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THE ALTERNATING CURRENT POWER SYSTEM X/R RATIO AND ITS EFFECT ON THREE PHASE BRIDGE RECTIFIER OPERATION

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ABSTRACT

A complete mathematical solution for the controlled three phase bridge rectifier circuit including the effects of ac reactance is abundantly available in the power electronics literature. This paper presents a generalization of this mathematical solution by including the effects of ac supply resistance -- a parameter that is usually neglected in rectifier analysis due to the predominance of reactance limited 60 Hz ac supplies. A mathematical treatment is given beginning from fundamentals and concludes with a FORTRAN solver for the general three phase controlled bridge circuit with a large dc inductive load. The analyses covers the range of x/r (ac reactance/ac resistance) ratios from zero to infinity. The x/r ranges where ac resistance has significant effects on the commutation angle and on the dc voltage are identified.

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Variable Definitions

V_d	-	Time Average DC Voltage at Rectifier Terminals (volts)
I_d	-	DC Load Current (Amps)
E_{LL}	-	AC rms Line-to-Line Thevinin Voltage Behind AC impedance (Volts)
I_L	-	AC rms Line Current (Amps)
L	-	AC Inductance Per Phase (Henries)
x	-	AC Reactance Per Phase (Ohms)
r	-	AC Resistance Per Phase (Ohms)
τ	-	AC System Time Constant (Sec)
ω	-	AC System Angular Frequency (Radians/Sec)
α	-	SCR Firing Angle (Radians)
u	-	SCR Commutation Angle (Radians)
η	-	Ratio of AC Reactance Per Phase to AC Resistance Per Phase (Dimensionless)
t	-	Time (Sec)
θ	-	Angular Time (Radians)
i_1	-	Instantaneous SCR Commutating Turn-On Current (Amps)
i_2	-	Instantaneous SCR Commutating Turn-Off Current (Amps)

1.0 INTRODUCTION

1.1 Purpose

The purpose of this study is to present the mathematical relationship between the variables that describe a 3-phase bridge rectifier with a large inductive dc load. Although many references describe this circuit in detail, there is little information available on the effects of resistance in the ac lines. The mathematical treatment of the bridge rectifier presented here accounts for the effects of ac line inductance and ac line resistance.

The issue of ac line resistance is usually neglected in most rectifier literature because it is often assumed that the ratio of ac line reactance to ac line resistance (x/r ratio) is large. In this situation, the rectifier circuit characteristics of regulation and commutation are almost entirely determined by the reactive portion of the ac impedance. This is usually the case in high voltage circuits because the required spacing between conductors in lines and transformer windings contributes to a high x/r ratio. However, in low voltage rectifier circuits the x/r for triangularized cable may be less than 2 for cables in the 250 - 1000 MCM range at 60 Hz. Triangularized 4/0 cable may have an x/r as low as 1/2. Three phase rectifiers in the 0 - 2 kW range are usually driven by a transformer with an x/r ratio between 0.2 and 1.5. In rectifier circuits where the overall x/r is this low, the classical reactance limited rectifier equations may given commutation angle predictions of more than 50% error.

In this study the equations for a rectifier driven by a reactive - resistive ac system are derived. The range of x/r where resistance plays a significant role in circuit characteristics is then identified.

1.2 Procedure

The procedure for solving the 3-phase bridge rectifier circuit involves circuit analysis and mathematics. Circuit analysis is used for two purposes: first, to construct the fundamental differential equation that describes the switching of ac line current from one SCR to another SCR (commutation); and secondly, circuit analysis is used to determine the integration constant in the solution to this differential equation. Laplace transform mathematics is used to solve the differential equation. The final mathematical results are coded into a FORTRAN 77 program to study rectifier behavior as a function of ac source impedance.

2.0 CIRCUIT ANALYSIS

2.1 Basic Rectifier Operation

The circuit of the rectifier under study is shown in Fig. 1. This analysis is based on steady state operation. The ac line current waveforms are shown in Fig. 2 for a predominately inductive ac supply. The instantaneous dc voltage waveform is constructed from the rectified sine curves shown in Fig. 3. An increase in ac line inductance or resistance will result in a reduction in the average dc voltage for a fixed dc load current. The dc load current may be held constant by appropriately decreasing the dc load resistance as the ac impedance is increased. An increase in ac resistance results in a decrease in the commutation angle as opposed to an increase in commutation angle that results from an increase in ac line inductance. This is because an increase in ac resistance effectively shortens the time constant of the ac system.

2.2 The Commutating Circuit

The mathematical modeling of the commutation process forms the basis of rectifier analysis. The commutation circuit for two arbitrary SCR's is shown in Fig. 4. The KVL (Kirkoff voltage law) around the commutating loop gives the differential equation:

$$\sqrt{2}\, E_{LL} \sin \omega t - 2L \frac{di}{dt} - 2ir + I_d r = 0 \qquad (1)$$

The commutating current is subject to the conditions

$$i(\alpha) = 0 \qquad (2)$$

$$i(\alpha + u) = I_d \qquad (3)$$

where α and u are the SCR firing angle and commutation angle, respectively. The dc load current I_d is constant because of the large inductive dc load so that

$$\frac{dI_d}{dt} = 0 \qquad (4)$$

3.0 THE MATHEMATICAL SOLUTION

3.1 Solution of the Fundamental Differential Equation

The commutating loop expression in Equation (1) can be expressed as

$$\frac{di}{dt} + \frac{\omega}{(x/r)} i - \frac{\sqrt{2}}{2} \frac{E_{LL}}{L} \sin \omega t - \frac{I_d}{2} \frac{\omega}{(x/r)} = 0 . \qquad (5)$$

Taking the Laplace transform gives:

$$sI(s) - C_1 + \frac{\omega}{(x/r)} I(s) - \frac{\sqrt{2}}{2} \frac{E_{LL}}{L} \left(\frac{\omega}{s^2 + \omega^2}\right) - \frac{I_d}{2} \frac{\omega}{(x/r)} \frac{1}{s} = 0 \qquad (6)$$

where

$$C_1 = i\,(o). \qquad (7)$$

Solving for I(s) gives

$$I(s) = \frac{\sqrt{2}}{2} \frac{E_{LL}}{L} \omega \frac{1}{(s + j\omega)(s - j\omega)(s + 1/\tau)} + \frac{I_d}{2\tau} \frac{1}{s(s + 1/\tau)} + C_1/(s + 1/\tau). \qquad (8)$$

A partial fraction expansion of Equation (8) gives:

$$I(s) = \frac{\sqrt{2}}{4} \frac{E_{LL}}{\omega L} \left(\frac{\omega}{\omega^2 + 1/\tau^2}\right) \left\{\frac{-\omega + j\ 1/\tau}{s + j\omega} + \frac{-\omega - j\ 1/\tau}{s - j\omega} + \frac{2\omega}{s + 1/\tau}\right\} + \frac{1}{2} I_d \left(\frac{1}{s} - \frac{1}{s + 1/\tau}\right) + \frac{C_1}{s + 1/\tau} \qquad (9)$$

The inverse Laplace transform of Equation (9) gives:

$$i(t) = \frac{\sqrt{2}}{4} \frac{E_{LL}}{\omega L} \left(\frac{\omega}{\omega^2 + 1/\tau^2}\right) \left\{2\sqrt{\omega^2 + 1/\tau^2} \sin\left[\omega t + \tan^{-1}(-\omega\tau)\right] + 2\,\omega e^{-t/\tau}\right\} + \frac{1}{2} I_d \left(1 - e^{-t/\tau}\right) + C_1\, e^{-t/\tau} \qquad (10)$$

Using the variable definitions on Page iv and expressing the current in terms of electrical angular time, Equation (10) can be written:

$$i(\theta) = \frac{\sqrt{2}}{2}\,\frac{E_{LL}}{x}\,\left(\frac{1+\eta^2}{\eta^2}\right)\left\{\sqrt{1+1/\eta^2}\,\sin\,(\theta - \tan^{-1}\eta) + e^{-\theta/\eta}\right\} + \frac{1}{2}\,I_d\,(1 - e^{-\theta/\eta}) + C_1\,e^{-\theta/\eta} \qquad (11)$$

Equation (11) is the solution to the commutation loop differential equation [Equation (1)].

