

## The Circular RFQ Storage Ring

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**Abstract.** This paper presents a novel idea of storage ring for the accumulation of intense beams of light and heavy ions at low energy. The new concept is a natural development of the combined features used in a conventional storage ring and an ion trap, and is basically a linear RFQ bend on itself. In summary the advantages are: smaller beam dimensions, higher beam intensity, and a more compact storage device.

## INTRODUCTION

There is need to develop compact storage rings for the accumulation of low-energy beams of ions for a variety of applications; namely: molecular and atomic physics, solid state, chemical-physics, astrophysics, and other more exotic applications like crystalline beams and ion fusion for energy production. Several experimental apparatus can be conceived making use of these compact storage rings, for instance: colliding beams circulating in two intersecting storage rings, collision of a stored beam of ions with an internal target, head-on collision with an electron beam or a X-ray beam from a synchrotron radiation source. Typical requirements are high beam intensity and density. Also, very small energy spreads are sought which can be achieved with cooling techniques like electron and laser cooling.

In this report we describe a novel idea of storage ring for the accumulation of intense and dense beams of light and heavy ions in a more compact structure. The concept takes advantages of established principles of operation of conventional low-energy storage rings, ion traps, and RFQ's. The proposed new storage ring is basically a circular RFQ bend on itself and closed mechanically. Instead of quadrupole magnets, focusing of the particles is provided by the rf field of the structure. Since electrically the structure is not closed on itself, it is expected that ion beams can be stored at intensities and densities higher than those achieved in conventional storage rings.

There is a limit on the intensity that can be stored in a conventional storage ring like ASTRID. This is given by the space-charge tune-depression  $\Delta v$  given by the formula

$$\Delta v = N Q^2 r_0 b / 2 A \beta^2 \gamma^3 \varepsilon \quad (1)$$

where  $r_0 = 1.535 \times 10^{-18}$  m is the classical proton radius,  $Q$  the ion charge state,  $A$  its mass number,  $\beta$  and  $\gamma$  respectively the velocity and energy relativistic factors,  $b$  is a bunching factor, that is the ratio of beam peak current to average current,  $N$  the number of ions stored, and, finally,  $\varepsilon$  is the beam emittance, that is the area occupied by the beam in the  $(x, x')$ -phase space (do not forget to include the factor  $\pi$ ). For practical purposes the tune-depression  $\Delta v$  cannot exceed a value at most as large as 0.5, but that more typically is taken at around 0.2. This limit is understood to be set by the presence of unavoidable random magnet imperfections which cause the lowering of the ring periodicity to essentially a unit, and the creation of stopbands around the half-integral tune values which cannot be crossed without substantial beam losses.

To simplify our concepts we shall consider below the case of beams of protons ( $Q = A = 1$ ) completely debunched ( $b = 1$ ) at the kinetic energy of 100 keV ( $\beta = 0.0146$ ,  $B\rho = 45.7$  kG-cm) and a tune-depression  $\Delta v = 0.2$ . From Equation (1) we derive the transverse phase-space density at the space-charge limit

$$D = N / \varepsilon < 2 \beta^2 \gamma^3 \Delta v / r_0 \quad (2)$$

For our case  $D = 5.6 \times 10^{13} \text{ m}^{-1}$ . To be noticed that the phase-space density of Equation (2) does not depend on the dimension of the storage ring, but only on the beam energy.

A more relevant parameter to several experimental applications is the actual beam transverse density in the physical space  $D_S = N / S$ , where  $S$  is the beam cross-section area. For "circular" beams  $S \sim \pi a^2$ , where  $a$  is the radius of the beam cross-section. There is a relation between the beam radius  $a$  and the emittance  $\varepsilon$ , so that

$$S = \beta_L \varepsilon \quad (3)$$

and

$$D_S = D / \beta_L \quad (4)$$

The amplitude lattice function  $\beta_L$  for ASTRID is plotted in Fig. 2. The average value is about 5 meter, so that for our example the physical density that can be achieved at the space charge limit is  $D_S = 1.1 \times 10^9 \text{ cm}^{-2}$ . In a conventional storage ring, the amplitude lattice function  $\beta_L$  is a measure of the strength of focusing. The stronger the focusing, the smaller is the amplitude lattice function, and the smaller is the beam cross-section. The focusing strength is increased by placing quadrupoles closer to each other. Actually the average value of  $\beta_L$  is given by the length of the focusing period: the distance

## A CONVENTIONAL STORAGE RING

An example of a low-energy and small storage ring is ASTRID (1), used for the accumulation of light and heavy ions and for the demonstration of Laser Cooling. The ring magnetic rigidity is 1.87 T-m, from which is then possible to estimate the maximum beam energy for different ion species. The circumference is about 40 m. As shown in Fig. 1, the ring is made of eight dipole magnets which bend the beam trajectory on a circular and closed orbit and sixteen quadrupole magnets for transverse focusing. The quadrupoles are arranged in eight doublets. There are four periods each with a straight section of about 4 meter length free of magnets. As an example, beam of ions  ${}^7\text{Li}^+$  and  ${}^{24}\text{Mg}^+$  have been stored at the energy of 100 keV for Laser Cooling experiments. Stored currents were in the 1-10  $\mu\text{A}$  range.

