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THE BOOSTER STOPBAND CORRECTION SYSTEM - 1997

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1 Introduction

The Booster Stopband Correction System has undergone a number of changes since it was last documented [1] in 1993. Windings have been added to existing magnets, power supplies have been added and rearranged, and the controlling code, Stopband-Correct, has evolved considerably. The new windings produce skew sextupoles, octupoles, and skew octupoles, and the Stopband-Correct code now can incorporate the values of the main dipole field, B, and its time derivative, dB/dt, in the calculation of the required corrections. What follows is an up-to-date list containing a detailed description of each correction. The operation of the function generators for the corrections and the archiving process are also described. A brief review of the Theory of Sum and Difference Resonances and the application of the theory to the Booster resonances are included in an Appendix.

2 Quadrupole Corrections

2.1 Purpose

The purpose of these corrections is to compensate the harmonics due to random or systematic quadrupole field errors in the Booster which excite the half-integer resonances $2Q_x = 9$ and $2Q_y = 9$. Each of these resonances must be correctable without affecting the correction of the other and without altering the machine tunes or introducing unwanted harmonics.

2.2 Magnets Used

Each superperiod of the Booster contains eight quadrupoles which are labled QVX1, QHX2, QVX3, QHX4, QVX5, QHX6, QVX7, and QHX8, where QH and QV denote horizontal and vertical focusing quadrupoles, and X refers to superperiod A, B, C, D, E, or F. These are the main quadrupoles of the Booster lattice. Each quadrupole has a main winding consisting of 5 turns, a tune-control winding consisting of 1 turn, and an auxiliary winding consisting of 2 turns. The auxiliary windings are used for the half-integer resonance correction and are refered to here by the names of the quadrupoles on which they are wound. The mechanical and electromagnetic characteristics of the quadrupoles are documented in Refs. [2, 3]. The measured integrated strengths of the QH and QV quadrupoles are, respectively, 1.834 and 1.877 gauss per ampere-turn.

The auxiliary windings on the quads are connected together to form four strings which are labeled QVSTR1, QHSTR1, QVSTR2, and QHSTR2. Each string consists of the auxiliary windings of 12 of the quads connected in series as indicated below. The '+' and '−' signs indicate the polarity of each auxiliary winding in the string.

QVSTR1: +QVA1 +QVA7 −QVB1 −QVB7 +QVC1 +QVC7 −QVD1 −QVD7 +QVE1 +QVE7 −QVF1 −QVF7.

QHSTR1: +QHA2 +QHA8 −QHB2 −QHB8 +QHC2 +QHC8 −QHD2 −QHD8 +QHE2 +QHE8 −QHF2 −QHF8.

QVSTR2: +QVA3 −QVA5 −QVB3 +QVB5 +QVC3 −QVC5 −QVD3 +QVD5 +QVE3 −QVE5 −QVF3 +QVF5.

QHSTR2: +QHA4 −QHA6 −QHB4 +QHB6 +QHC4 −QHC6 −QHD4 +QHD6 +QHE4 −QHE6 −QHF4 +QHF6.

The magnets in each string are connected together with number 2 (AWG) cable. The resistance of each string, including the resistance of the windings, is 0.624 ohms; the estimated inductance is 0.77 mH. The net EMF induced in each string by the currents in the main windings and tune-control windings is zero.

2.3 Power Supplies

Each string is connected to its own programable power supply which is bipolar and can deliver a maximum current of 20 Amps at a maximum of 30 volts. Each 2-turn auxiliary winding therefore provides a maximum excitation of 40 ampere-turns. This is 0.35% of the maximum excitation of 11500 ampere-turns provided by each 5-turn main winding during proton extraction at 1.5 GeV. Since the resistance of each string is 0.624 ohms, the minimum voltage required at 20 Amps is 12.5 volts. The current in each string may be ramped at a rate of at most 3 Amps/ms. The four power supplies are located in building 930A and are controlled via the CDC.BCOR.STOP-BAND controller.

2.4 Excitation Scheme

The division of the 48 auxiliary windings into four strings with the indicated magnets and polarities ensures that no 10θ, 5θ, 4θ, or 0θ harmonic components are produced by any of the quadrupole strings. If the four strings QVSTR1, QHSTR1, QVSTR2, QHSTR2 are excited respectively with currents J_1, J_2, J_3, and J_4, then, as shown in the Appendix and Ref. [4], the $2Q_x = 9$ and $2Q_y = 9$ resonance excitation components $\cos 9x$, $\sin 9x$, $\cos 9y$, and $\sin 9y$ produced by the currents are given by

$$\begin{pmatrix} \cos 9x \\ \sin 9x \\ \cos 9y \\ \sin 9y \end{pmatrix} = N\left(\frac{C}{P}\right)\mathbf{MQ}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{1}$$

where

$$\mathbf{M} = \begin{pmatrix} 28.139 & -76.062 & -37.769 & -126.518 \\ 36.050 & 127.317 & 27.678 & -77.381 \\ 124.432 & -9.590 & -85.089 & -45.423 \\ 85.027 & 45.418 & 124.537 & -9.564 \end{pmatrix}, \tag{2}$$

$$\mathbf{Q} = \begin{pmatrix} Q_V & 0 & 0 & 0 \\ 0 & Q_H & 0 & 0 \\ 0 & 0 & Q_V & 0 \\ 0 & 0 & 0 & Q_H \end{pmatrix}, \tag{3}$$

$Q_H = 3.668 \times 10^{-4}$ and $Q_V = 3.754 \times 10^{-4}$ tesla/Amp are the integrated quadrupole strengths produced by each 2-turn auxiliary winding on the

QH and QV quadrupoles, P is the momentum in units of GeV/c, $C = 0.299792458$, and the normalization constant, $N = 10^5/(2\pi)$, is such that $\sqrt{C^2 + S^2}$ (where C and S are the cos and sin components of the correction) is 10^5 times the resonance stopband widths produced by the correction scheme. The matrix elements of $\mathbf{M}$ have been computed using the MAD code and are given in units of meters. The currents required to produce a given set of excitation components are obtained by inverting equation (1). Thus

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} = \frac{1}{N}\left(\frac{P}{C}\right)\mathbf{Q}^{-1}\mathbf{M}^{-1}\begin{pmatrix} \cos 9x \\ \sin 9x \\ \cos 9y \\ \sin 9y \end{pmatrix}. \tag{4}$$

Defining vectors

$$\mathbf{C}_0 = \begin{pmatrix} \cos 9x \\ \sin 9x \\ \cos 9y \\ \sin 9y \end{pmatrix}, \quad \mathbf{J}_0 = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{5}$$

and four-by-four matrix

$$\mathbf{R} = N\left(\frac{C}{P}\right)\mathbf{MQ}, \tag{6}$$

we can write equations (1) and (4) in the more compact form

$$\mathbf{C}_0 = \mathbf{RJ}_0, \quad \mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0. \tag{7}$$

In the application code, Stopband-Correct, the user may specifiy values for the components of $\mathbf{C}_0$ at various times during the magnetic cycle and the code then calculates the corresponding currents given by $\mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0$. The user may also specify the currents at various times during the magnetic cycle, in which case the code calculates the resonance components given by $\mathbf{C}_0 = \mathbf{RJ}_0$.

2.5 Incorporation of B and dB/dt Dependence

Measurements [5–7] have shown that currents required to correct the $2Q_x = 9$, $2Q_y = 9$ and other resonances at various times during the magnetic cycle depend linearly on the main dipole field, B, and its time derivative dB/dt. Since the values of B and dB/dt throughout the magnetic cycle are available in the computer's memory, a new feature has been added to the Stopband-Correct code which incorporates these values in the calculation of the currents and resonance components. To show how this is done we introduce the four-dimensional vectors

$$\mathbf{J}_r = \mathbf{R}^{-1}\mathbf{C}_r, \quad \mathbf{J}_b = \mathbf{R}^{-1}\mathbf{C}_b, \quad \mathbf{J}_t = \mathbf{R}^{-1}\mathbf{C}_t, \tag{8}$$

where $\mathbf{R}$ is given by (6) and $\mathbf{C}_r$, $\mathbf{C}_b$, $\mathbf{C}_t$ are four-dimensional vectors of constant parameters which are specified and tuned by the user. The total currents to be delivered to the quadrupole strings are then given by

$$\mathbf{J}(t) = \mathbf{J}_0(t) + \mathbf{J}_r + \mathbf{J}_b B(t) + \mathbf{J}_t \dot{B}(t), \quad (\dot{B} = dB/dt) \tag{9}$$

where t is the time during the magnetic cycle. The corresponding resonance excitation components are given by

$$\mathbf{C}(t) = \mathbf{R}\mathbf{J}(t) = \mathbf{C}_0(t) + \mathbf{C}_r + \mathbf{C}_b B(t) + \mathbf{C}_t \dot{B}(t). \tag{10}$$

Thus by specifying values for the parameters contained in the vectors $\mathbf{C}_r$, $\mathbf{C}_b$, and $\mathbf{C}_t$, the user can achieve the desired linear dependence on B and dB/dt. If these parameters are set to zero, then $\mathbf{J}(t) = \mathbf{J}_0(t)$, $\mathbf{C}(t) = \mathbf{C}_0(t)$, and the currents and resonance excitation components are given by equations (7) as in the old version of the Stopband-Correct code. If, on the other hand, the components of $\mathbf{C}_0(t)$ are zero throughout the magnetic cycle, then $\mathbf{C}(t)$ and $\mathbf{J}(t)$ are completely specified by $\mathbf{C}_r$, $\mathbf{C}_b$, and $\mathbf{C}_t$. Both $\mathbf{C}_0(t)$ and the parameters contained in $\mathbf{C}_r$, $\mathbf{C}_b$, and $\mathbf{C}_t$ can be nonzero, of course, and this gives the user a considerable amount of flexibility in specifying the resonance excitation components and currents throughout the magnetic cycle.

