# WHERE ARE THE AGS MAGNETS? 

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## U.S. Department of Energy <br> USDOE Office of Science (SC)

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## Introduction

Since we are planning to resurvey the AGS both horizontally and vertically for what is probably the first time since its conversion, and since we are exploring the possibilities of installing the AGS in some computer modelling programs other than BEAM, it seems appropriate to review the layout of the AGS magnets. There are three basic documents which describe the layout:

1. "Location of Magnets", Lloyd Smith, ADDIR LS-2, May 14, 1956
2. "Location of Magnets", M.H. Blewett, A.I.D., May 28, 1957
3. "Standard Survey Data", O.S. Reading and M. Buchanen, August 20, 1957.

Smith gives the prescription, Blewett recalculates with higher precision, and Reading tabulates the final version, which is incorporated into the AGS Standards Book. In this note we restate the prescription given by Smith, bringing out explicitly the approximations he made. Since Blewett does not show her calculations, we must infer what she did, and since our results are in very good agreement, we conclude that our prescription is the same as the one used to layout the AGS.

In the appendices we develop the formulas to be used, specify the parameters of the AGS, restate the as built parameters, briefly consider the question of the AGS circumference, evaluate some standard AGS drawings which we believe have a slight error, and consider the BEAM program, which we also believe is slightly in error. This note was largely prompted by the discrepancies between the BEAM program and the AGS Standards Book.

## Overview

Consider a point, which is essentially at the center of the AGS, which we shall call the Center of the Reference Circle. Imagine a circle drawn around this point at a radius of 430.45 feet. The 24 Primary Monuments are located on this circle at 15 degree intervals. Imagine a second circle drawn around this point at a radius of 421.45 feet. The 72 Secondary Monuments are located on this circle at varying angular intervals. Since the AGS magnets are placed essentially above the Secondary Monument Circle, this radius, 421.45 feet, is often called the Orbit Radius. It is important to note that this radius is only approximately the orbit radius. Appendix IV discusses in detail the various possible radii of the AGS.

For a coordinate system encompassing several magnets we sha11 choose as the x axis a primary chord, running between two primary monuments. The $y$ axis is in the plane of the ring, perpendicular to the x axis, and directed away from the center of the ring. Figure 1 shows this geometry. The figure shows 10 magnets, one half of a superperiod, which is generated by reflecting the pattern of the five magnets in the right half of the picture through the center line to the left. A superperiod consists of two half superperiods, the ring of 12 superperiods. We need calculate only the five magnets shown in the right half of the figure.

Each magnet is located in the Primary Monument Coordinate System by means of two survey sockets which are located on top of the magnet, above the magnet center line and three inches in from the ends of the steel. There are three local coordinate systems in use.

1. The SOCKET SYSTEM is used by the surveyors to locate elements in the straight sections and is based on a line connecting the socket on one magnet to the socket on the next.
2. The BEAM SYSTEM is used in the computer program BEAM. For each magnet its $x$-axis is the socket line, its $y$-axis is perpendicular to the $x$-axis directed outward, and its origin is the intersection of the $x$-axis with the upstream end of the magnet stee1. In a straight section the $x$-axis is the line
connecting the points where the $x$-axes of the neighboring magnets intersect the ends of the magnet steel.
3. The THEORETICAL BEAM CENTER LINE SYSTEM seems to be falling into disuse, but we shall try to revive it in a subsequent paper.

Figure 2a shows the magnet geometry, and indicates the magnet effective length, L, which we take to be exactly 4.000 inches longer than the magnet steel, which is taken to be exactly 75.000 inches for short magnets and 90.000 inches for long magnets. Figure 2 b shows a straight section. There are three lengths: $\mathrm{L}_{1}$ equals 28.000 inches, $\mathrm{L}_{2}$ equals 64.000 inches, and $\mathrm{L}_{3}$ equals 123.9907 inches.

## Detailed Calculation

Tracking an orbit through a combined function magnet can be a very complex operation. In Appendix I we consider the appropriate formulas and reduce them to the very simple results given in Table AI-4. These are the formulas used to lay out the AGS. In this system and in this approximation when we are considering the central orbit, each magnet looks like a dipole magnet, bending the orbit by an amount proportional to the magnet length with the orbit going symmetrically through the magnet. Figure 3 shows schematically an orbit through a magnet.

