## The 1985 Horizontal Survey

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## Collider Accelerator Department

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TECHNICAL NOTE
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Part I. Monuments
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## 1. INTRODUCTION

During 1985 a partial survey of the radial position of the AGS magnets was done. The radial survey was interrupted to allow the crews to do a vertical realignment - there was not the time and manpower to do both before the AGS turn-on in the fall of 1985 . This report shows what has been accomplished so far:

The primary reference for the radial position is the set of 24 control stations, or monuments, placed evenly around the ring, about 110 inches outside the beam line at each 10 foot straight section. These monuments are on 20 foot steel pipes isolated from the floor; but they have not proved to be stable over the long term either horizontally or vertically (1). The magnet stands are on the "pile caps", which are supported by four 50 foot piles and are also isolated from the floor. The pile caps may be more stable than the primary monuments, but they under the AGS ring and are poorly accessible, and do not have any horizontal survey references on them. (The pile caps do, however, have a vertical reference on them.)

The magnets themselves have survey sockets on top, near each end above the pole face centerline of the magnet. These sockets are measured with respect to the primary monuments to give the radial and azimuthal position of the magnet.

A series of horizontal surveys was done during the construction and early operation of the AGS, starting in 1959. The monuments and magnets were remeasured, and the magnets realigned, in 1962. This present survey is apparently the first complete (assuming that it will be completed) radial survey since then.

## 2. SURVEY

The primary monuments are evenly spaced around the ring on a 5165.4 inch (430.45') radius circle. The monuments are identified with one or two letters telling their position in the ring. For example, LA is near magnets L-20 and $A-1$, and $A$ is near $A-10$ and $A-11$. One monument, $F G$, is obstructed by the SEB line, and a temporary monument, denoted $\mathrm{FG}^{\prime}$, is used instead in the traverse to determine the monument locations. The permanent monument $F G_{\text {, }}$ which is needed for getting the magnet positions, is determined by angle and length from monument $G$.

The top of each monument consists of a disk with a bushed hole which is used to locate the survey instruments. After the initial survey, in 1958, the original disks were replaced by disks with holes offeet to put them at the desired locations. Thus the monuments were all originally at their "ideal" positions on a regular 24-sided polygon (within survey errors), and any real differences from that now, or in the 1962 survey, must be due to monument motion.

For the initial surveys of the monuments, the magnets were not yet in place, and from each monument it was possible to sight to two adjacent monuments on each side. Thus there was a large degree of redundancy in the original measurements; with completely measured triangles, three angles and three sides, formed at each three consecutive monuments. Such redundancy is no longer possible, because with the magnets in place, only the immediately adjacent monuments are visible to each other. Thus the present monument survey consists only of the distances between adjacent monuments, and the angle at each vertex.

The angles were measured with a Wild $T 3$ transit, taking five sets of readings, reading three crosshairs in each set, giving 15 measurements which are averaged. The surveyors expect an accuracy of $.6-7$ seconds of arc; the manufacturer's literature claims a standard deviation of 0.5 second. The 15 independent measurements of each angle have an rms of typically about 6 sec; thus the average may be expected to have a standard deviation of $06 / \sqrt{15}$ or about. 15 seconds, but claiming such accuracy does not appear to be warranted. All angles were later reduced by 0.3 sec to correct for a miscentering of the instrument (the ball which locates the instrument in the hole in the monument was not centered on the axis of the instrument). After this correction, the sum of the 24 angles differed from 360 degrees by 3.1 seconds, which is consistent with an rms error of 3.1/ $24=0.63$ seconds in each.

Although the survey group was prepared to measure the distances between monuments with a laser interferometer, there was not sufficient time to do thisy so the distances were measured with an invar tape instead. One intermonument distance was measured with the interferometer, and this was used to calibrate the tape in the morning and afternoon of each of the four days for this job. These eight calibrations of the tape have an rms spread of .014", which is an order of magnitude larger than would be expected from the temperature variations. The average of these eight tape calibrations was then used to calculate the intermonument distances. Three of the distances ( $\mathrm{F}-\mathrm{FG}$ ), FG' $\mathrm{G}_{3}$ and $(\mathrm{EH}-\mathrm{H})$ were measured with a surveyors tape measure instead and are thus subject to larger errors.

