Transverse Damping Algorithms

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Abstract

Damping algorithms are discussed with respect to their useful be-
tatron tune range and thier damping time constants. The algorithms
are based on the beam position information from a single PUE location
and make use of information from up to three turns. It is shown that
it is possible to find algorithms for efficient damping at most betatron
tune values which also include fast closed orbit suppression. All algo-
rithms can be implemented using a simple non-recursive digital filter
design.

1 1-Turn Algorithm

The Booster and AGS transverse damper use one pick-up electrode (PUE) to measure the bunch position and a strip line at the same location to apply a correcting kick to the same bunch an the following turn. The magnitude of the kick has to be determined such that the coherent dipole motion of the bunch is reduced. The amount of coherent dipole motion of the bunch is expressed in terms of the Courant-Snyder invariant:

$$\epsilon = \pi \left(\gamma x_0^2 + 2\alpha x_0 x_0' + \beta x_0'^2 \right) \tag{1}$$

where x_0 and x_0' are the position and angle of the bunch at the time when the kick is applied. The change of ϵ for a kick θ is then

$$\triangle\epsilon = 2\pi \left(\alpha x_0 + \beta x_0' \right) \tag{2}$$

x_0 and x_0' can be obtained from the PUE information of previous turns. Using just the last turn information $\begin{pmatrix} x_{-1} \\ x_{-1}' \end{pmatrix}$, of which of course only x_{-1} can be measured, gives:

$$\begin{pmatrix} x_0 \\ x_0' \end{pmatrix} = \begin{pmatrix} \cos\left(2\pi\nu\right) + \alpha\sin\left(2\pi\nu\right) & \beta\sin\left(2\pi\nu\right) \\ -\gamma\sin\left(2\pi\nu\right) & \cos\left(2\pi\nu\right) - \alpha\sin\left(2\pi\nu\right) \end{pmatrix} \begin{pmatrix} x_{-1} \\ x_{-1}' \end{pmatrix} \tag{3}$$

Inserting into eq. 2 then gives:

$$\triangle\epsilon = 2\pi\theta \left[\left(\alpha\cos\left(2\pi\nu\right) + \sin\left(2\pi\nu\right) \right) x_{-1} + \left(\beta\cos\left(2\pi\nu\right) \right) x_{-1}' \right] \tag{4}$$

With $\begin{pmatrix} x_{-1} \\ x_{-1}' \end{pmatrix}$ expressed in terms of ϵ and phase ψ,

$$\begin{aligned} x_{-1} &= \sqrt{\tfrac{\beta\epsilon}{\pi}} \cos\psi \\ x_{-1}' &= \sqrt{\tfrac{\epsilon}{\pi\beta}} \left(-\alpha\cos\psi - \sin\psi \right) \end{aligned} \tag{5}$$

and a kick that is proportional to $x_{-1}(\theta = kx_{-1})$ the change of ϵ becomes:

$$\begin{aligned} \triangle\epsilon &= 2k\beta\epsilon \left[\left(\alpha\cos\left(2\pi\nu\right) + \sin\left(2\pi\nu\right) \right)\cos\psi - \cos\left(2\pi\nu\right)\left(\alpha\cos\psi + \sin\psi \right) \right] \times \\ &\quad \cos\psi \\ &= 2k\beta\epsilon \left[\sin\left(2\pi\nu\right)\cos\psi - \cos\left(2\pi\nu\right)\sin\psi \right] \cos\psi \end{aligned} \tag{6}$$

or, after averaging over the angle variable,

$$\langle \triangle \epsilon \rangle = \frac{1}{2\pi} \int_0^{2\pi} \triangle \epsilon d\psi = k\beta\epsilon \sin\left(2\pi\nu\right) \qquad (7)$$

Clearly maximum damping is achieved for a fractional betatron tune of $n/4$. In this case the damping time constant is $k\beta\epsilon$.

