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BEAM LINE RAY TRACING

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Introduction

This technical note reviews how one may trace a beam through a beam transport line from the known magnetic element transport matrices. It shows that only in a rare case may one calculate the beam position in a downstream portion of a line from the known input position and the matrix product of the elements between the input and output. A set of corollaries are obtained that one needs only to remember to trace a beam thru any beam line. Procedures are given to treat any type of magnetic element offset, tilt, or rotation in the beam line.

The Drift Space

To illustrate the procedure necessary to trace a beam thru different magnetic elements, the drift space will be studied thoroughly. One may argue correctly that this procedure is not necessary for simple lines, but it is the only way to treat bending magnets with edge focussing, gradient magnets, or magnets with horizontal or vertical offsets.

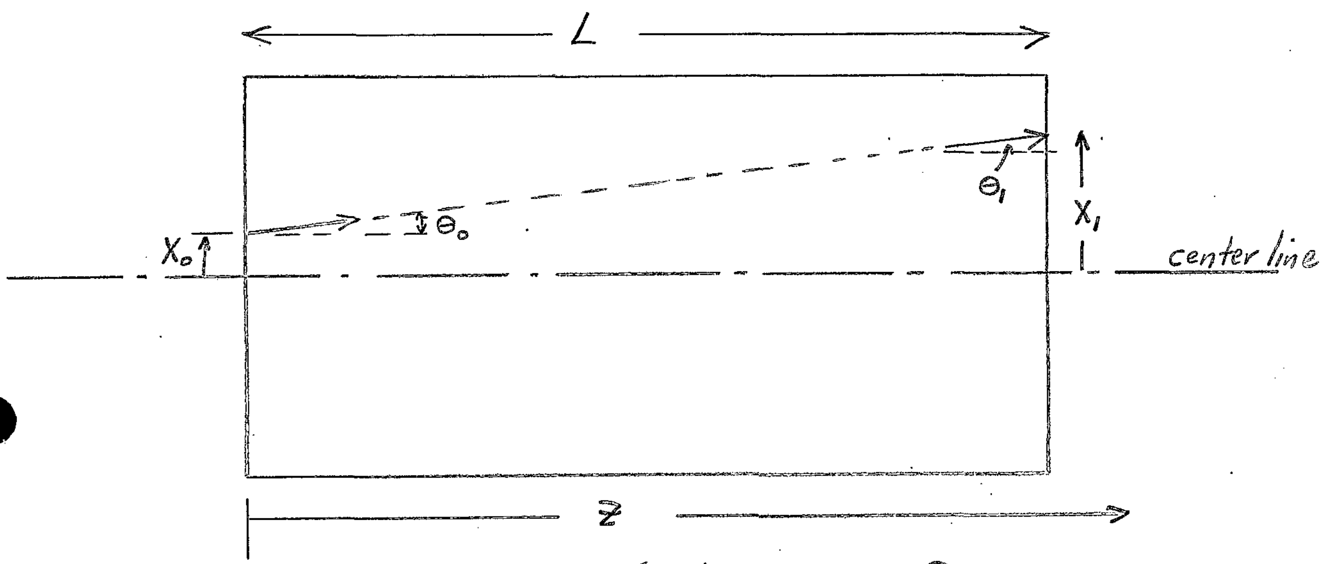


FIGURE 1 -- DRIFT SPACE

Figure 1 shows a drift space of length L. By definition no forces are exerted on the particles in a drift space, and as a result, the particles travel in a straight line. Initially only two dimensions will be used:

Defining:

χ_o ---- x displacement of the beam from the center line of the drift space at the entrance to the drift space (inches).

θ_o ---- angle, measured with respect to the center line of the drift space at the input, (radians)

One may easily write;

$$\chi_1 = \chi_o + L \tan \theta_o \tag{1}$$

$$\theta_1 = \theta_o$$

where χ_1 and θ_1 are the values at the output of the drift space.

This value is exact and can be used for ray tracing. However, in a transport line one is also interested in the beam size as well as the beam position. In fact, in beam line design, one is more interested in beam size since this determines the beam line apertures.

It has been shown in beam line design that if all the elements are described with linear matrices, a simple matrix equation can be used to find beam sizes in a lossless beam line. As a result, one always tries to express equations like equation 1 in matrix form. It is not necessary for ray tracing, but it is necessary for beam size determinations.

Matrix form requires a form as:

$$\chi_1 = A\chi_o + B\theta_o \tag{2}$$

$$\theta_1 = C\chi_o + D\theta_o$$

where A, B, C, and D are independent of χ_o and θ_o.

One can see that equation 1 can be put in the matrix form if:

$$\tan \theta_o \approx \theta_o \tag{3}$$

This occurs for small angles.

Thus the well known transport matrix form for a drift space becomes:

$$\chi_1 = 1\chi_o + L\theta_o \quad \text{inches} \tag{4}$$
$$\theta_1 = 0\chi_o + 1\theta_o \quad \text{radians}$$

If milliradians are used, as is common:

$$\chi_1 = 1\chi_o + {}^{L}/_{1000} \quad \theta_o \tag{5}$$
$$\theta_1 = 0\chi_o + 1\theta_o$$

Corollary #1 can now be stated:

#1 The transport matrix equation representation of a magnetic element is not the exact description of the element. It is only a simplification.

The distance and angle for the drift space are measured with respect to the center line of the drift space. This center line will now be referred to as the element centroid as in standard transport practice.

Corollary #2 can now be stated:

#2 To use the transport matrix, one must know where the centroid is for that matrix. Knowing only the transport matrix is insufficient information to solve beam ray tracing problems.

Matrix form and matrix multiplication can be used to transport a particle or a beam from the input of a magnetic element to the output of the magnetic element.

Equation 4 can be written in matrix form as:

$$\begin{bmatrix} VX1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} VX0 \end{bmatrix} \tag{6}$$

where:

$\begin{bmatrix} VX0 \end{bmatrix}$ is the vector describing the input parameters:

For Figure 1:

$$\begin{bmatrix} VX0 \end{bmatrix} = \begin{bmatrix} \chi_o \\ \theta_o \end{bmatrix} \tag{7}$$

The input vector [VXO] can also be a 5 dimensional vector:

$$[VXO] = \begin{bmatrix} \chi_o \\ \theta_o \\ Y_o \\ \phi_o \\ \delta_o \end{bmatrix} \tag{8}$$

where χ_o, θ_o, Y_o, ϕ_o, and δ_o have standard transport definitions.

χ_o -- horizontal displacement of input ray, in inches, with respect to the assumed central trajectory or centroid.

θ_o -- the angle that this input ray makes in the horizontal plane with respect to the central trajectory or centroid.

Y_o -- vertical displacement of input ray with respect to central trajectory or centroid.

ϕ_o -- the angle that this input ray makes in vertical plane with respect to central trajectory or centroid.

δ_o -- $\Delta p/p$ = fractional momentum deviation (%) of this input ray and the assumed central trajectory or centroid.

If a 5 dimensional vector is used, the transport matrix for the magnetic element also becomes a 5x5 square matrix, R.

$$[R] = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \tag{9}$$

The output vector then is a 5 dimensional vector.

The standard vector equation for a drift space or any other magnetic element is the following well known equation:

$$[VX1] = [R] \times [VXO] \tag{10}$$

It should be emphasized that this gives the vector [VX1], at the end of the magnetic element knowing the vector [VX0] at the input of the element. The centroid and the [R] matrix must be known. Nothing is known about the beam outside this element.

The Beam Pipe Element

A new transport line element will now be introduced to keep track of element centroids, called a Beam Pipe element. All beam line instrumentation such as swics or flags are assumed to be located only in the Beam Pipe element. All commonly known transport elements are separated from each other by Beam Pipe elements. Thus, two drift spaces are not allowed to be connected. They must be connected thru a Beam Pipe element.

A Beam Pipe element has a unity matrix. If the input to the Beam Pipe is the vector [VBP0], the output is the vector [VBP1] and:

[VBP1] = [VBP0] Equation (11)

always in the same Beam Pipe element.

The centroid of the Beam Pipe element is the center line of the beam pipe.

A third corollary may be stated:

#3 All magnetic elements in a transport line are separated from each other by the unity matrix Beam Pipe element.

To show the procedure for solving beam steering problems, several examples using a drift space will be used.

Example #1

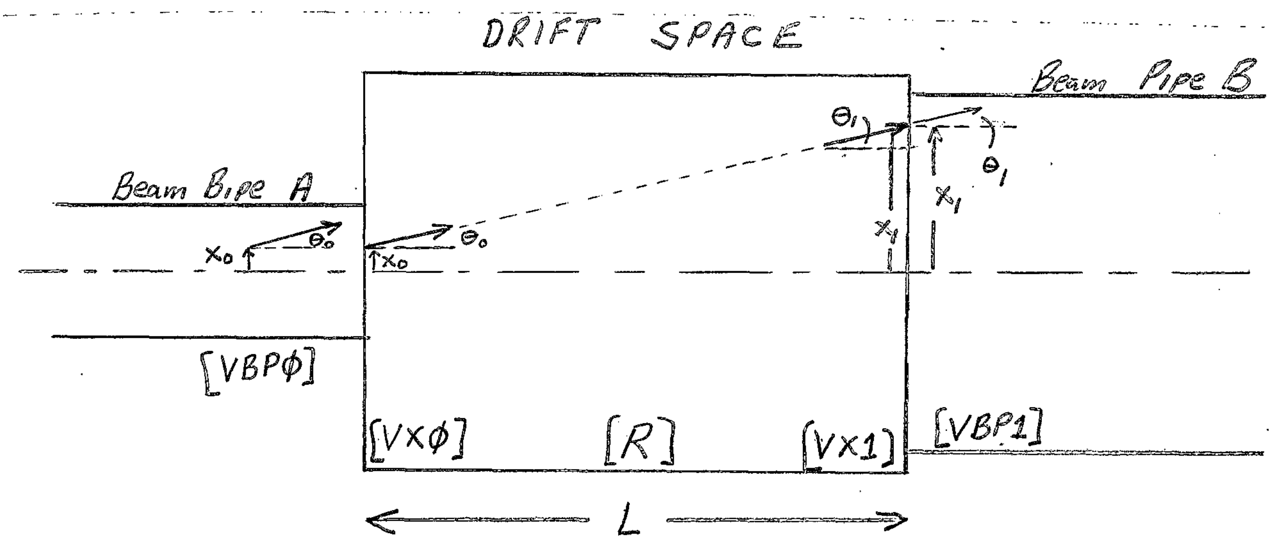


FIGURE 2

In the Beam Pipe A (12)

$$[VBP0] = \begin{bmatrix} X_o \\ \theta_o \end{bmatrix}$$ in the horizontal plane only

Transforming this vector to the drift space, the vector at the input of the drift space is [VX0].

$$[VX0] = [VBP0] = \begin{bmatrix} X_o \\ \theta_o \end{bmatrix} \tag{13}$$

Moving thru the drift space using the transport matrix [R]

$$[VX1] = [R] \times [VX0] \tag{14}$$

Transforming this vector to beam pipe B

$$[VBP1] = [VX1] = \begin{bmatrix} X_1 \\ \theta_1 \end{bmatrix} \tag{15}$$

This trivial example illustrates the four important vectors [VBP0], [VX0], [VX1] and [VBP1].