In what follows, the random errors in angle and length are estimated to be sigma-angle $=0.6 \mathrm{sec}$ sigma-length $=0.014 \mathrm{inch}$
except for the distances measured with the tape measure, which are estimated to have errors twice the above.

## 3. ANALYSIS

The 24-sided figure determined by a traverse around the ring, measuring angles and lengths, will in general not close on itself, and the measured quantities must be adjusted slightly to give a closed figure. A least squares fit should give the best (i.e., most probable) result. If the deviations from a perfectly symmetrical figure are small, the computation of the fit can be done in a relatively simple way $(1,2)$, but in our case the symmetry is ruined by the use of the temporary monument instead of FG. A general least squares fit for a traverse like ours, which has a very low level of redundancy, is also quite simple, requiring only a three-by-three matrix equation. This is shown in Appendiu $A$, along with a comparison with other methods of achieving closure.

Using a least squares fit should give fitted values which are closer to the true values than were the original measuremente. However, the improvement
to be expected in a case like this, where the degree of overdetermination is small, is slight. With 48 measurements, we have for the expected value of the sum of errors squared (where the sum ranges over the 24 angles and 24 lengths)

$$
\left\langle\sum\left(x_{\text {true }}-x_{\text {measured }}\right)^{2} / \sigma^{2}\right\rangle=48
$$

where the true values, of course, are not known. After a fit with only three degrees of freedom, we have, for the sum of residuals squared,

$$
\left\langle\sum\left(x_{\text {fit }}-x_{\text {measured }}\right)^{2} / \sigma^{2}\right\rangle=3
$$

These fitted values still have errors with an expected value

$$
\left\langle\sum\left(x_{\text {true }}-x_{\text {fit }}\right)^{2} / \sigma^{2}\right\rangle=48-3=45
$$

which is not much better than the measured values. (This relationship is explained in Appendix A). The real virtue of the fitted values $i s$ that they describe a closed figure, without a discontinuity between the starting and ending points. As is shown in Appendix A, other methods of enforcing closure on the data give answers which are slightly "different" but probably not significantly "less correct".

## 4. FESULTS

After adjusting for angle closure, the monument data fails to close by 0.210 inches $(d x=-.20 \%, d y=.017)$. The least squares fit to close the figure gives a chisquare of 9.44 , compared to an expected value of 3 for the number of degrees of freedom here. The discrepancy may be due to bad luck ( $2 \%$ confidence level), an underestimate of the random errors, or mistakes (blunders) in the data. Mistakes, if any, are most likely in the length data, since the angles were measured multiple times and averaged.

Table 1 gives the data, and Table 2 thefitted coordinates of the monuments, making the calculation to use permanent monument FG. Also shown are deviations of the monuments from their ideal positions, in both $x-y$ and polar coordinates. The absolute coordinates of the monuments are not, of course, determined by this survey. The survey data only gives a 24 -sided polygon, which must. be oriented using some extra criteria. The coordinate system has been chosen here by translating and rotating the 24 -sided polygon, so that the average of the deviations from the ideal monument positions in $k, y$, and azimuth are zero. (Or, to visualize it another way, the centroid of the 24 points is put at $(0,0)$, and the figure rotated to make the points lie as close as possible to rays from the center at 15 degree multiples from "east"). The average radial deviation of the points reflects a change in the "radius" of the monument figure. It is, of course, real, and can not be made zero by any choice of coordinate system. Figure 1 shows these radial deviations. The average radial position is $0.113^{\prime \prime}$ inside the original positions on the ideal figure. This much deviation could be due to a systematic undermeasurement of the internonument lengths by

$$
.113^{\prime \prime} \times 2 \pi / 24=.030^{\prime \prime}
$$

Such a systematic effect, which would have to come from the tape calibration with the laser interferometer; is considered unlikely by the surveyors.