2 2-Turn Algorithm

In order to get optimum damping for betatron values different from $n/4$ information from two turns can be used to determine both x_0 and x_0':

$$\begin{pmatrix} x_{-1} \\ x_{-2} \end{pmatrix} = \begin{pmatrix} \cos\left(2\pi\nu\right) - \alpha\sin\left(2\pi\nu\right) & -\beta\sin\left(2\pi\nu\right) \\ \cos\left(4\pi\nu\right) - \alpha\sin\left(4\pi\nu\right) & -\beta\sin\left(4\pi\nu\right) \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \qquad (8)$$

or

$$\begin{pmatrix} x_0 \\ x_0' \end{pmatrix} = \frac{1}{\beta\sin(2\pi\nu)} \times$$
$$\begin{pmatrix} \beta\sin\left(4\pi\nu\right) & -\beta\sin\left(2\pi\nu\right) \\ \cos\left(4\pi\nu\right) - \alpha\sin\left(4\pi\nu\right) & -\cos\left(2\pi\nu\right) + \alpha\sin\left(2\pi\nu\right) \end{pmatrix} \begin{pmatrix} x_{-1} \\ x_{-2} \end{pmatrix}$$
$$(9)$$

After inserting in eq. 2 this gives

$$\triangle\epsilon = \frac{2\pi\theta}{\sin\left(2\pi\nu\right)} \left(\cos\left(4\pi\nu\right) x_{-1} - \cos\left(2\pi\nu\right) x_{-2}\right) \qquad (10)$$

Therefore for optimum damping the kick θ has to be chosen as

$$\begin{aligned} \theta &= \frac{k}{\sin(2\pi\nu)} \left(\cos\left(4\pi\nu\right) x_{-1} - \cos\left(2\pi\nu\right) x_{-2}\right) \\ &= k\left(w_{-1}\left(\nu\right) x_{-1} + w_{-2}\left(\nu\right) x_{-2}\right) \end{aligned} \qquad (11)$$

In this case the maximum damping rate is always $k\beta$. In Eq. 11 I also introduced the weights $w_i\left(\nu\right)$ that need to be used in a digital filter design. The weights are plotted in Fig. 1. Such a scheme is being implemented in the Booster damping system[1] using the quadrupole Gauss clocks to give the tune value, which in turn selects the two weights from a look-up table to be used in a non-recursive 2-turn digital notch filter.

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3 3-Turn Algorithm

With the information from 3 turns it is also possible to suppress the closed orbit information which is the same every turn and would just reduce the dynamic range of the damping system. The closed orbit contribution to x_0 and x_0' was ignored in the calculation so far. A close orbit subtraction determination of x_0 and x_0' can be obtained from the differences:

$$\begin{pmatrix} x_{-1} - x_{-2} \\ x_{-1} - x_{-3} \end{pmatrix} = \begin{pmatrix} c\,(2,4) - \alpha s\,(2,4) & -\beta s\,(2,4) \\ c\,(2,6) - \alpha s\,(2,6) & -\beta s\,(2,6) \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \quad (12)$$

or

$$\begin{pmatrix} x_0 \\ x_0' \end{pmatrix} = \frac{1}{\beta(2\sin(2\pi\nu) - \sin(4\pi\nu))} \times$$
$$\begin{pmatrix} \beta s_{2,6}\,(\nu) & -\beta s_{2,4}\,(\nu) \\ c_{2,6}\,(\nu) - \alpha s_{2,6}\,(\nu) & -c_{2,4}\,(\nu) + \alpha s_{2,4}\,(\nu) \end{pmatrix} \begin{pmatrix} x_{-1} - x_{-2} \\ x_{-1} - x_{-3} \end{pmatrix} \quad (13)$$

where

$$c_{n,m}\,(\nu) = \cos\,(n\pi\nu) - \cos\,(m\pi\nu) \quad \text{and} \quad s_{n,m}\,(\nu) = \sin\,(n\pi\nu) - \sin\,(m\pi\nu) \quad (14)$$