Example #2

Consider that the drift space is offset from the beam pipe in the horizontal direction by the distance Δ.

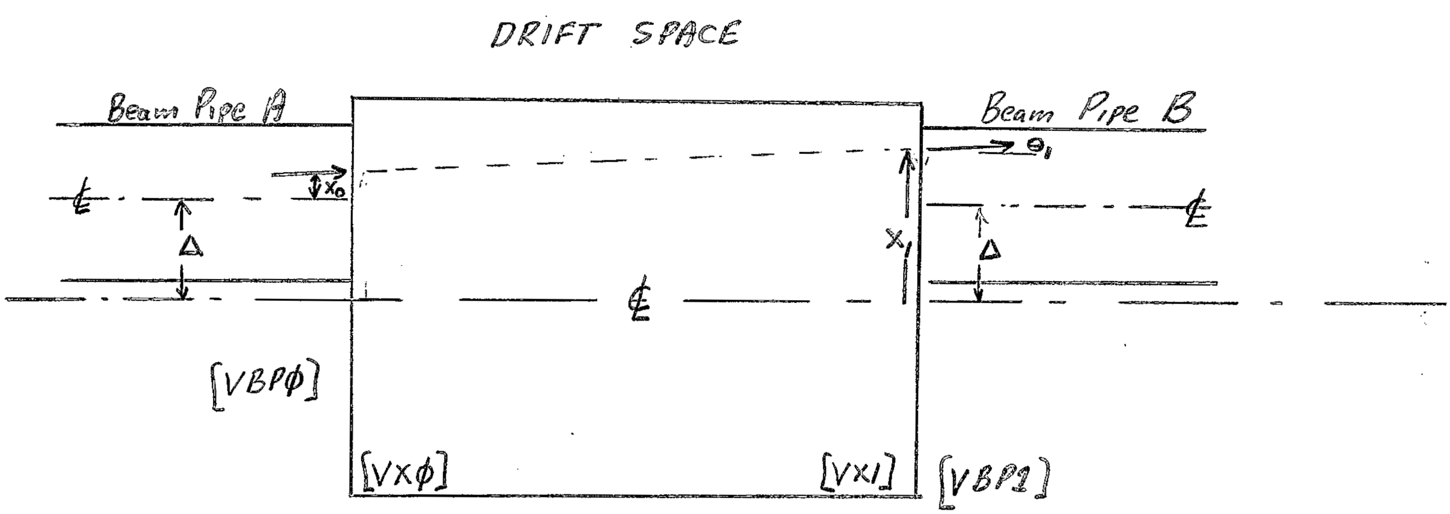


FIGURE 3

In the Beam Pipe A

$$[VBP0]_A = \begin{bmatrix} X_o \\ \theta_o \end{bmatrix} \tag{16}$$

Because of the offset:

$$[VX0] = \begin{bmatrix} X_o + \Delta \\ \theta_o \end{bmatrix} \tag{17}$$

$$[VX1] = [R] \times [VX0] = [R] \times \begin{bmatrix} X_o + \Delta \\ \theta_o \end{bmatrix} = \begin{bmatrix} X_1 \\ \theta_1 \end{bmatrix} \tag{18}$$

$$[VBP1] = \begin{bmatrix} X_1 - \Delta \\ \theta_1 \end{bmatrix} \tag{19}$$

The output vector in beam pipe B is also shifted from the [VX1] vector at the output of the drift space.

It is true that since certain elements of the drift space [R] matrix are unity or zero, that these four steps could be combined. However, in general, one must always follow these four steps for each magnetic element.

Corollary #4 can now be stated:

#4 To trace a beam from the input beam pipe to the output beam pipe, four vectors [VBP0], [VX0], [VX1], and [VBP1] must be found for each element in the beam line.

Example #3

Consider that the element is rotated in the horizontal plane by an angle α.

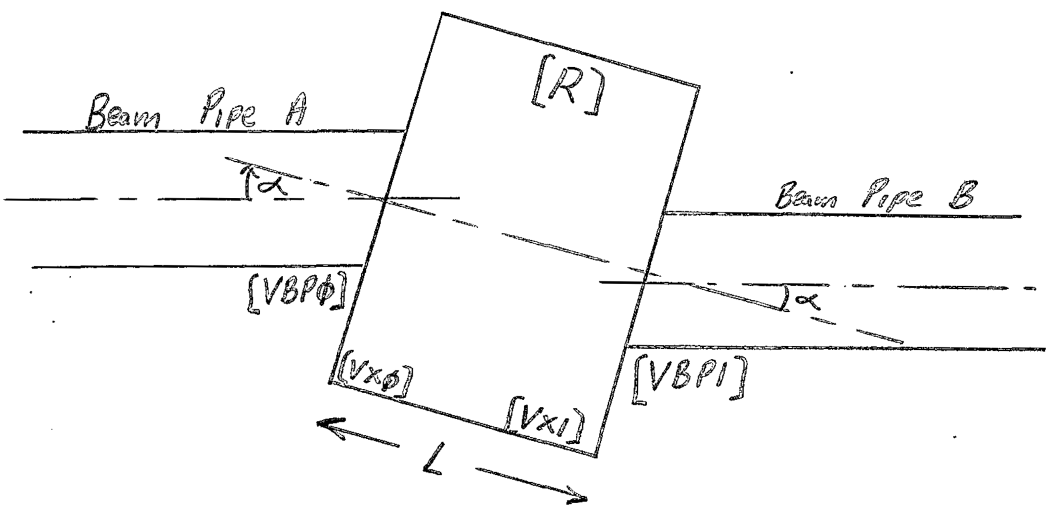


FIGURE 4

Actually, the important fact is not that the element is rotated, but that the centroid of the element makes an angle α with the centroid of the beam pipe. The centroid in some elements, as in bending magnets, will move even though the physical magnet is not moved.

Following Corollary #4.

$$[VBP0] = \begin{bmatrix} \chi_o \\ \theta_o \end{bmatrix} \tag{20}$$

$$[VX0] = \begin{bmatrix} \chi_o \cos\alpha \\ \theta_o + \alpha \end{bmatrix} \tag{21}$$

For small angles a simplification may be made, but is not essential:

$$\cos\alpha = 1$$

$$[VX0] = \begin{bmatrix} \chi_o \\ \theta_o + \alpha \end{bmatrix} \tag{22}$$

$$[VX1] = [R] \times \begin{bmatrix} \chi_o \\ \theta_o + \alpha \end{bmatrix} = \begin{bmatrix} \chi_1 \\ \theta_1 \end{bmatrix} \tag{23}$$

$$[VBP1] = \begin{bmatrix} \chi_1/\cos\alpha \\ \theta_1 - \alpha \end{bmatrix} \quad \text{or for small } \alpha \tag{24}$$

$$[VBP1] = \begin{bmatrix} \chi_1 \\ \theta_1 - \alpha \end{bmatrix} \tag{25}$$

If the last equation is expanded:

$$\chi_{VBP1} = \chi_1$$

$$\theta_{VBP1} = \theta_1 - \alpha$$

one can see that this can not be expressed as a matrix equation, i.e., one can not write:

$$\chi_{VBP1} = A\chi_1 + B\theta_1$$

$$\theta_{VBP1} = C\chi_1 + D\theta_1$$

but must write

$$\theta_{VBP1} = C\chi_1 + D\theta_1 - \alpha$$

One can conclude that if the centroids for all the magnetic elements are not aligned, one can not use matrix multiplication to find the output vector from the product of several element matrices and the input vector. It is necessary to step thru the beam line element by element.

However, the equations for each of the elements do not have to be derived since the transport matrices for all elements have been evaluated. It is necessary to know where the centroid is for each of the elements so that one can move from the beam pipe into the element or from the element to the beam pipe.

The Quadrupole Matrix

A quadrupole is a four pole magnet as shown in Figure 5.

