

THE BUCKET AND TIPE BUNCH WITH A SECOND RF HARMONIC

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Technical Note

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THE BUCKET AND THE BUNCH WITH A 2.ND RF HARMONIC

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In the presence of a second RF harmonic the voltage acting on a particle is

$$(1) \quad eV = eV_1 \sin(\omega_{RF}t + \varphi_1) + eV_2 \sin(2\omega_{RF}t + \varphi_2).$$

With the positions

$$(2) \quad V_0 \equiv V_1, \quad \chi = \frac{V_2}{V_0}, \quad \phi = \omega_{RF}t + \varphi_1, \quad \Delta\varphi = \varphi_2 - 2\varphi_1,$$

the differential equation for the energy change during the acceleration becomes

$$(3) \quad \tau \frac{d}{dt}(\Delta E) = eV_0 \left\{ \sin \phi - \sin \phi_s + \chi [\sin(2\phi + \Delta\varphi) - \sin(2\phi_s + \Delta\varphi)] \right\}$$

with ϕ_s the synchronous phase, to be calculated by solving the transcendental Equation

$$(4) \quad \boxed{\frac{\Delta E_s}{eV_0} = \sin \phi_s + \chi \sin(2\phi_s + \Delta\varphi)}.$$

To find the bucket separatrix -the "bucket"- , let us start from the differential equation for the phase

$$(5) \quad \frac{d^2\phi}{dt^2} = \frac{2\pi}{\tau^2} \frac{h\eta}{\beta^2} \frac{eV_0}{E_s} f_b,$$

with f_b the function on the rhs of Eq. (3)

$$(6) \quad f_b = \sin \phi - \sin \phi_s + \chi [\sin(2\phi + \Delta\varphi) - \sin(2\phi_s + \Delta\varphi)],$$

and with the harmonic number h , the dispersion η , and the relativistic velocity β evaluated for the synchronous particle (we dropped the subscript "s").

An integral of motion is found by integration in the standard way¹

$$(7) \quad \int \frac{d^2\phi}{dt^2} \frac{d\phi}{dt} dt = \frac{2\pi}{\tau^2} \frac{h\eta}{\beta^2} \frac{eV_0}{E_s} \int f_b \frac{d\phi}{dt} dt.$$

The integration yields the following

$$(8) \quad \frac{K}{2} \left(\frac{\Delta E}{E_s} \right)^2 + eV_0 \tilde{b}(\phi, \phi_s) = H = \text{const},$$

with

$$(9) \quad K = 2\pi \frac{h\eta}{\beta^2} E_s,$$

and

$$(10) \quad \tilde{b}(\phi, \phi_s) = \cos \phi + \phi \sin \phi_s + \chi \left[\frac{1}{2} \cos(2\phi + \Delta\varphi) + \phi \sin(2\phi_s + \Delta\varphi) \right].$$

The constant H of Eq.(3) is the Hamiltonian of the system

$$(11) \quad H = \frac{1}{2} KW^2 + U,$$

and the bucket is found using the two fixed points ϕ_s and $\pi - \phi_s$ of the distribution²

$$(12) \quad H(\Delta E, \phi) = H(0, \pi - \phi_s).$$

The resulting bucket is

$$(13) \quad \boxed{b(\phi, \phi_s) = \cos \phi + \phi \sin \phi_s + \cos \phi_s - (\pi - \phi_s) \sin \phi_s + \chi \left[\frac{1}{2} \cos(2\phi + \Delta\varphi) + \phi \sin(2\phi_s + \Delta\varphi) - \cos(2\phi_s - \Delta\varphi) - (\pi - \phi_s) \sin(2\phi_s + \Delta\varphi) \right]}.$$

¹ D.A.Edwards and M.J.Syphers *An Introduction to the Physics of High Energy Accelerators*
Wiley, NY 1993, p.38

² J.M.Kats *Synchronous Particle and Bucket Dynamics* AGS/AD/89-1, BNL-52171, October 3, 1988

To accommodate the bunch in the bucket, we want to find the two extreme points of the separatrix, ϕ_1 and ϕ_2 ($\phi_2 < \phi_1$). With some manipulation of Eq. (13), a convenient form of the transcendental Equation to be solved to find these points, using as a parameter the bunch length $\Delta\phi$ can be written as

$$(14) \quad \boxed{\sin \psi = A - \frac{\chi}{4 \sin(\frac{1}{2} \Delta\phi)} \sin(2\psi + \Delta\phi)},$$

with the positions

$$(15) \quad \phi_1 = \psi + \frac{\Delta\phi}{2}, \quad \phi_2 = \phi_1 - \Delta\phi, \quad A = \frac{\frac{1}{2} \Delta\phi}{\sin(\frac{1}{2} \Delta\phi)} [\sin \phi_s + \chi \frac{1}{2} \Delta\phi \sin(2\phi_s + \Delta\phi)]$$