3 Skew Quadrupole Corrections

3.1 Purpose

The purpose of these corrections is to compensate the harmonics due to random or systematic skew quadrupole field errors in the Booster which excite the linear-coupling resonances $Q_x - Q_y = 0$ and $Q_x + Q_y = 9$. Each of these resonances must be correctable without affecting the correction of the other and without introducing unwanted harmonics.

3.2 Magnets Used

With the exception of locations C4, C6, D6 and F6, the Booster has a standard correction magnet centered 67.1 cm upstream of the center of each quadrupole. Each magnet consists of an iron core with windings that can be excited to produce a skew quadrupole and a horizontal or vertical dipole. (The inner winding on the core consists of a layer of 48 turns which produces the skew quadrupole; the outer winding consists of a 47-turn and a 5-turn layer which produce the dipole. The non-standard magnets at C4, C6, D6, and F6 have windings consisting of 88, 98, 88, and 100 turns respectively, which produce horizontal dipoles only.) The skew quadrupoles in the the correction magnets upstream of quadrupoles QV1, QH2, QV7, and QH8 in each superperiod are used for the correction of the $Q_x - Q_y = 0$ and $Q_x + Q_y = 9$ resonances. We label them QSX1, QSX2, QSX7, and QSX8, where X refers to the superperiod. The mechanical and electromagnetic characteristics of the skew quads are documented in Ref. [8]. Each skew quad has a measured integrated strength of 5.45 gauss/Amp.

The skew quads are connected together to form four strings which are labeled QSSTR1, QSSTR2, QSSTR3, and QSSTR4. Each string consists of six of the skew quads connected in series as indicated below. The skew quads in each string all have the same polarity.

QSSTR1: QSA1 QSA7 QSC1 QSC7 QSE1 QSE7

QSSTR2: QSA2 QSA8 QSC2 QSC8 QSE2 QSE8

QSSTR3: QSB1 QSB7 QSD1 QSD7 QSF1 QSF7

QSSTR4: QSB2 QSB8 QSD2 QSD8 QSF2 QSF8

The six skew quads in each string are connected together with number 2 (AWG) cable. The resistance of each string, including the resistance of the skew quads, is 0.771 ohms; the estimated inductance is 12 mH.

3.3 Power Supplies

Each of the four strings of skew quadrupoles is connected to its own programable power supply which is bipolar and can deliver a maximum current of 25 Amps at a maximum of 32 volts. Each skew quad therefore has a maximum integrated strength of 136 gauss. This is 0.65% of the maximum integrated strength of 2.1 tesla provided by each main quadrupole during proton extraction at 1.5 GeV/c. Since the resistance of each string is 0.771 ohms, the minimum voltage required at 25 Amps is 19 volts. The current in each string may be ramped at a rate of at most 3 Amps/ms. The four power supplies are located in building 930A and are controlled via the CDC.BCOR.STOP-BAND controller.

3.4 Excitation Scheme

The divison of the 24 skew quads into the four strings indicated above allows one to independently correct the $Q_x - Q_y = 0$ and $Q_x + Q_y = 9$ resonances without introducing any 4θ or 5θ harmonic components. If the four strings QSSTR1, QSSTR2, QSSTR3, and QSSTR4 are excited respectively with currents J_1, J_2, J_3, and J_4, then, as shown in the Appendix, the $Q_x - Q_y = 0$ and $Q_x + Q_y = 9$ resonance excitation components produced by the currents are given by

$$\begin{pmatrix} \cos 0xy \\ \sin 0xy \end{pmatrix} = \frac{N}{2}\left(\frac{CQ}{P}\right)\mathbf{N}_1 \begin{pmatrix} J_1 + J_3 \\ J_2 + J_4 \end{pmatrix} \tag{11}$$

and

$$\begin{pmatrix} \cos 9xy \\ \sin 9xy \end{pmatrix} = \frac{N}{2}\left(\frac{CQ}{P}\right)\mathbf{N}_2 \begin{pmatrix} J_1 - J_3 \\ J_2 - J_4 \end{pmatrix} \tag{12}$$

where

$$\mathbf{N}_1 = \begin{pmatrix} 88.000 & 88.000 \\ -9.700 & 9.700 \end{pmatrix}, \quad \mathbf{N}_2 = \begin{pmatrix} 71.556 & -14.017 \\ 43.519 & 82.487 \end{pmatrix}, \tag{13}$$

$Q = 5.45 \times 10^{-4}$ tesla/Amp is the integrated strength of each skew quad, P is the momentum in units of GeV/c, $C = 0.299792458$, and the normalization constant, $N = 10^5/(8\pi)$. The matrix elements of $\mathbf{N}_1$ and $\mathbf{N}_2$ have been computed using the MAD code and are given in units of meters. (The matrix elements of $\mathbf{N}_1$ have actually been adjusted so that if $\cos 0xy$ is the only nonzero component then $J_1 = J_2 = J_3 = J_4$.) The currents required to produce a given set of excitation components are obtained by inverting equations (11–12). Thus

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \frac{1}{N}\left(\frac{P}{CQ}\right)\left[\mathbf{N}_1^{-1}\begin{pmatrix} \cos 0xy \\ \sin 0xy \end{pmatrix} + \mathbf{N}_2^{-1}\begin{pmatrix} \cos 9xy \\ \sin 9xy \end{pmatrix}\right] \tag{14}$$

and

$$\begin{pmatrix} J_3 \\ J_4 \end{pmatrix} = \frac{1}{N}\left(\frac{P}{CQ}\right)\left[\mathbf{N}_1^{-1}\begin{pmatrix} \cos 0xy \\ \sin 0xy \end{pmatrix} - \mathbf{N}_2^{-1}\begin{pmatrix} \cos 9xy \\ \sin 9xy \end{pmatrix}\right]. \tag{15}$$

Defining vectors

$$\mathbf{C}_0 = \begin{pmatrix} \cos 0xy \\ \sin 0xy \\ \cos 9xy \\ \sin 9xy \end{pmatrix}, \quad \mathbf{J}_0 = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{16}$$

and four-by-four matrix

$$\mathbf{R} = \frac{N}{2}\left(\frac{CQ}{P}\right)\begin{pmatrix} \mathbf{N}_1 & \mathbf{N}_1 \\ \mathbf{N}_2 & -\mathbf{N}_2 \end{pmatrix}, \tag{17}$$

we can write equations (11–12) and (14–15) in the more compact form

$$\mathbf{C}_0 = \mathbf{R}\mathbf{J}_0, \quad \mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0. \tag{18}$$

In the application code, Stopband-Correct, the user may specifiy values for the components of $\mathbf{C}_0$ at various times during the magnetic cycle and the code then calculates the corresponding currents given by $\mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0$. The user may also specify the currents at various times during the magnetic cycle, in which case the code calculates the resonance components given by $\mathbf{C}_0 = \mathbf{R}\mathbf{J}_0$.

3.5 Incorporation of B and dB/dt Dependence

We define, as discussed in Section 2.5,

$$\mathbf{J}_r = \mathbf{R}^{-1}\mathbf{C}_r, \quad \mathbf{J}_b = \mathbf{R}^{-1}\mathbf{C}_b, \quad \mathbf{J}_t = \mathbf{R}^{-1}\mathbf{C}_t, \tag{19}$$

where $\mathbf{C}_r$, $\mathbf{C}_b$, $\mathbf{C}_t$ are four-dimensional vectors of constant parameters which are specified and tuned by the user. The total currents to be delivered to the skew quad strings are then given by

$$\mathbf{J}(t) = \mathbf{J}_0(t) + \mathbf{J}_r + \mathbf{J}_b B(t) + \mathbf{J}_t \dot{B}(t), \quad (\dot{B} = dB/dt) \tag{20}$$

where t is the time during the magnetic cycle. The corresponding resonance excitation components are given by

$$\mathbf{C}(t) = \mathbf{R}\mathbf{J}(t) = \mathbf{C}_0(t) + \mathbf{C}_r + \mathbf{C}_b B(t) + \mathbf{C}_t \dot{B}(t). \tag{21}$$

4 Sextupole Corrections

4.1 Purpose

The purpose of these corrections is to compensate the harmonics due to random or systematic sextupole field errors in the Booster which excite the normal third-integer resonances $3Q_x = 14$, $Q_x + 2Q_y = 14$, $3Q_x = 13$, and $Q_x + 2Q_y = 13$. Each of these resonances must be correctable without affecting the correction of the others and without altering the machine chromaticities or introducing unwanted harmonics.