The calculating procedure is, to quote Smith, "starting, for instance, with a straight section, we proceed in an arbitrary direction a distance equal to the effective length of that straight section, turn through an angle $\mathrm{L} / 2 \rho$, proceed along the aperture center line for a distance equal to the effective magnet length, turn through an angle $\mathrm{L} / 2 \rho$, proceed for a distance equal to the length of the next straight section, and so on." This is simple enough, the only complication being the effective length of the straight section which is developed in Fig. 4.

When the magnets are not identical, the different saggitas lead to some complications which must be treated carefully. Essentially the short magnets have a smaller saggita than the long magnets and are
displaced to the center of the ring by about 0.06 inches. This is detailed in Figure 5. Using the formulas and parameters specified in Table AI-4 and in Appendix II our results agree with Reading to a few tenths of a mill, as shown in Table 1 , indicating that we are following the same prescription. For completeness Appendix III reproduces the basic results of Reading from the AGS Standards Book. These are the numbers which we believe describe the actual layout of the AGS. They should be used by both the surveyors and by people generating programs. In this note we have deduced the prescription used to generate these numbers.

With the passage of time some of these numbers have been lost and errors have crept into our standard procedures. Appendix $V$ discusses several places we have found errors or differing approximations from the numbers presented in this note. The practical effects are inconsequential, but correcting these errors would give greater intellectual consistency to the AGS data.

Table 1

Design Values vs. Recalculated Values

|  | AGS Standards Book |  | $\begin{gathered} \text { "Standards" } \\ \text { minus } \\ \text { "This Note" } \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Element | $x$ (inches) | y (inches) | $\frac{\Delta x}{\left(10^{-4}\right.}$ | $\frac{\Delta y}{\text { inches })}$ |
| Center | 674.2200 | -63.4354 |  |  |
| $5^{\prime} \mathrm{SS}$ |  |  |  |  |
| Mag 1 | 706.2200 | -63.4633 | 0 | $+.64$ |
|  | 796.2112 | -64.7219 | 0 | -1.50 |
| Mag 2 | 824.2002 | -65.5046 | -. 50 | . 61 |
|  | 914.1211 | -69.2798 | . 15 | . 20 |
| Mag 3 | 978.0210 | -72.8564 | 0 | . 66 |
|  | 1067.8011 | -79.1435 | -. 50 | -. 4 |
| Mag 4 | 1095.6976 | -81.5494 | . 04 | . 1 |
|  | 1170.3548 | -88.7120 | 0 | $-.7$ |
| Mag 5 | 1198.1935 | -91.7131 | . 22 | 1.75 |
|  | 1272.6617 | -100.6280 | -. 40 | -1.1 |




Figure 2a


Figure 2b


Figure 3


Figure 4


Figure 5

Formulas for Tracking a Particle Through a Combined Function Magnet

The motion of a charged particle in a magnetic field is governed by the Lorentz equation:

$$
\frac{d}{d t}(\overrightarrow{m v})=e \vec{v} \times \vec{B}
$$

AI. 1

In order to study the central orbit in an accelerator we need only consider the case where the particle is moving in the horizontal plane and the field has only a vertical component in that plane. Figure 3 shows the geometry. Then following Steffen (High Energy Beam Optics, Klaus G. Steffen, Wiley, New York, 1965) we find the complete and exact equation for this case to be:

$$
y^{\prime \prime}=-\frac{e}{p} B_{z}(x, y) \quad\left\{1+y^{\prime 2}\right\} 3 / 2
$$

$$
\mathrm{p}=\mathrm{mv}
$$

$$
y^{\prime}=\frac{d y}{d x}
$$

We shall assume simple hard edge magnets. For a central trajectory we need not consider focusing effects at the entrance and exit of the magnet. We can assume that our magnets are perfect and therefore we shall not consider imperfections in the magnetic fields. We are dealing with the central ray and therefore need not consider off momentum trajectories. Therefore Equation AI. 2 contains everything we need to evaluate.
AI-2

For a combined function magnet:

$$
\begin{gathered}
B_{z}(x, y)=B_{o}\left\{1-\frac{n y}{\rho}\right\} \\
\frac{1}{\rho}=\frac{e B_{o}}{p} \\
n=-\frac{\rho}{B_{o}} \frac{d B}{d y}
\end{gathered}
$$

The field index, $n$, is negative for a focusing magnet and positive for a defocusing magnet. Since keeping track of the sign calls for some attention on the part of the reader, we choose at this time to replace n with $\lambda$, defined as:

$$
\lambda=\frac{\rho}{\sqrt{-n}}
$$

for a focusing magnet; and

$$
\lambda=\frac{\rho}{\sqrt{n}}
$$

for a defocussing magnet. Henceforward our parameters are:
$\rho$, the bending radius, about 3000 inches;
$\lambda$, which turns out to be the wavelength of the betatron
function divided by two pi, about 200 inches;
L, the magnet effective length, about 100 inches.