Eq.(4) for the synchronous phase and Eq.(14) for the bucket extreme points can be solved numerically. A very efficient and fast routine for this task is the Newton-Raphson iteration, based on the simple algorithm

$$(16) \quad x_{n+1} = x_n - \frac{f}{f'},$$

that solves the Equation $f \approx 0$ - f' is the derivative of f . A convenient starting point for the NR iteration, for Eq. (4) is the synchronous phase with no second harmonic

$$(17) \quad \phi_s^{(1)} = \arcsin \frac{\Delta E_s}{eV_0}$$

(actually a point slightly beyond ϕ_s). A starting point for solving Eq. (14) is

$$(18) \quad \phi_1^{(1)} = \frac{1}{2} \Delta\phi + \arcsin \left(\frac{\frac{1}{2} \Delta\phi}{\sin \frac{1}{2} \Delta\phi} \sin \phi_s \right).$$

Indeed, if $\chi = 0$, the exact solutions of Eqs. (4) and (14) are given by Eqs. (17) and (18), respectively.

Examples of how the synchronous phase is changing by adding a second harmonic to the RF field are shown in Figures 1 and 2. The stable phase is found as the (first positive) zero point of these curves. Figures 3, 4 and 5 show buckets. Parameters are:

$$\frac{eV_0}{\Delta E_s} = 2; \quad \chi = -.75, -.5, -.25, 0, .25, .5, .75.$$

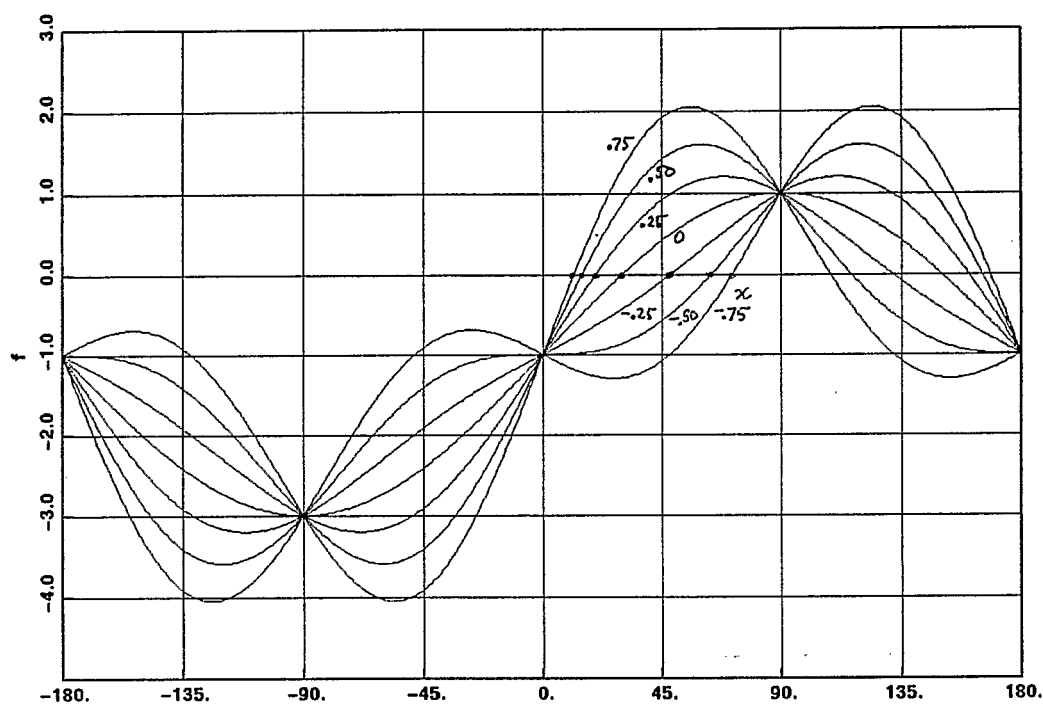


Fig. 1. Synchronous phase ($f=0$). $\Delta\phi=0^\circ$.