Also shown in Figure 1 are the radial deviations determined in the 1962 survey. The present deviations are larger by a factor of approximately four. Figure 2 shows the 1962 deviations magnified by a factor of four, compared with
the present. There is an indication that in the region of the ring away from the external lines, which have been extensively changed since 1962, the monument motion which occurred from 1960 to 1962 has continued and is now about four times as great. There are some caveats to this interpretation. Monuments FG, $G$, $G H, H$, and IJ were apparently given new offeet dieks after the 1962 survey so their motion started over from 'zero' again then. The monument at IJ showed a very large motion in the 1962 survey, presumably due to the removal of the adjacent wall for the construction of the conjunction area; it has behaved like its neighbors since then.

Figure 3 shows the $x$ and $y$ deviations for the present data, added to a figure from ref. 1 which gave results for the 1762 and older data. (Figure B12 shows the present deviations at a smaller scale and is perhaps less confusing). The present deviations are substantially larger than before. It should be remembered that the absolute origin is arbitrary for each survey. Thus the motion of a single monument (or one measuring mistake) will cause an apparent motion of all monuments.

Since the radial survey of the magnets depends on the monument survey, it is crucial that the monuments be correct. The present data give a disconcertingly high chisquare, suggestive of a possible blunder somewhere. But this survey, unlike the original surveys before the magnets were installed, has no constrained subsets of the data which can be independently checked to allow detection and isolation of a mistake. The weakest elements in the present survey are the lengths between monuments, which unlike the angles, were measured only once, and not with the most carefull techniques. (Although note that the three lengths measured with the tape measure are essentially perpendicular to the closure error and thus are probably not the culprits). Although a Monte Carlo analysis in Appendix B shows that, even with the relatively high random errors assumed here for the length measurementsy the angle errors are the dominant contributors to all but the very low harmonics, that does not mean we are tolerant of mistakes in the length measurements. When the remainder of the magnet offset measurements are made, it should be worthwhile to repeat the intermonument length measurements, with more care for accuracy, and measuring each more than once so mistakes can be detected.

Appendix A. Closure of survey data

## A1. CLOSUPE

Since three more numbers from outside the survey are needed to orient the ring - for example, the two coordinates of the first point and the bearing to the second - the redundancy in the 48 measurements is only three. or, as another way of looking at it, the shape of the 24 -gon can be determined by measuring 23 gides and the 22 angles beween them, leaving the last gide and two angles as redundant measurements. Ideally; this overdetermined set of measurements will be used to give a least squares fit for the positions of the monuments. This Appendik shows such a least squares fit, done in a way which does not require the handing of large matrix equations. Also shown is a comparison with other surveyors methods of closing the traverse. These methods have the virtue of being computationally simple enough to be done with pocket calculators, or, in years past, by hand.

Starting at point 1 and applying the measured distances and angles around the $N$-sided monument figure brings us back to point $\mathrm{N}+1$, which should be the same as point 1. The error in closure consiets of $d X$ and $d Y$, the coordinate errores, and d $\phi$, the difference between the sum of the turning angles and $360^{\circ}$. The measurements are adjusted to eliminate these closure errors using what are called "closure rules': In all cases, the angles are first adjusted to eliminate the angle error by subtracting $d \phi / \mathrm{N}$ from all measured angles; this is not actually necessary in the full least squares fit because the fit will take care of angle closure too. Then the $d X$ and $d y$ closure errors are eliminated by these rules:

1. Compass Rule(4). Each leg of the traverse consists of $a d x$ and a dy and has a length $1^{2}=d x^{2}+d y^{2}$. The sum of all lengths $i=L$. Each $d x$ is adjusted by $d x^{\prime}=d x-d X \cdot 1 / L$, and similarly for $d y$.
2. Transit Rule(4). Like Compass Fule, except the fractional dx correction for each leg, instead of being 1/L, is $\mid d x /$ sumldy, and similarly for dy.
3. Crandall Fule(4). This is a least squares adjustment, but only the lengthe and not the angles are adjusted, soit is valid only if the angle data are of much higher quality than the length data. This limited least squares fit is linear and leads to only two simultaneous equations, and is therefore easy to do.
4. Least Squares. (Derived below).