3.2 Circuit Constraints and Final Solution For The Commutation Angle

The solution for the commutating current $i(\theta)$ in Equation (11) is subjected to the constraints

$$i(\alpha) = 0 \qquad (12)$$

$$i(\alpha + u) = I_d \qquad (13)$$

Substitution of Equation (12) into Equation (11) gives the integration constant as

$$C_1 = -\frac{\sqrt{2}}{2}\,\frac{E_{LL}}{x}\,\left(\frac{1+\eta^2}{\eta^2}\right)\left\{\sqrt{1+1/\eta^2}\,\sin\,(\alpha - \tan^{-1}\eta) + e^{-\alpha/\eta}\right\} e^{\alpha/\eta} - \frac{1}{2}\,I_d\,(e^{\alpha/\eta} - 1) \qquad (14)$$

Substitution of Equation (13) into Equation (11) and using the expression for C_1 in Equation (14) gives:

$$\sqrt{1+1/\eta^2}\,\sin\,(\alpha + u - \tan^{-1}\eta) + e^{-(\alpha+u)/\eta} + \frac{\sqrt{2}}{2}\,\frac{xI_d}{E_{LL}}\left(\frac{\eta^2}{1+\eta^2}\right)\cdot$$

$$(1 - e^{-(\alpha+u)/\eta}) = \frac{\sqrt{2}\;xI_d}{E_{LL}}\left(\frac{\eta^2}{1+\eta^2}\right) + \left\{\sqrt{1+1/\eta^2}\,\sin\,(\alpha - \tan^{-1}\eta\right\}$$

$$e^{-\alpha/\eta} + \frac{\sqrt{2}}{2}\,\frac{xI_d}{E_{LL}}\left(\frac{\eta^2}{1+\eta^2}\right)(e^{-u/\eta} - e^{-(\alpha+u)/\eta}) \qquad (15)$$

Rearranging of these terms gives a final expression in the commutation angle u in terms of α, E_{LL}, x, η, and I_d:

$$\sqrt{1 + 1/\eta^2}\ \{\sin(\alpha + u - \tan^{-1}\eta) - e^{-u/\eta} \sin(\alpha - \tan^{-1}\eta)\} - \frac{\sqrt{2}}{2} \frac{xI_d}{E_{LL}} \left(\frac{\eta^2}{1 + \eta^2}\right) (1 + e^{-u/\eta}) = 0 \tag{16}$$

Equation (16) is one of the two key equations that define the solution to the rectifier circuit. The second equation involves the average dc voltage and is derived in the following section.

3.3 Derivation of the DC Voltage Equation

The dc voltage is calculated by taking the mean of the instantaneous dc voltage waveform over one cycle of fundamental rectifier ripple frequency (360 Hz). Each cycle is composed of a commutation interval and a conduction interval. Referring to Fig. 3 the commutation interval is shown for $\pi/3 + \alpha \leq \theta \leq \pi/3 + \alpha + u$. In the latter interval, one SCR in each polarity is in full conduction with the dc load current with no commutations in progress. A detail of one cycle of dc voltage waveform and the commutation circuit that determines this waveform is shown in Fig. 5. The commutation interval in Fig. 5 takes place during $\alpha \leq \theta \leq \alpha + u$. The illustration shows that the instantaneous dc voltage during commutation is

$$e_1 = e_0 + \frac{1}{2}(e_2 - e_0) - \frac{3}{2} r I_d \tag{17}$$

and during conduction the voltage is

$$e_3 = e_2 - 2r I_d \tag{18}$$

The dc load voltage is the mean of e_1 and e_3 over the commutation and conduction intervals:

$$V_d = \frac{1}{\pi/3} \left\{ \int_{\pi/3 + \alpha}^{\pi/3 + \alpha + u} e_1(\theta)d\theta + \int_{\pi/3 + \alpha + u}^{2\pi + \alpha} e_3(\theta)d\theta \right\} \tag{19}$$

From Fig. 5 the line voltages are

$$e_0(\theta) = \sqrt{2}\, E_{LL} \sin(\theta + \pi/3) \tag{20}$$

$$e_2(\theta) = \sqrt{2}\, E_{LL} \sin\theta \tag{21}$$

Substitution of Equations (20) and (21) into Equations (17) and (18) gives:

$$e_1(\theta) = \frac{\sqrt{2}}{2} E_{LL} \sin\theta + \frac{\sqrt{2}}{2} E_{LL} \sin(\theta + \pi/3) - \frac{3}{2} rI_d \tag{22}$$

$$e_3(\theta) = \sqrt{2}\, E_{LL} \sin\theta - 2rI_d \tag{23}$$

Integration of Equation (19) then gives

$$V_d = \frac{3}{\pi\sqrt{2}} E_{LL} \left\{\cos\alpha + \cos(\alpha + u)\right\} - \frac{3}{2\pi} \frac{xI_d}{\eta} \left(\frac{4\pi}{3} - u\right) \tag{24}$$

Equation (24) expresses the time average dc rectifier voltage as a function of the dc load current and circuit parameters. Equations (16) and (24) constitute the solution to the 3-phase controlled rectifier circuit driven by a reactive-resistive ac supply.

The derivation of the ac rms line current is very tedious because of the complexity of the commutating current expression in Equation (11). This expression must be squared and integrated over the commutating interval in the first steps of the derivation. This work is omitted here, however, approximate expressions for the ac rms current are given for the purely reactive case in standard references [1].

4.0 CIRCUIT CHARACTERISTICS AND CHECKS

4.1 A Mathematical Check

The final rectifier Equations (16) and (24) must satisfy a limit requirement. These two equations should simplify to the well known standard formulae for a 3-phase bridge rectifier driven by purely reactive system. The limit of interest is the AC system x/r ratio approaching infinity ($\eta \to \infty$). As this limit is approached, Equation (16) becomes

$$\cos(\alpha + u) = \cos\alpha - \frac{\sqrt{2}\; xI_d}{E_{LL}} \tag{25}$$

As $\eta \to \infty$ Equation (24) becomes

$$V_d = \frac{3}{\pi\sqrt{2}} E_{LL} \{\cos\alpha + \cos(\alpha + u)\} \tag{26}$$

Equations (25) and (26) appear in various forms in the literature on 3-phase bridge rectifier circuits [2] [3].

4.2 A FORTRAN Solution and the Effect of the x/r Ratio

The solution equations for the rectifier circuit Equations (16) and (24) were coded into a FORTRAN 77 program that calls an IMSL (International Mathematics and Statistics Library) solving routine to produce numerical solutions. Equations (16) and (24) are defined in a total of seven variables so that a knowledge of any five variables fixes the values of the remaining two unknowns. The user supplies the main program (REC1.FOR) with information about which two of the seven variables are designated as the unknowns. Numerical data for the five known variables are supplied along with estimates of the two unknown varibles. REC1.FOR then calls the IMSL nonlinear simultaneous equation solver NEQNF to solve the system of two equations [4]. A listing of REC1.FOR with input and output files is shown in Appendix A. In the example run, a rectifier is driven by a 24.7 V 3-phase supply with an x/r ratio of 1.8 and a dc current of 3700 A. The program is directed to solve for the dc voltage (ans.: 29.9 V) and the commutation angle (ans.: 21.9 degrees). If the ac system is made predominately reactive by setting x/r = 50, the new solution for the dc voltage and commutation angle is 31.2 V and 29.2 degrees, respectively. Several runs were made on the computer to determine the behavior of the rectifier circuit as the x/r ratio is changed. It is found that the general solution of the rectifier circuit is roughly independent of the x/r ratio for values greater than five. For x/r < 5 the parameter that is most effected is the commutation angle. An increase in ac resistance shortens the time constant of the ac system - in effect making the ac system "stronger" in terms of response. The SCRs, therefore require less time to commutate for the same dc load current. Fig. 6 shows a graphical compilation of computer runs where the commutation angle and dc voltage are plotted as a function of x/r ratio while holding the dc load current constant.