Again inserting into Eq. 2 gives:

$$\triangle\epsilon = \frac{2\pi\theta}{2\sin\,(2\pi\nu) - \sin\,(4\pi\nu)} \left(c_{6,4}\,(\nu)\,x_{-1} + c_{2,6}\,(\nu)\,x_{-2} + c_{4,2}\,(\nu)\,x_{-3}\right) \quad (15)$$

and optimum damping is therefore achieved for

$$\begin{aligned} \theta &= \frac{k}{2\sin(2\pi\nu) - \sin(4\pi\nu)} \left(c_{6,4}\,(\nu)\,x_{-1} + c_{2,6}\,(\nu)\,x_{-2} + c_{4,2}\,(\nu)\,x_{-3}\right) \\ &= k\,(w_{-1}\,(\nu)\,x_{-1} + w_{-2}\,(\nu)\,x_{-2} + w_{-3}\,(\nu)\,x_{-3}) \end{aligned} \quad (16)$$

Note that for both the 2-turn and the 3-turn algorithm the coefficients for the digital notch filter are divergent for an integer tune value. This is to be expected since for integer tune phase and amplitude of the betatron motion are indistinguishable and it is also impossible to subtract a closed orbit. For the 2-turn algorithm the coefficients are diverging linearly and for the 3-turn algorithm they diverge quadraticly. This can also be seen from Fig. 2 which shows the three weights as a function of betatron tune. Note that the sum of the three weights is zero which ensures closed orbit suppression.

4 Closed Orbit Suppression using 2 Turns

With the information of 3-turns available one can also achieve damping with closed orbit suppression by using only the differences of the position measurements of two out of the three turns. The damping will not be optimal for all tune values but can still be adequate over a limited range. The situation is therefore similar to the 1-turn algorithm with the additional advantage of fast closed orbit suppression.

I will first treat damping with the following kick:

$$\theta = \frac{k}{2}\left(x_{-1} - x_{-2}\right) \tag{17}$$

For the other two possibilities

$$\begin{aligned} \theta &= \frac{k}{2}\left(x_{-1} - x_{-3}\right) \\ \theta &= \frac{k}{2}\left(x_{-2} - x_{-3}\right) \end{aligned} \tag{18}$$

the derivations are similar and only the results will be listed.

In terms ϵ and phase ψ we have

$$\begin{aligned} x_{-1} &= \sqrt{\frac{\beta\epsilon}{\pi}}\left(\cos\left(2\pi\nu\right)\cos\psi - \sin\left(2\pi\nu\right)\sin\psi\right) \\ x_{-2} &= \sqrt{\frac{\beta\epsilon}{\pi}}\cos\psi \\ x_{-3} &= \sqrt{\frac{\beta\epsilon}{\pi}}\left(\cos\left(2\pi\nu\right)\cos\psi + \sin\left(2\pi\nu\right)\sin\psi\right) \end{aligned} \tag{19}$$

Inserting Eq. 19 and Eq. 17 into Eq. 13 gives:

$$\begin{aligned} \triangle\epsilon = \; & k\beta\epsilon\left(\sin\left(4\pi\nu\right)\cos\psi + \cos\left(4\pi\nu\right)\sin\psi\right) \times \\ & \left(\left(1 - \cos\left(2\pi\nu\right)\right)\cos\psi + \sin\left(2\pi\nu\right)\sin\psi\right) \end{aligned} \tag{20}$$

After averaging over the angle variable we get:

$$\langle\triangle\epsilon\rangle = k\beta\epsilon\frac{s_{2,4}\left(\nu\right)}{2} \tag{21}$$

Corresponding calculations for the other two possibilities give:

$$\begin{aligned} \theta = \frac{k}{2}\left(x_{-1} - x_{-3}\right): \quad & \langle\triangle\epsilon\rangle = k\beta\epsilon\frac{s_{2,6}\left(\nu\right)}{2} \\ \theta = \frac{k}{2}\left(x_{-2} - x_{-3}\right): \quad & \langle\triangle\epsilon\rangle = k\beta\epsilon\frac{s_{4,6}\left(\nu\right)}{2} \end{aligned} \tag{22}$$

Clearly the damping rate is always smaller than what was obtained for optimal damping. The relative damping rates compared with optimal damping are shown in Fig. 3. Depending on the betatron tune the difference which gives the best damping rate can be chosen. This scheme is implemented in the AGS transverse damper.