All these quantities are positive. For a focussing magnet Eq. AI. 2 becomes:

$$
y^{\prime \prime}=-\frac{1}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y\right)\left\{1+y^{\prime}\right\} 3 / 2
$$

and for a defocussing magnet it is:

$$
y^{\prime \prime}=-\frac{1}{\rho}\left(1-\frac{\rho}{\lambda^{2}} y\right)\left\{1+y^{\prime}\right\} 3 / 2
$$

The solution to either of these is a mess. However the slope of the orbit, $y^{\prime}$, in a magnet is about 0.01 radians. The square of this is $10^{-4}$, which can be ignored compared to one. A tedious evaluation suggests that including this term gives effects on the orbit of the order of one percent. In any case we shall drop the term in $y^{\prime}$ from all further consideration. We shall refer to this as Approximation 1 , and estimate the errors introduced by this approximation as lying between $10^{-2}$ and $10^{-4}$. Since we have no evidence that they were taken into account in the original layout of the AGS, which is what we are trying to reproduce, we can safely ignore them. To this approximation we have for a focusing magnet:

$$
y^{\prime \prime}=-\frac{1}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y\right)
$$

The solution is:

$$
\begin{gathered}
y=-\frac{\lambda^{2}}{\rho}+\frac{\lambda^{2}}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y_{o}\right) \cos \frac{x}{\lambda}+\lambda y_{0}^{\prime} \sin \frac{x}{\lambda} \\
y^{\prime}=-\frac{\lambda}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y_{o}\right) \sin \frac{x}{\lambda}+y_{o}^{\prime} \cos \frac{x}{\lambda}
\end{gathered}
$$

where $y_{o}$ and $y_{o}^{\prime}$ are the position and the tangent of the angle of the orbit at the entrance to the magnet. Taking the second derivative confirms this solution, which is given by Smith (LS-2) and is derived
by Bruck (Circular Particle Accelerators, Henri Bruck, LA-TR-72-10) for a curved magnet.

We now have two equations relating four unknowns, $y$ and $y$-prime at the entrance ( $x=0$ ) and the exit ( $x=L$ ) of the magnet. If we require that the entrance and exit angles be equal but of opposite sign:

$$
y_{0}^{\prime}=-y^{\prime}(\mathrm{L})
$$

this is a third requirement, leading to the relationships:

$$
\begin{gathered}
y_{o}^{\prime}=\frac{\lambda}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y_{o}\right) \tan \frac{L}{2 \lambda} \\
y(L)=y_{o}
\end{gathered}
$$

Note that by making the entrance and exit angles equal, the entrance and exit positions are equal. The beam position at the magnet center is:

$$
y(L / 2)=\frac{\lambda^{2}}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y_{0}\right) \frac{1}{\cos (L / 2 \lambda)}-\frac{\lambda^{2}}{\rho}
$$

and the saggita is:

$$
y(L / 2)-y_{0}=\frac{\lambda^{2}}{\rho}\left(1+\frac{\rho}{\lambda^{2}} y_{0}\right)\left\{\frac{1-\cos (L / 2 \lambda)}{\cos (L / 2 \lambda)}\right\}
$$

Tables AI-1 and AI-2 summarize these results for focussing and defocussing magnets. These results are exact except for Approximation 1. They seem very reasonable. There is a small correction to the bend due to where the particle enters the magnet, the sign of which depends on whether the magnet is focussing or defocussing, and there is a second order difference in the total bend, again depending on magnet type.