4.2 Magnets Used

Each superperiod of the Booster contains eight sextupoles which are labled SVX1, SHX2, SVX3, SHX4, SVX5, SHX6, SVX7, and SHX8, where SH and SV denote, respectively, sextupoles located near horizontal and vertical beta maximums, and X refers to superperiod A, B, C, D, E, or F. These are the main sextupoles of the Booster lattice used to adjust the machine chromaticities. The centers of the sextupoles are located 55.2 cm upstream of the centers of the quadrupoles—i.e. the center of SVX1 is 55.2 cm upstream of the center of QVX1, and so on. Each sextupole has a main winding consisting of 8 turns, a monitor winding consisting of one turn, and an auxiliary winding consisting of either one or two turns. The mechanical and electromagnetic characteristics of the sextupoles are documented in Ref. [9]. The measured integrated strength of each sextupole is 1.643×10^{-3} tesla/m per ampere-turn. The auxiliary windings are used for the normal third-integer resonance correction and are refered to here by the names of the sextupoles on which they are wound. (The monitor windings are used to produce the ninth-harmonic sextupole correction discussed in Section 6.)

The auxiliary windings on the sextupoles are connected togther to form eight strings which are labeled SVSTR1, SHSTR1, SVSTR2, SHSTR2, SVSTR3, SHSTR3, SVSTR4, SHSTR4. Each string consists of the auxiliary windings of 6 of the sextupoles connected in series as indicated below. The '+' and '–' signs indicate the polarity of each winding in the string, and '/2' indicates auxiliary windings with half as many turns as the others—i.e. with just one turn.

SVSTR1: +SVA1 –SVA3 –SVC1/2 +SVC3/2 –SVE1/2 +SVE3/2

SHSTR1: +SHA2 –SHA4 –SHC2/2 +SHC4/2 –SHE2/2 +SHE4/2

SVSTR2: +SVA5 –SVA7 –SVC5/2 +SVC7/2 –SVE5/2 +SVE7/2

SHSTR2: +SHA6 –SHA8 –SHC6/2 +SHC8/2 –SHE6/2 +SHE8/2

SVSTR3: +SVD1 –SVD3 –SVF1/2 +SVF3/2 –SVB1/2 +SVB3/2

SHSTR3: +SHD2 –SHD4 –SHF2/2 +SHF4/2 –SHB2/2 +SHB4/2

SVSTR4: +SVD5 –SVD7 –SVF5/2 +SVF7/2 –SVB5/2 +SVB7/2

SHSTR4: +SHD6 –SHD8 –SHF6/2 +SHF8/2 –SHB6/2 +SHB8/2

The 6 auxiliary windings in each string are connected together with number 2 (AWG) cable. The total resistance of each series string, including the resistance of the windings, is 0.3066 ohms. The estimated inductance of each string is 0.003 mH. The net EMF induced in each string by the currents in the main windings of the magnets is zero.

4.3 Power Supplies

Each of the eight strings of auxiliary windings is connected to its own programable power supply which is bipolar and can deliver a maximum current of 50 Amps at a maximum of 25 volts. Each 2-turn auxiliary winding therefore provides a maximum excitation of 100 ampere-turns. This is 5% of the maximum excitation of 2000 ampere-turns provided by each 8-turn main winding. Since the resistance of each string is 0.3066 ohms, the mimimum voltage required at 50 Amps is 15.33 volts. The current in each string may be ramped at a rate of at most 3 Amps/ms. The eight power supplies are located in building 930A and are controlled via the CDC.BCOR.STOP-BAND controller.

4.4 Excitation Scheme

The division of the 48 auxiliary windings into eight strings with the indicated magnets and polarities ensures that no 14θ, 10θ, 9θ, 4θ or 0θ harmonic components are produced during correction of the $3Q_x = 13$ and $Q_x + 2Q_y = 13$ resonances, and no 13θ, 9θ, 5θ, or 0θ harmonic components are produced during correction of the $3Q_x = 14$ and $Q_x + 2Q_y = 14$ resonances. If the eight strings SVSTR1, SHSTR1,..., SHSTR4 are excited respectively with currents J_1, J_2,..., J_8, then, as shown in the Appendix and Ref. [4], the $3Q_x = 14$ and $Q_x + 2Q_y = 14$ resonance excitation components $\cos 14x$, $\sin 14x$, $\cos 14xy$, and $\sin 14xy$ produced by the currents are given by

$$\begin{pmatrix} \cos 14x \\ \sin 14x \\ \cos 14xy \\ \sin 14xy \end{pmatrix} = \frac{N}{2}\left(\frac{CS}{P}\right)\mathbf{M}_{14}\begin{pmatrix} J_1+J_5 \\ J_2+J_6 \\ J_3+J_7 \\ J_4+J_8 \end{pmatrix} \tag{22}$$

and the $3Q_x = 13$ and $Q_x + 2Q_y = 13$ resonance excitation components $\cos 13x$, $\sin 13x$, $\cos 13xy$, and $\sin 13xy$ are given by

$$\begin{pmatrix} \cos 13x \\ \sin 13x \\ \cos 13xy \\ \sin 13xy \end{pmatrix} = \frac{N}{2}\left(\frac{CS}{P}\right)\mathbf{M}_{13}\begin{pmatrix} J_1-J_5 \\ J_2-J_6 \\ J_3-J_7 \\ J_4-J_8 \end{pmatrix} \tag{23}$$

where $S = 3.286 \times 10^{-3}$ tesla/m per Amp is the integrated sextupole strength produced by each 2-turn auxiliary winding, P is the momentum in units of GeV/c, $C = 0.299792458$, the normalization constant $N = 125$,

$$\mathbf{M}_{14} = \begin{pmatrix} 39.017 & -230.433 & -8.606 & -191.932 \\ 34.454 & 60.248 & 56.746 & -147.730 \\ 125.164 & -59.570 & 0.214 & -88.629 \\ 75.279 & 66.529 & 150.993 & -14.219 \end{pmatrix} \tag{24}$$

and

$$\mathbf{M}_{13} = \begin{pmatrix} 52.718 & -211.124 & 35.220 & -252.319 \\ 33.732 & 140.525 & 59.665 & 54.026 \\ 148.590 & -46.760 & 103.567 & -83.961 \\ 62.797 & 92.117 & 131.038 & 61.137 \end{pmatrix}. \tag{25}$$

The matrix elements of $\mathbf{M}_{13}$ and $\mathbf{M}_{14}$ have been calculated using the MAD code and are given in units of $m^{3/2}$. The currents required to produce a given set of excitation components are obtained by inverting equations (22–23). Thus

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} = \frac{1}{N}\left(\frac{P}{CS}\right)\left[\mathbf{M}_{14}^{-1}\begin{pmatrix} \cos 14x \\ \sin 14x \\ \cos 14xy \\ \sin 14xy \end{pmatrix} + \mathbf{M}_{13}^{-1}\begin{pmatrix} \cos 13x \\ \sin 13x \\ \cos 13xy \\ \sin 13xy \end{pmatrix}\right], \tag{26}$$

and

$$\begin{pmatrix} J_5 \\ J_6 \\ J_7 \\ J_8 \end{pmatrix} = \frac{1}{N}\left(\frac{P}{CS}\right)\left[\mathbf{M}_{14}^{-1}\begin{pmatrix} \cos 14x \\ \sin 14x \\ \cos 14xy \\ \sin 14xy \end{pmatrix} - \mathbf{M}_{13}^{-1}\begin{pmatrix} \cos 13x \\ \sin 13x \\ \cos 13xy \\ \sin 13xy \end{pmatrix}\right]. \tag{27}$$

Defining vectors

$$\mathbf{C}_0 = \begin{pmatrix} \cos 14x \\ \sin 14x \\ \cos 14xy \\ \sin 14xy \\ \cos 13x \\ \sin 13x \\ \cos 13xy \\ \sin 13xy \end{pmatrix}, \quad \mathbf{J}_0 = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \\ J_7 \\ J_8 \end{pmatrix} \tag{28}$$

and eight-by-eight matrix

$$\mathbf{R} = \frac{N}{2}\left(\frac{CS}{P}\right)\begin{pmatrix} \mathbf{M}_{14} & \mathbf{M}_{14} \\ \mathbf{M}_{13} & -\mathbf{M}_{13} \end{pmatrix}, \tag{29}$$

we can write equations (22–23) and (26–27) in the more compact form

$$\mathbf{C}_0 = \mathbf{R}\mathbf{J}_0, \quad \mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0. \tag{30}$$

In the application code, Stopband-Correct, the user may specifiy values for the components of $\mathbf{C}_0$ at various times during the magnetic cycle and the code then calculates the corresponding currents given by $\mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0$. The user may also specify the currents at various times during the magnetic cycle, in which case the code calculates the resonance components given by $\mathbf{C}_0 = \mathbf{R}\mathbf{J}_0$.

4.5 Incorporation of B and dB/dt Dependence

We define, as discussed in Section 2.5,

$$\mathbf{J}_r = \mathbf{R}^{-1}\mathbf{C}_r, \quad \mathbf{J}_b = \mathbf{R}^{-1}\mathbf{C}_b, \quad \mathbf{J}_t = \mathbf{R}^{-1}\mathbf{C}_t, \tag{31}$$

where $\mathbf{C}_r$, $\mathbf{C}_b$, $\mathbf{C}_t$ are now eight-dimensional vectors of constant parameters which are specified and tuned by the user. The total currents to be delivered to the sextupole strings are then given by

$$\mathbf{J}(t) = \mathbf{J}_0(t) + \mathbf{J}_r + \mathbf{J}_b B(t) + \mathbf{J}_t \dot{B}(t), \quad (\dot{B} = dB/dt) \tag{32}$$

where t is the time during the magnetic cycle. The corresponding resonance excitation components are given by

$$\mathbf{C}(t) = \mathbf{R}\mathbf{J}(t) = \mathbf{C}_0(t) + \mathbf{C}_r + \mathbf{C}_b B(t) + \mathbf{C}_t \dot{B}(t). \tag{33}$$

5 Skew Sextupole Corrections

5.1 Purpose

The purpose of these corrections is to compensate the harmonics due to random or systematic skew sextupole field errors in the Booster which excite the skew third-integer resonances $Q_y + 2Q_x = 14$ and $3Q_y = 14$. Each of these resonances must be correctable without affecting the correction of the other and without introducing unwanted harmonics.