All these methods are relatively easy to implement on a spreadsheet, which is being used to calculate the monument and magnet positions from the survey data. All provide a geometrically correct closed figure. However, the transit rule is not rotationally invariant - its adjustments depend on the choice of the $x$ and $y$ directions - so it is not likely to be a good choice. Figure Al shows the radial deviations of the monuments from their ideal positions, using the four methods of closure. Figure $A 2$ shows the radial differences relative to that for the least squares fit. The origin in each of the four cases has been chosen independently to cancel the average $k$, $y$, and azimuth deviations from the ideal monument figure. Thus the apparent sine wave difference between the Crandall and least squares solutions, which looks like it could be due to a simple displacement of one system, is not - there are azimuth shifts causing it. The difference in radii between the various closure rules is less than
V.025 inchy which is significant, but much less than the radial deviations themselves.

## Az. LEAST GLARES FIT

Figure $A 弓$ shows a simplified traverse where N=4. The lengths if and turning angles $\phi_{i}$ are measured with estimated errors $s_{i}$ and $e_{i}$ respectively. The direction of each leg is given by

$$
\begin{equation*}
\theta_{i}=\theta_{0}+\sum_{j=1}^{i} \phi_{j} \tag{Al}
\end{equation*}
$$

where $\theta_{0}$ is an arbitrary direction, chosen (with $x_{1}$ and $y_{1}$ ) to orient the figure as desired.

The closure errors are

$$
\begin{align*}
& X \equiv x_{N+1}-x_{1}  \tag{ARa}\\
& Y \equiv Y_{N+1}-Y_{1}  \tag{ARb}\\
& T \equiv 2 \pi-\sum_{1}^{N} \phi_{i} \tag{ARC}
\end{align*}
$$

We want an adjusted set of lengths and angles, $l_{i}^{\prime}$ and $\phi_{i}^{\prime}$, that eliminate the closure errors: Denote the length and angle residuals by a and a:

$$
\begin{align*}
& d_{i}=\ell_{i}^{\prime}-l_{i}  \tag{ASa}\\
& a_{i}=\phi_{i}^{!}-\phi_{i} \tag{ABb}
\end{align*}
$$

The conditions on the $d^{\prime} \leq$ and $a^{\prime} \equiv$ to close the figure are nonlinear but if they are small we can write a linear approximation. It is clear from Figure A3, by considering the effect of $d i$ or $a i$ on the point N+1, that closure requires

$$
\begin{align*}
& X=\sum_{i}\left(d_{i} \cos \theta_{i}+a_{i} y_{i}\right)  \tag{A4a}\\
& Y=\sum_{i}\left(d_{i} \sin \theta_{i}-a_{i} x_{i}\right)  \tag{ALb}\\
& T=\sum_{i} a_{i} \tag{AMD}
\end{align*}
$$

The least squares solution requires minimizing the "chisquare";

$$
\begin{equation*}
\psi=\sum\left(\frac{d_{i}}{s_{i}}\right)^{2}+\sum\left(\frac{a_{i}}{e_{i}}\right)^{2} \tag{AS}
\end{equation*}
$$

subject to the constraints A4. This can be done by using Lagrange multipliers,
minimizing a modified chisquare,

$$
\begin{align*}
\psi^{\prime} & =\psi+A\left[X-\sum\left(d_{i} \cos \theta_{i}+a_{i} y_{i}\right)\right] \\
& +B\left[Y-\sum\left(d_{i} \sin \theta_{i}-a_{i} x_{i}\right)\right]+C\left[T-\sum a_{i}\right] \tag{Ab}
\end{align*}
$$

where $A, B$, and $C$ are the Lagrange multipliers: This is minimized by requiring