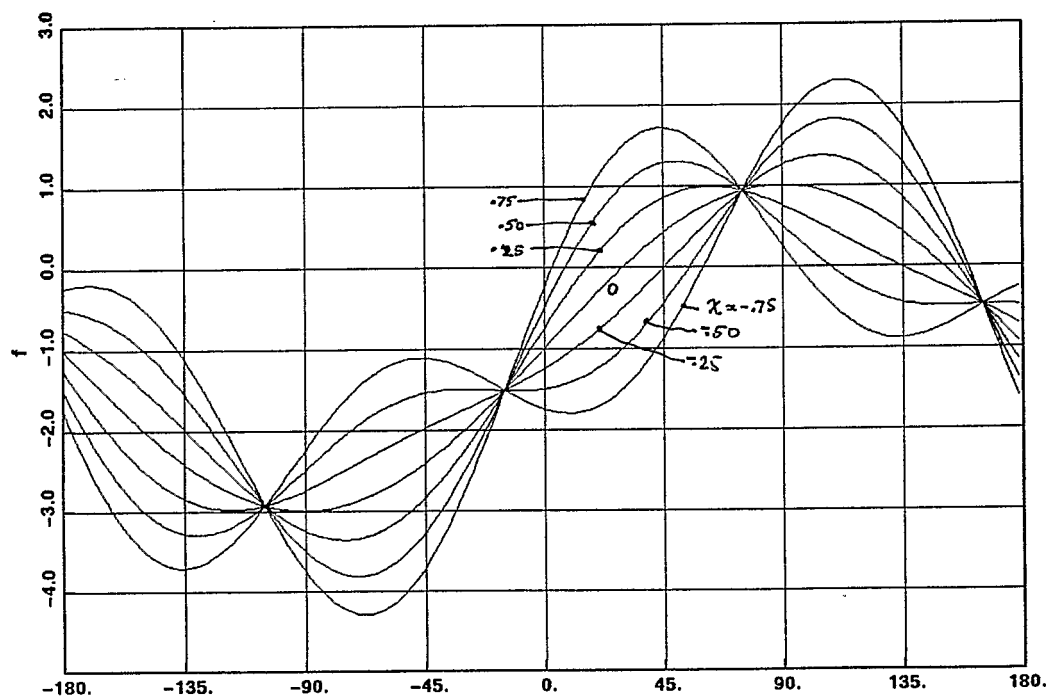


Fig. 2. Synchronous phase ($f=0$). $\Delta\phi=30^\circ$.

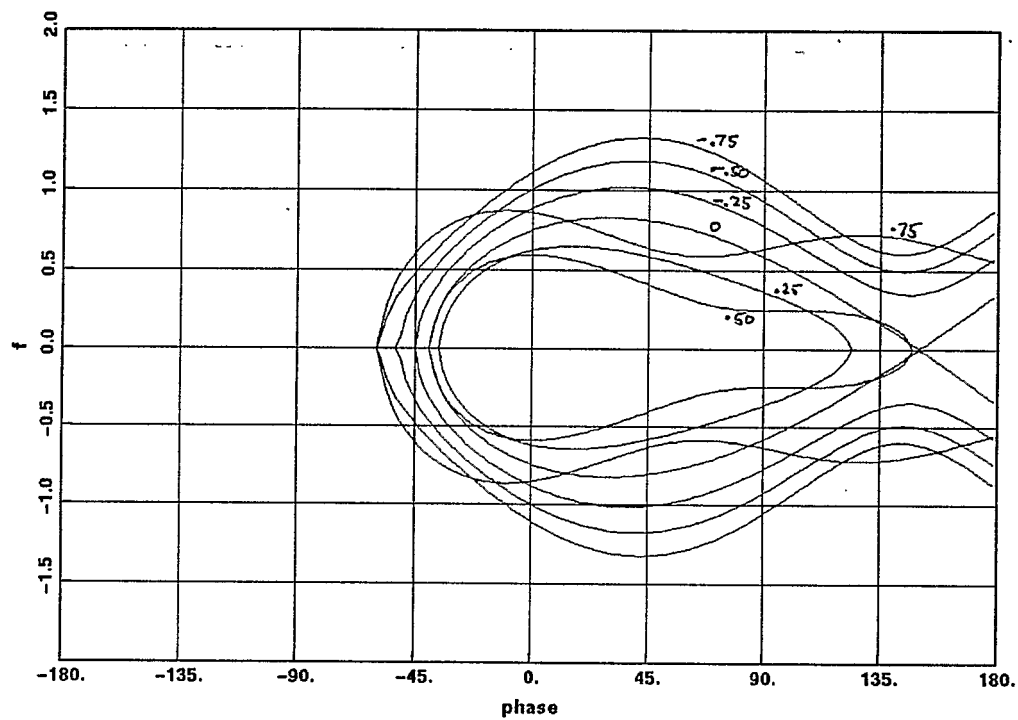


Fig. 3. Buckets: $\Delta\phi = -30^\circ$.