5 Non-Proportional Damping

So far we have assumed that the applied kick is always proportional to the calculated correction. This leads to exponential damping with the emittance decreasing with the turn number as

$$\epsilon(n) = \epsilon_0 \exp\left(-\frac{n}{n_p}\right) \tag{23}$$

with

$$n_p = \frac{1}{k\beta} \tag{24}$$

for optimum damping. For non-optimal damping n_p would need to increased according to Eqs. 7,22 or 23. Alternatively the kick can always be at maximum amplitude and only its sign depends on the sign of the correction. This amounts to running at such a high loop gain that the output is always in saturation. In this case the averaging over the angle variable has to be changed from

$$\frac{1}{2\pi}\int_0^{2\pi} \cos^2\psi \, d\psi = \frac{1}{2} \tag{25}$$

to

$$\frac{1}{2\pi}\int_0^{2\pi} |\cos\psi| \, d\psi = \frac{2}{\pi}. \tag{26}$$

The average change of ϵ is then

$$\langle \triangle\epsilon \rangle = -4\theta_{\max}\sqrt{\frac{\beta\epsilon}{\pi}} \tag{27}$$

The solution of this difference equation is a parabolic dependence on the turn number:

$$\epsilon(n) = \epsilon_0\left(1 - \frac{n}{n_0}\right)^2 \tag{28}$$

with

$$n_0 = \sqrt{\frac{\epsilon_0\pi}{\beta}}\,\frac{1}{2\theta_{\max}} \tag{29}$$

To compare with proportional damping we assume that the maximum kick is applied at beginning for proportional damping:

$$\theta_{\max} = k\sqrt{\frac{\beta\epsilon_0}{\pi}} \tag{30}$$

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which gives

$$\epsilon\left(n\right) = \epsilon_0 \left(1 - \frac{2n}{\pi n_p}\right)^2 \tag{31}$$

Fig. 4 shows the time evolution of ϵ/ϵ_0 for both proportional damping and saturated (bang-bang) damping. Saturated damping clearly gives faster damping[2].

6 Figure Captions

Fig. 1: Optimal weights for the 2-turn algorithm as a function of the fractional betatron tune. The solid line is $w_{-1}\left(\nu\right)$ and the dashed line is $w_{-2}\left(\nu\right)$.

Fig. 2: Optimal weights for the 3-turn algorithm as a function of the fractional betatron tune. The solid line is $w_{-1}\left(\nu\right)$, the dashed line is $w_{-2}\left(\nu\right)$, and the dotted-dashed line is $w_{-3}\left(\nu\right)$.

Fig. 3: The reduction of the damping rate for taking the difference of only two out of the three turn position information is plotted as a function of the betatron tune. The solid line shows the damping rate for $x_{-1} - x_{-2}$, the dashed line for $x_{-1} - x_{-3}$, and the dotted-dashed line for $x_{-2} - x_{-3}$.

Fig. 4: Evolution of the invariant ϵ as a function of turn number for proportional damping (solid line) and saturated damping (dashed line).n_p is damping rate turn number for proportional damping.

References

[1] Results from the AGS Booster transverse damper, D.Russo et al., 1993 Particle Accelerator Conf., Washington D.C.