We still have one free parameter and could choose $y_{o}$ to center the beam transversely in the magnet, but a slightly different choice offers a nice simplification and was adopted by the original designers of the AGS. We choose the mean bending radius, $\rho$, around the ring to be given by the total effective magnet bending length divided by two pi:

$$
\rho=\frac{\Sigma \mathrm{L}}{2 \pi}
$$

We choose the bending in each magnet to be the magnet effective length divided by $\rho$.

$$
\theta=\frac{L}{\rho}
$$

This is very natural for a string of dipoles, but is a somewhat special case for a combined function machine. This is the fourth requirement on our system of four unknowns so all the parameters are fixed. Table AI-3 shows the results for $y_{0}$. In laying out the AGS only the first order terms of these expressions were used. The final results are given in Table AI-4. The central orbit was laid out so that the focussing and defocussing magnets had the same bends and looked essentially like dipoles. The central orbit is very simple. However, off momentum orbits become complicated since the bending in the magnet does not depend on just the magnet length. In order to achieve this simplicity the beam is moved about 0.05 inches inward in the magnets. That is:

$$
y_{o}=-\frac{1}{12} \frac{L^{2}}{\rho}
$$

rather than:

$$
y_{o}=\frac{1}{2}(\text { Saggita })=-\frac{1}{16} \frac{L^{2}}{\rho}
$$

For completeness we show in Table AI-5 the results of keeping two terms in the expansion. The focussing and defocussing magnets are now displaced by a few mills, a difference much too small to be of any practical importance, but which does show up when careful calculations are done with the BEAM program.

## Table AI-1

## Basic Orbit Equations

## A. Focussing Magnet

$$
\begin{gathered}
y(x)=-\frac{\lambda^{2}}{\rho}+\frac{\lambda^{2}}{\rho}\left\{1+\frac{\rho}{\lambda^{2}} y_{0}\right\} \cos (x / \lambda)+\lambda y_{0}^{\prime} \sin (x / \lambda) \\
y^{\prime}(x)=-\frac{\lambda}{\rho}\left\{1+\frac{\rho}{\lambda^{2}} y_{0}\right\} \sin (x / \lambda)+y_{0}^{\prime} \cos (x / \lambda)
\end{gathered}
$$

B. Defocussing Magnet

$$
\begin{aligned}
& y(x)=\frac{\lambda^{2}}{\rho}-\frac{\lambda^{2}}{\rho}\left\{1-\frac{\rho}{\lambda^{2}} y_{0}\right\} \cosh (x / \lambda)+\lambda y_{0}^{\prime} \sinh (x / \lambda) \\
& y^{\prime}(x)=-\frac{\lambda}{\rho}\left\{1-\frac{\rho}{\lambda^{2}} y_{0}\right\}_{0} \sinh (x / \lambda)+y_{0}^{\prime} \cosh (x / \lambda)
\end{aligned}
$$

Require:

$$
y^{\prime}(L)=-y_{0}^{\prime}
$$

Then:
A. Focussing Magnet

$$
\begin{gathered}
y(L)=y_{0} \\
y_{0}^{\prime}=\frac{\lambda}{\rho}\left\{1+\frac{\rho}{\lambda^{2}} y_{o}\right\}_{0} \tan (L / 2 \lambda) \\
\text { Saggita }=y(L / 2)-y_{0} \\
=\frac{\lambda^{2}}{\rho} \quad\left\{1+\frac{\rho}{\lambda^{2}} y_{0}\right\} \quad\left\{\frac{1-\cos (L / 2 \lambda)}{\cos (L / 2 \lambda)}\right\}
\end{gathered}
$$

B. Defocussing Magnet

$$
\begin{gathered}
y(L)=y_{o} \\
y_{o}^{\prime}=\frac{\lambda}{\rho}\left\{1-\frac{\rho}{\lambda^{2}} y_{o}\right\} \tanh (L / 2 \lambda) \\
\text { Saggita }=y(L / 2)-y_{o} \\
=\frac{\lambda^{2}}{\rho}\left\{1-\frac{\rho}{\lambda^{2}} y_{o}\right\} \quad\left\{\frac{\cosh (L / 2 \lambda)-1}{\cosh (L / 2 \lambda)}\right\}
\end{gathered}
$$

Require:

$$
y_{o}^{\prime}=\tan (L / 2 \rho)
$$

Then:
A. Focussing Magnet

$$
y_{0}=\frac{\lambda^{2}}{\rho}\left\{\frac{\rho \tan (\mathrm{~L} / 2 \rho)}{\lambda \tan (\mathrm{L} / 2 \lambda)}-1\right\}
$$

B. Defocussing Magnet

$$
\mathrm{y}_{\mathrm{o}}=\frac{\lambda^{2}}{\rho}\left\{1-\frac{\rho \tan (\mathrm{L} / 2 \rho)}{\lambda \tanh (\mathrm{L} / 2 \lambda)}\right\}
$$

