E_s} = 2; \quad \chi = -.75, \, -.5, \, -.25, \, 0, \, .25, \, .5, \, .75.$

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Fig. 4. Buckets: $\Delta \phi = 0^{\circ}$.

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