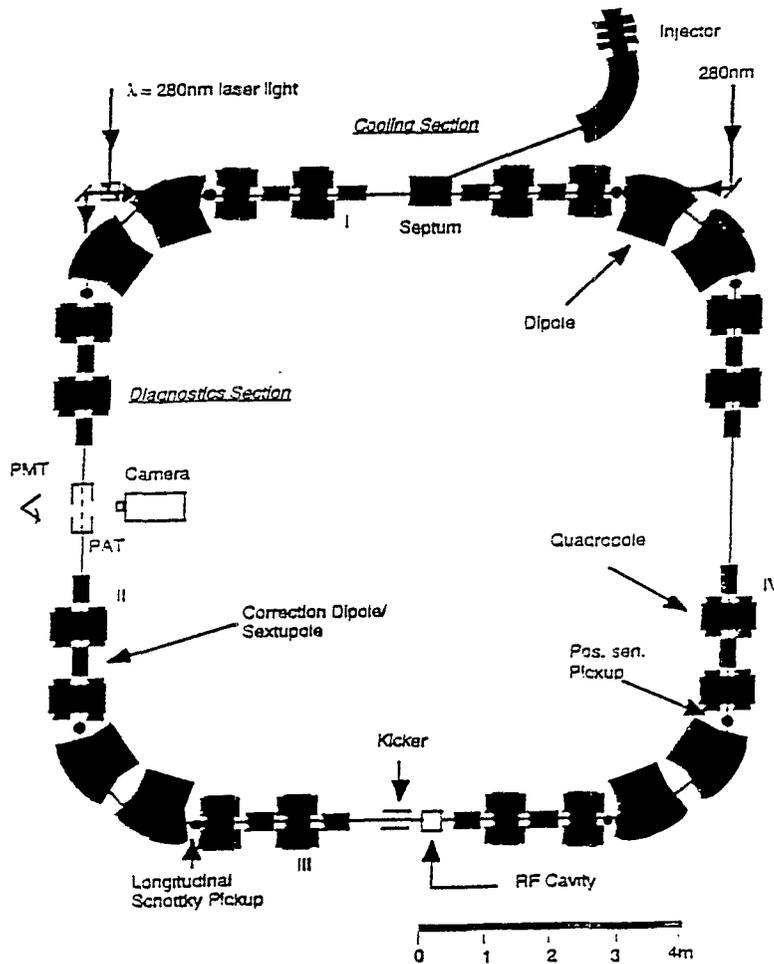


FIGURE 1. The ASTRID Storage Ring

between doublets in ASTRID, or half of the cell length in a FODO structure. Unfortunately, in a conventional storage ring like ASTRID, the average value of  $\beta_L$  around the ring can hardly be less than few meters; in fact, quadrupoles have a significant length and space between quadrupoles is required to accommodate several functions, for instance, bending the beam trajectory.

In conclusion, the physical density that can be achieved in a conventional low-energy storage ring is limited, first, by the largest amount of space-charge tune-depression according to Equations (1 and 2) and, second, by the strength of focusing according to Equations (3 and 4).