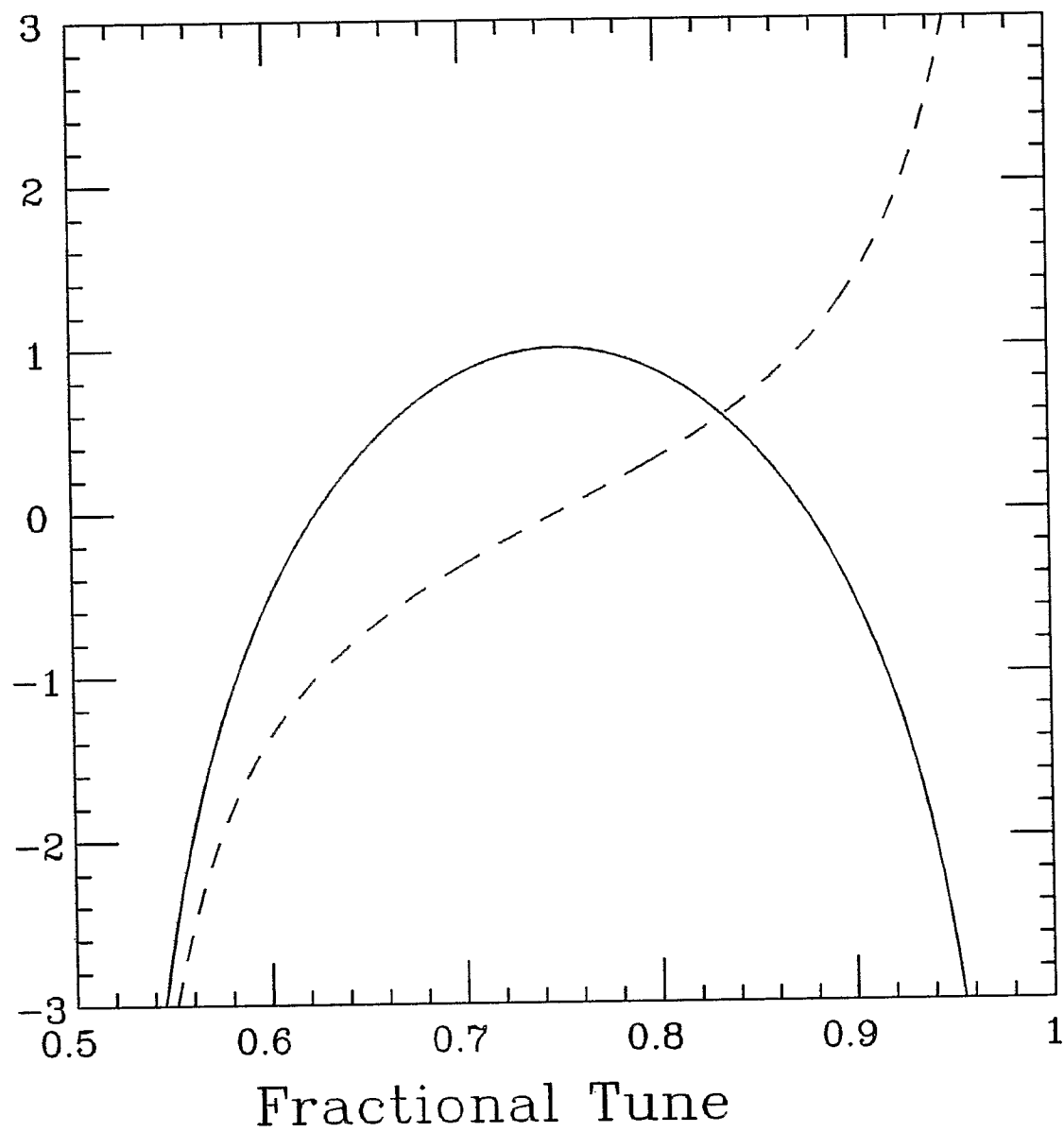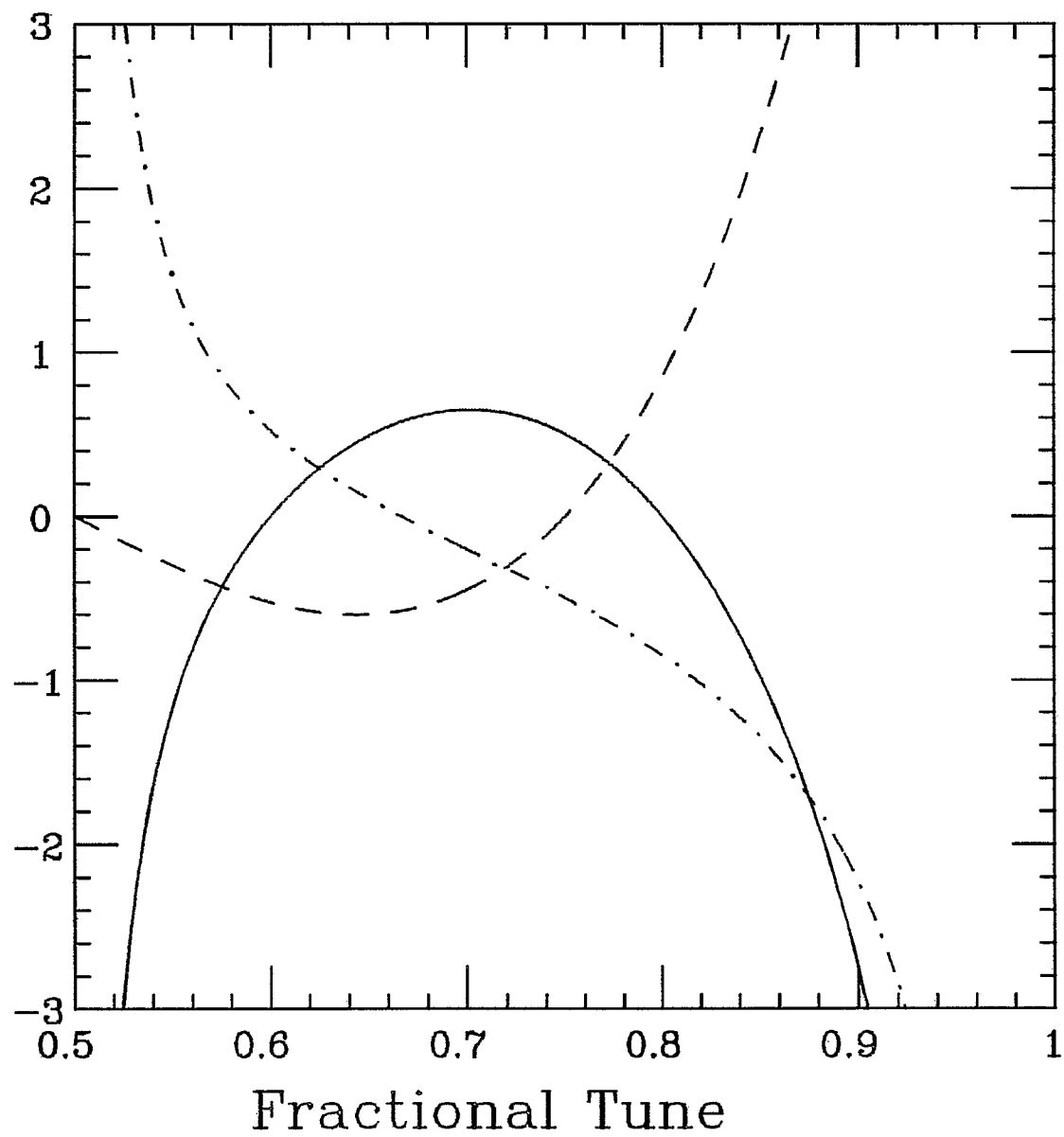