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NEUTRINO HORN WITH ECCENTRIC INNER CONDUCTOR

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INTRODUCTION

A neutrino horn is basically a coaxial transmission line constructed of air filled concentric conducting cylinders or cones. Axial electric current flows on the inner diameter of the outer conductor. The front end is short-circuited which enables the outside of the inner conductor to carry the return current in the opposite direction.

When the cylindrical axis of the inner and outer conductors are not coincident, see Figure I, the result is an unbalanced force on the inner conductor. Due to rapid pulsing of the neutrino horn this flexural force could produce an oscillation about the equilibrium position. This vibration, if of significant magnitude, could render the device dynamically unstable. The purpose of this note is to document the equation used to evaluate the force per unit length caused by conductor eccentricity.

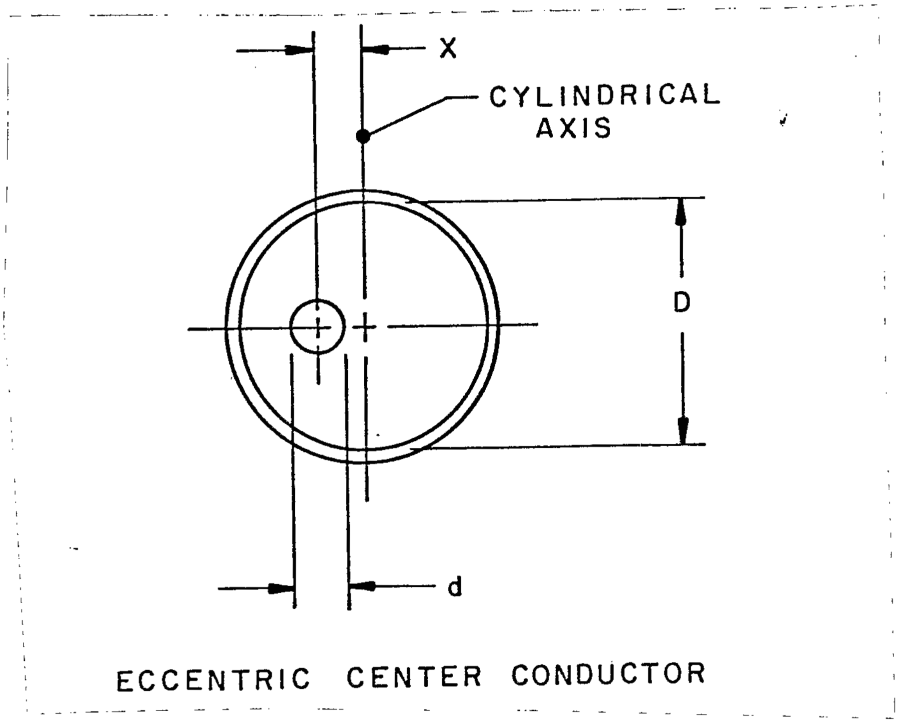

FIGURE I

NOTATION

W	STORED ENERGY IN THE MAGNETIC FIELD	JOULES
X	DISPLACEMENT	METERS
F	FORCE	NEWTONS; POUNDS
I	CURRENT	AMPERES
Z_c	CHARACTERISTIC IMPEDANCE	OHMS
L	INDUCTANCE/UNIT LENGTH	HENRY/CM
C	CAPACITANCE/UNIT LENGTH	FARAD/CM
v	VELOCITY OF LIGHT (3×10^{10})	CM/SECOND
ε	PERMITTIVITY (DIELECTRIC CONSTANT) 8.85×10^{-14}	FARAD/CM
D	INNER DIAMETER OF THE OUTER CONDUCTOR	METERS
d	OUTER DIAMETER OF THE INNER CONDUCTOR	METERS
ℓ	LENGTH	CENTIMETER

MATHEMATICAL CONCEPT

Force can be defined by the partial differential equation

$$F = -\frac{\partial W}{\partial X} \quad (1)$$

The stored energy W in the magnetic field of a self-inductance L due to a current I flowing is

$$W = \frac{1}{2} L I^2$$

Taking the partial derivative yields

$$\frac{\partial W}{\partial X} = \frac{1}{2} I^2 \frac{\partial L}{\partial X}$$

Substituting in equation (1) gives

$$F = -\frac{1}{2} I^2 \frac{\partial L}{\partial X} \quad (2)$$

The following argument is based on a method similar to the one developed by Brown,[1] based on an equation for the capacitance per unit length between two eccentric cylinders derived by Smythe.[2]

The characteristic impedance of a coaxial transmission line is

$$Z_c = \sqrt{\frac{L}{C}} \quad (3)$$

The velocity of a wave on the line is

$$v = \frac{1}{\sqrt{LC}} \quad (4)$$

The characteristic impedance is then

$$Z_c = \frac{1}{vC} \tag{5}$$

The equation for the capacitance per unit length (farads per centimeter) of an eccentric coaxial line derived by Smythe is, expressed in terms of radii,[3]

$$C = \frac{2\pi\varepsilon}{\cosh^{-1}\left(\frac{b^2+a^2-X^2}{2ba}\right)} \tag{6}$$

Let

$$U = \left(\frac{b^2+a^2-X^2}{2ba}\right)$$

Then U expressed in terms of diameters is

$$U = \frac{1}{2}\left[\left(\frac{D}{d}\right) + \left(\frac{d}{D}\right) - \left(\frac{4X^2}{dD}\right)\right]$$