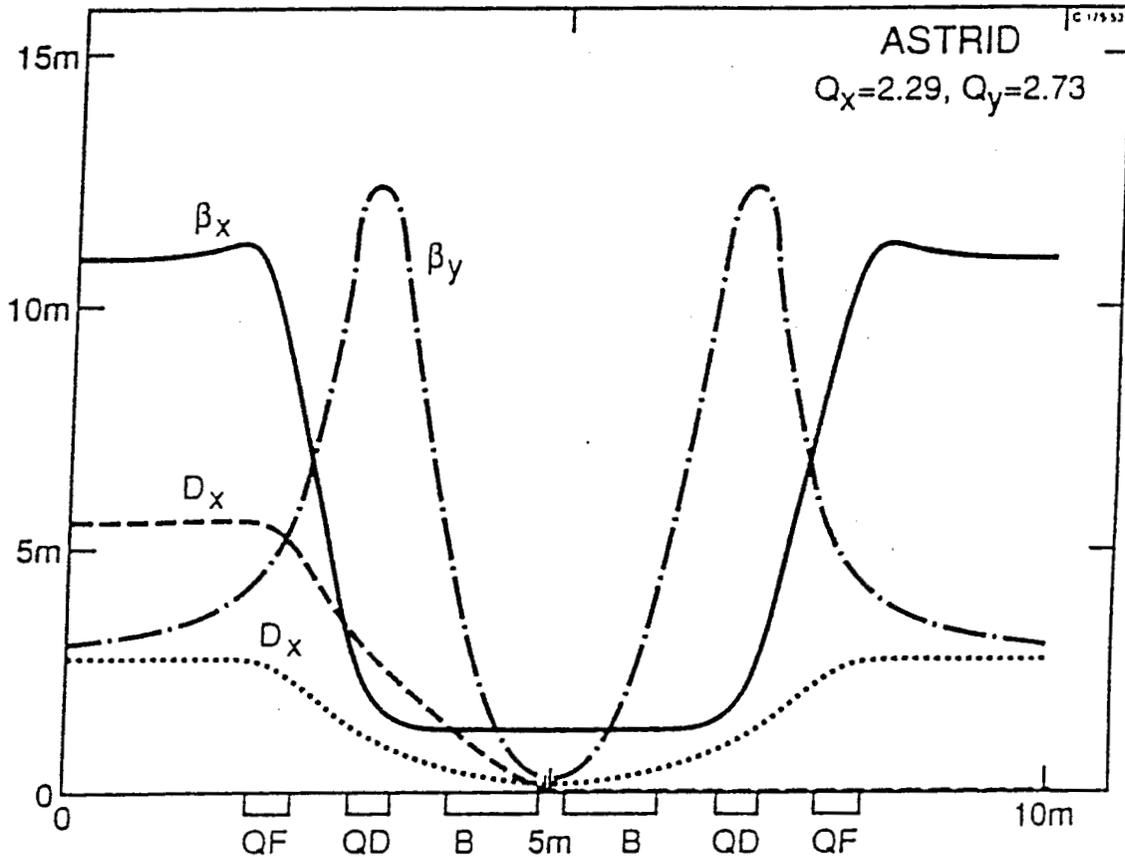


FIGURE 2. The Lattice Functions in ASTRID

## THE LINEAR RADIO-FREQUENCY QUADRUPOLE

Another accelerator device is the Radio-Frequency Quadrupole (RFQ) which is used at the front end of a Linac. The device accelerates quickly the beam from an ion source to energies large enough when space-charge effects are considerably reduced, and provides simultaneously focusing of the transverse motion of the particles. The RFQ is not a magnetic device, but employs an alternating rf electric field for both the acceleration and the focusing of the particles. It is a straight waveguide, as shown in Fig. 3, with four internal metallic rods. An rf field at the frequency  $f = \omega/2\pi$  is applied between diagonally opposite electrodes, as shown in Fig. 4, to generate in the opening a quadrupolar oscillating field of the same frequency. The motion of the particle will apparently modulate the actual field sequentially in time, creating the equivalent of an alternating focusing transverse field with periodicity  $L = \beta\lambda$ , where  $\beta$  is the particle velocity, and  $\lambda = 2\pi c/\omega$  is the rf wavelength. Acceleration is provided by an axial oscillating electric field which is introduced by shaping the four electrodes with corrugations of the same length  $L$ , as shown in Fig. 3. As a consequence of acceleration, the beam, that is supposedly entering the RFQ with no time structure, will leave the RFQ at the other end bunched. If acceleration is not required, but only transverse focusing, which is the case of interest of our considerations here, the corrugation of the electrodes is not required, since the axial field is also not required, and the electrodes will appear just straight, as shown in Fig. 5. In this case the excitation is not an rf wave travelling down the waveguide structure, but just a stationary oscillating rf field between the pairs of electrodes.