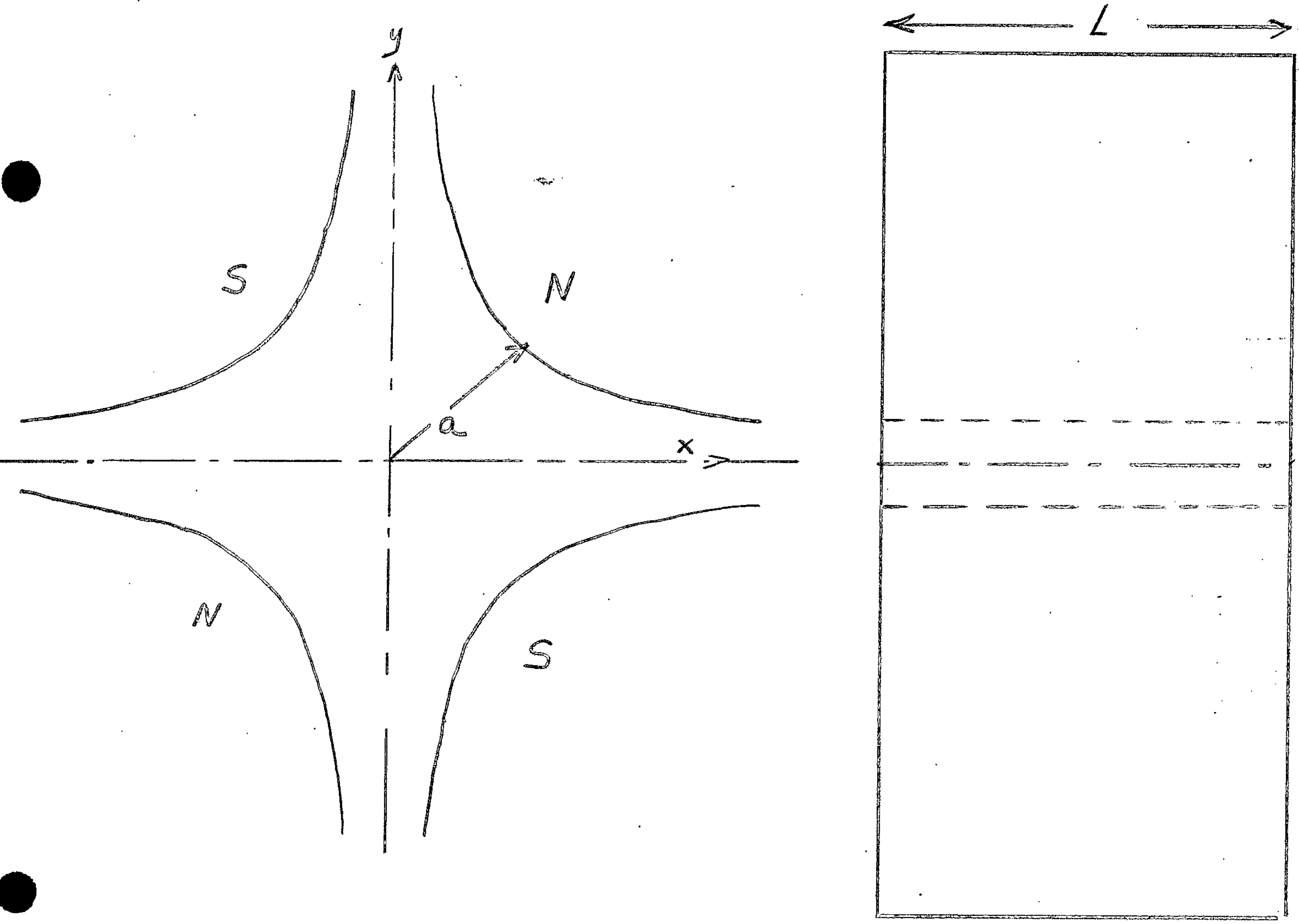


FIGURE 5

For a pure quadrupole field:

$$B_x = \frac{B_o y}{a} \qquad B_y = \frac{B_o X}{a}$$

where B_o is the field at the pole and a is the pole radius

The length of the quad is L.

If one knows the input vector to the quad, [VX0], one can solve for the output vector [VX1] using the above field equations. One must define a centroid for the quadrupole and, for ray tracing, this can be located anywhere within the quad, within reason. For example, the physical bottom of the quad could be the centroid location. However, as discussed for the drift space, it is desired to be able to express the output vector as a product of an input vector and an element matrix. This is necessary for beam size determination but not for beam steering. Penner has found that if the centroid of the magnet is the optical or physical center of the ideal quad, the expression relating the output vector to the input vector is a square matrix ... the commonly known Quadrupole Transport Matrix.

$$[VX1] = [R] \times [VX0] \quad \text{where} \tag{9}$$

$$R = \begin{bmatrix} \cos(KL) & \frac{1}{K}\sin(KL) & 0 & 0 & 0 \\ -K\sin(KL) & \cos(KL) & 0 & 0 & 0 \\ 0 & 0 & \cosh(KL) & \frac{1}{K}\sinh(KL) & 0 \\ 0 & 0 & K\sinh(KL) & \cosh(KL) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K = \sqrt{\frac{B_o/a}{(P/e)}}$$

(P/e) = rigidity = 1313.24 P with P in Gev/c and $\frac{B_o}{a}$ in KG/in.

L - length in inches.

The centroid is located in the center of the quad.

Following Corollary #3, the output beam can be found for any location of the quad. If the quad axis is offset from the beam pipe axis, [VXO] is found from the offset and [VBPO]. The quad can also be rotated or tipped. It is only necessary to know the distance between the centroid of the beam pipe and the quad centroid.

The Dipole Matrix

The most confusing element for beam line tracing was found to be the simple dipole. The reason for this confusion was a lack of knowledge of where the centroid for a dipole actually is. To transform the Beam Pipe vector, [VBPO] into the input element vector, [VXO], one needs to know the location of the Beam Pipe centroid and the dipole magnet centroid.

The wedge magnet matrix was first found by Penner. His derivation will now be outlined so that the most essential results can be shown. He first assumed a wedge magnet and then added corrections for a rectangular magnet. His definition of a wedge magnet is a magnet in which the central trajectory or centroid enters and leaves the magnet perpendicular to the face of the magnet. This is shown in Figure 6. The magnet field is assumed constant and in the vertical direction. As a result when the beam enters the magnet, it follows the arc of a circle in the radial or horizontal plane with a radius of curvature of ρ.

It should be emphasized that this is not a difficult problem if only beam ray tracing is considered. The center of the coordinate system or the centroid could be taken as the bottom of the magnet and the output ray location and angle could be found. The problem, however, is to be able to express the output angle and position as a matrix product of the input angle and position. Penner solved this problem by measuring distance and angle from a special ray called a central trajectory or centroid. It should be apparent that as the current changes in the magnet, the bend angle α will change. Thus, if the components of [VBPO] are fixed, the components of the [VXO] vector will change. This was the reason for introducing the Beam Pipe element.

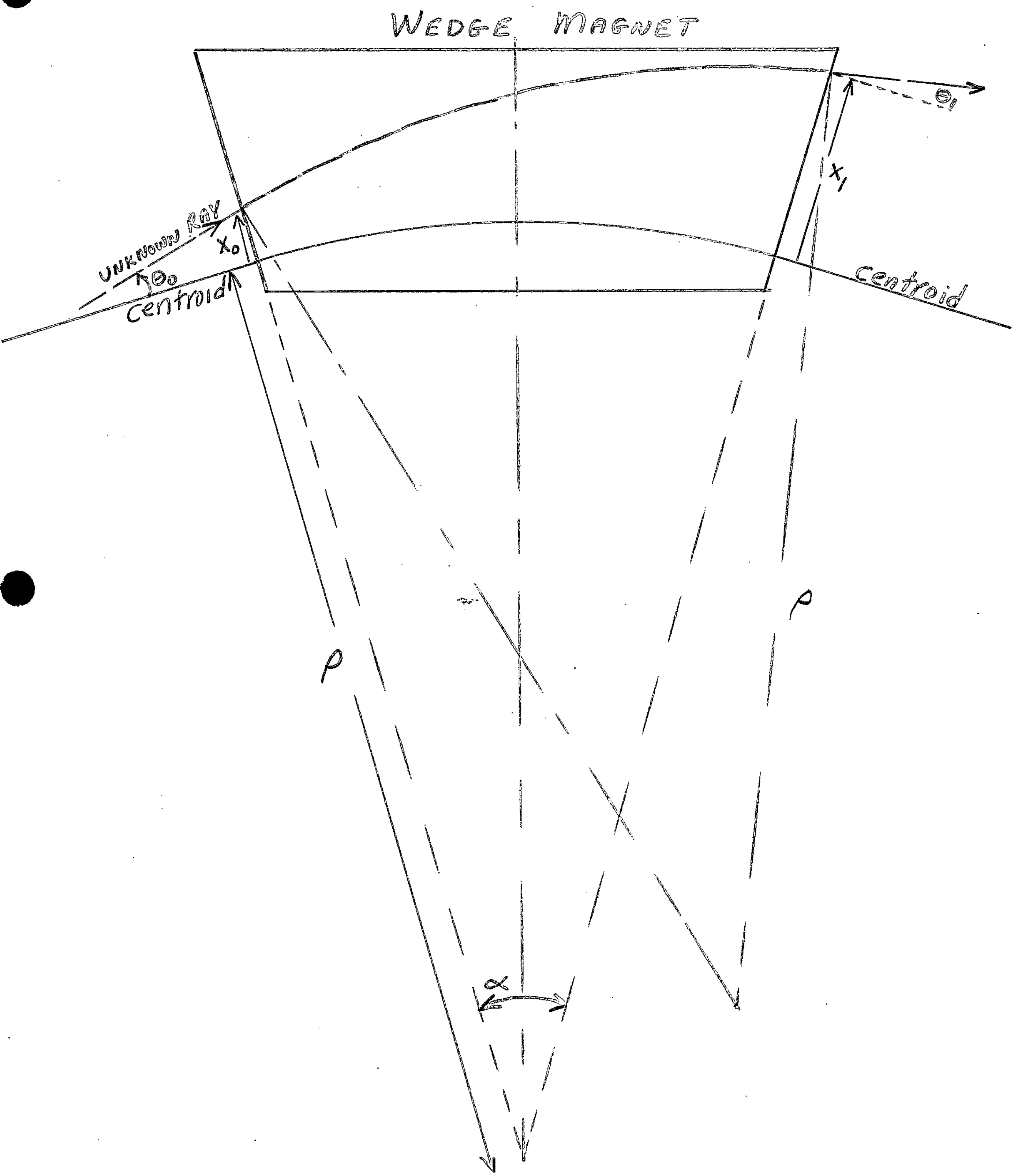

WEDGE MAGNET
UNKNOWN RAY
X_0
θ_0
centroid
θ_1
x_1
centroid
ρ
ρ
α

FIGURE 6

Figure 6 shows the components of [VXO] which is the vector at the input of the wedge dipole

$$VXO = \begin{bmatrix} \chi_0 \\ \theta_o \end{bmatrix}$$

One wants the components χ_1 and θ_1 of the output vector. Penner assumes that the unknown ray can also have a slightly different momentum, P + ΔP, from the central ray or centroid. If the effective length of the magnet is Leff (inches) and the field is B (kilogauss) then:

$$2 \sin \frac{\alpha}{2} = \frac{BLeff}{(P/e)}$$

where (P/e) is the rigidity of the particles in (Kilogauss-inches).

$$\rho = \frac{Leff}{\alpha} \quad \text{for small bending angles.}$$

$$\frac{\Delta P}{P} = \frac{\Delta\rho}{\rho}$$

Following Penner's procedure, and correcting the misprint in his equation 23, one obtains the exact results:

$$\text{Sin } \theta_1 = \frac{(\rho + \Delta\rho) \sin (\theta_o + \alpha) - (\rho + X_o) \sin \alpha}{(\rho + \Delta\rho)}$$

and

$$\chi_1 = \chi_o \cos \alpha - \rho \cos (\alpha + \theta_o) - \Delta\rho \cos (\alpha + \theta_o) +$$

$$+ \Delta\rho \cos \theta_1 + \rho (\cos \alpha + \cos \theta_1 - 1)$$

Note that θ_1 must be found before χ_1 can be evaluated.