[2] Tevatron Studies Report, G.Jackson, D.McConnell, B.Fellenz, E.Raka, S.P.Yamin, AGS/AD/Tech Note No. 369 (1992)
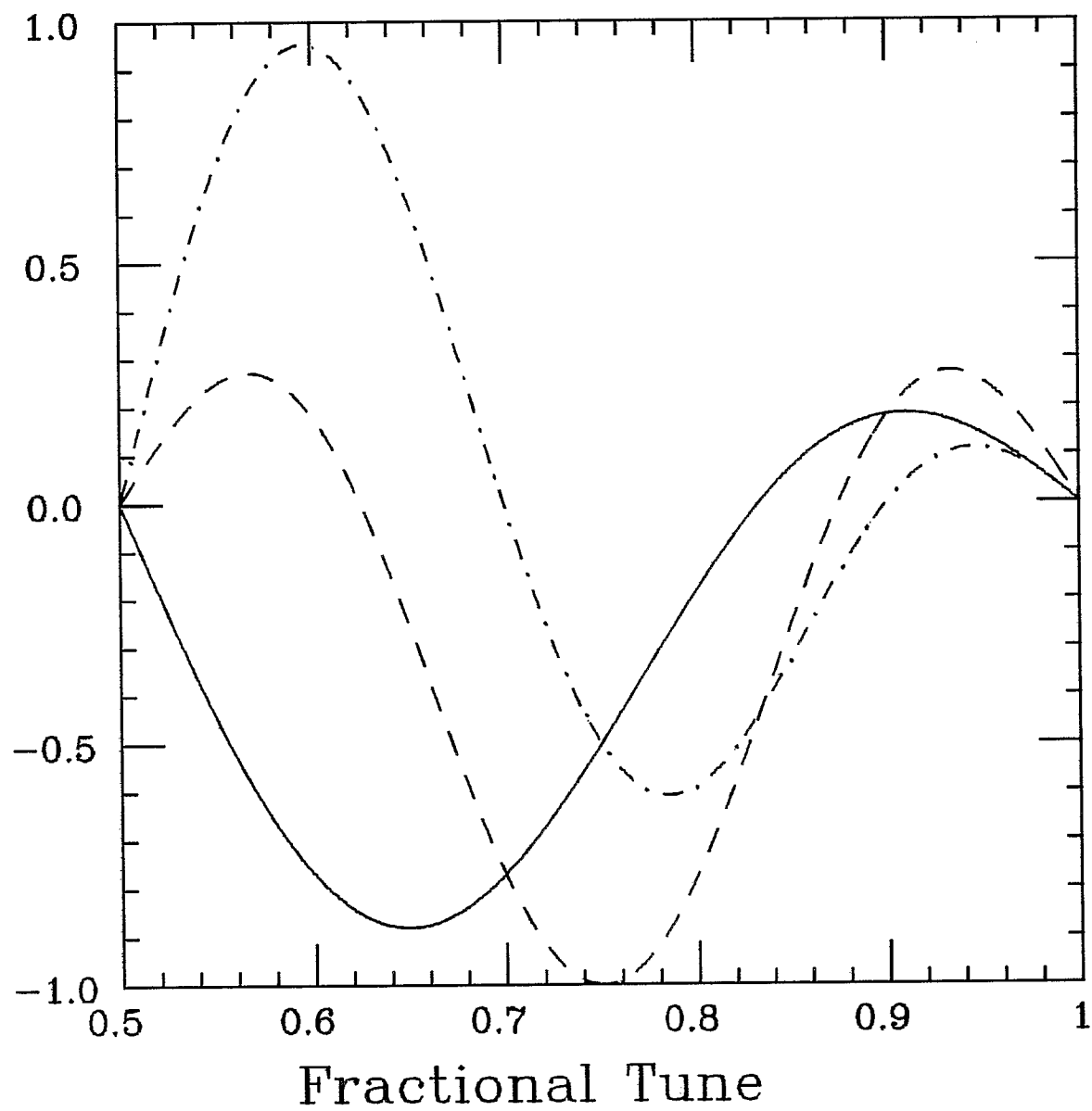