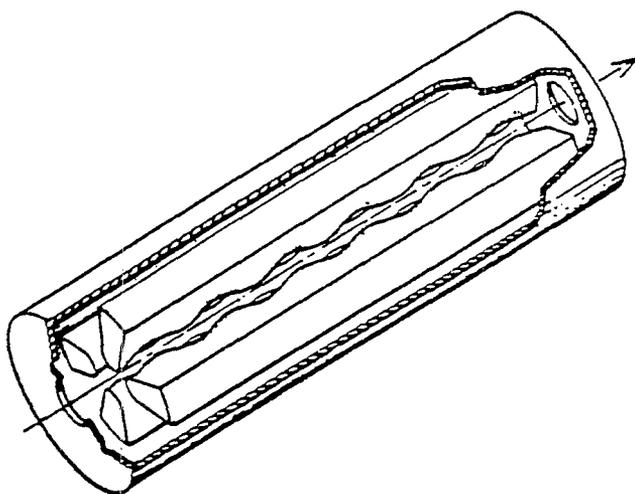
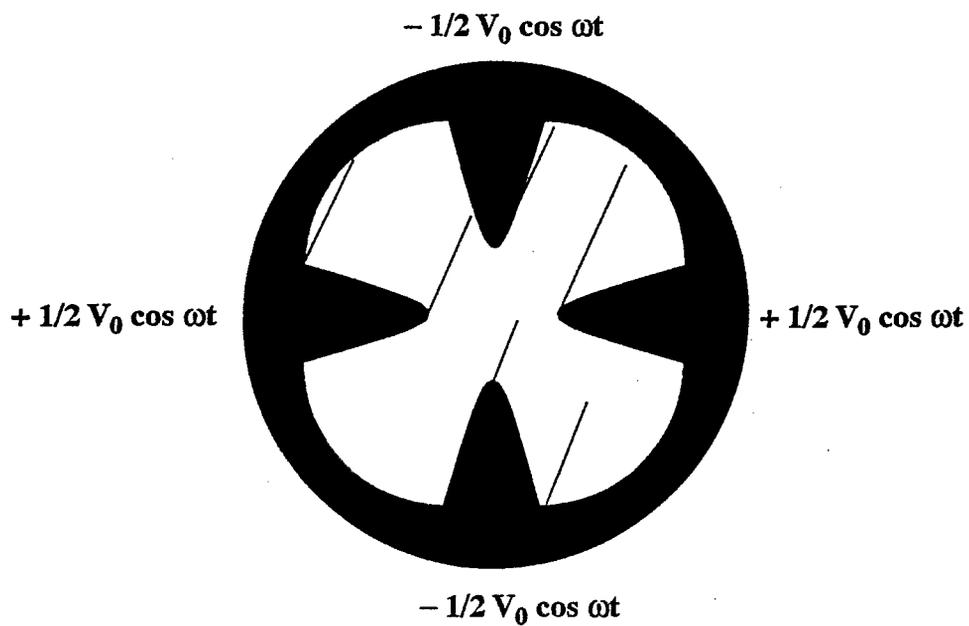
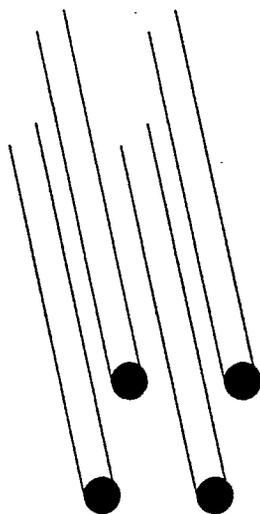


FIGURE 3. Four-Vane RFQ (Linear) Corrugated Structure



**FIGURE 4.** Cross-section of a RFQ



**FIGURE 5.** Four-Vane RFQ without corrugation for Transverse Focusing only

In the case without acceleration, the equation of motion (2) of a particle of mass at rest  $m$  and electric charge  $e$  is

$$m \frac{d^2 x}{dt^2} = (eV_0 / b^2) x \cos(\omega t) \quad (5)$$

where  $x$  is the lateral displacement, horizontal or vertical,  $2b$  the internal diameter, and  $V_0$  the peak rf voltage. Introduce the dimensionless RFQ parameter

$$B_0 = (eV_0 / mc^2) (\lambda / b)^2 \quad (6)$$

with the substitutions  $B_0 = 2\pi^2 q$ ,  $z = \beta ct$ , and  $\theta = \pi z / L$ , Equation (5) transforms to

$$\frac{d^2 x}{d\theta^2} - 2q x \cos(2\theta) = 0 \quad (7)$$

which clearly shows the alternating behavior of the focusing of the device with periodicity  $L = \beta\lambda$ . The phase advance per period depends only on the RFQ parameter  $B_0$ . For example, a phase advance of  $90^\circ$  per period  $L$  is obtained with  $B_0 = 6.812$  or  $q = 0.345$ .

In our example of a proton beam with the kinetic energy of 100 keV, adopting an rf field at 200 MHz, that is  $\lambda = 150$  cm, the periodicity of the focusing is  $L = 2.2$  cm which is considerably shorter than the one can be obtained in a conventional storage ring. Correspondingly also the average value of the amplitude lattice function  $\beta_L$  is significantly reduced since  $\beta_L \sim L$ . Thus with the same beam emittance and energy the beam size is also considerably smaller when compared to the beam dimension in a conventional storage ring.

There is another interesting feature of the RFQ. It is an open structure, both mechanically and electrically, where the beam enters one end and leaves the other. There is no global periodicity involved, like in the magnetic storage ring. There are no intrinsic imperfection resonances to worry about. It is thus legitimate to expect that considerably higher beam intensities can be transported since the conventional storage ring space-charge limitation Equation (1) does not apply to a RFQ.