These equations are sufficient for ray tracing but can be simplified for matrix calculations. If one assumes small angles and a small momentum difference:

$$\theta_o << 1$$

$$\chi_o << \rho$$

$$\Delta\rho << \rho$$

$$\theta_1 << 1$$

Then one obtains the standard transport matrix equation for a wedge magnet.

$$\chi_1 = (\cos\alpha)\ \chi_o + (\rho\sin\alpha)\ \theta_o + (1-\cos\alpha)\ \Delta\rho$$

$$\theta_1 = \frac{-\sin\alpha}{\rho}\ \chi_o + (\cos\alpha)\ \theta_o + \frac{\sin\alpha}{\rho}\ \Delta\rho$$

In terms of the momentum deviation, the standard matrix becomes:

$$R = \begin{bmatrix} \cos\alpha & \rho\sin\alpha & \rho(1-\cos\alpha) \\ \frac{-\sin\alpha}{\rho} & \cos\alpha & \sin\alpha \\ 0 & 0 & 1 \end{bmatrix}$$

Wedge Magnet Matrix in the Bend Plane

The wedge magnet vertical or non-bend plane matrix is the drift space matrix.

The Transport Program Manual gives the transport matrix for other elements also. There are provisions for printing these matrices with the Transport Program. The important fact for ray tracing is that these are all available and that one can easily obtain the output vector of the element, [VX1] from the input vector [VX0]. The sometimes difficult and important problem is the obtaining of the transform from the Beam Pipe input, [VBP0] to [VX0] and the transform from the element output, [VX1] to the output Beam Pipe [VBP1]. One may note that the rectangular magnet has a matrix in the horizontal plane of a drift space so that these transformations can be simplified. However, these transformations must be done for any element in which the matrix is not a simple drift space matrix ... i.e., vertical focusing in a dipole or gradient fields in a dipole.

Tracing A Beam Thru A Dipole

Tracing a beam thru a dipole is similar to tracing a beam thru a drift space except that the centroid is curved rather than a straight line. Different types of dipoles exist in beam lines. The common trim dipole bends the beam a few milliradians and is used for steering corrections. This can be considered a wedge dipole for beam tracing problems. A dipole that bends a beam a few degrees is usually considered a symmetrical rectangular dipole. The matrix for this dipole includes edge effects which produce vertical focussing for a horizontal dipole. It is symmetrical because the magnet is located so that the input Beam Pipe element makes the same angle with the input magnet face as the output Beam Pipe element makes with the output magnet face.

Figure 7 shows a trim dipole lying in the beam line with the current adjusted to bend a beam α milliradians.

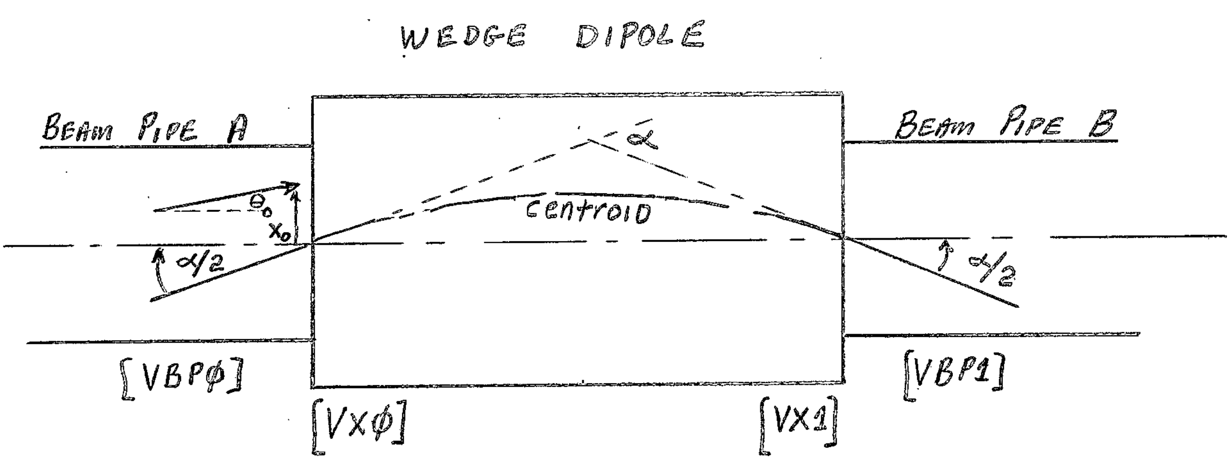


FIGURE 7

The centroid for the dipole is an arc of a circle. For convenience it can be assumed to intersect at the centroid of the Beam Pipe A and Beam Pipe B. Thus:

$$[VX0] = \begin{bmatrix} \chi_o \cos\alpha/2 \\ \theta_o - \alpha/2 \end{bmatrix} = \begin{bmatrix} \chi_o \\ \theta_o - \alpha/2 \end{bmatrix} \quad \text{for small angles}$$

$$[VX1] = [R] \times [VX0] = \begin{bmatrix} \chi_1 \\ \theta_1 \end{bmatrix}$$

Transforming out to Beam Pipe B

$$[VBP1] = \begin{bmatrix} \chi_1 \cos\alpha/2 \\ \theta_1 - \alpha/2 \end{bmatrix} = \begin{bmatrix} \chi_1 \\ \theta_1 - \alpha/2 \end{bmatrix}$$

Note that if the dipole matrix is a drift space, then

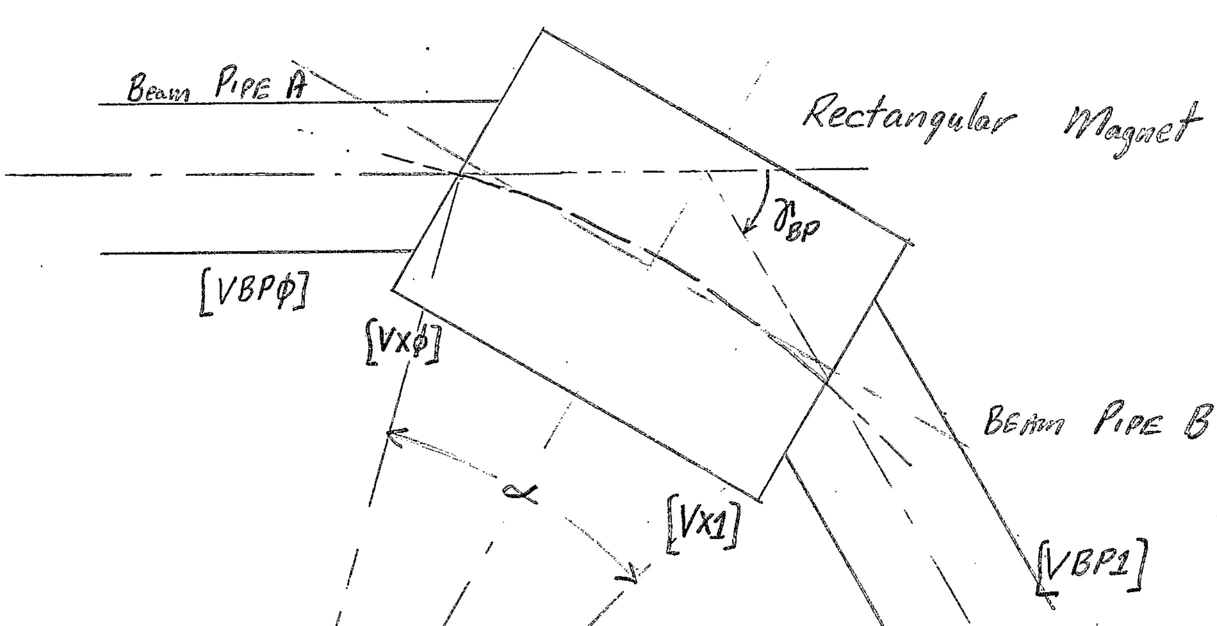
$$\theta_1 = \theta_o - \alpha/2$$

and $VBP1(\theta) = \theta_1 - \alpha/2 = \theta_o - \alpha$.

The output Beam Pipe angle differs from the input Beam Pipe angle by the bend angle of the magnet α. This is only true if the matrix element is equal to a drift space.

For the rectangular magnet, symmetrically located, one must include the bend angle between the two Beam Pipe elements.

FIGURE 8



Assume that the angle between Beam Pipe A and Beam Pipe B is γ_{BP}. This angle is fixed since it is determined by the beam line layout when the beam line is designed. The bend angle for the magnet is α and varies with the current in the magnet. Given the input vector in Beam Pipe A is [VBP0]

$$[VBP0] = \begin{bmatrix} \chi_o \\ \theta_o \end{bmatrix}$$

Transforming this to the entrance to the magnet, one must measure this vector from the centroid in the magnet. One can see that both γ_{BP} and α must be included.