## Table AI-4

Formulas Used to Design AGS

$$
\begin{gathered}
y_{o}^{\prime}=-y^{\prime}(L)=\frac{L}{2 \rho} \\
y_{o}=y(L)=-\frac{1}{12} \frac{L^{2}}{\rho} \\
\text { Saggita }=\frac{1}{8} \frac{L^{2}}{\rho}
\end{gathered}
$$

## Formulas Keeping 2 Terms

A. Focussing Magnet

$$
\begin{aligned}
& y_{o}^{\prime}=-y^{\prime}(L)=\frac{L}{2 \rho}\left\{1+\frac{1}{3} \frac{L}{2 \rho}\right\} \\
& y_{o}=y(L)=-\frac{1}{12} \frac{L^{2}}{\rho}\left\{1-\frac{\lambda^{2}}{\rho^{2}}\right\} \\
& \text { Saggita }=\frac{1}{8} \frac{L^{2}}{\rho}\left\{1+\frac{1}{24} \frac{L^{2}}{\lambda^{2}}\right\}
\end{aligned}
$$

B. Defocussing Magnet

$$
\begin{aligned}
& y_{o}^{\prime}=-y^{\prime}(L)=\frac{L}{2 \rho}\left\{1+\frac{1}{3} \frac{L}{2 \rho}\right\} \\
& y_{o}=y(L)=-\frac{1}{12} \frac{L^{2}}{\rho}\left\{1+\frac{\lambda^{2}}{\rho^{2}}\right\} \\
& \text { Saggita }=\frac{1}{8} \frac{L^{2}}{\rho^{2}}\left\{1-\frac{1}{24} \frac{L^{2}}{\lambda^{2}}\right\}
\end{aligned}
$$

## Appendix

## AGS Layout Parameters

| Parameter | Long Magnet | Short Magnet | Units |
| :---: | :---: | :---: | :---: |
| Number | 144 | 96 |  |
| Steel Length | 90.0000 | 75.0000 | inches |
| Effective Length, L | 94.0000 | 79.0000 | inches |
| Total Bending Length | 211.20 .0000 |  | inches |
| Bending Radius, $\rho$ | 3361.3524 |  | inches |
| Bend per Magnet, $\theta$ | 27.9649 | 23.5024 | mr |
| 1/2 Bend, $\Theta / 2$ | 13.9825 | 11.7512 | mr |
| $\mathrm{y}_{\mathrm{o}}^{\prime}, \tan (\theta / 2)$ | 0.0139834 | 0.0117517 |  |
| 1/2 Bend, $\theta / 2$ | 48.068 | 40.398 | minutes |
| $\text { Saggita, } L^{2} / 8 \rho$ | 0.3286 | 0.2321 | inches |
| $\mathrm{y}_{\mathrm{o}}, \mathrm{L}^{2} / 12 \mathrm{p}$ | -0.2191 | -0.1547 | inches |
| $y(L / 2)$ | 0.1095 | 0.0774 | inches |
| $y_{0} \text { (Long) }-y_{o} \text { (Short) }$ | 0.0643 |  | inches |
| $\delta$ | 2.681 |  | mr |
| $\frac{\theta}{2} \pm \delta$ | 16.663 | 9.071 | mr |
| Straight Sections |  |  |  |
| $\mathrm{L}_{1}$ |  |  | inches |
| $\mathrm{L}_{2}$ |  |  | inches |
| $\mathrm{L}_{3}$ |  |  | inches |

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suघject...-Standard_Survey.-Data $\qquad$
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STANDARD SURVEY DATA

- The standard survey data consists of the following information:

Primary chord rectangular coordinates and primary station polar coordinatea of secondary pedestal stations (Table 2).

Primary chord rectangular coordinates and primary station polar coordinatess of top locating sockets of magnets; $3^{\prime \prime}$ in from ends of magnets (Table 3).

Primary chord intersections of radials from magnet locating sockets (Table 4)
Radial distances from intersection with primary chord and angle along radiala to magnet sockets (Table 4).

Distances between end locating sockets of adjacent magnets (Table 5).
Rectangular coordinates along secondary chords between pedestal stations of locating pin sockets at base of magnets, $2^{\prime \prime}$ in from end cf magat (Table 6).