Space charge detracts of course from the focusing strength an amount  $\Delta$  proportional to the beam density (2). The equation of motion modifies as follows

$$\frac{d^2 x}{d\theta^2} - 2q x \cos(2\theta) - \Delta x = 0 \quad (8)$$

where, for a uniform transverse and longitudinal distribution,

$$\Delta = 2 N r_0 \lambda^2 / (\pi^2 a^2 C) \quad (9)$$

with  $a$  the beam radius. There are  $N$  particles traversing at one time the length  $C$  of the RFQ. The maximum intensity that can be transported corresponds to a depression of the

phase advance per period to about  $45^\circ$ . This yields  $\Delta \sim 0.044$ . For our example, taking the RFQ length to be  $C = 40$  m (the circumference of ASTRID), we have the physical beam density limit  $D_S = N / \pi a^2 = 2.5 \times 10^{13} \text{ cm}^{-2}$ , that is four orders of magnitude larger than that it can be obtained in a conventional storage ring like ASTRID.

## AN ION TRAP

This is shown in Fig. 6. It is also referred to as a quadrupole storage ring (3). The diameter of the device is of only 12 cm. It is made of four annular electrodes with the cross-section as shown in the same figure. The internal diameter is 5 mm. An electrostatic voltage is applied between each pair of diagonally opposing electrodes. This generates a constant radial electric field which vanishes at the center and increases about linearly with the distance from the main axis. An atomic gas of the desired ion species is diffused in the region between the electrodes. An electron gun ionizes the atoms, and the resulting ions are trapped transversely in the small storage ring, oscillating around the circular main axis under the effect of the restoring forces of the electrostatic quadrupole. There is no beam in this configuration like in the previous more conventional magnetic storage ring, or in the linear Radio-Frequency Quadrupole, since the ions do not drift azimuthally along the main axis. The particles adjust their mutual longitudinal distance by Coulomb interaction, whereas the transverse interaction is compensated by the external quadrupolar forces. In a similar trap, ions of  $^{24}\text{Mg}^+$  were cooled transversely with a laser beam. A crystalline formation was then observed were the ions arrange rigidly with respect to each other at a distance of about  $20 \mu\text{m}$  (see Fig. 7).

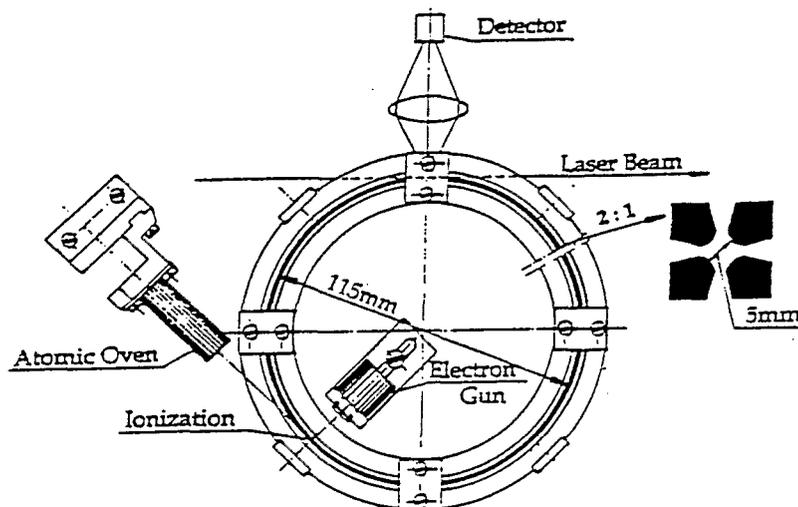


FIGURE 6. The Ion Trap. (The Quadrupole Storage Ring).