4.3 Phase Shifted Rectifier Circuits

Resistance in the ac lines seems to play a special role in determining commutation angles in the case of phase shifted rectifiers. The resistance referred to here is that of the main feeder circuit that supplies a set of phase shifting rectifier transformers. Specifically, it is suspected that the commutation angle of a rectifier is partially dependent on the amount of phase shift. It is clear that the solution Equations (16) and (24) cannot be the exact solutions for a rectifier that is phase shifted but may only be good approximations. A formal mathematical analysis of such a circuit showing how the feeder resistance effectively couples the rectifier circuits appears to require a considerable effort. In place of this approach, the following discussion is presented only to identify an effect on commutation that appears to be unique to phase shifted rectifiers.

In order to obtain dc power supplies with a pulse number greater than six, several 3-phase bridge rectifiers can be connected in series where the ac circuits that drive each rectifier are phase displaced from each other. The phase shifting is accomplished with various types of transformer connection schemes that are connected to one primary 3-phase circuit -- either from a synchronous generator or the utility line. This is standard practice for the main magnet power supplies of the AGS, the AGS Booster, and many other smaller dc supplies. A simple example of a 12-pulse power supply is shown in Fig. 7 with two rectifier transformers shifting their circuits +15 and -15 degrees, respectively. Assuming that the dc outputs are connected in series and that the commutation angle is less than 30 degrees, it is then established that there is only one commutation in progress at any given time among both rectifiers. This means that when one rectifier has a commutation in progress the other rectifier currents are in a dc state. These dc currents cause IR drops in two of the three phases in the main 3-phase feeder circuit resistances. It appears as if these IR drops can either add to or subtract from the commutating voltage depending on whether the phase shift is ahead of or behind the primary ac supply. This would effectively make the commutation angle a function of the phase shift angle along with the other variables such as load current, firing angle, ac voltage, and ac impedance.

The IR drops in the commutating circuit that are phase shift dependent are identified in the circuit diagram in Fig. 8. This illustration shows the 3-phase equivalent circuit of the 12 pulse power supply in Fig. 7. The commutating loop equation is written in the standard manner with the primary ac voltages and the phase shifting voltages all represented by one source during he commutation interval. The resulting KVL around the commutating loop written in Fig. 8 appears normal except for the last term. The polarity of this term appears to depend on whether the rectifier circuit is shifted ahead or behind the primary ac source. Also, if the phase shift is zero, the commutation equation assumes the standard form as in Equation (1).

If the commutation angle is partially dependent on the phase shift angle, then it follows that such a power supply must produce subharmonic ripple voltages on the dc side even if the ac supply is perfectly balanced in voltage magnitude, phase, and impedance. In the case of a series connection of 3-phase bridges, a 360 Hz ripple voltage and its integer multiples would have to be present in some magnitude regardless of the pulse number of the whole power supply. In the case of the power supply in Fig. 7, the fundamental ripple voltage is 720 Hz, however, the coupling effect of the resistance in the primary feeder would cause a 360 Hz ripple voltage to appear at the dc terminals. This is a consequence of the commutation angles of the two rectifiers being different. The magnitude of the subharmonic ripple voltage would depend on the reactance and resistance of the main feeder circuit and the individual rectifier circuits. A hypothetical zero resistance primary feeder circuit would eliminate the coupling effect and all associated subharmonics below the fundamental ripple frequency at the dc terminals.

5.0 CONCLUSIONS

The exact solution for the 3-phase bridge rectifier is expressed in Equations (16) and (24). These expressions are derived for a large inductive dc load and account for ac supply reactance and resistance. Numerical solutions for practical SCR bridge rectifier problems can be obtained by coding these equations into FORTRAN and using computer library solvers (IMSL) to find the required zeros. Any PC mathematical package that can solve two simultaneous transcendental equations for any two for the seven variables would also be sufficient. If the ac x/r ratio is greater than 5, computer work is unnecessary as Equations (16) and (24) can be solved by hand. For $x/r < 5$ simplifications or approximations are not apparent and computer work seems to be required. The solutions of these rectifier equations may be used for voltage, current, and power calculations or to determine commutating reactance and ac resistance from accurate ac and dc line meter readings. Commutating angle and SCR firing angle may also be computed and used as input for dc voltage and current ripple calculation routines.