5.2 Magnets Used

The skew sextupole correction magnets are produced by special windings on the cores of the correction magnets located at B1, B2, B7, B8, E1, E2, E7, and E8. The measured integrated strength (as per John Jackson) of each skew sextupole is 8.79×10^{-3} tesla/m per Amp. The magnets are labeled SSVB1, SSHB2, SSVB7, SSHB8, SSVE1, SSHE2, SSVE7, and SSHE8, and are connected together to form four series strings labeled SSVSTR1, SSHSTR1, SSVSTR2, SSHSTR2. Each string consists of two skew sextupoles connected in series as indicated below.

SSVSTR1: +SSVB1 +SSVE1

SSHSTR1: +SSHB2 +SSHE2

SSVSTR2: +SSVB7 +SSVE7

SSHSTR2: +SSHB8 +SSHE8

The '+' signs indicate that the two magnets in each string have the same polarity.

5.3 Power Supplies

Each of the four strings of skew sextupoles is connected to its own programable power supply which is bipolar and can deliver a maximum current of 20 Amps at a maximum of 30 volts. Each skew sextupole therefore has a maximum integrated strength of 0.176 tesla/m. This is 5.3% of the maximum integrated strength of 3.3 tesla/m provided by the main winding of each normal sextupole. The current in each string may be ramped at a rate of at most 3 Amps/ms. The four power supplies are located in building 930A and are controlled via the CDC.BCOR.ORB-BC controller.

5.4 Excitation Scheme

Since each of the strings listed above consists of two magnets connected in series with the same polarity and separated by 180°, only even harmonics of the skew sextupole field are produced when each string is excited. In particular, the 14th harmonic necessary for the correction of the $Q_y + 2Q_x = 14$ and $3Q_y = 14$ resonances is produced, but the potentially harmful 5th and 9th harmonics are not. If the four strings SSVSTR1, SSHSTR1, SSVSTR2, SSHSTR2 are excited, respectively, with currents J_1, J_2, J_3, and J_4, then, as shown in the Appendix, the $3Q_y = 14$ and $Q_y + 2Q_x = 14$ resonance excitation components $\cos 14y$, $\sin 14y$, $\cos 14yx$, and $\sin 14yx$ produced by the currents are given by

$$\begin{pmatrix} \cos 14y \\ \sin 14y \\ \cos 14yx \\ \sin 14yx \end{pmatrix} = N\left(\frac{CS}{P}\right)\mathbf{M}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{34}$$

where

$$\mathbf{M} = \begin{pmatrix} 80.307 & -2.078 & 13.125 & 20.593 \\ 12.438 & 20.673 & -79.676 & 2.270 \\ 30.537 & -31.211 & 8.043 & 40.569 \\ 8.751 & 38.387 & -30.605 & 29.905 \end{pmatrix}, \tag{35}$$

$S = 8.79 \times 10^{-3}$ tesla/m per Amp is the integrated strength produced by each skew sextupole, P is the momentum in units of GeV/c, $C = 0.299792458$, and the normalization constant $N = 300$. The matrix elements of M have been calculated using the MAD code and are given in units of $m^{3/2}$. The currents required to produce a given set of excitation components are obtained by inverting equation (34). Thus

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} = \frac{1}{N}\left(\frac{P}{CS}\right)\mathbf{M}^{-1}\begin{pmatrix} \cos 14y \\ \sin 14y \\ \cos 14yx \\ \sin 14yx \end{pmatrix}. \tag{36}$$

Defining vectors

$$\mathbf{C}_0 = \begin{pmatrix} \cos 14y \\ \sin 14y \\ \cos 14yx \\ \sin 14yx \end{pmatrix}, \quad \mathbf{J}_0 = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{37}$$

and four-by-four matrix

$$\mathbf{R} = N\left(\frac{CS}{P}\right)\mathbf{M}, \tag{38}$$

we can write equations (34) and (36) in the more compact form

$$\mathbf{C}_0 = \mathbf{R}\mathbf{J}_0, \quad \mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0. \tag{39}$$

In the application code, Stopband-Correct, the user may specifiy values for the components of $\mathbf{C}_0$ at various times during the magnetic cycle and the code then calculates the corresponding currents given by $\mathbf{J}_0 = \mathbf{R}^{-1}\mathbf{C}_0$. The user may also specify the currents at various times during the magnetic cycle, in which case the code calculates the resonance components given by $\mathbf{C}_0 = \mathbf{R}\mathbf{J}_0$.

5.5 Incorporation of B and dB/dt Dependence

We define, as discussed in Section 2.5,

$$\mathbf{J}_r = \mathbf{R}^{-1}\mathbf{C}_r, \quad \mathbf{J}_b = \mathbf{R}^{-1}\mathbf{C}_b, \quad \mathbf{J}_t = \mathbf{R}^{-1}\mathbf{C}_t, \tag{40}$$

where $\mathbf{C}_r$, $\mathbf{C}_b$, $\mathbf{C}_t$ are four-dimensional vectors of constant parameters which are specified and tuned by the user. The total currents to be delivered to the skew sextupole strings are then given by

$$\mathbf{J}(t) = \mathbf{J}_0(t) + \mathbf{J}_r + \mathbf{J}_b B(t) + \mathbf{J}_t \dot{B}(t), \quad (\dot{B} = dB/dt) \tag{41}$$

where t is the time during the magnetic cycle. The corresponding resonance excitation components are given by

$$\mathbf{C}(t) = \mathbf{R}\mathbf{J}(t) = \mathbf{C}_0(t) + \mathbf{C}_r + \mathbf{C}_b B(t) + \mathbf{C}_t \dot{B}(t). \tag{42}$$

6 Ninth-Harmonic Sextupole Correction

6.1 Purpose

During the course of studies carried out in 1993–94 [5–7] it was found that the quadrupole corrections could not completely eliminate the beam loss observed as the $2Q_x = 9$ and $2Q_y = 9$ resonances were crossed. As discussed in Ref. [10], this is due to the momentum spread of the beam and can be compensated by introducing a ninth-harmonic sextupole field around the machine. The Ninth-Harmonic Sextupole Correction provides the desired sextupole field without altering the machine chromaticities or introducing unwanted harmonics.

6.2 Magnets Used

The one-turn monitor windings on the Booster sextupoles (see Section 4.2) are used for this correction. These are connected together to form four series strings labeled SVSTR91, SHSTR91, SVSTR92, and SHSTR92. Each string consists of 12 monitor windings connected in series as indicated below. The '+' and '−' signs indicate the polarities of the windings in each string.

SVSTR91: +SVA1 +SVA7 −SVB1 −SVB7 +SVC1 +SVC7 −SVD1 −SVD7 +SVE1 +SVE7 −SVF1 −SVF7

SHSTR91: +SHA2 +SHA8 −SHB2 −SHB8 +SHC2 +SHC8 −SHD2 −SHD8 +SHE2 +SHE8 −SHF2 −SHF8

SVSTR92: +SVA3 −SVA5 −SVB3 +SVB5 +SVC3 −SVC5 −SVD3 +SVD5 +SVE3 −SVE5 −SVF3 +SVF5

SHSTR92: +SHA4 −SHA6 −SHB4 +SHB6 +SHC4 −SHC6 −SHD4 +SHD6 +SHE4 −SHE6 −SHF4 +SHF6

The net EMF induced in each string by the currents in the main windings of the sextupoles is zero.

6.3 Power Supplies

Each string is connected to its own programable power supply which is bipolar and can deliver a maximum current of 50 Amps at a maximum of

25 volts. The current in each string may be ramped at a rate of at most 3 Amps/ms. The four power supplies are located in building 930A and are controlled via the CDC.BCOR.ORB-CD controller.

6.4 Excitation Scheme

The division of the 48 monitor windings into four strings with the indicated magnets and polarities ensures that no 14θ, 13θ, 10θ, 5θ, 4θ, or 0θ harmonic components are produced by any of the strings. If the four strings SVSTR91, SHSTR91, SVSTR92, SHSTR92 are excited with currents J_1, J_2, J_3, and J_4 respectively, then, as shown in the Appendix, the $2Q_x = 9$ and $2Q_y = 9$ resonance excitation components $\cos 9x$, $\sin 9x$, $\cos 9y$, and $\sin 9y$ produced by the currents (for a beam particle with $\delta P/P = 1$) are given by

$$\begin{pmatrix} \cos 9x \\ \sin 9x \\ \cos 9y \\ \sin 9y \end{pmatrix} = N \left(\frac{CS}{P}\right) \mathbf{M} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{43}$$

where

$$\mathbf{M} = \begin{pmatrix} 38.121 & -84.505 & -45.315 & -328.007 \\ 39.371 & 163.197 & 61.313 & -165.612 \\ 122.551 & 1.125 & -90.596 & -136.651 \\ 94.352 & 68.390 & 187.432 & 5.708 \end{pmatrix}, \tag{44}$$

$S = 1.643 \times 10^{-3}$ tesla/m per Amp is the integrated strength produced by each monitor winding, P is the momentum in units of GeV/c, $C = 0.299792458$, and N is a normalization constant. The matrix elements of $\mathbf{M}$ have been calculated using the MAD code and are given in units of m^2. The currents required to produce a given set of excitation components are obtained by inverting equation (43). Thus

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} = \frac{1}{N} \left(\frac{P}{CS}\right) \mathbf{M}^{-1} \begin{pmatrix} \cos 9x \\ \sin 9x \\ \cos 9y \\ \sin 9y \end{pmatrix} \tag{45}$$

where

$$\mathbf{M}^{-1} = \frac{1}{1000}\begin{pmatrix} -1.513 & -2.336 & 6.613 & 3.595 \\ -3.115 & 5.390 & 0.856 & -2.102 \\ 1.980 & -0.744 & -3.674 & 4.282 \\ -2.696 & -1.557 & 1.056 & 0.368 \end{pmatrix}. \tag{46}$$

The Ninth-Harmonic Sextupole Correction is not yet a part of the code Stopband-Correct, but the currents in the four strings can be programed using the Function Editor program.