In simplified form equation (6) is

$$C = \frac{2\pi\varepsilon}{\cosh^{-1} U} \tag{7}$$

Substituting equation (7) in equation (5) yields

$$Z_c = \frac{1}{v} \cdot \frac{\cosh^{-1} U}{2\pi\varepsilon} \tag{8}$$

Solving equation (3) for L and substituting equation (8) yields an expression for the inductance per centimeter,

$$L = Z_c^2 \; C = \left(\frac{1}{v} \cdot \frac{\cosh^{-1}U}{2\pi\varepsilon}\right)^2 \left(\frac{2\pi\varepsilon}{\cosh^{-1}U}\right)$$

$$L = \frac{1}{v^2} \left(\frac{\cosh^{-1}U}{2\pi\varepsilon}\right) = \frac{1}{v} \left(60 \cosh^{-1}U\right)$$

$$L = 0.2 \times 10^{-8} \cosh^{-1}U \tag{9}$$

Let

$$k = 0.2 \times 10^{-8}$$

Then

$$L = k \cosh^{-1}U$$

$$\cosh\left(\frac{L}{k}\right) = U \tag{10}$$

$$\cosh\left(\frac{L}{k}\right) = \cosh(\psi) = U = f(X)$$

$$\frac{\partial}{\partial X} \cosh(\psi) = \sinh(\psi) \frac{\partial \psi}{\partial X} = \frac{\partial}{\partial X} f(X)$$

$$\frac{1}{k} \sinh\left(\frac{L}{k}\right) \frac{\partial L}{\partial X} = \frac{1}{2} \left(- \frac{8X}{dD}\right)$$

$$\frac{\partial L}{\partial X} = - \frac{4X}{dD} \cdot \frac{k}{\sinh\left(\frac{L}{k}\right)} \tag{11}$$

Knowing that

$$\cosh^2 x - \sinh^2 x = 1$$

Then

$$\sinh x = \sqrt{\cosh^2 x - 1}$$

$$\sinh \left(\frac{L}{k}\right) = \sqrt{\cosh^2 \left(\frac{L}{k}\right) - 1}$$

From equation (10) we know

$$\cosh \left(\frac{L}{k}\right) = U$$

Then

$$\sinh \left(\frac{L}{k}\right) = \sqrt{U^2 - 1}$$

Equation (11) becomes

$$\frac{\partial L}{\partial X} = -\frac{4X}{dD} \cdot \frac{k}{\sqrt{U^2 - 1}}$$

Substituting in equation (2) yields the force per centimeter length

$$\frac{F}{\ell} = -\frac{1}{2} I^2 \left(-\frac{4X}{dD}\right) \left(\frac{k}{\sqrt{U^2-1}}\right)$$

$$\frac{F}{\ell} = \frac{0.4 \times 10^{-8} I^2 X}{dD} \cdot \frac{1}{\sqrt{U^2-1}} \qquad (12)$$

<u>NUMERICAL ANALYSIS</u>

Assume $I = 30 \times 10^4$ amperes and equation (12) becomes

$$\frac{F}{\ell} = 360 \cdot \frac{X}{dD} \cdot \frac{1}{\sqrt{U^2 - 1}} \qquad \text{(nt/cm)}$$

Where

$$U = \frac{1}{2}\left[\left(\frac{D}{d}\right) + \left(\frac{d}{D}\right) - \left(\frac{4X^2}{dD}\right)\right]$$

Using the above expressions for F/ℓ and U, the values presented in Table I were calculated. The quantities X, d and D are expressed in meters.

X(M)	F/ℓ d=0.019 D=0.064	F/ℓ d=0.076 D=0.147	F/ℓ d=0.127 D=0.203	F/ℓ d=0.127 D=0.300
0.00100	84.038428	14.401088	7.029792	3.634527
0.00089	74.779111	12.816501	6.256401	3.234700
0.00076	63.843030	10.944016	5.342445	2.762190
0.00063	52.913235	9.071726	4.428535	2.289693
0.00051	42.828841	7.343602	3.584962	1.853550
0.00038	31.908104	5.471593	2.671121	1.381069
0.00030	25.189280	4.319637	2.108770	1.090315
0.00020	16.792039	2.879733	1.405840	0.726875
0.00010	8.395775	1.439859	0.702918	0.363437

<u>TABLE I</u>

Converting the first two left hand columns to units of inches and pounds/inch we obtain,

X (inches)	F/ℓ (#/in) d=0.750 D=2.500
0.040	7.437
0.035	6.618
0.030	5.650
0.025	4.683
0.020	3.790
0.015	2.824
0.012	2.229
0.008	1.486
0.004	0.743

CONCLUSIONS

From the computer printout (Table I) note that the slope within each geometric situation is constant (the function changes uniformly), therefore, written in point slope form we have

$$\frac{F}{\ell} = mX + b$$

where

$$m = \text{SLOPE}$$

Substituting in equation (12) X = 0, yields $F/\ell = 0$, therefore b = 0, and the above equation becomes

$$\frac{F}{\ell} = mX$$

For each geometric configuration we need only compute one case. Then substituting in the above slope equation, additional values of F/ℓ for any given eccentricity can be computed.

The on-site machine shop practices enables them to maintain conductor straightness well within the 12 to 15 mil range. The smaller diameters, most critical situation, have larger wall thicknesses (a compressive pressure develops during pulsing) which makes it easier for the machine shop to maintain straightness. In addition, the inner conductor is assembled with a prescribed amount of tension in the axial direction. Small amounts of conductor eccentricity will not reduce the structural integrity of the neutrino horn.

ACKNOWLEDGEMENTS

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FOOT NOTES

1. Brown, George H., "Impedance Determination of Eccentric Lines," Electronics, Vol. 15, 1942, p. 49.
2. Smythe, William R., Static and Dynamic Electricity, McGraw-HIll Book Co., New York, Third Edition, 1968, pp. 76-78.
3. Ibid., Equation 5, p.78.

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