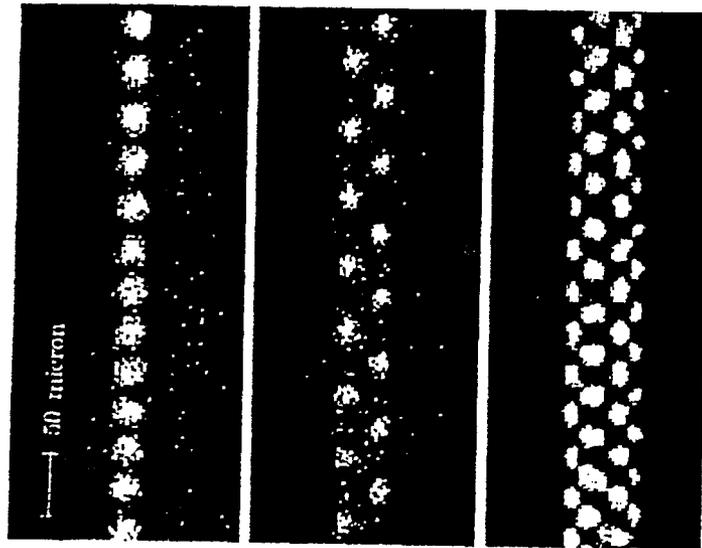


FIGURE 7. Observed Crystalline Structures in an Ion Trap

## THE CIRCULAR RADIO-FREQUENCY QUADRUPOLE STORAGE RING (CRFQ)

The Circular Radio-Frequency Quadrupole (CRFQ) is a compact storage device which includes features of all the devices described above. It is a storage ring where beams of light and heavy ions (including protons and negative ions) can circulate at constant speed corresponding to energies comparable to those used for ASTRID. Like in the Ion Trap, focussing is provided by four annular electrodes. Like in the linear RFQ, rf oscillating voltages are applied between the electrodes to provide transverse alternating focusing of motion over a short period of about few centimeters. From this point of view, the CRFQ resembles the linear RFQ without acceleration, and without, therefore, corrugated vanes. The CRFQ is actually a linear RFQ curved and closed mechanically on itself as shown in Figs 8 and 9. The CRFQ can be used for a variety of applications where very intense and dense beams of ions are required. Some of these applications are in common with both the ASTRID storage ring and the Ion Trap. They range from molecular, atomic and ion physics, to Laser Cooling with the formation of Crystalline Beams, and to the demonstration of Intersecting and Colliding Beams for the study of Nuclear Fusion of light ions.

Below we shall describe typical parameters and performance of a CRFQ where a single beam of protons is circulating at the kinetic energy of 100 keV. In particular we shall explain how it is possible to store in this device ion beams with considerably higher intensity and density, well beyond the capability of conventional storage rings. We shall also describe how ions can be kept circulating in the device.

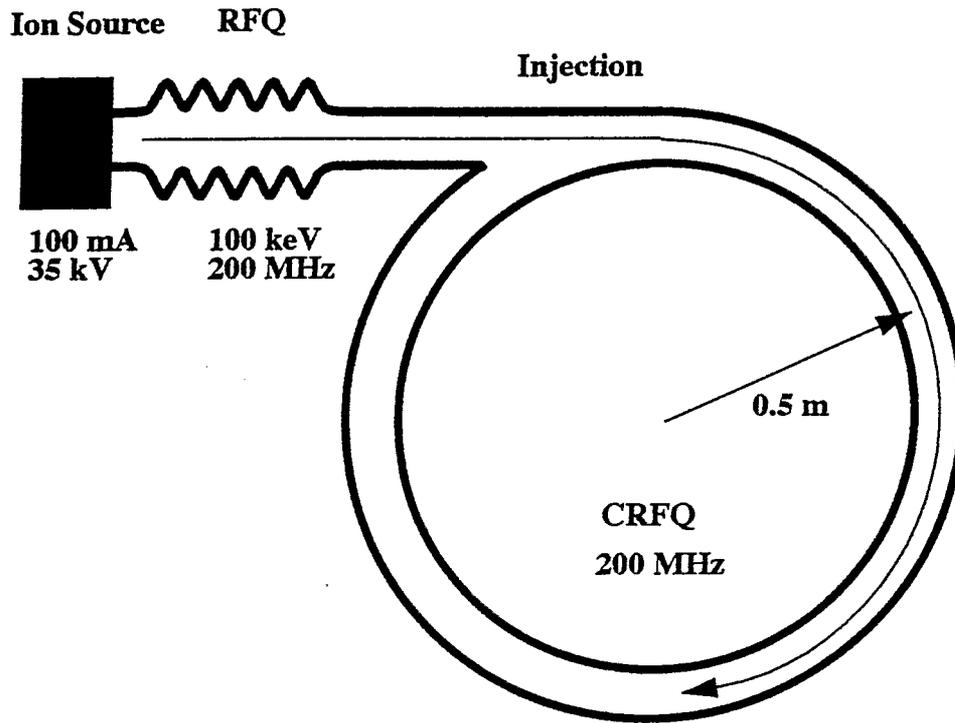


FIGURE 8. Schematic layout of a Circular Radio-Frequency Storage Ring (CRFQ)

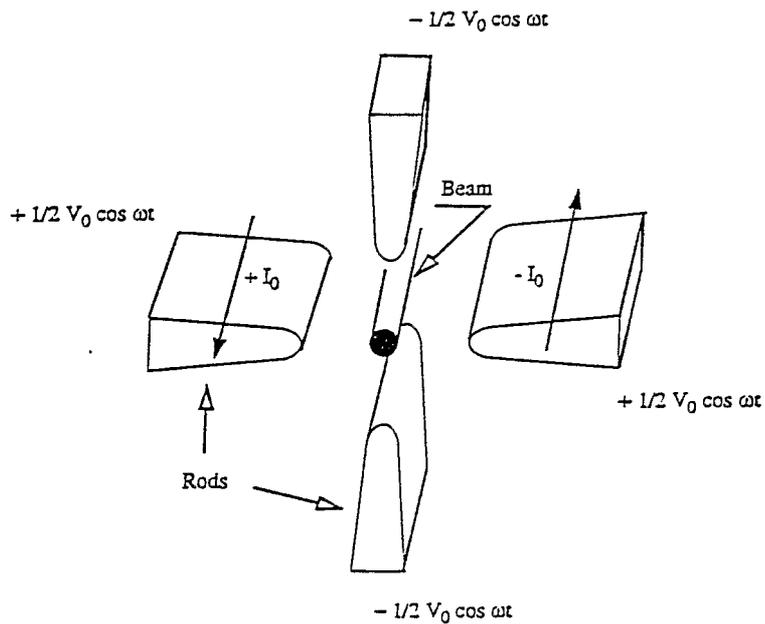


FIGURE 9. Cross-section view of the CRFQ. A transverse rf field is excited between the electrodes. The rf excitation is as shown. DC current  $I_0$  flows along the outer electrodes generating a magnetic field to bend the beam circulating in the region inside.