7 Ninth-Harmonic Skew Sextupole Correction

7.1 Purpose

During the course of studies carried out in 1993–94 [5–7] it was found that the skew quadrupole corrections could not completely eliminate the beam loss observed as the $Q_x + Q_y = 9$ resonance was crossed. As discussed in Ref. [10], this is due to the momentum spread of the beam and can be compensated by introducing a ninth-harmonic skew sextupole field around the machine. The Ninth-Harmonic Skew Sextupole Correction provides the desired sextupole field.

7.2 Magnets Used

The skew sextupole correction magnets are produced by special windings on the cores of the correction magnets located at A3, A7, D3 and D7. The measured integrated strength (as per John Jackson) of each skew sextupole is 8.79×10^{-3} tesla/m per Amp. The magnets are labeled SSVA3, SSVA7, SSVD3, and SSVD7, and are connected together to form two series strings labeled SSVSTR91 and SSVSTR92. Each string consists of two skew sextupoles connected in series as indicated below.

SSVSTR91: +SSVA3 −SSVD3

SSVSTR92: +SSVA7 −SSVD7

The '+' and '−' signs indicate the polarities of the magnets in each string.

7.3 Power Supplies

Each string is connected to its own programable power supply which is bipolar and can deliver a maximum current of 20 Amps at a maximum of 30 volts. The current in each string may be ramped at a rate of at most 3 Amps/ms. The two power supplies are located in building 914 and are controlled via the CDC.BCOR.ORB-FA controller.

7.4 Excitation Scheme

Since each string consists of two magnets connected in series with opposite polarity and separated by 180°, only odd harmonics of the skew sextupole field are produced when each string is excited. In particular the desired 9th harmonic and potentially harmful 5th harmonic are produced. If the two strings SSVSTR91 and SSHSTR92 are excited with currents J_1 and J_2 respectively, then, as shown in the Appendix, the $Q_x + Q_y = 9$ resonance excitation components, $\cos 9xy$ and $\sin 9xy$, produced by the currents (for a beam particle with $\delta P/P = 1$) are given by

$$\begin{pmatrix} \cos 9xy \\ \sin 9xy \end{pmatrix} = N \left(\frac{CS}{P}\right) \mathrm{M} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} \tag{47}$$

where

$$\mathrm{M} = \begin{pmatrix} -15.258 & 13.509 \\ 10.318 & 18.324 \end{pmatrix}, \tag{48}$$

$S = 8.79 \times 10^{-3}$ tesla/m per Amp is the integrated strength produced by each skew sextupole, P is the momentum in units of GeV/c, $C = 0.299792458$, and N is a normalization constant. The matrix elements of M have been calculated using the MAD code and are given in units of m^2. The currents required to produce a given set of excitation components are obtained by inverting equation (47). Thus

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \frac{1}{N} \left(\frac{P}{CS}\right) \mathrm{M}^{-1} \begin{pmatrix} \cos 9xy \\ \sin 9xy \end{pmatrix} \tag{49}$$

where

$$\mathrm{M}^{-1} = \frac{1}{100} \begin{pmatrix} -4.374 & 3.224 \\ 2.463 & 3.642 \end{pmatrix}. \tag{50}$$

The Ninth-Harmonic Skew Sextupole Correction is not yet a part of the code Stopband-Correct, but the currents in the two strings can be programed using the Function Editor program.

8 Octupole and Skew Octupole Correction

Octupole and skew octupole magnets are produced by special windings on the cores of the correction magnets at A4 and D4. At each location there is a winding that produces an octupole and a winding that produces a skew octupole. The octupoles are labeled OHA4 and OHD4, and the skew octupoles are labeled OSHA4 and OSHD4. The integrated strengths (as per John Jackson) of the octupole and skew octupole are 0.028 and 0.084 T/m^2 per Amp. The magnets can be connected together (using connectors at A4 and D4) to form either one of two series strings, OCTSTR and SOCTSTR, as indicated below. The '+' and '$\pm$' signs indicate the relative polarities of the magnets.

OCTSTR: + OHA4 $\pm$ OHD4

SOCTSTR: + OSHA4 $\pm$ OSHD4

The relative polarities can be changed by a simple swap of cables at A4 or D4. If the polarities of the magnets in a string are the same (opposite), the string produces only even (odd) harmonics.

Although not yet observed, the $4Q_x = 19$, $4Q_y = 19$, $2Q_x + 2Q_y = 19$, and $2Q_x - 2Q_y = 0$ resonances can in principle be excited with the octupoles, and the $Q_x + 3Q_y = 19$, $3Q_x + Q_y = 19$, $3Q_x - Q_y = 10$, and $3Q_y - Q_x = 10$ resonances can be excited with the skew octupoles.

The Octupole and Skew Octupole Corrections are not yet a part of the code Stopband-Correct, but the current in the string of magnets can be programed using the Function Editor program. The power supply for the string is located in building 914 and is controlled via the CDC.BCOR.ORB-FA controller.

9 Function Generator Operation

The current in each string of correction magnets is programed with a 400 point function generator which has the same name as the corrector string.

The 400 points are assigned values determined by the equations for $\mathbf{J}(t)$ given in the previous sections and are clocked at a user-specified rate of either one point per ms or one point every two ms. The first and second of the 400 points are always assigned value zero (regardless of the values specified by $\mathbf{J}(t)$ for these points). The first point is clocked out at T_0 and the function is held at value zero until a start pulse is received at the user-specified time $\mathrm{T}_{\mathrm{start}}$. When the start pulse is received, the second and subsequent points are clocked out at the specified rate of either 1000 or 500 Hz. The points are clocked out until the function generator receives a stop pulse at time, $\mathrm{T}_{\mathrm{stop}}$, which is specified by the user and must be at least 25 ms before T_0 for the next Booster cycle. The value of the point to be clocked out at time $\mathrm{T}_{\mathrm{stop}}$ is also assigned value zero (regardless of the value specified by $\mathbf{J}(t)$). If the values of the points are such that the currents in the corrector strings would have to be ramped at a rate of more than 3 Amps/ms, then the values are modified by an algorithm (developed by John Morris) in the Stopband-Correct code which requires that the currents reach the desired values as quickly as possible without exceeding the 3 Amp/ms ramp rate.

10 Archiving

When the corrections in the Stopband-Correct program are saved in the "Live File", "Library" or "Archive", the parameters saved are the 400-point functions contained in the vectors $\mathbf{J}_0(t)$ and $\mathbf{J}(t)$, the constants contained in the vectors $\mathbf{J}_r$, $\mathbf{J}_b$ and $\mathbf{J}_t$, and the values of $B(t)$ and $\dot{B}(t)$. ($\mathbf{C}_0$, $\mathbf{C}_r$, $\mathbf{C}_b$, and $\mathbf{C}_t$ are also saved, but are not actually used upon retrieval.) Upon retrieval of the saved parameters, the program first calculates

$$\mathbf{C}_0(t) = \mathbf{R}\mathbf{J}_0(t), \quad \mathbf{C}_r = \mathbf{R}\mathbf{J}_r, \quad \mathbf{C}_b = \mathbf{R}\mathbf{J}_b, \quad \mathbf{C}_t = \mathbf{R}\mathbf{J}_t. \tag{51}$$

If $B(t)$ and $\dot{B}(t)$ have not changed, it then calculates

$$\mathbf{C}(t) = \mathbf{C}_0(t) + \mathbf{C}_r + \mathbf{C}_b B(t) + \mathbf{C}_t \dot{B}(t) \tag{52}$$

and if one "Loads" the corrections at this point, the 400-point functions are assigned values determined by the retrieved vector $\mathbf{J}(t)$. If the values of $B(t)$ and $\dot{B}(t)$ have changed, the user is given the option of having $\mathbf{J}(t)$ and $\mathbf{C}(t)$ recalculated using the new values of $B(t)$ and $\dot{B}(t)$.

11 Appendix: Sum and Difference Resonances

A detailed discussion of the theory of sum and difference resonances may be found in Ref. [11]. Following are some basic results.