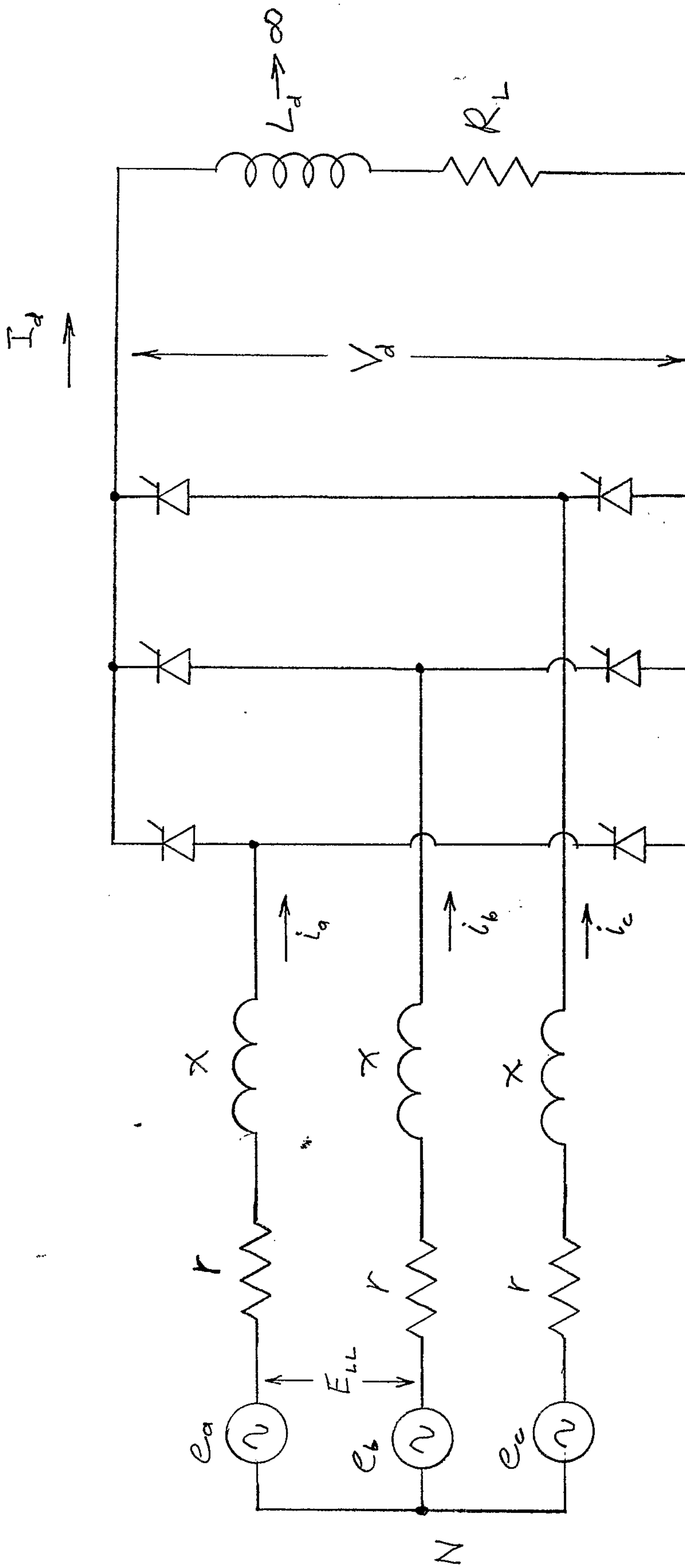


Fig. 1 Rectifier Circuit Topology

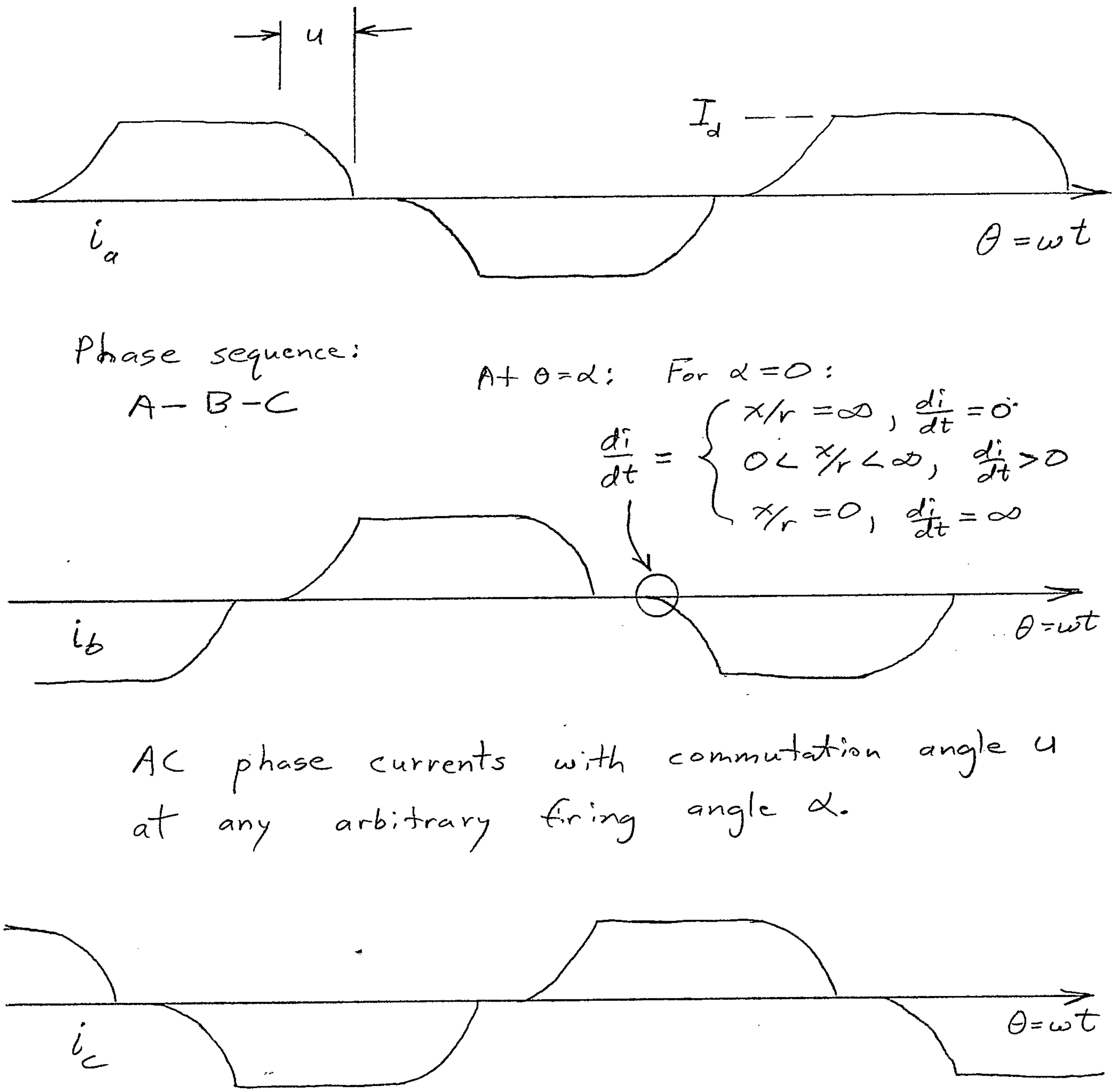
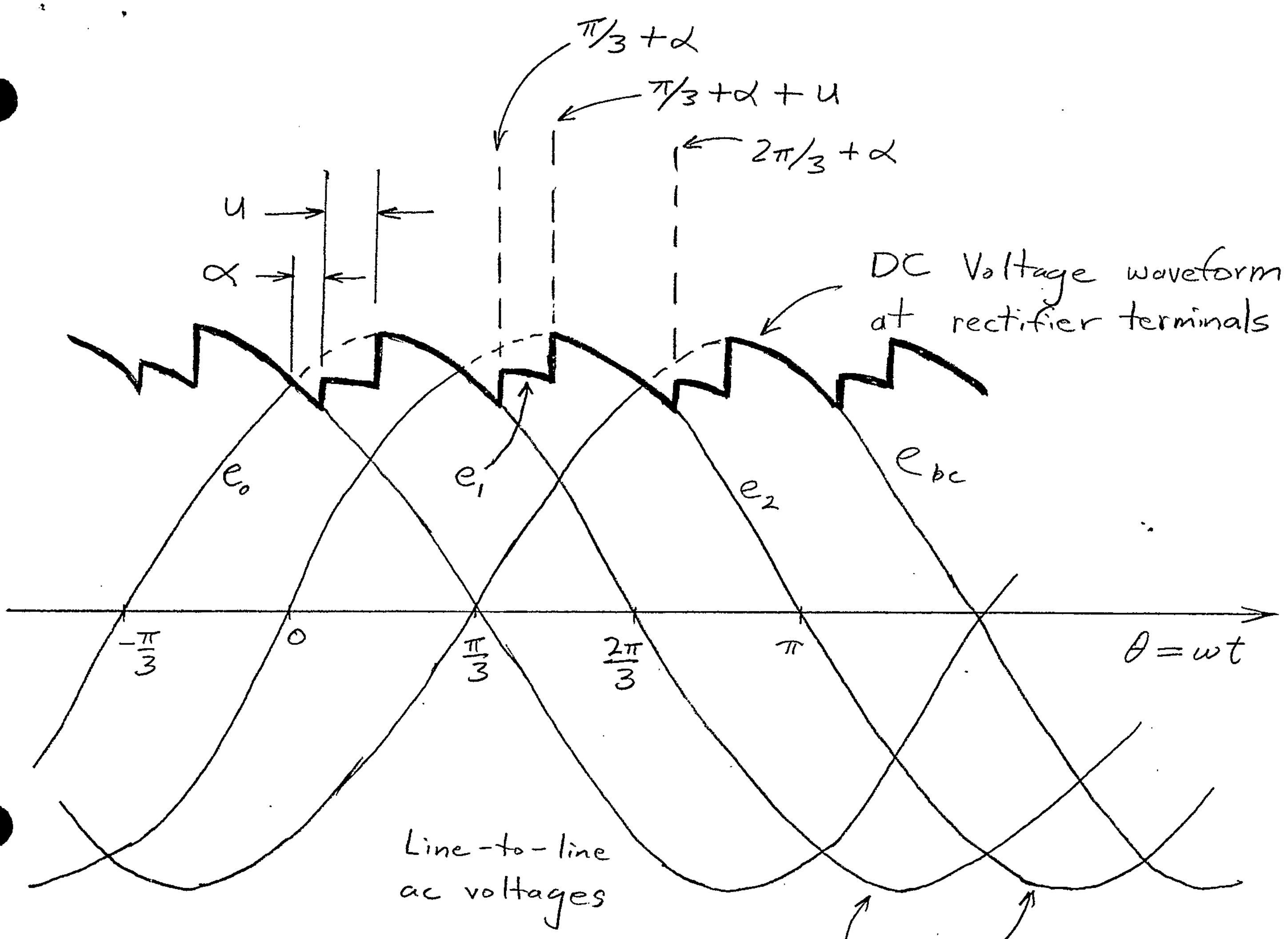