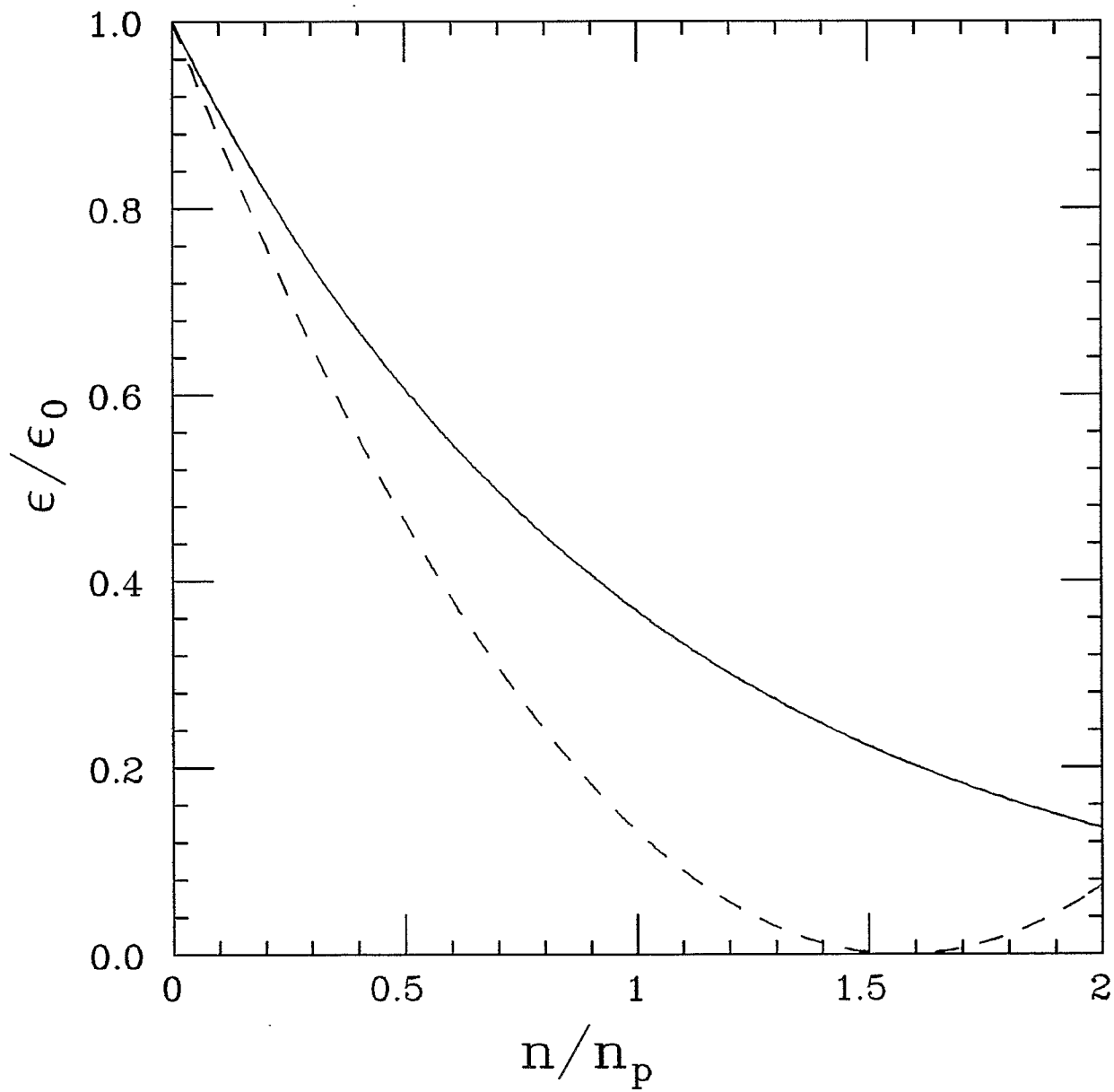
Figure 1

Figure 2

Figure 3

Figure 4