The resonance is defined by the equation

$$mQ_x + nQ_y = N, \tag{53}$$

where Q_x and Q_y are the horizontal and vertical tunes, and m, n, and N are integers. If m and n have opposite signs, the resonance is called a difference resonance; otherwise it is called a sum resonance. The order, l, of the resonance is

$$l = |m| + |n|. \tag{54}$$

If the tunes are sufficiently close to the resonance, i.e. if they are within the resonance stopband, and if n is even (odd), the resonance will be excited by the Nth harmonic, in azimuth θ, of the normal (skew) $2l$-pole fields present in the machine. The resonance condition (53) arises from the first-order perturbation treatment of the vector potential terms $x^{|m|}y^{|n|}$ associated with the $2l$-pole field. The width of the stopband is proportional to the strength of the $2l$-pole field, and for resonances of order 3 and higher also depends on the amplitudes, J_x and J_y, of the betatron oscillations. If the tunes are near the resonance, and if $m \neq 0$, $n \neq 0$, then the quantity

$$C = nJ_x - mJ_y \tag{55}$$

is a constant of the motion. For the case of the difference resonances this implies that the amplitudes are bounded; for sum resonances the amplitudes can increase without bound. In the presence of space-charge forces the amplitudes are generally bounded as discussed in Ref. [12].

11.1 Excitation Coefficients

The excitation coefficient for the resonance (53) is of the form

$$\kappa = \frac{C}{p}\int_0^{2\pi r} \beta_x^{|m/2|}\beta_y^{|n/2|}K(s)e^{i\psi}ds, \tag{56}$$

where C is a normalization constant, p is the momentum,

$$\psi(s) = m(\mu_x - Q_x\theta) + n(\mu_y - Q_y\theta) + N\theta, \tag{57}$$

$$\mu_x(s) = \int_0^s \frac{ds'}{\beta_x(s')}, \quad \mu_y(s) = \int_0^s \frac{ds'}{\beta_y(s')}, \quad K(s) = \frac{\partial^{(l-1)} B}{\partial x^{(l-1)}}, \tag{58}$$

and $B = B_y(B_x)$ if n is even (odd).

11.2 Resonance Correction

For the correction of the $mQ_x + nQ_y = N$ resonance we require

$$\kappa_0 + \kappa = 0 \tag{59}$$

where κ_0 is the excitation coefficient due to imperfections in the machine, and κ is produced by the resonance correction scheme. For a set of point correction elements we have

$$\kappa = C \frac{K}{p} \sum_j I_j \beta_{xj}^{|m/2|} \beta_{yj}^{|n/2|} e^{i\psi_j}, \tag{60}$$

where C is a normalization constant, K is the integrated strength per unit current of each correction element, I_j is the current in the jth element, and

$$\beta_{xj} = \beta_x(s_j), \quad \beta_{yj} = \beta_y(s_j), \quad \psi_j = \psi(s_j). \tag{61}$$

If the correction elements occupy the same relative positions in each of P superperiods, and we let q and p denote the qth correction element in the pth superperiod, then (60) becomes

$$\kappa = C \frac{K}{p} \sum_{p=1}^{P} \sum_q I_{pq} \beta_{xq}^{|m/2|} \beta_{yq}^{|n/2|} e^{i\psi_{pq}}, \tag{62}$$

where

$$\psi_{pq} = \psi_q + 2\pi N(p-1)/P. \tag{63}$$

Now, if we choose

$$I_{pq} = I_q \cos \frac{2\pi M}{P}(p-1) = I_q f_p, \tag{64}$$

as in the correction schemes proposed in Ref. [13], then (62) becomes

$$\kappa = C \frac{K}{p} S_{MN} \sum_q I_q \beta_{xq}^{|m/2|} \beta_{yq}^{|n/2|} e^{i\psi_q}, \tag{65}$$

where

$$S_{MN} = \sum_{p=1}^{P} e^{i2\pi N(p-1)/P} \cos\frac{2\pi M}{P}(p-1) = \frac{1}{2}\sum_{p=1}^{P}(X^{p-1}+Y^{p-1}), \qquad (66)$$

$$X = e^{i2\pi(N+M)/P}, \quad Y = e^{i2\pi(N-M)/P}. \qquad (67)$$

Here we see that since

$$(Z-1)\sum_{p=1}^{P} Z^{p-1} = Z^P - 1, \qquad (68)$$

S_{MN} is zero unless $N \pm M = PI$, where I is any integer. Thus S_{MN} serves as a filter for unwanted harmonics. For the case of the Booster, $P = 6$, and we find the following values for S_{MN} and f_p:

M	N	S_{MN}	U_{MN}	V_{MN}
14	14	3	3/2	3/2
13	13	3	3/2	3/2
13	5	3	3/2	3/2
14	4	3	3/2	3/2
9	9	6	3	3
0	0	6	3	3
13	14	0	3/2	−3/2
14	13	0	3/2	−3/2
14	5	0	3/2	−3/2
13	4	0	3/2	−3/2
9	0	0	3	−3
0	9	0	3	−3

and

M	f_1	f_2	f_3	f_4	f_5	f_6
14	1	−1/2	−1/2	1	−1/2	−1/2
13	1	1/2	−1/2	−1	−1/2	1/2
9	1	−1	1	−1	1	−1
0	1	1	1	1	1	1

where

$$S_{MN} = U_{MN} + V_{MN}, \quad U_{MN} = \sum_{p=1}^{P/2} e^{i2\pi N(2p-2)/P} \cos\frac{2\pi M}{P}(2p-2). \quad (69)$$

11.3 Correction of Resonances $2Q_x = 9$ and $2Q_y = 9$

The $2Q_x = 9$ and $2Q_y = 9$ resonances are corrected with the four strings of quadrupoles listed in Section 2.2. Since these strings are wired so that the auxiliary windings on the quads in superperiods B, D, F have polarity opposite those in superperiods A, C, E, f_p will have the values tabulated for the case $M = 9$. The currents in each superperiod are given by (64) with $M = 9$ and $I_1 = I_7$, $I_2 = I_8$, $I_3 = -I_5$, $I_4 = -I_6$. If J_1, J_2, J_3, J_4 are the currents in quadrupoles 1, 2, 3, 4 of superperiod A, then (65) becomes

$$\begin{pmatrix} CX \\ SX \\ CY \\ SY \end{pmatrix} = \left(\frac{C}{p}\right) \mathbf{MQ} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{70}$$

where $\mathbf{Q}$ is given by (3), CX and SX are the real and imaginary parts of the excitation coefficient for the $2Q_x = 9$ resonance, CY and SY are the real and imaginary parts of the excitation coefficient for the $2Q_y = 9$ resonance, and

$$\mathbf{M} = 6(\mathbf{M}_a + \mathbf{M}_b), \tag{71}$$

$$\mathbf{M}_a = \begin{pmatrix} \beta_{x1}C_{x1} & \beta_{x2}C_{x2} & \beta_{x3}C_{x3} & \beta_{x4}C_{x4} \\ \beta_{x1}S_{x1} & \beta_{x2}S_{x2} & \beta_{x3}S_{x3} & \beta_{x4}S_{x4} \\ \beta_{y1}C_{y1} & \beta_{y2}C_{y2} & \beta_{y3}C_{y3} & \beta_{y4}C_{y4} \\ \beta_{y1}S_{y1} & \beta_{y2}S_{y2} & \beta_{y3}S_{y3} & \beta_{y4}S_{y4} \end{pmatrix}, \tag{72}$$

$$\mathbf{M}_b = \begin{pmatrix} \beta_{x7}C_{x7} & \beta_{x8}C_{x8} & -\beta_{x5}C_{x5} & -\beta_{x6}C_{x6} \\ \beta_{x7}S_{x7} & \beta_{x8}S_{x8} & -\beta_{x5}S_{x5} & -\beta_{x6}S_{x6} \\ \beta_{y7}C_{y7} & \beta_{y8}C_{y8} & -\beta_{y5}C_{y5} & -\beta_{y6}C_{y6} \\ \beta_{y7}S_{y7} & \beta_{y8}S_{y8} & -\beta_{y5}S_{y5} & -\beta_{y6}S_{y6} \end{pmatrix}. \tag{73}$$

Here

$$C_{xj} = \cos\psi_{xj}, \quad S_{xj} = \sin\psi_{xj}, \quad C_{yj} = \cos\psi_{yj}, \quad S_{yj} = \sin\psi_{yj}, \tag{74}$$

$$\psi_{xj} = 2\mu_{xj} + (N - 2Q_x)\theta_j, \quad \psi_{yj} = 2\mu_{yj} + (N - 2Q_y)\theta_j, \tag{75}$$

and $N = 9$.