Fig. 2 AC Current Waveforms



$e_0 = -e_{bc}$ $\qquad$ $e_{ab} = \sqrt{2}\, E_{LL} \sin\theta$

α = SCR firing angle

u = Commutation angle

$e_0 = e_{cb} = \sqrt{2}\, E_{LL} \sin(\theta + \pi/3)$

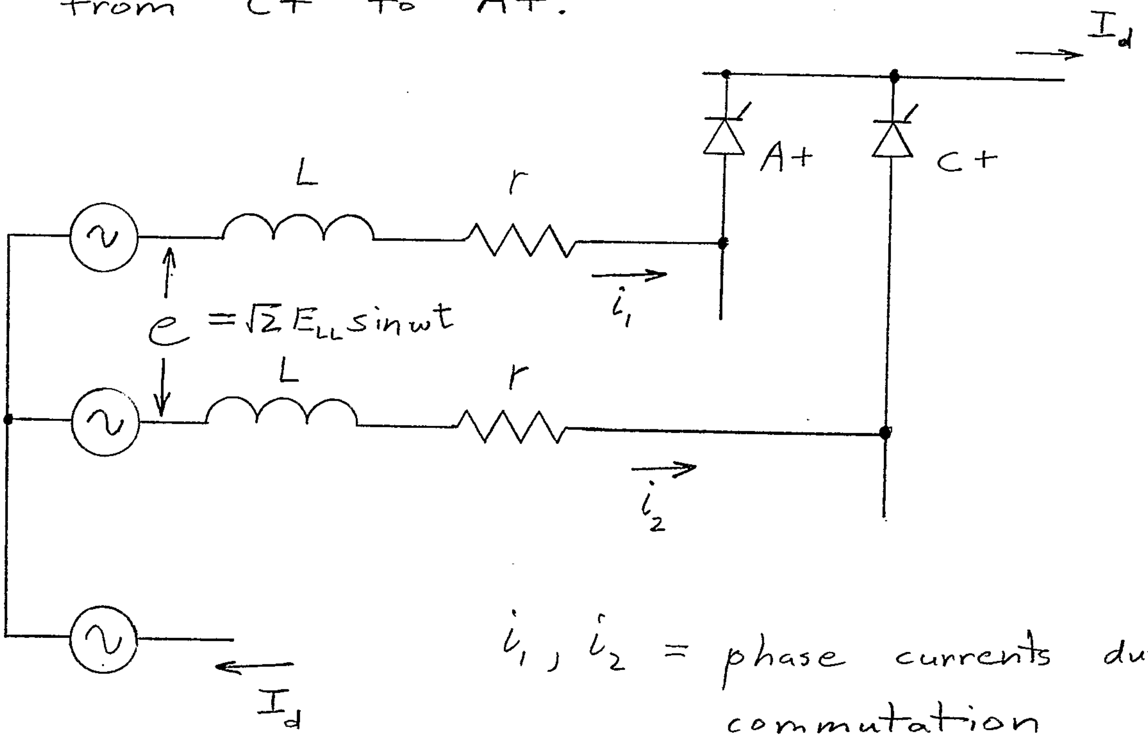
Phase sequence AB–BC–CA

$$e_{ab} = \sqrt{2}\, E_{LL} \sin\theta$$
$$e_{bc} = \sqrt{2}\, E_{LL} \sin(\theta - 120^\circ)$$
$$e_{ca} = \sqrt{2}\, E_{LL} \sin(\theta + 120^\circ)$$

} Line-to-line ac voltages

Fig. 3 DC Voltage Waveform at Rectifier Terminals

SCR current commutates from C+ to A+.



i_1, i_2 = phase currents during commutation

$$I_d = \text{constant} \qquad i_1(\alpha) = 0 \qquad i_2(\alpha) = I_d$$

$$i_1 + i_2 = I_d \qquad i_1(\alpha + u) = I_d \qquad i_2(\alpha + u) = 0$$

$$X = \omega L$$

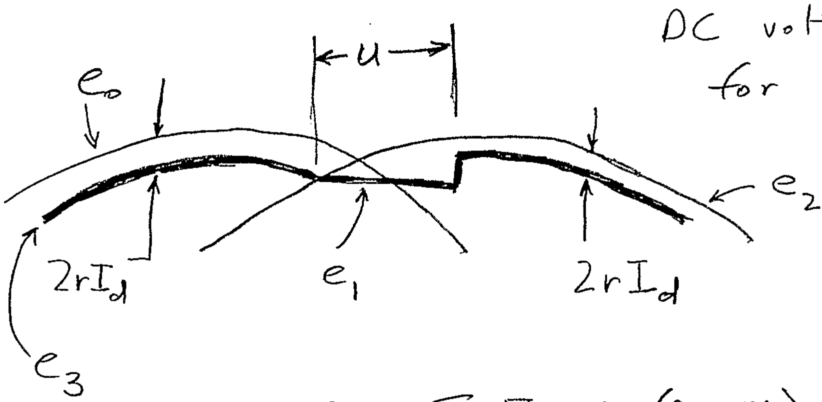
Commutation differential equation:

$$\sqrt{2} E_{LL} \sin \omega t - L \frac{di_1}{dt} + L \frac{di_2}{dt} - i_1 r + i_2 r = 0$$

Set $i = i_1$:

$$\sqrt{2} E_{LL} \sin \omega t - 2L \frac{di}{dt} - 2ir + I_d r = 0$$

Fig. 4 Commutation Circuit

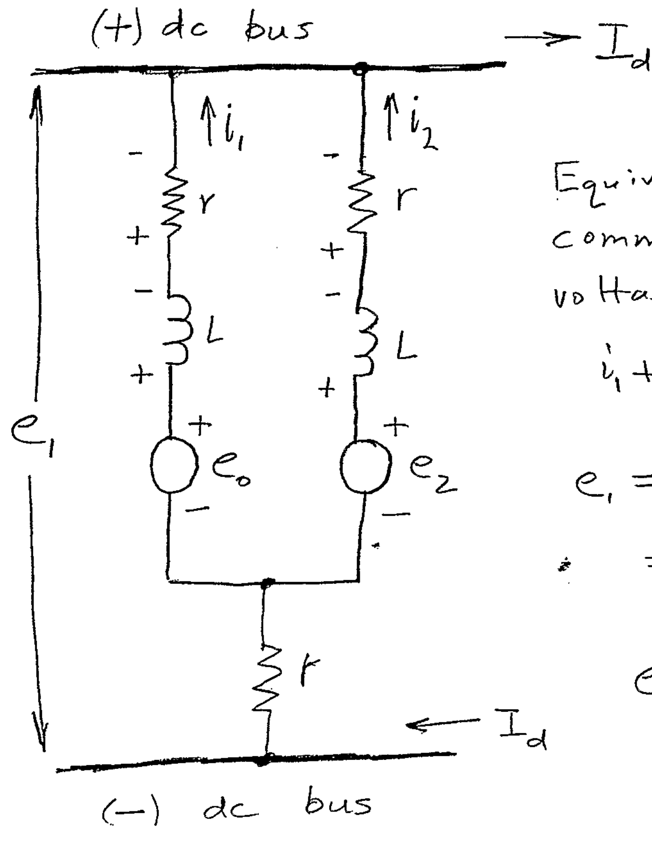


DC voltage waveform for $\alpha = 0$.

e_o, e_2 = ac Thevenin line-to-line voltages

$$e_o = \sqrt{2}\, E_{LL} \sin(\theta + \pi/3)$$

$$e_2 = \sqrt{2}\, E_{LL} \sin\theta$$



Equivalent circuit during commutation used to derive voltage e_1 during $\alpha \leq \theta \leq \alpha + u$.