## EQUATIONS OF MOTION AND CONFINEMENT IN THE CRFQ

It is required to keep the ions on a circular orbit, centered with the azimuthal axis of the CRFQ. This can be accomplished in two ways. The CRFQ can be located inside and on the midplane of a pure 360° circular dipole magnet. Or, electric currents can be made to flow along and in the opposite directions of the two horizontal electrodes as shown in Fig. 9. The electric currents generate then a bending field in the internal region of the CRFQ. In the following we shall adopt the latter solution. Obviously the bending field is to be adjusted to match a reference ion momentum  $p_0$ .

We shall assume in this section that the major radius of the CRFQ, that is the radius of curvature  $R$ , is much larger than the focusing period  $L = \beta\lambda$ , and that this is in turn larger than the internal diameter  $2b$ . We can then neglect the contribution of the curvature to the rf focusing field. The equation of motion in vertical direction  $y$  is then still given by Equations (5 to 7) as in the linear RFQ

$$d^2 y / d\theta^2 - 2q y \cos(2\theta) = 0 \quad (10)$$

There is in addition, in the radial plane, an extra focusing term due to the curvature of the trajectory, namely

$$d^2 x / d\theta^2 + 2q x \cos(2\theta) + x \beta^2 \lambda^2 / \pi R^2 = \delta \beta^2 \lambda^2 / \pi R \quad (11)$$

where  $\delta$  is the particle relative momentum deviation from the reference value  $p_0$ . The last term is also the source of dispersion. Both the contribution to the focusing of the curvature as well the dispersion are very small when compared to the focusing from the main  $q$ -factor, and, therefore, will be ignored in the following.

In the case the beam is completely debunched and no longitudinal forces are applied, the azimuthal motion of the ions is an angular precession movement at the angular frequency

$$\omega_0 = 2\pi f_0 = \beta c / R. \quad (12)$$

All the considerations made for the linear RFQ apply also to the CRFQ. For instance, the alternating focusing period is  $L = \beta\lambda$ , and the  $q$  parameter, related to the RFQ parameter  $B_0$  of Equation (6) by the relation  $B_0 = 2\pi^2 q$ , determines the phase advance per period and the amplitude lattice function  $\beta_L \sim L$ . As an example, the phase advance of 90° requires  $q = 0.345$ .

Thus, the Circular Radio-Frequency Storage Ring has the advantage to provide a very short focusing alternating period of only few centimeters, considerably smaller than that it can be obtained in a conventional storage ring like ASTRID.

## SPACE-CHARGE LIMITATIONS IN THE CRFQ

The other major advantage for the use of the CRFQ storage ring is the larger beam intensity that it can be stored when compared to a conventional storage ring of the same energy. In a conventional storage ring, like ASTRID, we have seen that there is a limit caused by the Space-Charge Tune-Depression  $\Delta v$  that for practical purposes cannot exceed a value at most as large as 0.5. This limit is understood to be set by the presence of random magnet imperfections which cause the lowering of the ring periodicity to essentially unit, and the creation of stopbands around half-integral tune values which cannot be crossed without substantial beam losses.

In the CRFQ the situation is very different and we believe identical to that encountered in the linear RFQ. First of all, the focusing periodicity is very high and cannot be broken down by magnetic imperfections that do not exist. Possible errors on the focusing can of course be introduced because of the limited accuracy of the placement of the electrodes, but these errors are of different nature and similar to those investigated in a linear RFQ. Secondly, the structure, though mechanically is closed on itself, is electrically open. Turn after turn, the CRFQ is just like a long transport (unless the total betatron tune per revolution is exactly an integer, a situation that is to be avoided, as resonances are in this case created). One can then make the analogy with the linear RFQ where, as we have seen, the Space-Charge limit is caused by lowering the phase advance per period from  $90^\circ$  down to  $45^\circ$  (or less), below which the particle motion may become unstable.