11.4 Correction of Resonances $Q_x - Q_y = 0$ and $Q_x + Q_y = 9$

These resonances are corrected with the four strings of skew quadrupoles listed in Section 3.2. (Only the skew quads in sections 1, 2, 7, 8 of each superperiod are used.) These strings can be excited so that f_p will have the values tabulated for the case $M = 0$ or $M = 9$. The currents in each superperiod are given by (64) with $M = 0$ or $M = 9$ and $I_1 = I_7$, $I_2 = I_8$. If J_1 and J_2 are the currents in skew quads 1 and 2 of superperiod A, and J_3 and J_4 are the currents in skew quads 1 and 2 of superperiod B, then (65) becomes

$$\begin{pmatrix} C_0 \\ S_0 \end{pmatrix} = \frac{C}{2}\left(\frac{Q}{p}\right)\mathbf{N}_1 \begin{pmatrix} J_1 + J_3 \\ J_2 + J_4 \end{pmatrix} \tag{76}$$

and

$$\begin{pmatrix} C_9 \\ S_9 \end{pmatrix} = \frac{C}{2}\left(\frac{Q}{p}\right)\mathbf{N}_2 \begin{pmatrix} J_1 - J_3 \\ J_2 - J_4 \end{pmatrix} \tag{77}$$

where C_0 and S_0 are the real and imaginary parts of the excitation coefficient for the $Q_x - Q_y = 0$ resonance, C_9 and S_9 are the real and imaginary parts of the excitation coefficient for the $Q_x + Q_y = 9$ resonance, and

$$\mathbf{N}_1 = 6\begin{pmatrix} \beta_{x1}^{1/2}\beta_{y1}^{1/2}C_{11} + \beta_{x7}^{1/2}\beta_{y7}^{1/2}C_{17} & \beta_{x2}^{1/2}\beta_{y2}^{1/2}C_{12} + \beta_{x8}^{1/2}\beta_{y8}^{1/2}C_{18} \\ \beta_{x1}^{1/2}\beta_{y1}^{1/2}S_{11} + \beta_{x7}^{1/2}\beta_{y7}^{1/2}S_{17} & \beta_{x2}^{1/2}\beta_{y2}^{1/2}S_{12} + \beta_{x8}^{1/2}\beta_{y8}^{1/2}S_{18} \end{pmatrix}, \tag{78}$$

$$\mathbf{N}_2 = 6\begin{pmatrix} \beta_{x1}^{1/2}\beta_{y1}^{1/2}C_{21} + \beta_{x7}^{1/2}\beta_{y7}^{1/2}C_{27} & \beta_{x2}^{1/2}\beta_{y2}^{1/2}C_{22} + \beta_{x8}^{1/2}\beta_{y8}^{1/2}C_{28} \\ \beta_{x1}^{1/2}\beta_{y1}^{1/2}S_{21} + \beta_{x7}^{1/2}\beta_{y7}^{1/2}S_{27} & \beta_{x2}^{1/2}\beta_{y2}^{1/2}S_{22} + \beta_{x8}^{1/2}\beta_{y8}^{1/2}S_{28} \end{pmatrix}. \tag{79}$$

Here

$$C_{nj} = \cos\psi_{nj}, \quad S_{nj} = \sin\psi_{nj}, \tag{80}$$

$$\psi_{1j} = \mu_{xj} - \mu_{yj} + (Q_y - Q_x)\theta_j, \quad \psi_{2j} = \mu_{xj} + \mu_{yj} + (9 - Q_x - Q_y)\theta_j. \tag{81}$$

11.5 Correction of Resonances $3Q_x = 14$, $Q_x + 2Q_y = 14$, $3Q_x = 13$, and $Q_x + 2Q_y = 13$

These resonances are corrected with the eight strings of sextupoles listed in Section 4.2. Since the auxiliary windings on the sextupoles in superperiods B, C, E, F have half as many turns as those in superperiods A and D, and are wired with the opposite polarity, f_p will have the values tabulated for the cases $M = 14$ or $M = 13$. For the correction of resonances $3Q_x = 14$ and $Q_x + 2Q_y = 14$, the currents in each superperiod are given by (64) with $M = 14$ and $I_3 = -I_1$, $I_4 = -I_2$, $I_7 = -I_5$, $I_8 = -I_6$. If J_1, J_2, J_3, J_4 are the currents in sextupoles 1, 2, 5, 6 of superperiod A, and J_5, J_6, J_7, J_8, are the currents in sextupoles 1, 2, 5, 6 of superperiod D, then (65) becomes

$$\begin{pmatrix} CX_{14} \\ SX_{14} \\ CY_{14} \\ SY_{14} \end{pmatrix} = \frac{C}{2}\left(\frac{S}{p}\right)\mathbf{M}_{14}\begin{pmatrix} J_1 + J_5 \\ J_2 + J_6 \\ J_3 + J_7 \\ J_4 + J_8 \end{pmatrix}, \tag{82}$$

where CX_{14} and SX_{14} are the real and imaginary parts of the excitation coefficient for the $3Q_x = 14$ resonance, CY_{14} and SY_{14} are the real and imaginary parts of $-1/3$ times the excitation coefficient for the $Q_x + 2Q_y = 14$ resonance, and

$$\mathbf{M}_{14} = 3(\mathbf{M}_a - \mathbf{M}_b), \tag{83}$$

$$\mathbf{M}_a = \begin{pmatrix} \beta_{x1}^{3/2}C_{x1} & \beta_{x2}^{3/2}C_{x2} & \beta_{x5}^{3/2}C_{x5} & \beta_{x6}^{3/2}C_{x6} \\ \beta_{x1}^{3/2}S_{x1} & \beta_{x2}^{3/2}S_{x2} & \beta_{x5}^{3/2}S_{x5} & \beta_{x6}^{3/2}S_{x6} \\ \beta_{x1}^{1/2}\beta_{y1}C_{y1} & \beta_{x2}^{1/2}\beta_{y2}C_{y2} & \beta_{x5}^{1/2}\beta_{y5}C_{y5} & \beta_{x6}^{1/2}\beta_{y6}C_{y6} \\ \beta_{x1}^{1/2}\beta_{y1}S_{y1} & \beta_{x2}^{1/2}\beta_{y2}S_{y2} & \beta_{x5}^{1/2}\beta_{y5}S_{y5} & \beta_{x6}^{1/2}\beta_{y6}S_{y6} \end{pmatrix}, \tag{84}$$

$$\mathbf{M}_b = \begin{pmatrix} \beta_{x3}^{3/2}C_{x3} & \beta_{x4}^{3/2}C_{x4} & \beta_{x7}^{3/2}C_{x7} & \beta_{x8}^{3/2}C_{x8} \\ \beta_{x3}^{3/2}S_{x3} & \beta_{x4}^{3/2}S_{x4} & \beta_{x7}^{3/2}S_{x7} & \beta_{x8}^{3/2}S_{x8} \\ \beta_{x3}^{1/2}\beta_{y3}C_{y3} & \beta_{x4}^{1/2}\beta_{y4}C_{y4} & \beta_{x7}^{1/2}\beta_{y7}C_{y7} & \beta_{x8}^{1/2}\beta_{y8}C_{y8} \\ \beta_{x3}^{1/2}\beta_{y3}S_{y3} & \beta_{x4}^{1/2}\beta_{y4}S_{y4} & \beta_{x7}^{1/2}\beta_{y7}S_{y7} & \beta_{x8}^{1/2}\beta_{y8}S_{y8} \end{pmatrix}. \tag{85}$$

Here

$$C_{xj} = \cos\psi_{xj}, \quad S_{xj} = \sin\psi_{xj}, \quad C_{yj} = \cos\psi_{yj}, \quad S_{yj} = \sin\psi_{yj}, \tag{86}$$

$$\psi_{xj} = 3\mu_{xj} + (N - 3Q_x)\theta_j, \quad \psi_{yj} = \mu_{xj} + 2\mu_{yj} + (N - Q_x - 2Q_y)\theta_j, \tag{87}$$

and $N = 14$. Similarly, for the correction of the $3Q_x = 13$ and $Q_x + 2Q_y = 13$ resonances we have

$$\begin{pmatrix} CX_{13} \\ SX_{13} \\ CY_{13} \\ SY_{13} \end{pmatrix} = \frac{C}{2}\left(\frac{S}{p}\right)\mathbf{M}_{13}\begin{pmatrix} J_1 - J_5 \\ J_2 - J_6 \\ J_3 - J_7 \\ J_4 - J_8 \end{pmatrix}, \tag{88}$$

where $\mathbf{M}_{13}$ is given by (83–87) with $N = 13$.

Now, since $S_{MN} = 3$ for $(M, N) = (14, 4)$ and $(13, 5)$, correction of the $3Q_x = 14$ and $Q_x + 2Q_y = 14$ resonances can produce a nonzero excitation coefficient for the $2Q_y - Q_x = 4$ resonance, and correction of the $3Q_x = 13$ and $Q_x + 2Q_y = 13$ resonances can produce a nonzero excitation coefficient for the $2Q_y - Q_x = 5$ resonance. Evaluating (65) for the $2Q_y - Q_x = 4$ resonance we find

$$\begin{pmatrix} CXY_4 \\ SXY_4 \end{pmatrix} = \frac{C}{2}\left(\frac{S}{p}\right)\mathbf{M}_4 \begin{pmatrix} J_1 + J_5 \\ J_2 + J_6 \\ J_3 + J_7 \\ J_4 + J_8 \end{pmatrix}, \tag{89}$$

where CXY_4 and SXY_4 are the real and imaginary parts of $-1/3$ times the excitation coefficient for the $2Q_y - Q_x = 4$ resonance, and

$$\mathbf{M}_4 = 3(\mathbf{M}_a - \mathbf{M}_b), \tag{90}$$

$$\mathbf{M}_a = \begin{pmatrix} \beta_{x1}^{1/2}\beta_{y1}C_1 & \beta_{x2}^{1/2}\beta_{y2}C_2 & \beta_{x5}^{1/2}\beta_{y5}C_5 & \beta_{x6}^{1/2}\beta_{y6}C_6 \\ \beta_{x1}^{1/2}\beta_{y1}S_1 & \beta_{x2}^{1/2}\beta_{y2}S_2 & \beta_{x5}^{1/2}\beta_{y5}S_5 & \beta_{x6}^{1/2}\beta_{y6}S_6 \end{pmatrix}, \tag{91}$$

$$\mathbf{M}_b = \begin{pmatrix} \beta_{x3}^{1/2}\beta_{y3}C_3 & \beta_{x4}^{1/2}\beta_{y4}C_4 & \beta_{x7}^{1/2}\beta_{y7}C_7 & \beta_{x8}^{1/2}\beta_{y8}C_8 \\ \beta_{x3}^{1/2}\beta_{y3}S_3 & \beta_{x4}^{1/2}\beta_{y4}S_4 & \beta_{x7}^{1/2}\beta_{y7}S_7 & \beta_{x8}^{1/2}\beta_{y8}S_8 \end{pmatrix}. \tag{92}$$

Here

$$C_j = \cos\psi_j, \quad S_j = \sin\psi_j, \quad \psi_j = 2\mu_{yj} - \mu_{xj} + (N - 2Q_y + Q_x)\theta_j, \tag{93}$$

and $N = 4$. Similarly, for the $2Q_y - Q_x = 5$ resonance we have

$$\begin{pmatrix} CXY_5 \\ SXY_5 \end{pmatrix} = \frac{C}{2}\left(\frac{S}{p}\right)\mathbf{M}_5 \begin{pmatrix} J_1 - J_5 \\ J_2 - J_6 \\ J_3 - J_7 \\ J_4 - J_8 \end{pmatrix}, \tag{94}$$

where $\mathbf{M}_5$ is given by (90–93) with $N = 5$.