$$i_1 + i_2 = I_d$$

$$e_1 = -rI_d + e_o - L\frac{di_1}{dt} - ri_1$$

$$= rI_d + e_2 - L\frac{di_2}{dt} - ri_2$$

$$e_1 = e_o + \tfrac{1}{2}(e_2 - e_o) - \tfrac{3}{2} rI_d$$

during commutation

$e_3 = e_2 - 2rI_d$ during single SCR conduction

Fig. 5 DC Voltage Derivation

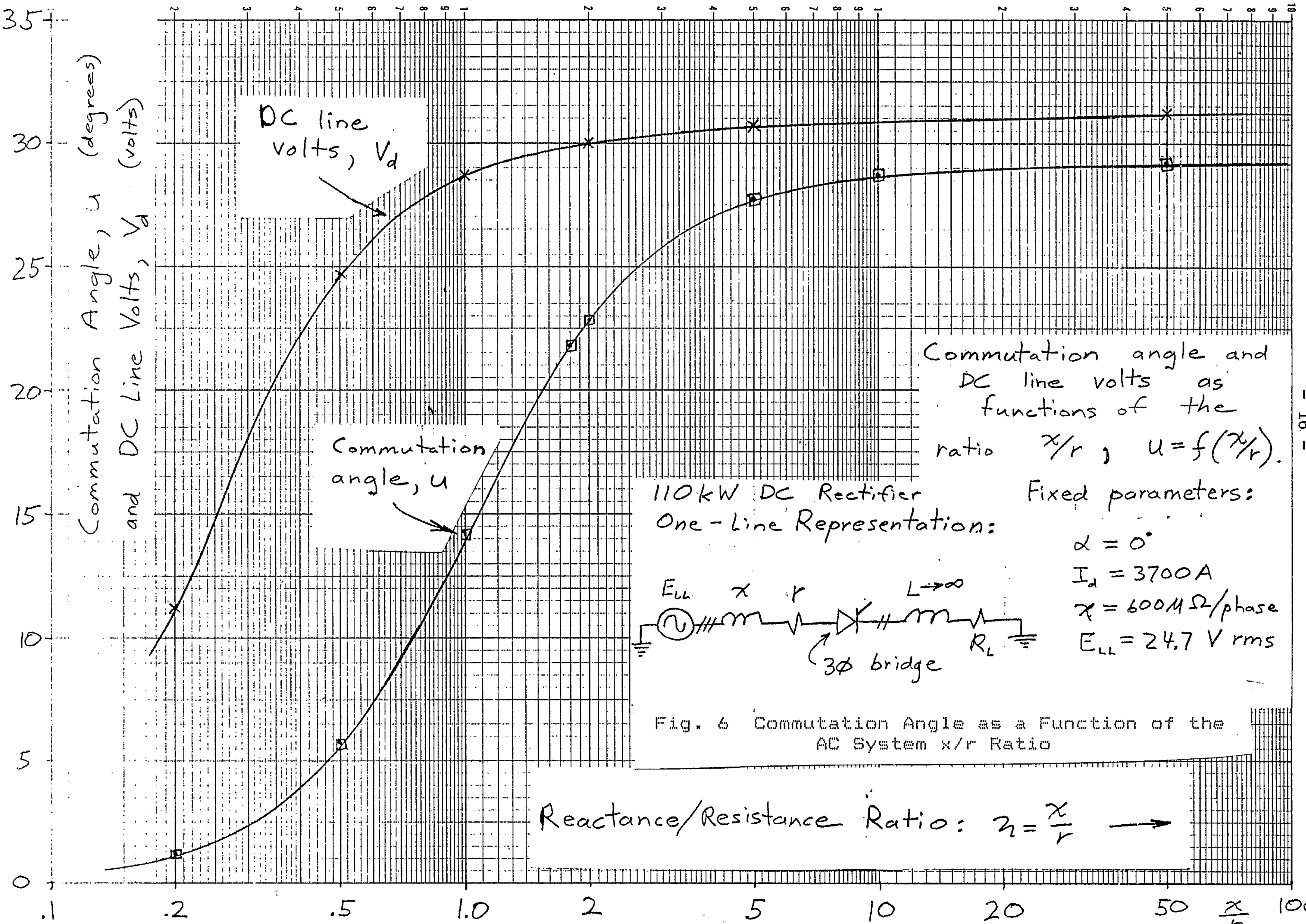


Fig. 6 Commutation Angle as a Function of the AC System x/r Ratio

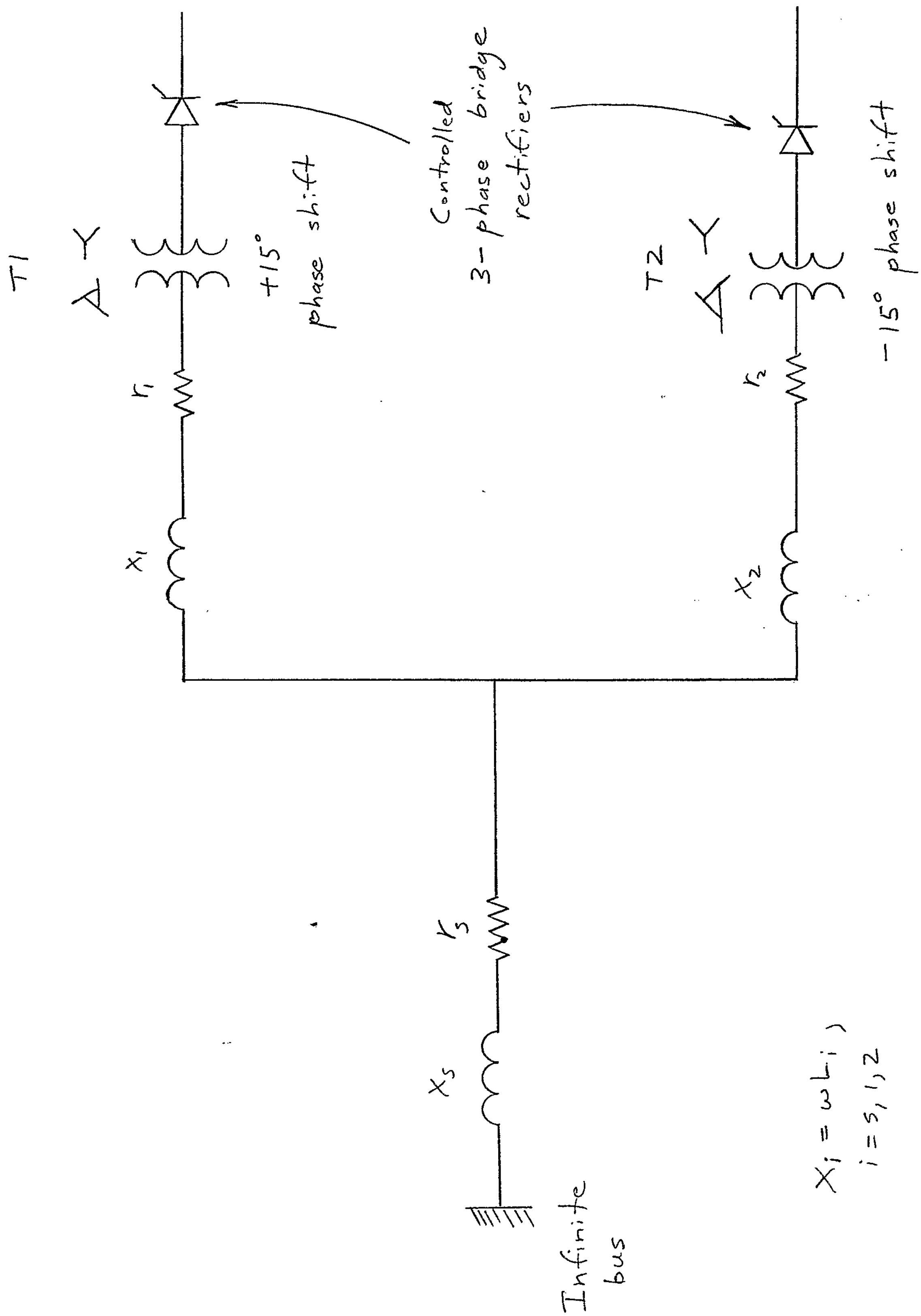
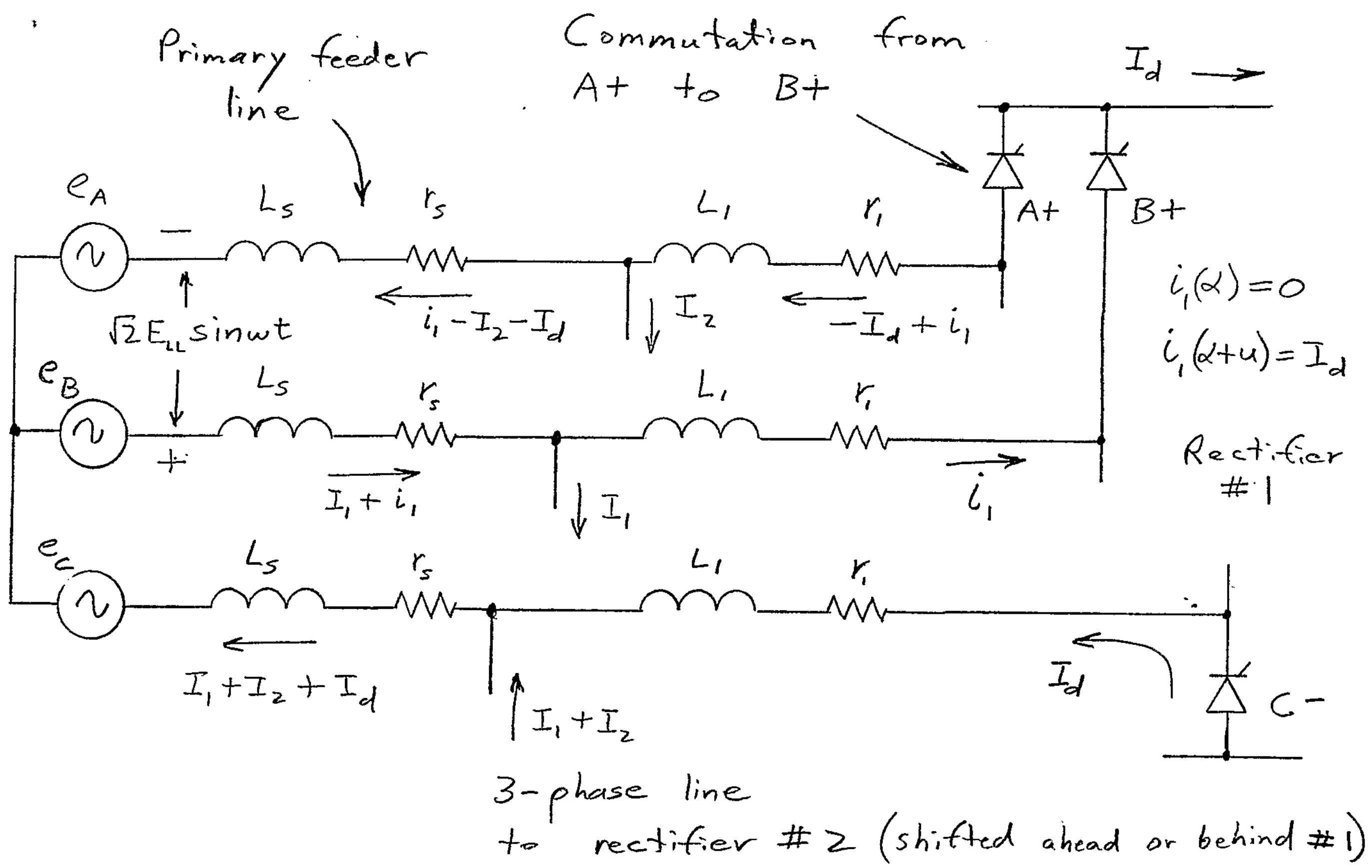