In computing these, we used the table and figure on page 2 of "Location of Magm nets", M. H. Blewett, May 28, 1957. These, slightly changed, are reproduced belorf as Figure 1 and Table 1.

430.45' Radius Reqerance Circle.

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## TABIF 1：Location of Magnets：

$\theta$ is the angular distance．from the center of the 10＇straight sections（the position of the primary survey momuments）．
$\Delta R$ is the radial distance from the reference circle（radius $=421.45 \mathrm{ft} .=5057.400 \mathrm{im}$ ）

X，Y，are rectangular coordinates with the straight line between primary monuments as the axis of abscissae．Negative $Y$ is Inside the chord．
Primary Primary Chord Primary Mon．

| DESCRIPTION | PT．NO． | $\theta$（Degrees） | $\triangle \mathrm{R}$（Inches） | X（Inches） | Y（Inches） |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Center of $10^{\circ}$ Straight Section | 1 | 0.000000 | －1．5707 | － | $\cdots$ |
| ．68－3＂Magnet＊ | $\stackrel{2}{3}$ | $\begin{aligned} & 0.702547 \\ & 1.552375 \end{aligned}$ | $\begin{aligned} & -1.2785 \\ & -0.6837 \end{aligned}$ | $\begin{array}{r} 75.7785 \\ 150.2465 \end{array}$ | $\begin{array}{r} -100.6284 \\ -91.7131 \end{array}$ |
| － $6^{+}-3^{\prime \prime}$ Magnet | $\begin{array}{r} 4 \\ 5 \end{array}$ | $\begin{aligned} & 1.869621 \\ & 2.719356 \end{aligned}$ | $\begin{aligned} & -0.5058 \\ & -0.1463 \end{aligned}$ | $\begin{aligned} & 178.0852 \\ & 52.7424 \end{aligned}$ | $\begin{aligned} & -88.7120 \\ & -81.5494 \end{aligned}$ |
| $7^{1}-6^{\prime \prime}$ Magnet ${ }^{*}$ | $16$ | $\begin{aligned} & 3.036571 \\ & 4.056194 \end{aligned}$ | $\begin{aligned} & 0.0038 \\ & 0.0857 \end{aligned}$ | $\begin{aligned} & 280.6389 \\ & 370.4190 \end{aligned}$ | $-79.1435$ <br> －72．8564 |
| 7＊－5＂Magnet | $\begin{aligned} & 8 \\ & 9 \end{aligned}$ | $\begin{aligned} & 4.781237 \\ & 5.800788 \end{aligned}$ | $\begin{aligned} & 0.2234 . \\ & 0.5288 \end{aligned}$ | $\begin{aligned} & 434.3189 \\ & 524.2398 \end{aligned}$ | $\begin{array}{r} -69.2788 \\ -65.5046 \end{array}$ |
| $\cdots \quad 7^{\prime}-67$ liagnet | $\begin{aligned} & 10 \\ & 11 \end{aligned}$ | $\begin{aligned} & 6.117968 \\ & 7.137499 \end{aligned}$ | $\begin{aligned} & 0.5588 \\ & 0.4472 \end{aligned}$ | $\begin{aligned} & 552.2288 \\ & 642.2200 \end{aligned}$ | $\begin{array}{r} -64.7219 \\ -63.4633 \end{array}$ |
| Center of 5＇Straight Section | 12 | 7.500000 | 0.3739 | 674.2200 | －63．4354 |
| 7：－6＂Magnet | $\frac{11}{10}$ | $\begin{aligned} & 7.862501 \\ & 8.882032 \end{aligned}$ | $\begin{aligned} & 0.4472 \\ & 0.5588 \end{aligned}$ | $\begin{aligned} & 706.2200 \\ & 796.2112 \end{aligned}$ | $\begin{aligned} & -63.4635 \\ & -64.7219 \end{aligned}$ |
| 7＊－6＂Magnet | $\begin{aligned} & 9 \\ & 8 \end{aligned}$ | $\begin{array}{r} 9.199212 \\ 10.218763 \end{array}$ | $\begin{aligned} & 0.5288 \\ & 0.2234 \end{aligned}$ | $\begin{aligned} & 824.2002 \\ & 914.1211 \end{aligned}$ | $\begin{aligned} & -65.5046 \\ & -69.2788 \end{aligned}$ |
| 7＇－6＇Magnet | $\begin{aligned} & 7 \\ & 6 \end{aligned}$ | $\begin{aligned} & 10.943806 \\ & 11.963429 \end{aligned}$ | $\begin{aligned} & 0.0857 \\ & 0.0038 \end{aligned}$ | $\begin{array}{r} 978.0210 \\ 1067.8011 \end{array}$ | $\begin{aligned} & -72.8564 \\ & -79.1435 \end{aligned}$ |
| $6^{\prime}-3^{\prime \prime}$ Magnet | $\begin{aligned} & 5 \\ & 4 \end{aligned}$ | $\begin{aligned} & 126880644 \\ & 13.130379 \end{aligned}$ | $\begin{aligned} & -0.1463 \\ & -0.5058 \end{aligned}$ | $1095.6976$ | $\begin{aligned} & -81.5494 \\ & -88.7120 \end{aligned}$ |
| $6^{1}-3^{\prime \prime}$ Magnet | $\begin{aligned} & 3 \\ & 2 \end{aligned}$ | $\begin{aligned} & 13.447625 \\ & 14.297453 \end{aligned}$ | $\begin{array}{r} -0.5837 \\ -1.2785 \end{array}$ | $1198.1935$ | $\begin{array}{r} -91.713 \lambda \\ -100.6284 \end{array}$ |
| Center of 101 Straight Section | 1 | 15.000000 | －1．5707 | － | － |