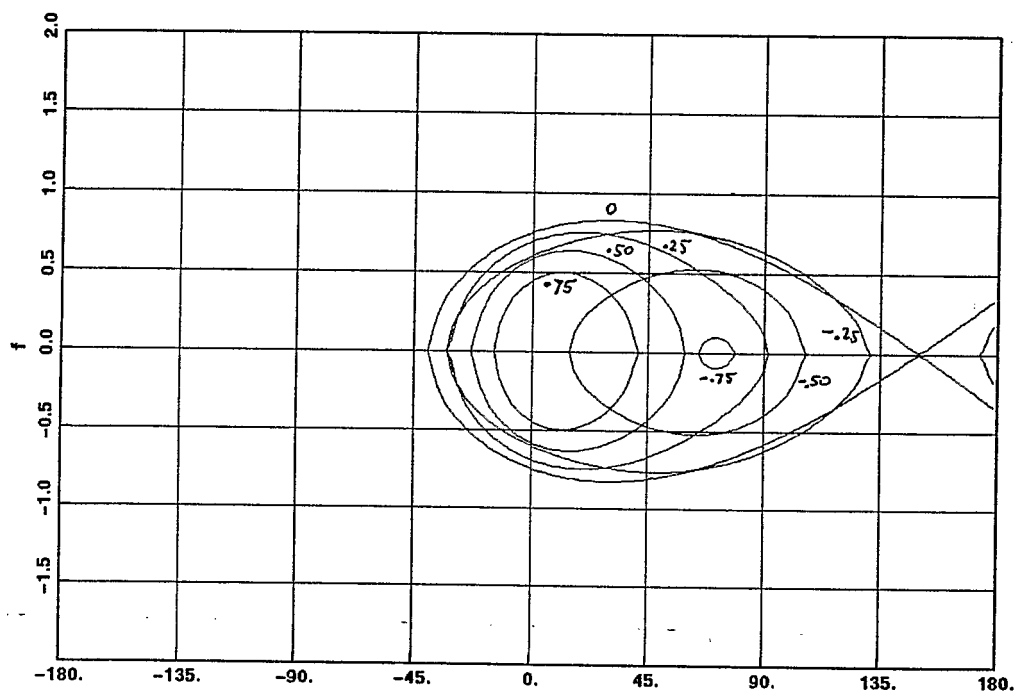


Fig. 4. Buckets: $\Delta\phi = 0^\circ$.

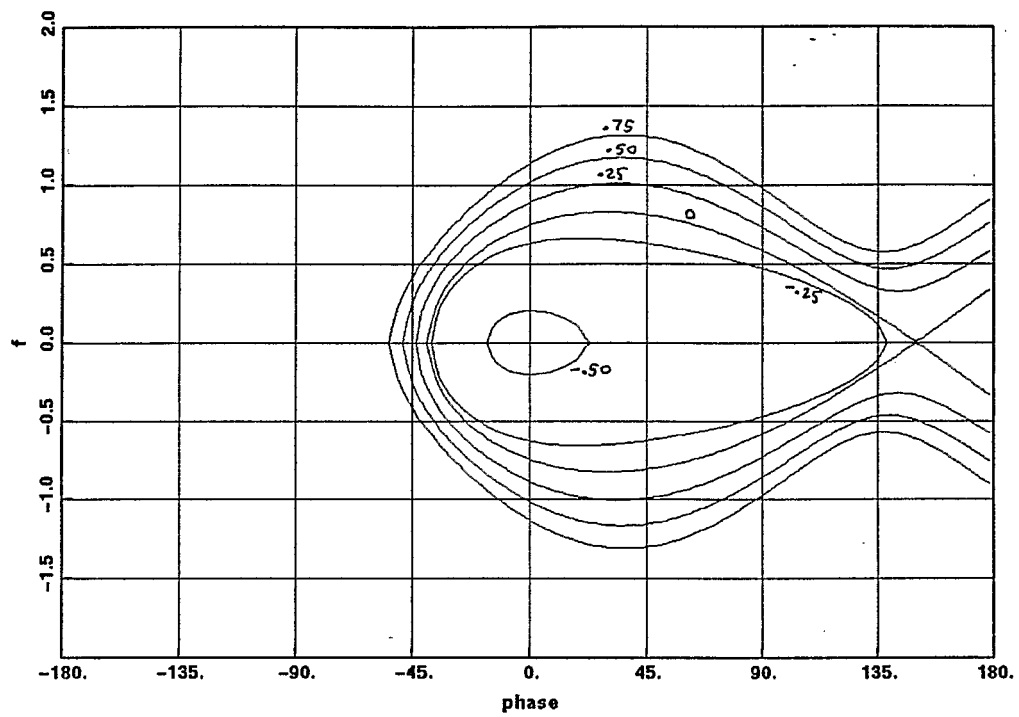


Fig. 5. Buckets: $\Delta\phi = 30^\circ$.