Fig. 7 One Line Diagram For a 12 Pulse DC Power Supply



Commutation Equation: (switch from A+ to B+):

$$\sqrt{2}\,E_{LL}\sin\omega t - 2(L_s+L_1)\frac{di_1}{dt} - 2(r_s+r_1)i_1 + (r_s+r_1)I_d + r_s(I_2-I_1) = 0$$

where the last term depends on phase shift:

$$r_s(I_2-I_1) = \begin{cases} r_s I_d, & \text{shifted ahead, } I_2 = I_d,\ I_1 = 0 \\ -r_s I_d, & \text{shifted behind, } I_2 = 0,\ I_1 = I_d \end{cases}$$

If both rectifiers are in phase:

$$\sqrt{2}\,E_{LL}\sin\omega t - 2(2L_s+L_1)\frac{di_1}{dt} - 2(2r_s+r_1)i_1 + (r_1+2r_s)I_d = 0$$

Fig. 8 Commutation Circuit Model For a 12 Pulse DC Power Supply

APPENDIX

FORTRAN bridge rectifier solver REC1.FOR with example runs.

```
C REC1.FOR  -- (6/21/88)  LAST REV (8/10/88)
C REC1.FOR  -- RECTIFIER SOLVER PROGRAM -- CALCULATES
C THE SOLUTION FOR A 3-PHASE 6-PULSE BRIDGE RECTIFIER CIRCUIT.
C AN INFINITE INDUCTIVE DC LOAD IS ASSUMED.  REC1 DESCRIBES THE
C RECTIFIER CIRCUIT WITH 7 PARAMETERS:
C
C   X(1) = VD -- DC LINE VOLTS (VOLTS)
C   X(2) = ELL -- AC RMS LINE-TO-LINE THEVININ VOLTAGE (VOLTS)
C   X(3) = A -- SCR FIRING ANGLE (DEGREES)
C   X(4) = U -- SCR COMMUTATION ANGLE (DEGREES)
C   X(5) = X -- AC REACTANCE PER PHASE (OHMS)
C   X(6) = ID -- DC LINE CURRENT (AMPERES)
C   X(7) = NU -- RATIO OF AC LINE REACTANCE PER PHASE TO
C                AC LINE RESISTANCE PER PHASE (NU = X/R)
C
C REC1 CONTAINS TWO INDEPENDENT NONLINEAR EQUATIONS IN THESE
C 7 VARIABLES THAT RESULT FROM RECTIFIER CIRCUIT ANALYSIS.
C ANY 2 OF THESE VARIBLES MAY BE SPECIFIED AS THE OUTPUT.
C THE SPECIFIED VARIABLES ARE THE 2 UNKNOWNS FOR REC1 TO
C SOLVE.  THE REMAINING 5 VARIABLES ARE THEN REQUIRED AS
C INPUT.  REC1 CALLS THE IMSL NONLINEAR SOLVER NEQNF TO SOLVE
C THE SYSTEM OF EQUATIONS.
C
C ALL RECTIFIER SOLUTIONS ARE PRACTICALLY INDEPENDENT OF
C X(7) (X/R) FOR X/R > 5.  FOR CIRCUITS KNOWN TO BE
C PREDOMINATELY REACTIVE X(7) MAY BE SET EQUAL TO 20.
C
C
      PARAMETER (N=7)
      INTEGER LBL1(7)/'VD','ELL','A','U',
     -'X','ID','NU'/
      COMPLEX*16 LBL2(7)/'DC LINE VOLTS',
     -'AC RMS L-L VOLTS','DEG,FIRING ANGLE',
     -'DEG, COMM. ANGLE','OHMS, REACTANCE','DC LINE AMPERES',
     -'AC SYS X/R RATIO'/
      INTEGER TITLE1(80),TITLE2(80)
      REAL FCN,FNORM,X(N),XGUESS(N)
      EXTERNAL FCN,NEQNF
      COMMON /GROUP1/ P(7),IO1,IO2
      NAMELIST /RADAR/NZ
      OPEN (5,FILE='REC1.DATA',STATUS='OLD')
      OPEN (6,FILE='REC1.OUT',STATUS='NEW')
      OPEN (9,FILE='REC1.NMLST',STATUS='NEW')
      NZ=0
C ---------------------------------------------------------------------
      NZ=NZ+1
      WRITE (9,RADAR)
      READ (5,130) TITLE1
      READ (5,130) TITLE2
130   FORMAT (80A1)
C READ TWO OUTPUT VARIABLE INTEGER SUBSCRIPTS
      READ (5,*) IO1,IO2
C READ MAX ITERATIONS AND CONVERGENCE CRITERIA
      READ (5,*) ITMAX,ERRREL
C READ CIRCUIT DATA AND ESTIMATES FOR OUTPUT VARIABLES
      DO 12 J=1,N
      READ (5,*) XGUESS(J)
```

```
C CONVERT ANGLES FROM DEGREES TO RADIANS
      IF (J.EQ.3.OR.J.EQ.4) XGUESS(J)=.01745*XGUESS(J)
      IF (XGUESS(J).LT.1.E-05) XGUESS(J)=1.E-05
      P(J)=XGUESS(J)
12    CONTINUE
C ---------------------------------------------------------------------
      NZ=NZ+1
      WRITE (9,RADAR)
C
C CALL NONLINEAR SYSTEM SOLVER NEQNF FROM IMSL MATHEMATICS LIBRARY
C
      CALL NEQNF(FCN,ERRREL,N,ITMAX,XGUESS,X,FNORM)
C
C ---------------------------------------------------------------------
      NZ=NZ+1
      WRITE (9,RADAR)
C
C WRITE OUTPUT TO REC1.OUT
C
C CONVERT ANGLES FROM RADIANS TO DEGREES
      DO 28 J=3,4
      P(J)=57.2958*P(J)
28    X(J)=57.2958*X(J)
      WRITE (6,21)
21    FORMAT (////,3X,'REC1.FOR -- 3-PHASE BRIDGE RECTIFIER
     - CIRCUIT ANALYSIS')
      WRITE (6,23)
23    FORMAT (///)
      WRITE (6,130) TITLE1
      WRITE (6,130) TITLE2
      WRITE (6,133)
133   FORMAT (///,2X,'INPUT VARIABLES:')
      DO 100 J=1,N
      IF (J.NE.IO1.AND.J.NE.IO2) WRITE (6,145)
     -LBL1(J),P(J),LBL2(J)
100   CONTINUE
145   FORMAT (10X,A4,'= ',E11.4,3X,2A8)
      WRITE (6,150)
150   FORMAT (///,2X,'OUTPUT VARIABLE ESTIMATES:')
      WRITE (6,160) IO1,LBL1(IO1),P(IO1),LBL2(IO1)
      WRITE (6,160) IO2,LBL1(IO2),P(IO2),LBL2(IO2)
160   FORMAT (/,1X,I1,8X,A4,'= ',E11.4,3X,2A8)
      WRITE (6,169) N,ITMAX,ERRREL
169   FORMAT (//,3X,'N = ',I1,3X,'ITMAX = ',I3,
     -3X,'ERRREL = ',F8.7)
      WRITE (6,170)
170   FORMAT (////,3X,'3-PH, 6-PULSE  RECTIFIER CIRCUIT SOLUTION:',/)
      DO 29 J=1,N
      IF (J.EQ.IO1.OR.J.EQ.IO2) GOTO 37
      WRITE (6,177) LBL1(J),X(J),LBL2(J)
      GOTO 29
37    WRITE (6,178) LBL1(J),X(J),LBL2(J)
178   FORMAT (2X,'SOLN:',3X,A4,'= ',E11.4,3X,2A8)
29    CONTINUE
177   FORMAT (10X,A4,'= ',E11.4,3X,2A8)
      WRITE (6,39) FNORM
39    FORMAT (//,10X,'FNORM = ',E11.4)
```

```
C
      ENDFILE(6)
      ENDFILE(9)
      CLOSE(5)
      CLOSE(6)
      CLOSE(9)
      CALL EXIT
      END
```