Neglecting again the curvature of the ring, to take into account the finite beam intensity, the equations of motion are to be modified according to Equation (8) with the space-charge parameter  $\Delta$  given again by Equation (9), where  $C = 2\pi R$  is now the circumference of the ring, and  $N$  is the total number of particles stored. The same limit for space charge  $\Delta \sim 0.044$  is assumed. If the CRFQ storage ring has the same circumference of ASTRID (40 m) the physical beam density that can be accepted is again

$$D_S = N / \pi a^2 = 2.5 \times 10^{13} \text{ cm}^{-2}.$$

### AN EXAMPLE OF CRFQ

The main purpose of the following example of CRFQ is the construction of a prototype device to demonstrate the two basic principles: (i) that it is possible to achieve very short alternating focusing structure with a period of few centimeters, and (ii) that it is possible to store ion beams at intensities higher than those that can be achieved in conventional storage rings. We shall take again a proton beam at 100 keV and an rf of 200 MHz. The ring radius is  $R = 50$  cm. The major parameters are listed in Table 1. The transverse electric field is around 21 MV/m, which at the chosen frequency, is just below one kilpatrick unit, that is below the expected surface limit.

To bend the beam trajectory, one needs a magnetic field of 0.92 kG which can be obtained by letting 1.7 kA current flow on the electrodes on the midplane of the CRFQ.

The physical density which corresponds to the space-charge limit (RFQ) is  $N_S = 2 \times 10^{12} \text{ cm}^{-2}$ . With a beam normalized emittance of  $1 \pi \text{ cm mrad}$ , the average beam radius at the space-charge limit is  $a = 6 \text{ mm}$ , so that about  $2 \times 10^{12}$  protons can be stored in the ring. This is about 20 times larger than the conventional limit given by Equation (1) with  $\Delta v = 0.2$ . Thus an experiment can be easily done to verify that indeed it is possible to store considerable more current in the CRFQ storage ring than in a conventional storage ring.

Table 1 shows that the circulating current at the RFQ space-charge limit is 460 mA, and only 20 mA if the conventional limit of Equation (1) should apply. An ion source, operating at a very low duty cycle, is certainly capable to produce a beam pulse of about  $0.5 \mu\text{s}$  duration in excess of 100 mA.

The pulse duration is shorter than the revolution period so that only one turn need to be injected in the CRFQ. The actual implementation of the single turn injection in the CRFQ device is a topic that still needs to be studied.

The ion source can be placed on a platform at 35 kV. The energy difference to 100 keV can be obtained by accelerating the beam in a short linear RFQ operating also at 200 MHz. At the end of the acceleration, the vanes of the linear RFQ are no longer corrugated, and only focusing is then provided. This linear section would then merge with and match to the entrance of the CRFQ.

The experiment can be performed at a very low duty cycle, for example, with a beam pulse injected and stored in the CRFQ every few minutes. There is thus no much beam power involved and the beam itself can directly be disposed by turning off the bending field. It would be also very useful to learn how to extract in a single turn and in a desired direction the stored beam.

In conclusion, the experimental setup includes an Ion Source, a linear RFQ, a matching section between the RFQ and the CRFQ, an injection system to be developed, a 200-MHz rf source, a 2-kA dc-current source to bend the trajectory, and, of course, the CRFQ itself. The device is to be complemented with a vacuum, a beam diagnostic system, and control. One needs to study and to determine the construction tolerances of the four annular electrodes, the engineering of the grounded enclosure, and the rf and dc powering of the electrodes.

## CONCLUSION

We have described a new concept of Storage Ring for low-energy ion beams. The principle of operation of the new device is similar to that of an ordinary RFQ, except that it is mechanically bent on itself. It is then possible to achieve very short alternating focusing periods and also to store considerable higher beam intensities well beyond the ordinary space-charge limit of conventional storage rings.

We have also outlined the design of a prototype that can be used for the demonstration of the new concept.

**TABLE 1.** Parameters of the proposed CRFQ

Kinetic Energy	100		keV
$\beta$	0.0146		
Magnetic Rigidity	45.7		kG - cm
Major Radius, R	50		cm
Minor Radius, b	7.5		mm
rf frequency, f	200		MHz
rf wavelength, $\lambda$	150		cm
RFQ parameter, $B_0$	6.812		
q	0.345		
peak rf Voltage, $V_0$	160		kVolt
Electric Field	21.33		MeV/m
Periodicity, $L = \beta\lambda$	2.2		cm
Space-Charge $\Delta$	0.0	0.044	
Phase Advance / period	90°	45°	
No. of periods per turn	143.562		
Tune / revolution	35.9076	17.9806	
$\beta$ - min	1.98	4.02	cm
$\beta$ - max	2.85	5.69	cm
Bending Field	914		Gauss
d.c. current	1714		Amperes
Revolution Frequency, $f_0$	1.393		MHz
Revolution Period, $T_0$	0.718		$\mu$ s
Density Limit, $D_S$	$2.0 \times 10^{12}$		$\text{cm}^{-2}$
Normal Emittance	1.0		$\pi$ cm mrad
a-min	3.68	5.24	mm
a-max	4.42	6.24	mm
max No. of Protons, N		$2.07 \times 10^{12}$	
max. Beam Current		460	mAmp

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