Assuming, for simplicity, only 2 components for [VBP0]; and

$$VX0(\chi) = VBP0(\chi) \Big/ \cos\left(\frac{\gamma_{BP} - \alpha}{2}\right)$$

$$VX0(\theta) = VBP0(\theta) - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2}$$

For small angles:

$$VX0(\chi) = VBP0(\chi) = \chi_o$$

$$VX0(\theta) = VBP0(\theta) - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} = \theta_o - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2}$$

or in vector notation:

$$[VX0] = \begin{bmatrix} \chi_o \\ \theta_o - \alpha/2 + \frac{\gamma_{BP}}{2} \end{bmatrix}$$

$$[VX1] = [R] \times [VX0] = \begin{bmatrix} \chi_1 \\ \theta_1 \end{bmatrix}$$

And transforming to Beam Pipe B

$$[VBP1] = \begin{bmatrix} \chi_1 \cos\left(\frac{\gamma_{BP}}{2} - \frac{\alpha}{2}\right) \\ \theta_1 - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} \end{bmatrix} = \begin{bmatrix} \chi_1 \\ \theta_1 - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} \end{bmatrix}$$

One can see that the vector [VBP1] can not be expressed as a matrix product of different matrices and the input vector as long as sum and difference terms exist. Thus, only in the rare case that γ_{BP} and α are equal can the output vector be expressed as:

$$[VBP1] = [R] \times [VBP0]$$

This case occurs if the bending magnet current is adjusted so that the centroid bends the same amount as the beam pipe. If trim magnets are in the line and the beam pipe does not bend, then only if the current is zero can the output be expressed as a vector product of the input.

Note that no limitations are put on the [R] matrix. The only requirement is to know the location of the centroid of the [R] matrix. For this reason the [R] matrix could be a rectangular dipole or pitching magnet, a gradient dipole, or a tilted or rotated dipole.

The observer should note that this is not a new technique. If one were to trace a beam thru magnetic elements by integrating the magnetic fields, the same procedure would be necessary. The difference is that it is assumed that the element [R] matrix gives the element output location vector from the input location vector.

Conclusion

This technical note outlines a procedure for tracing a beam through any beam line. This is necessary to solve beam steering problems. The most important result is that the commonly known Transport Matrices can be used to step thru each element providing the centroid is known for each element. It is not possible to multiply or combine matrices to find the output vector in terms of the input vector due to shifts in element centroids. The results can be summarized in four corollaries.

#1 The transport matrix equation representation of a magnetic element is not the exact description of the element. It is only a simplification.

#2 To use the transport matrix, one must know where the centroid is for that matrix. Knowing only the transport matrix is insufficient information to solve beam ray tracing problems.

#3 All magnetic elements, including drift spaces, in a beam transport line are separated from each other by the unity matrix Beam Pipe element.

#4 To trace a beam from the input Beam Pipe to the output Beam Pipe, four vectors [VBPO], [VXO], [VX1], and [VBP1] must be found for each element in the beam line.

Appendix A gives an example using this procedure for the upstream part of the U line.

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References:

1. K.L. Brown et al "TRANSPORT, A Computer Program for Designing Charge Particle Beam Transport Systems," SLAC-91, Rev. 2, May 1977

2. S. Penner "Calculations of Properties of Magnetic Deflection Systems," "The Review of Scientific Instruments," Vol. 32, #2, p150-160, Feb. 1961

3. J.F. Ryan "The QTUNE Program .. The AGS Extracted Beam Transport Program," AGS Tech Note #181, Aug. 4, 1982

APPENDIX A -- A Beam Line Ray Tracing Example

As an example of this procedure, a beam will be traced through the first several magnets in the U line. The beam line is shown in Figure A1.

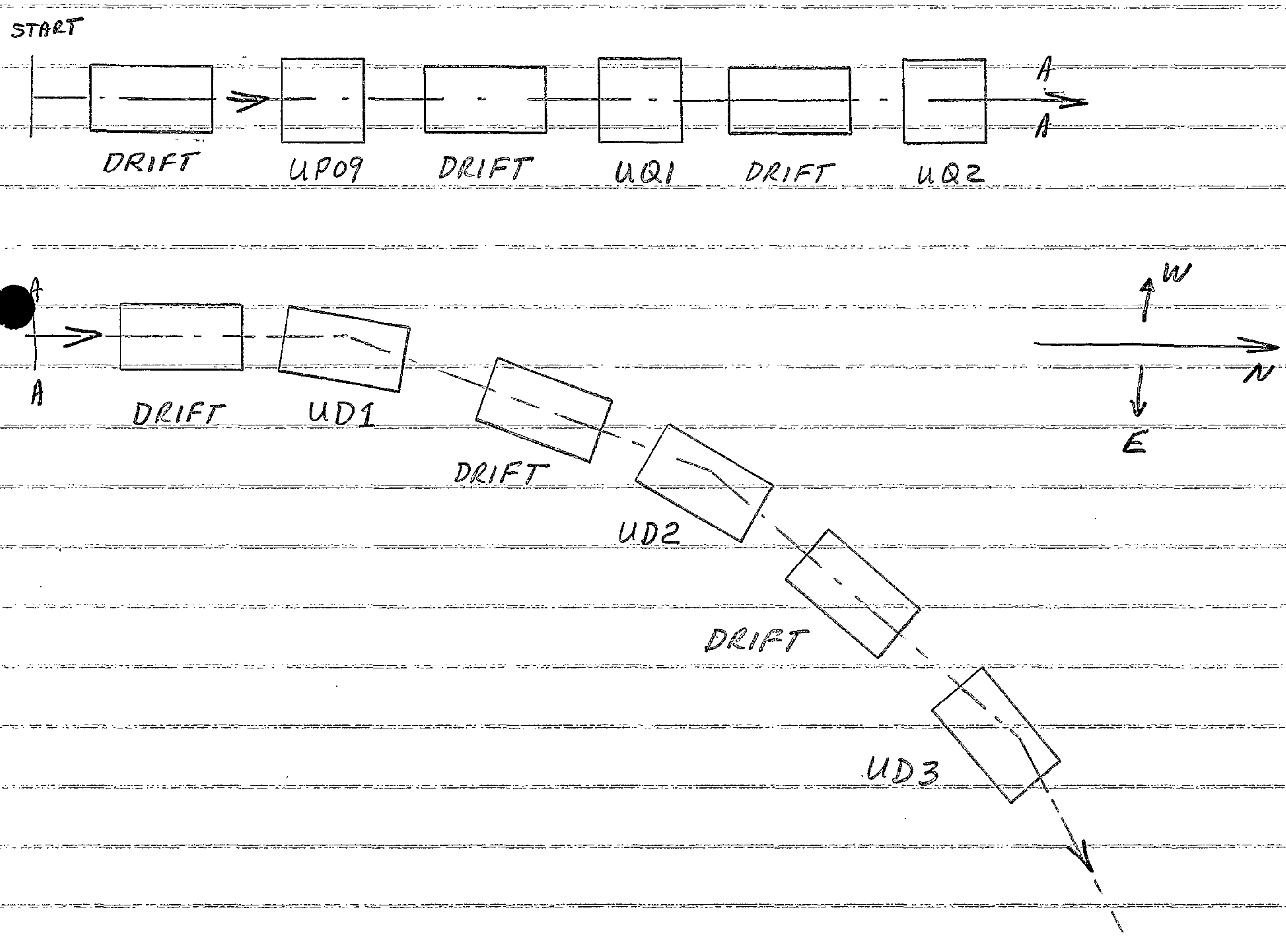

FIGURE A1 -- UPSTREAM U LINE

The characteristics of this beam line are given in Figure A2, lines 1-18.

The first element is a 95.045 inch drift followed by a wedge pitching magnet, UP09. This magnet bends the beam down by 0.03 degrees (GEL = 0.03 and positive indicates a bend down). Following UP09 is a 85.78 inch drift space. UQ1 follows which is a horizontally focussing quad with a gradient of 7.98839 KG/inch. This magnet is assumed offset in the positive horizontal direction by 0.1 inch (XSHIFT = 0.1). A 19.5 inch drift follows UQ1. This is followed by a vertically focussing quad UQ2 with a gradient of 7.69255 KG/inch. It is assumed that this magnet is offset horizontally and vertically by 0.1 inch (XSHIFT = YSHIFT = 0.1). This magnet is followed by a 18.794 inch drift. This drift is then followed by three symmetrically located rectangular dipole magnets UD1,2,3 separated by 18.1 inch drift spaces. These dipoles bend the beam 1.44220 degrees each in the horizontal direction. The beam pipe bends 1.41664 degrees in each of these magnets. It is assumed that the beam at the start can be described by an initial vector START.

$$\text{START} = \begin{bmatrix} 0.1\ \text{inch} & \chi \\ 0.1\ \text{mr} & \chi \\ 0.1\ \text{inch} & y \\ 0.1\ \text{mr} & y \\ 0\ \% & \end{bmatrix} \qquad \begin{array}{l} \text{positive is west} \\ \text{positive is west} \\ \text{positive is up} \\ \text{positive is up} \\ \ \end{array}$$

The computer printout of Figures A2-A5 step-by-step traces the beam through this beam line.

line 20	The 5 components of the initial starting vector are printed.
line 21	The element is a drift (blank). DL should be neglected.
line 22	Transforming the starting vector into the beam pipe, one obtains [VBP0].
line 23	Since this is a drift, [VX0] = [VBP0].
line 24	The element length, DZ = 95.045 inch.
lines 25-30	The [R] matrix for this drift space.
line 32	The vector [VX1] at the output of the drift [VX1] = [R] x [VX0]
line 33	Transforming the [VX1] vector into the beam pipe to obtain [VBP1].
line 36	[VBP1] repeated indicating the end of the element.
line 40	The beam pipe vector at the beginning of UP09 is the same as the beam pipe vector at the end of the previous element - [VBP0] = [VBP1].
line 41	Transforming [VBP0] into the pitching magnet to obtain [VX0]. Note that a positive iny direction is up and this magnet bends down. The beam pipe does not bend. α = .03 deg = 0.5236 mr. Note that the mry component is changed by $\alpha/2$.
line 50	The vector [VX1] obtained from [VX0] and the [R] matrix for the wedge magnet.
line 51	Transforming out to the beam pipe, the mry component is changed by $\alpha/2$ again. (see Figure 7 on Wedge Magnet)
line 58	The [VBP1] vector for the pitching magnet becomes the [VBP0] vector for the next drift space.
line 59	For a drift space, [VX0] = [VBP0].