PROGRAM SECTIONS

	Name	Bytes	Attributes
0	$CODE	1077	PIC CON REL LCL SHR EXE RD NOWRT LONG
1	$PDATA	370	PIC CON REL LCL SHR NOEXE RD NOWRT LONG
2	$LOCAL	1060	PIC CON REL LCL NOSHR NOEXE RD WRT QUAD
3	GROUP1	36	PIC OVR REL GBL SHR NOEXE RD WRT LONG
	Total Space Allocated	2543	

ENTRY POINTS

Address	Type	Name
0-00000000		REC1$MAIN

VARIABLES

Address	Type	Name	Address	Type	Name	Address	Type	Name	Address	Type	Name
2-00000350	R*4	ERRREL	2-00000344	R*4	FNORM	3-0000001C	I*4	IO1	3-00000020	I*4	IO2
2-0000034C	I*4	ITMAX	**	I*4	J	2-00000348	I*4	NZ			

ARRAYS

Address	Type	Name	Bytes	Dimensions
2-00000070	I*4	LBL1	28	(7)
2-00000000	C*16	LBL2	112	(7)
3-00000000	R*4	P	28	(7)
2-0000008C	I*4	TITLE1	320	(80)
2-000001CC	I*4	TITLE2	320	(80)
2-0000030C	R*4	X	28	(7)
2-00000328	R*4	XGUESS	28	(7)

```
C
C
      SUBROUTINE FCN(X,F,N)
      REAL SIN,ATAN,EXP,SQRT
      REAL X(N),F(N)
      INTEGER N,IO1,IO2
      INTRINSIC SIN,ATAN,EXP,SQRT
      COMMON /GROUP1/ P(7),IO1,IO2
C
C THE SYSTEM OF 2 NONLINEAR EQUATIONS IS CONTAINED
C IN F(IO1) AND F(IO2)
C
      A1=.6752*X(2)*(COS(X(3))+COS(X(3)+X(4)))
      A2=.4775*X(5)*X(6)*(-X(4)+4.1888)/X(7)
C
      F(IO1)=X(1)-A1+A2
C
      B1=SQRT(1.+1./(X(7)*X(7)))
      B2=SIN(X(3)+X(4)-ATAN(X(7)))
      B3=EXP(-X(4)/X(7))*SIN(X(3)-ATAN(X(7)))
      B4=.7071*(X(5)*X(6)/X(2))*(X(7)*X(7)/(1.+X(7)*X(7)))
      B5=1.+EXP(-X(4)/X(7))
C
      F(IO2)=B1*(B2-B3)-B4*B5
C
C THE REMAINING 5 EQUATIONS ARE DETERMINED BY THE
C 5 INPUT VARIABLES:
C
      DO 11 I=1,N
      IF (I.NE.IO1.AND.I.NE.IO2) F(I)=(P(I)-X(I))/P(I)
11    CONTINUE
      RETURN
      END
```

```
  -- EXAMPLE RUN - SOLVE FOR VD AND U --
 RECTIFIER CIRCUIT SOLUTIONS
1 4             (TWO UNKNOWNS)
500 .000005
 28.                 1    DC VOLTS
 24.7                2    AC RMS VOLTS
 0.                  3    ALPHA
 15.                 4    U
 .0006               5    X
 3700.               6    DC AMPS
 1.8                 7    X/R RATIO
```

```
  REC1.FOR -- 3-PHASE BRIDGE RECTIFIER CIRCUIT ANALYSIS


 -- EXAMPLE RUN - SOLVE FOR VD AND U --
RECTIFIER CIRCUIT SOLUTIONS


 INPUT VARIABLES:
         ELL =  0.2470E+02   AC RMS L-L VOLTS
         A   =  0.5730E-03   DEG,FIRING ANGLE
         X   =  0.6000E-03   OHMS, REACTANCE
         ID  =  0.3700E+04   DC LINE AMPERES
         NU  =  0.1800E+01   AC SYS X/R RATIO


 OUTPUT VARIABLE ESTIMATES:

1        VD  =  0.2800E+02   DC LINE VOLTS

4        U   =  0.1500E+02   DEG, COMM. ANGLE

  N = 7   ITMAX = 500   ERRREL = .0000050


  3-PH, 6-PULSE  RECTIFIER CIRCUIT SOLUTION:

 SOLN:   VD  =  0.2991E+02   DC LINE VOLTS
         ELL =  0.2470E+02   AC RMS L-L VOLTS
         A   =  0.5730E-03   DEG,FIRING ANGLE
 SOLN:   U   =  0.2187E+02   DEG, COMM. ANGLE
         X   =  0.6000E-03   OHMS, REACTANCE
         ID  =  0.3700E+04   DC LINE AMPERES
         NU  =  0.1800E+01   AC SYS X/R RATIO

         FNORM =  0.7542E-09
```

```
 REC1.FOR -- 3-PHASE BRIDGE RECTIFIER CIRCUIT ANALYSIS


 -- EXAMPLE RUN - SOLVE FOR VD AND U --
RECTIFIER CIRCUIT SOLUTIONS


 INPUT VARIABLES:
         ELL =  0.2470E+02   AC RMS L-L VOLTS
         A   =  0.5730E-03   DEG,FIRING ANGLE
         X   =  0.6000E-03   OHMS, REACTANCE
         ID  =  0.3700E+04   DC LINE AMPERES
         NU  =  0.5000E+02   AC SYS X/R RATIO


 OUTPUT VARIABLE ESTIMATES:

1        VD  =  0.2800E+02   DC LINE VOLTS

4        U   =  0.1500E+02   DEG, COMM. ANGLE

  N = 7   ITMAX = 500   ERRREL = .0000050


  3-PH, 6-PULSE  RECTIFIER CIRCUIT SOLUTION:

 SOLN:   VD  =  0.3116E+02   DC LINE VOLTS
         ELL =  0.2470E+02   AC RMS L-L VOLTS
         A   =  0.5730E-03   DEG,FIRING ANGLE
 SOLN:   U   =  0.2917E+02   DEG, COMM. ANGLE
         X   =  0.6000E-03   OHMS, REACTANCE
         ID  =  0.3700E+04   DC LINE AMPERES
         NU  =  0.5000E+02   AC SYS X/R RATIO

         FNORM =  0.1096E-07
```

REFERENCES

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3. Kloss, Albert, A Basic Guide to Power Electronics, John Wiley & Sons, c. 1984, pp. 82 - 84.

4. IMSL Math/Library User's Manual -- Fortran Subroutines for Mathematical Applications, Version 1.0, c. 1987 by IMSL Inc., pp. 776 - 779.