$$
\frac{d \psi^{\prime}}{d x}=0
$$

for $x=d^{3} E_{3} a^{3} E_{3} A_{3} B_{3}$ and $C$. Taking the derivative with respect to the Lagrange multipliers just recovers the constraint equations A4. Taking the derivatives with respect to the residuals gives the conditions

$$
\begin{align*}
& \frac{d \psi^{\prime}}{d d_{i}}=2 \frac{d_{i}}{s_{i}^{2}}-2 A \cos \theta_{i}-2 B \sin \theta_{i}=0  \tag{A7a}\\
& \frac{d \psi^{\prime}}{d a_{i}}=2 \frac{a_{i}}{e_{i}^{2}}-2 A y_{i}+2 B x_{i}-2 C=0 \tag{AFb}
\end{align*}
$$

Solving for $d$ and a gives

$$
\begin{align*}
& d_{i}=s_{i}^{2}\left(A \cos \theta_{i}+B \sin \theta_{i}\right)  \tag{ABa}\\
& a_{i}=e_{i}^{2}\left(A y_{i}-B x_{i}+C\right) \tag{ABb}
\end{align*}
$$

Substitute these expressions into the constraint equations A4:

$$
\begin{align*}
X & =\sum s_{i}^{2}\left(A \cos \theta_{i}+B \sin \theta_{i}\right) \cos \theta_{i} \\
& +\sum e_{i}^{2}\left(A y_{i}-B x_{i}+C\right) y_{i}  \tag{Aaa}\\
Y & =\sum s_{i}^{2}\left(A \cos \theta_{i}+B \sin \theta_{i}\right) \sin \theta_{i} \\
& -\sum e_{i}^{2}\left(A y_{i}-B x_{i}+C\right) x_{i} \\
T & =\sum e_{i}^{2}\left(A y_{i}-B x_{i}+C\right)
\end{align*}
$$

(AFb)
(AFC)

Regroup to get three simultaneous equations to solve for $A, E_{\text {, }}$ and $\mathrm{C}:$

$$
\begin{align*}
& A[c c+y y]+B[s c-x y]+C[y]=X  \tag{A10a}\\
& A[s c-x y]+B[s s+x x]+C[-x]=Y \\
& A[y]+B[-x]+C[1]=T
\end{align*}
$$

(A10b)
(A10C)
where the coefficients in brackets [] are shorthand for the sums

$$
\begin{aligned}
& {[y y]=\sum e_{i}^{2} y_{i}^{2}} \\
& {[y]=\sum e_{i}^{2} y_{i}} \\
& {[s c]=\sum s_{i}^{2} \sin \theta_{i} \cos \theta_{i}} \\
& {[1]=\sum e_{i}^{2}} \\
& \text { etc. }
\end{aligned}
$$

The values of $A, B$ and $C$ determined from $A 10$ are then used in $A B$ (and AB) to give the adjusted lengths and angles.

The covariance matrix for $A, B$, and $C$ is the inverse of the "matrix" in A10. The errors and correlations on the monument coordinates could be obtained by propagating the errors on $A, B$, and $C$ through $A B$ and $A B$ to the adjusted lengths and angles, and then through the geometry equations to the coordinates. Instead, results from a Monte Carlo simulation will be presented in Appendix B.

How close is this adjusted set of values to the (unknown true values? If the estimates of the random errors are correct, then fusing $x$ and $s$ to mean the value and error for both lengths and angles

$$
\begin{align*}
& \left\langle D_{\text {ma }}\right\rangle=\left\langle\sum\left(x_{\text {weosured }}-x_{\text {djojucted }}\right)^{2} / s^{2}\right\rangle=3  \tag{A11a}\\
& \left\langle D_{m-t}\right\rangle=\left\langle\sum\left(x_{\text {measured }}-x_{\text {true }}\right)^{2} / s^{2}\right\rangle=48 \tag{A11b}
\end{align*}
$$

where the values 3 and 49 are the number of constraints and the number of variables, respectively: A measure of correctness of the adjusted values is