lines 74-81	The output vector [VBP1] for the drift space becomes the input vector [VBP0] for UQ1.
line 82	The quad is offset by 0.1 inch in the horiz. direction so that the INX component changes transforming from the beam pipe to the magnet.
lines 84-89	The horizontal focussing quad matrix.
line 92	To obtain [VBP1] from [VX1], the INX component is changed by the offset.
lines 95-153	The vectors are stepped through drift spaces and a vertically focussing quad offset horiz. and vertically.
line 157	The input beam pipe vector to UD1, a rectangular dipole.
line 158	To obtain the mrx component of [VX0], the MRX component of [VBP0] is changed by the bend angle of the magnet and the beam pipe. The magnet bends more than the beam pipe by 0.02556 deg or 0.44611 mr. The mrx component is changed by half this angle.
lines 160-164	The [R] matrix for UD1.
line 167	The output [VX1] vector. Note that the mry component of [VX1] is less than [VX0] illustrating vertical focussing.
line 168	Transforming [VX1] out to [VBP1] the MRX component is again changed by half the angle difference.
lines 169-end	The VBP1 vectors are found for each component by stepping through each element.

If one wanted to plot the x and y position for this example, the inx and iny would be plotted against the downstream z.

```
 # ELELBL MAGKIN ELELEN      ZUS       GEL     QPERC XBPIPE     YBPIPE     XSHIFT YSHIFT
 1          0     95.045     0.000    0.00000    0.0  0.00000    0.0        0.000  0.000
 2 UP09    -4     30.750    95.045    0.03000    0.0  0.00000    0.0        0.000  0.000
 3          0     85.780   125.795    0.00000    0.0  0.00000    0.0        0.000  0.000
 4 UQ1      1     37.500   211.575   -7.98839    0.0  0.00000    0.0        0.100  0.000
 5          0     19.500   249.075    0.00000    0.0  0.00000    0.0        0.000  0.000
 6 UQ2      1     37.500   268.575    7.69255    0.0  0.00000    0.0        0.100  0.100
 7          0     18.794   306.075    0.00000    0.0  0.00000    0.0        0.000  0.000
 8 UD1     -1     81.900   324.869    1.44220    0.0  1.41664    0.0        0.000  0.000
 9          0     18.100   406.769    0.00000    0.0  0.00000    0.0        0.000  0.000
10 UD2     -1     81.900   424.869    1.44220    0.0  1.41664    0.0        0.000  0.000
11          0     18.100   506.769    0.00000    0.0  0.00000    0.0        0.000  0.000
12 UD3     -1     81.900   524.869    1.44220    0.0  1.41664    0.0        0.000  0.000
13          0     10.450   606.769    0.00000    0.0  0.00000    0.0        0.000  0.000


 0.100000 INX  0.100000 MRX  0.100000 INY  0.100000 MRY 0.000 %    START
   ELEMENT=          UPSTREAM Z=      0.000     DL=     0.000
 0.100000 INX  0.100000 MRX  0.100000 INY  0.100000 MRY 0.000 %    VBP0
 0.100000 INX  0.100000 MRX  0.100000 INY  0.100000 MRY 0.000 %    VX0
 Z0=      0.000    DZ=     95.045    XZ=    95.045
                                1.00000   0.09505   0.00000   0.00000   0.00000
                                0.00000   1.00000   0.00000   0.00000   0.00000
           ===                  0.00000   0.00000   1.00000   0.09505   0.00000
                                0.00000   0.00000   0.00000   1.00000   0.00000
                                0.00000   0.00000   0.00000   0.00000   1.00000
                       0.000 INCHES
   ELEMENT=          DOWNSTREAM Z=    95.045
 0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %    VX1
 0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %    VBP1
-------------------------------

 0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %    VBP1
-------------------------------

   ELEMENT= UP09     UPSTREAM Z=     95.045     DL=     0.000
 0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %    VBP0
 0.109504 INX  0.100000 MRX  0.109504 INY -0.161799 MRY 0.000 %    VX0
 Z0=     95.045    DZ=     30.750    XZ=   125.795
                                1.00000   0.03075   0.00000   0.00000   0.00000
                                0.00000   1.00000   0.00000   0.00000   0.00000
 UP09      ===                  0.00000   0.00000   1.00000   0.03075   0.00008
                                0.00000   0.00000  -0.00001   1.00000   0.00524
                                0.00000   0.00000   0.00000   0.00000   1.00000
                      95.045 INCHES
   ELEMENT= UP09     DOWNSTREAM Z=   125.795
 0.112579 INX  0.100000 MRX  0.104529 INY -0.161800 MRY 0.000 %    VX1
 0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %    VBP1
-------------------------------

 0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %    VBP1
-------------------------------

   ELEMENT=          UPSTREAM Z=    125.795     DL=     0.000
 0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %    VBP0
 0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %    VX0
 Z0=    125.795    DZ=     85.780    XZ=   211.575
```

FIGURE 12

470

```
                             1.00000    0.08578    0.00000    0.00000    0.00000
                             0.00000    1.00000    0.00000    0.00000    0.00000
            ===              0.00000    0.00000    1.00000    0.08578    0.00000
                             0.00000    0.00000    0.00000    1.00000    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                      125.795 INCHES
   ELEMENT=             DOWNSTREAM Z=    211.575
 0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %     VX1
 0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %     VBP1
-----------------------------------

 0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %     VBP1
-----------------------------------

   ELEMENT= UQ1     UPSTREAM Z=    211.575   DL=     0.000
 0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %     VBP0
 0.021157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %     VX0
 Z0=    211.575    DZ=     37.500    XZ=    249.075
                             0.85801    0.03571    0.00000    0.00000    0.00000
                            -7.38805    0.85801    0.00000    0.00000    0.00000
 UQ1        ===              0.00000    0.00000    1.14904    0.03935    0.00000
                             0.00000    0.00000    8.14063    1.14904    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                      211.575 INCHES
   ELEMENT= UQ1         DOWNSTREAM Z=    249.075
 0.021724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %     VX1
 0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %     VBP1
-----------------------------------

 0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %     VBP1
-----------------------------------

   ELEMENT=             UPSTREAM Z=    249.075   DL=     0.000
 0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %     VBP0
 0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %     VX0
 Z0=    249.075    DZ=     19.500    XZ=    268.575
                             1.00000    0.01950    0.00000    0.00000    0.00000
                             0.00000    1.00000    0.00000    0.00000    0.00000
            ===              0.00000    0.00000    1.00000    0.01950    0.00000
                             0.00000    0.00000    0.00000    1.00000    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                      249.075 INCHES
   ELEMENT=             DOWNSTREAM Z=    268.575
 0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %     VX1
 0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %     VBP1
-----------------------------------

 0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %     VBP1
-----------------------------------

   ELEMENT= UQ2     UPSTREAM Z=    268.575   DL=     0.000
 0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %     VBP0
 0.020349 INX -0.070511 MRX -0.036977 INY  0.068399 MRY 0.000 %     VX0
 Z0=    268.575    DZ=     37.500    XZ=    306.075
                             1.14339    0.03928    0.00000    0.00000    0.00000
                             7.82535    1.14339    0.00000    0.00000    0.00000
 UQ2        ===              0.00000    0.00000    0.86315    0.03577    0.00000
                             0.00000    0.00000   -7.12749    0.86315    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                      268.575 INCHES
```

FIGURE A3

```
     ELEMENT= UQ2     DOWNSTREAM Z=    306.075
  0.020498 INX  0.078618 MRX -0.029469 INY  0.322588 MRY 0.000 %   VX1
  0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %   VBP1
 ----------------------------------------

  0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %   VBP1
 ----------------------------------------

     ELEMENT=             UPSTREAM Z=     306.075   DL=     0.000
  0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %   VBP0
  0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %   VX0
  Z0=     306.075    DZ=     18.794    XZ=   324.869
                             1.00000    0.01879    0.00000    0.00000    0.00000
                             0.00000    1.00000    0.00000    0.00000    0.00000
            ===              0.00000    0.00000    1.00000    0.01879    0.00000
                             0.00000    0.00000    0.00000    1.00000    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                  306.075 INCHES
     ELEMENT=             DOWNSTREAM Z=    324.869
  0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %   VX1
  0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %   VBP1
 ----------------------------------------

  0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %   VBP1
 ----------------------------------------

     ELEMENT= UD1     UPSTREAM Z=     324.869   DL=     0.000
  0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %   VBP0
  0.121975 INX -0.144435 MRX  0.076593 INY  0.322588 MRY 0.000 %   VX0
  Z0=     324.869    DZ=     81.900    XZ=   406.769
                             1.00000    0.08189    0.00000    0.00000    0.01031
                            -0.00000    1.00000    0.00000    0.00000    0.25172
  UD1       ===              0.00000    0.00000    0.99968    0.08190    0.00000
                             0.00000    0.00000   -0.00774    0.99968    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                  324.869 INCHES
     ELEMENT= UD1     DOWNSTREAM Z=    406.769
  0.110147 INX -0.144435 MRX  0.102989 INY  0.321894 MRY 0.000 %   VX1
  0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %   VBP1
 ----------------------------------------

  0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %   VBP1
 ----------------------------------------

     ELEMENT=             UPSTREAM Z=     406.769   DL=     0.000
  0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %   VBP0
  0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %   VX0
  Z0=     406.769    DZ=     18.100    XZ=   424.869
                             1.00000    0.01810    0.00000    0.00000    0.00000
                             0.00000    1.00000    0.00000    0.00000    0.00000
            ===              0.00000    0.00000    1.00000    0.01810    0.00000
                             0.00000    0.00000    0.00000    1.00000    0.00000
                             0.00000    0.00000    0.00000    0.00000    1.00000
                  406.769 INCHES
     ELEMENT=             DOWNSTREAM Z=    424.869
  0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %   VX1
  0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %   VBP1
 ----------------------------------------

  0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %   VBP1
 ----------------------------------------
```