11.6 Correction of Resonances $3Q_y = 14$ and $Q_y + 2Q_x = 14$

These resonances are corrected with the four strings of skew sextupoles listed in Section 5.2. Let $J_1, J_2, J_3, \ldots, J_8$ be the currents in skew sextupoles SSVB1, SSHB2, SSVB7, SSHB8, SSVE1, SSHE2, SSVE7, and SSHE8 respectively. Since the magnets are connected in series strings as indicated in Section 5.2, we have $J_5 = J_1$, $J_6 = J_2$, $J_7 = J_3$, $J_8 = J_4$, and it follows from (60) that

$$\begin{pmatrix} CY \\ SY \\ CX \\ SX \end{pmatrix} = \left(\frac{CS}{p}\right) \mathbf{M} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix}, \tag{95}$$

where CY and SY are the real and imaginary parts of the excitation coefficient for the $3Q_y = 14$ resonance, CX and SX are the real and imaginary parts of $-1/3$ times the excitation coefficient for the $Q_y + 2Q_x = 14$ resonance, and

$$\mathbf{M} = 2\mathbf{M}_a, \tag{96}$$

$$\mathbf{M}_a = \begin{pmatrix} \beta_{y1}^{3/2} C_{y1} & \beta_{y2}^{3/2} C_{y2} & \beta_{y7}^{3/2} C_{y7} & \beta_{y8}^{3/2} C_{y8} \\ \beta_{y1}^{3/2} S_{y1} & \beta_{y2}^{3/2} S_{y2} & \beta_{y7}^{3/2} S_{y7} & \beta_{y8}^{3/2} S_{y8} \\ \beta_{y1}^{1/2} \beta_{x1} C_{x1} & \beta_{y2}^{1/2} \beta_{x2} C_{x2} & \beta_{y7}^{1/2} \beta_{x7} C_{x7} & \beta_{y8}^{1/2} \beta_{x8} C_{x8} \\ \beta_{y1}^{1/2} \beta_{x1} S_{x1} & \beta_{y2}^{1/2} \beta_{x2} S_{x2} & \beta_{y7}^{1/2} \beta_{x7} S_{x7} & \beta_{y8}^{1/2} \beta_{x8} S_{x8} \end{pmatrix}. \tag{97}$$

Here

$$C_{yj} = \cos\psi_{yj}, \quad S_{yj} = \sin\psi_{yj}, \quad C_{xj} = \cos\psi_{xj}, \quad S_{xj} = \sin\psi_{xj}, \tag{98}$$

$$\psi_{yj} = 3\mu_{yj} + (N - 3Q_y)\theta_j, \quad \psi_{xj} = \mu_{yj} + 2\mu_{xj} + (N - Q_y - 2Q_x)\theta_j, \tag{99}$$

and $N = 14$.

11.7 The Ninth-Harmonic Sextupole Correction

These corrections consist of the four strings of sextupole monitor windings listed in Section 6.2. As shown in Ref. [10], the $2Q_x = 9$ and $2Q_y = 9$ resonance excitation coefficients for beam particles whose momentum differs from the central momentum of the beam by δp are given by (56) with

$$K(s) = -\frac{\delta p}{p} K_3(s) D(s). \tag{100}$$

Here $K_3(s)$ is the sextupole strength along the central trajectory and $D(s)$ is the periodic dispersion. Since the strings of sextupole monitor windings are wired in the same configuration as the strings of quadrupole auxiliary windings, the excitation coefficients will have the same form as equations (70). Thus, if J_1, J_2, J_3, J_4 are the currents in the monitor windings on sextupoles 1, 2, 3, 4 of superperiod A, then we have

$$\begin{pmatrix} CX \\ SX \\ CY \\ SY \end{pmatrix} = C\frac{\delta p}{p}\left(\frac{S}{p}\right)\mathbf{M}\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{pmatrix} \tag{101}$$

where CX and SX are the real and imaginary parts of the excitation coefficient for the $2Q_x = 9$ resonance, CY and SY are the real and imaginary parts of the excitation coefficient for the $2Q_y = 9$ resonance, S is the strength of the sextupoles, and

$$\mathbf{M} = 6(\mathbf{M}_a + \mathbf{M}_b), \tag{102}$$

$$\mathbf{M}_a = \begin{pmatrix} D_1\beta_{x1}C_{x1} & D_2\beta_{x2}C_{x2} & D_3\beta_{x3}C_{x3} & D_4\beta_{x4}C_{x4} \\ D_1\beta_{x1}S_{x1} & D_2\beta_{x2}S_{x2} & D_3\beta_{x3}S_{x3} & D_4\beta_{x4}S_{x4} \\ D_1\beta_{y1}C_{y1} & D_2\beta_{y2}C_{y2} & D_3\beta_{y3}C_{y3} & D_4\beta_{y4}C_{y4} \\ D_1\beta_{y1}S_{y1} & D_2\beta_{y2}S_{y2} & D_3\beta_{y3}S_{y3} & D_4\beta_{y4}S_{y4} \end{pmatrix}, \tag{103}$$

$$\mathbf{M}_b = \begin{pmatrix} D_7\beta_{x7}C_{x7} & D_8\beta_{x8}C_{x8} & -D_5\beta_{x5}C_{x5} & -D_6\beta_{x6}C_{x6} \\ D_7\beta_{x7}S_{x7} & D_8\beta_{x8}S_{x8} & -D_5\beta_{x5}S_{x5} & -D_6\beta_{x6}S_{x6} \\ D_7\beta_{y7}C_{y7} & D_8\beta_{y8}C_{y8} & -D_5\beta_{y5}C_{y5} & -D_6\beta_{y6}C_{y6} \\ D_7\beta_{y7}S_{y7} & D_8\beta_{y8}S_{y8} & -D_5\beta_{y5}S_{y5} & -D_6\beta_{y6}S_{y6} \end{pmatrix}. \tag{104}$$

Here

$$C_{xj} = \cos\psi_{xj}, \quad S_{xj} = \sin\psi_{xj}, \quad C_{yj} = \cos\psi_{yj}, \quad S_{yj} = \sin\psi_{yj}, \tag{105}$$

$$\psi_{xj} = 2\mu_{xj} + (N - 2Q_x)\theta_j, \quad \psi_{yj} = 2\mu_{yj} + (N - 2Q_y)\theta_j, \tag{106}$$

$D_j = D(s_j)$, and $N = 9$.

11.8 The Ninth-Harmonic Skew Sextupole Correction

These corrections consist of the two strings of skew sextupoles listed in Section 7.2. As shown in Ref. [10], the $Q_x + Q_y = 9$ resonance excitation coefficient for beam particles whose momentum differs from the central momentum of the beam by δp are given by (56) with

$$K(s) = -\frac{\delta p}{p}K_3(s)D(s). \tag{107}$$

Here $K_3(s)$ is the skew sextupole strength along the central trajectory and $D(s)$ is the periodic dispersion. Let J_1, J_2, J_3, J_4 be the currents in skew sextupoles SSVA3, SSVA7, SSVD3, SSVD7 respectively. Since the magnets are connected in series strings as indicated in Section 7.2, we have $J_3 = -J_1$, $J_4 = -J_2$, and it follows that

$$\begin{pmatrix} C_9 \\ S_9 \end{pmatrix} = C\frac{\delta p}{p}\left(\frac{S}{p}\right)\mathbf{M}\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} \tag{108}$$

where C_9 and S_9 are the real and imaginary parts of the excitation coefficient for the $Q_x + Q_y = 9$ resonance, S is the strength of the skew sextupoles, and

$$\mathbf{M} = 2\begin{pmatrix} D_3\beta_{x3}^{1/2}\beta_{y3}^{1/2}C_3 & D_7\beta_{x7}^{1/2}\beta_{y7}^{1/2}C_7 \\ D_3\beta_{x3}^{1/2}\beta_{y3}^{1/2}S_3 & D_7\beta_{x7}^{1/2}\beta_{y7}^{1/2}S_7 \end{pmatrix}. \tag{109}$$

Here

$$C_j = \cos\psi_j, \quad S_j = \sin\psi_j, \quad D_j = D(s_j), \tag{110}$$

$$\psi_j = \mu_{xj} + \mu_{yj} + (9 - Q_x - Q_y)\theta_j. \tag{111}$$

12 References

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