＊In the computation the short and long magnets were assumed to have length exactiy 75 t and 90 ！，respectively．

## Appendix IV

## The Radius of the AGS

Figure AIV-1 shows the radial displacement of the magnets from the secondary monument circle, which has a radius of 421.45 feet. On page X-B-1 of the AGS Standards Book, Courant specifies:

> "Circumference of equilibrium orbit $=3.17759 \times 10^{4}$ inches (obtained from the IBM 704 computations)"

Following the prescription in this note we can calculate the circumference, using the appropriate effective straight section lengths and the magnet arc lengths of 94.0030604 inches and 79.0018181 inches (for doing arithmetic it is easy to carry more decimal places than might actually be measured), to be 31775.246 inches. Table AIV-1 summarizes these three cases. Note that the radius found in this note is 0.1 inches less than the radius in the standards book. This difference could easily result from calculational problems or from differences in the definition of the equilibrium orbit. The possible equilibrium orbits will be discussed in a subsequent note. We expect an interesting set of measurements could be carried out to determine the circumference for various possible orbits.

|  | Standards Book | This Paper | Nominal <br> "Orbit Radiu |  |
| :---: | :---: | :---: | :---: | :---: |
| Circumference | 31775.9 | 31775.246 | 31776.581 | inches |
| Radius | 5057.29 | 5057.187 | 5057.40 | inches |
| Radius | 421.441 | 421.4323 | 421.45 | feet |
| Radius | 1.28 .4552 | 128.4526 | 128.4580 | meters |

MAGNET RADIAL DISPLACEMENT


Figure AIV-I

## Appendix V

## Discrepancies

Table AV-1 shows the bend angles determined in this note and those used in the computer program BEAM. Note that the angles used in the BEAM program have been rounded off to only four places, even though they are tabulated in the program as sines and cosines to nine places. The effect of this rounding is that the circle does not close by 1.5 milliradians. This probably has no effect on BEAM calculations, but does have an effect when these angles are being used for geometry calculations. For neatness it is probably desirable to fix these numbers at some point.

Figure AV-1 and Table AV-2 summarize some of the geometrical results. This information is essentially what is given on two AGS drawings (C-D05-M-864-2 and C-D05-M-865-2) often used for installing items in straight sections. The one significant error is that 865-2 shows the bend angle to be 36 minutes when it should be 40 minutes.

| As built | Beam |  |
| :--- | :--- | :--- |
| 13.9825 | 13.98 | mr |
| 11.7512 | 11.75 | mr |
| 16.6631 | 16.12 | mr |
| 9.0706 | 9.63 | mr |


| $L_{i}$ | 64.000 | 123.9907 |  |
| :--- | :--- | :--- | :--- |
| $\theta / 2$ | 13.9825 | 11.7512 | mr |
| $y_{o}^{\prime}=\tan \theta / 2$ | 13.9834 | 11.7517 | $10^{-3}$ |
| $\theta / 2$ | 48.068 | 40.398 | min |
| a | 42.0 | 35.2 | $10^{-3}$ in |
| b | 191.1 | 131.2 | $10^{-3}$ in |



Figure AV-1


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