FIGURE A4

```
    ELEMENT= UD2     UPSTREAM Z=   424.869   DL=     0.000
 0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %    VBP0
 0.103496 INX -0.590542 MRX  0.108815 INY  0.321894 MRY 0.000 %    VX0
 Z0=    424.869    DZ=     81.900    XZ=   506.769
                          1.00000    0.08189    0.00000    0.00000    0.01031
                         -0.00000    1.00000    0.00000    0.00000    0.25172
 UD2     ===              0.00000    0.00000    0.99968    0.08190    0.00000
                          0.00000    0.00000   -0.00774    0.99968    0.00000
                          0.00000    0.00000    0.00000    0.00000    1.00000
              424.869 INCHES
    ELEMENT= UD2     DOWNSTREAM Z=   506.769
 0.055136 INX -0.590542 MRX  0.135144 INY  0.320950 MRY 0.000 %    VX1
 0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VBP1
----------------------------------------

 0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VBP1
----------------------------------------

    ELEMENT=         UPSTREAM Z=   506.769   DL=     0.000
 0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VBP0
 0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VX0
 Z0=    506.769    DZ=     18.100    XZ=   524.869
                          1.00000    0.01810    0.00000    0.00000    0.00000
                          0.00000    1.00000    0.00000    0.00000    0.00000
         ===              0.00000    0.00000    1.00000    0.01810    0.00000
                          0.00000    0.00000    0.00000    1.00000    0.00000
                          0.00000    0.00000    0.00000    0.00000    1.00000
              506.769 INCHES
    ELEMENT=         DOWNSTREAM Z=   524.869
 0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VX1
 0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VBP1
----------------------------------------

 0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VBP1
----------------------------------------

    ELEMENT= UD3     UPSTREAM Z=   524.869   DL=     0.000
 0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VBP0
 0.040409 INX -1.036648 MRX  0.140953 INY  0.320950 MRY 0.000 %    VX0
 Z0=    524.869    DZ=     81.900    XZ=   606.769
                          1.00000    0.08189    0.00000    0.00000    0.01031
                          0.00000    1.00000    0.00000    0.00000    0.25172
 UD3     ===              0.00000    0.00000    0.99968    0.08190    0.00000
                          0.00000    0.00000   -0.00774    0.99968    0.00000
                          0.00000    0.00000    0.00000    0.00000    1.00000
              524.869 INCHES
    ELEMENT= UD3     DOWNSTREAM Z=   606.769
-0.044483 INX -1.036648 MRX  0.167194 INY  0.319758 MRY 0.000 %    VX1
-0.044483 INX -1.259701 MRX  0.167194 INY  0.319758 MRY 0.000 %    VBP1
----------------------------------------

-0.044483 INX -1.259701 MRX  0.167194 INY  0.319758 MRY 0.000 %    VBP1
----------------------------------------
```

FIGURE A5

```
 # ELELBL MAGKIN ELELEN      ZUS        GEL    QPERC XBPIPE    YBPIPE      XSHIFT YSHIFT
   1          0   95.045    0.000   0.00000    0.0  0.00000    0.0         0.000  0.000
   2 UP09    -4   30.750   95.045   0.03000    0.0  0.00000    0.0         0.000  0.000
   3          0   85.780  125.795   0.00000    0.0  0.00000    0.0         0.000  0.000
   4 UQ1      1   37.500  211.575  -7.98839    0.0  0.00000    0.0         0.100  0.000
   5          0   19.500  249.075   0.00000    0.0  0.00000    0.0         0.000  0.000
   6 UQ2      1   37.500  268.575   7.69255    0.0  0.00000    0.0         0.100  0.100
   7          0   18.794  306.075   0.00000    0.0  0.00000    0.0         0.000  0.000
   8 UD1     -1   81.900  324.869   1.44220    0.0  1.41664    0.0         0.000  0.000
   9          0   18.100  406.769   0.00000    0.0  0.00000    0.0         0.000  0.000
  10 UD2     -1   81.900  424.869   1.44220    0.0  1.41664    0.0         0.000  0.000
  11          0   18.100  506.769   0.00000    0.0  0.00000    0.0         0.000  0.000
  12 UD3     -1   81.900  524.869   1.44220    0.0  1.41664    0.0         0.000  0.000
  13          0   10.450  606.769   0.00000    0.0  0.00000    0.0         0.000  0.000


    0.100000 INX  0.100000 MRX  0.100000 INY  0.100000 MRY 0.000 %   START
      ELEMENT=          UPSTREAM Z=     0.000    DL=     0.000
    0.100000 INX  0.100000 MRX  0.100000 INY  0.100000 MRY 0.000 %   VBP0
    0.100000 INX  0.100000 MRX  0.100000 INY  0.100000 MRY 0.000 %   VX0
    Z0=      0.000    DZ=    95.045    XZ=     95.045
                            1.00000    0.09505    0.00000    0.00000    0.00000
                            0.00000    1.00000    0.00000    0.00000    0.00000
            ===             0.00000    0.00000    1.00000    0.09505    0.00000
                            0.00000    0.00000    0.00000    1.00000    0.00000
                            0.00000    0.00000    0.00000    0.00000    1.00000
                         0.000 INCHES
      ELEMENT=          DOWNSTREAM Z=    95.045
    0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %   VX1
    0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %   VBP1
   ---------------------------------


    0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %   VBP1
   ---------------------------------


      ELEMENT= UP09     UPSTREAM Z=    95.045    DL=     0.000
    0.109504 INX  0.100000 MRX  0.109504 INY  0.100000 MRY 0.000 %   VBP0
    0.109504 INX  0.100000 MRX  0.109504 INY -0.161799 MRY 0.000 %   VX0
    Z0=     95.045    DZ=    30.750    XZ=    125.795
                            1.00000    0.03075    0.00000    0.00000    0.00000
                            0.00000    1.00000    0.00000    0.00000    0.00000
    UP09    ===             0.00000    0.00000    1.00000    0.03075    0.00008
                            0.00000    0.00000   -0.00001    1.00000    0.00524
                            0.00000    0.00000    0.00000    0.00000    1.00000
                        95.045 INCHES
      ELEMENT= UP09     DOWNSTREAM Z=   125.795
    0.112579 INX  0.100000 MRX  0.104529 INY -0.161800 MRY 0.000 %   VX1
    0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %   VBP1
   ---------------------------------


    0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %   VBP1
   ---------------------------------


      ELEMENT=          UPSTREAM Z=   125.795    DL=     0.000
    0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %   VBP0
    0.112579 INX  0.100000 MRX  0.104529 INY -0.423600 MRY 0.000 %   VX0
    Z0=    125.795    DZ=    85.780    XZ=    211.575
```

FIGURE A2

```
                                1.00000   0.08578   0.00000   [illegible]0000   0.00000
                                0.00000   1.00000   0.00000   0.00000   0.00000
               ===              0.00000   0.00000   1.00000   0.08578   0.00000
                                0.00000   0.00000   0.00000   1.00000   0.00000
                                0.00000   0.00000   0.00000   0.00000   1.00000
                        125.795 INCHES
      ELEMENT=             DOWNSTREAM Z=   211.575
   0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %    VX1
   0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %    VBP1
   ------------------------------------------

   0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %    VBP1
   ------------------------------------------

      ELEMENT= UQ1         UPSTREAM Z=    211.575   DL=     0.000
   0.121157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %    VBP0
   0.021157 INX  0.100000 MRX  0.068193 INY -0.423600 MRY 0.000 %    VX0
   Z0=    211.575     DZ=     37.500    XZ=    249.075
                                0.85801   0.03571   0.00000   0.00000   0.00000
                               -7.38805   0.85801   0.00000   0.00000   0.00000
   UQ1     ===                  0.00000   0.00000   1.14904   0.03935   0.00000
                                0.00000   0.00000   8.14063   1.14904   0.00000
                                0.00000   0.00000   0.00000   0.00000   1.00000
                        211.575 INCHES
      ELEMENT= UQ1         DOWNSTREAM Z=   249.075
   0.021724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %    VX1
   0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %    VBP1
   ------------------------------------------

   0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %    VBP1
   ------------------------------------------

      ELEMENT=             UPSTREAM Z=    249.075   DL=     0.000
   0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %    VBP0
   0.121724 INX -0.070511 MRX  0.061690 INY  0.068399 MRY 0.000 %    VX0
   Z0=    249.075     DZ=     19.500    XZ=    268.575
                                1.00000   0.01950   0.00000   0.00000   0.00000
                                0.00000   1.00000   0.00000   0.00000   0.00000
               ===              0.00000   0.00000   1.00000   0.01950   0.00000
                                0.00000   0.00000   0.00000   1.00000   0.00000
                                0.00000   0.00000   0.00000   0.00000   1.00000
                        249.075 INCHES
      ELEMENT=             DOWNSTREAM Z=   268.575
   0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %    VX1
   0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %    VBP1
   ------------------------------------------

   0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %    VBP1
   ------------------------------------------

      ELEMENT= UQ2         UPSTREAM Z=    268.575   DL=     0.000
   0.120349 INX -0.070511 MRX  0.063023 INY  0.068399 MRY 0.000 %    VBP0
   0.020349 INX -0.070511 MRX -0.036977 INY  0.068399 MRY 0.000 %    VX0
   Z0=    268.575     DZ=     37.500    XZ=    306.075
                                1.14339   0.03928   0.00000   0.00000   0.00000
                                7.82535   1.14339   0.00000   0.00000   0.00000
   UQ2     ===                  0.00000   0.00000   0.86315   0.03577   0.00000
                                0.00000   0.00000  -7.12749   0.86315   0.00000
                                0.00000   0.00000   0.00000   0.00000   1.00000
                        268.575 INCHES
```

FIGURE A3

```
      ELEMENT= UQ2     DOWNSTREAM Z=    306.075
   0.020498 INX  0.078618 MRX -0.029469 INY  0.322588 MRY 0.000 %    VX1
   0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %    VBP1
 ----------------------------------------------

   0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %    VBP1
 ----------------------------------------------

      ELEMENT=          UPSTREAM Z=    306.075    DL=      0.000
   0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %    VBP0
   0.120498 INX  0.078618 MRX  0.070531 INY  0.322588 MRY 0.000 %    VX0
   Z0=    306.075    DZ=     18.794    XZ=   324.869
                         1.00000    0.01879    0.00000    0.00000    0.00000
                         0.00000    1.00000    0.00000    0.00000    0.00000
           ===           0.00000    0.00000    1.00000    0.01879    0.00000
                         0.00000    0.00000    0.00000    1.00000    0.00000
                         0.00000    0.00000    0.00000    0.00000    1.00000
                 306.075 INCHES
      ELEMENT=          DOWNSTREAM Z=    324.869
   0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %    VX1
   0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %    VBP1
 ----------------------------------------------

   0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %    VBP1
 ----------------------------------------------

      ELEMENT= UD1      UPSTREAM Z=    324.869    DL=      0.000
   0.121975 INX  0.078618 MRX  0.076593 INY  0.322588 MRY 0.000 %    VBP0
   0.121975 INX -0.144435 MRX  0.076593 INY  0.322588 MRY 0.000 %    VX0
   Z0=    324.869    DZ=     81.900    XZ=   406.769
                         1.00000    0.08189    0.00000    0.00000    0.01031
                        -0.00000    1.00000    0.00000    0.00000    0.25172
   UD1     ===           0.00000    0.00000    0.99968    0.08190    0.00000
                         0.00000    0.00000   -0.00774    0.99968    0.00000
                         0.00000    0.00000    0.00000    0.00000    1.00000
                 324.869 INCHES
      ELEMENT= UD1      DOWNSTREAM Z=    406.769
   0.110147 INX -0.144435 MRX  0.102989 INY  0.321894 MRY 0.000 %    VX1
   0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %    VBP1
 ----------------------------------------------

   0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %    VBP1
 ----------------------------------------------

      ELEMENT=          UPSTREAM Z=    406.769    DL=      0.000
   0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %    VBP0
   0.110147 INX -0.367488 MRX  0.102989 INY  0.321894 MRY 0.000 %    VX0
   Z0=    406.769    DZ=     18.100    XZ=   424.869
                         1.00000    0.01810    0.00000    0.00000    0.00000
                         0.00000    1.00000    0.00000    0.00000    0.00000
           ===           0.00000    0.00000    1.00000    0.01810    0.00000
                         0.00000    0.00000    0.00000    1.00000    0.00000
                         0.00000    0.00000    0.00000    0.00000    1.00000
                 406.769 INCHES
      ELEMENT=          DOWNSTREAM Z=    424.869
   0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %    VX1
   0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %    VBP1
 ----------------------------------------------

   0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %    VBP1
 ----------------------------------------------
```

FIGURE A4

```
    ELEMENT= UD2    UPSTREAM Z=   424.869   DL=     0.000
  0.103496 INX -0.367488 MRX  0.108815 INY  0.321894 MRY 0.000 %    VBP0
  0.103496 INX -0.590542 MRX  0.108815 INY  0.321894 MRY 0.000 %    VX0
  Z0=   424.869   DZ=    81.900   XZ=   506.769
                              1.00000    0.08189    0.00000    0.00000    0.01031
                             -0.00000    1.00000    0.00000    0.00000    0.25172
  UD2    ===                  0.00000    0.00000    0.99968    0.08190    0.00000
                              0.00000    0.00000   -0.00774    0.99968    0.00000
                              0.00000    0.00000    0.00000    0.00000    1.00000
                  424.869 INCHES
    ELEMENT= UD2    DOWNSTREAM Z=   506.769
  0.055136 INX -0.590542 MRX  0.135144 INY  0.320950 MRY 0.000 %    VX1
  0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VBP1
------------------------------------------------

  0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VBP1
------------------------------------------------

    ELEMENT=        UPSTREAM Z=   506.769   DL=     0.000
  0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VBP0
  0.055136 INX -0.813595 MRX  0.135144 INY  0.320950 MRY 0.000 %    VX0
  Z0=   506.769   DZ=    18.100   XZ=   524.869
                              1.00000    0.01810    0.00000    0.00000    0.00000
                              0.00000    1.00000    0.00000    0.00000    0.00000
         ===                  0.00000    0.00000    1.00000    0.01810    0.00000
                              0.00000    0.00000    0.00000    1.00000    0.00000
                              0.00000    0.00000    0.00000    0.00000    1.00000
                  506.769 INCHES
    ELEMENT=        DOWNSTREAM Z=   524.869
  0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VX1
  0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VBP1
------------------------------------------------

  0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VBP1
------------------------------------------------

    ELEMENT= UD3    UPSTREAM Z=   524.869   DL=     0.000
  0.040409 INX -0.813595 MRX  0.140953 INY  0.320950 MRY 0.000 %    VBP0
  0.040409 INX -1.036648 MRX  0.140953 INY  0.320950 MRY 0.000 %    VX0
  Z0=   524.869   DZ=    81.900   XZ=   606.769
                              1.00000    0.08189    0.00000    0.00000    0.01031
                              0.00000    1.00000    0.00000    0.00000    0.25172
  UD3    ===                  0.00000    0.00000    0.99968    0.08190    0.00000
                              0.00000    0.00000   -0.00774    0.99968    0.00000
                              0.00000    0.00000    0.00000    0.00000    1.00000
                  524.869 INCHES
    ELEMENT= UD3    DOWNSTREAM Z=   606.769
 -0.044483 INX -1.036648 MRX  0.167194 INY  0.319758 MRY 0.000 %    VX1
 -0.044483 INX -1.259701 MRX  0.167194 INY  0.319758 MRY 0.000 %    VBP1
------------------------------------------------

 -0.044483 INX -1.259701 MRX  0.167194 INY  0.319758 MRY 0.000 %    VBP1
------------------------------------------------
```

FIGURE A5

APPENDIX A -- A Beam Line Ray Tracing Example

As an example of this procedure, a beam will be traced through the first several magnets in the U line. The beam line is shown in Figure A1.

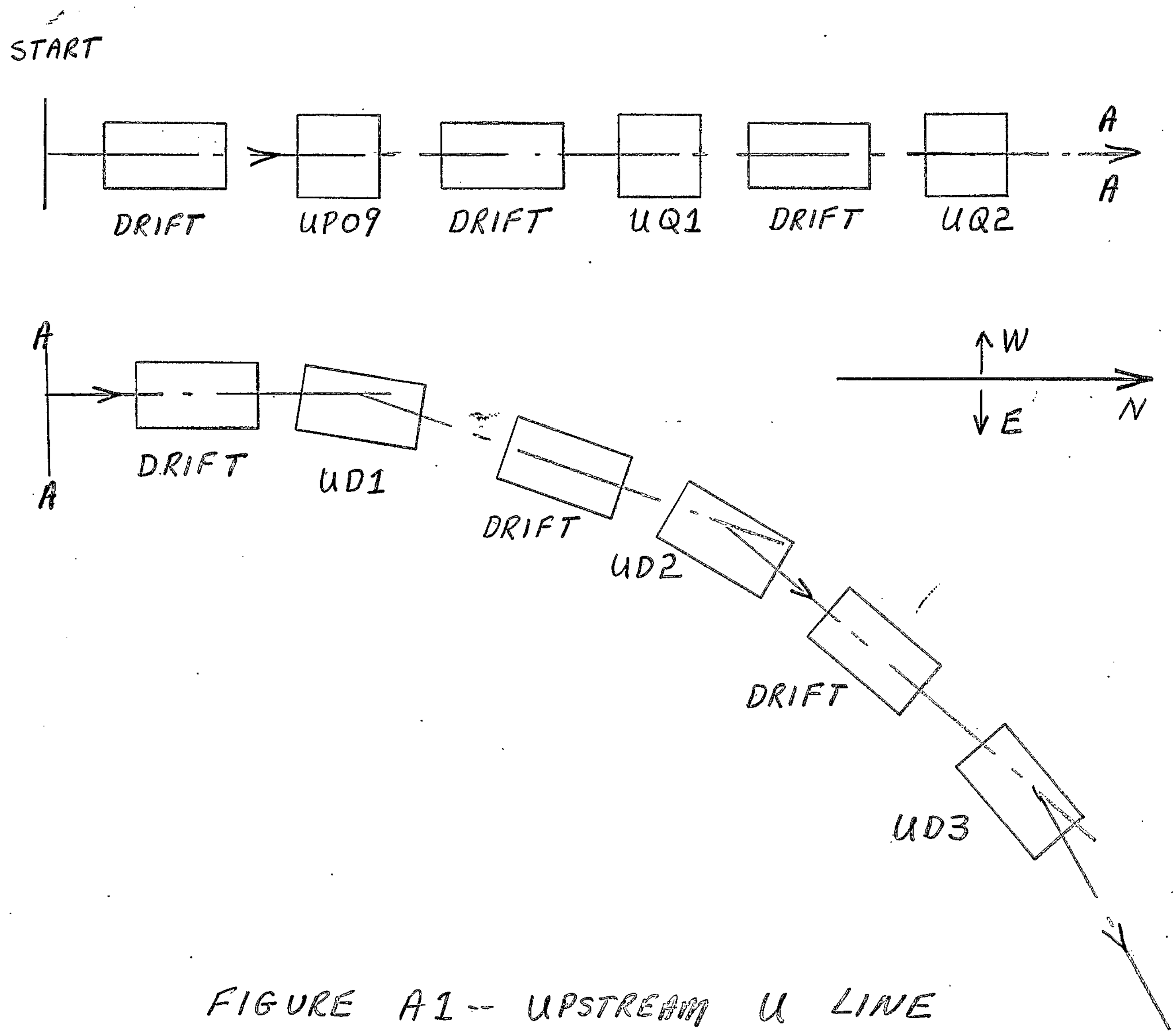

FIGURE A1-- UPSTREAM U LINE