A little algebra shows that

$$
\begin{align*}
& D_{a-t}=D_{m-t}-D_{m-a} \\
& +\sum\left(x_{\text {meas }}-x_{a d_{j}}\right)\left(x_{a d_{j}}-x_{\text {true }}\right) / s^{2} \tag{AlB}
\end{align*}
$$

As earlier, use d and a for length and angle differences (adjusted - measured), and use d' and a' for the differences (adjusted - true). Then, using equation AB for $d$ and $a$, the last term may be rewritten

$$
\begin{align*}
& \sum d_{i} d_{i}^{2} / s_{i}^{2}+\sum a_{i} a_{i}^{\prime} / e_{i}^{2} \\
& =\sum\left(A \cos \theta_{i}+B \sin \theta_{i}\right) d_{i}^{\prime}+\sum\left(A y_{i}-B x_{i}+C\right) a_{i}^{\prime} \\
& =A \sum\left(d_{i}^{\prime} \cos \theta_{i}+Q_{i}^{\prime} y_{i}\right) \\
& +B \sum\left(d_{i}^{\prime} \sin \theta_{i}-Q_{i}^{\prime} x_{i}\right)  \tag{A14}\\
& +C \sum a_{i}^{\prime}
\end{align*}
$$

Since both the adjusted and the true values satisfy the constraints, the $A, B$, and C terms vanish, by equation A 4 , and thus

$$
\begin{equation*}
D_{0 .-t}=D_{m-t}-D_{m-a} \tag{AIS}
\end{equation*}
$$

Note that Ais is true not only for expected values, but for actual values in each individual case.

Appendix b. MONTE CARLO OF SURVEY TECHNIGUE
B1. MONTE CARLO
Simulated sets of survey data were created by adding random gaussian (normal) deviates to the lengths and angles of the ideal monument figure (1348.44 in. and 15 degrees). The gaussians were generated from the formula

$$
\begin{align*}
& g_{i}=\sqrt{-2 \ln r_{i}} \cdot \cos \left(2 \pi r_{i+1}\right)  \tag{B1a}\\
& g_{i+1}=\sqrt{-2 \ln r_{i}} \cdot \sin \left(2 \pi r_{i+1}\right) \tag{E1b}
\end{align*}
$$

where $r$ is a random number with a uniform distribution from 0 to 1 , and $g$ is a random number with a gaussian distribution with mean=0 and sigma=1. (This formula is in common use but its derivation is not obvious; perhaps a note should be published showing why it works). Note that two independent gaussians are generated from each pair of uniform random numbers.

This simulated data was then "closed" by the same methods as the real data. The least squares fit on the average gave the answer closest to the true values, although occassionally one of the other methods was better. Figures B1 - BS show the distributions of the weighted sum of squares for each of the three differences between the true, measured, and adjusted values. These should be chi-square distributions with 3 (for measured-adjusted), 48 (measured-true), and 45 (adjusted-true) degrees of freedom.

All Monte Carlo results here use the ideal monument figure (i.e., equal sides and 15 degree angles) as the starting point.

## B2. FOURTEF ANALYSIS

Since the magnets are surveyed relative to the monuments, the errors in the monuments will propagate to the magnets and thus affect the beam. Thus it is interesting to see the harmonic content of the survey errors. For each of the 250 Monte Carlo runs, the radial deviations were Fourier analysed, with the average amplitudes of the components shown in Figure B4. The first harmonic corresponds only to a uniform shift of all monuments and will have no effect on the machine; it is nonzero here because the center was not redefined for each of the Monte Carlo runs. The distribution of amplitudes for the 9th harmonic is shown in Figure B5.

Separate Monte Carlos were run with only length errors, or only angle errors, with the results included in the Figure E4. With the sigmas used here, the length and angle error have comparable contributions to the lowest harmonics, but the relative effect of the length errors decreases at higher harmonics. For the 9th harmonic the average amplitudes are

$$
\begin{array}{lll}
\text { sig-len } & \text { sig-ang } & \text { <amp.-9th> } \\
0.014 \text { in } & 0.6 \text { sec } & 0.000498 \text { in } \\
0.014 & \text { none } & 0.000218 \\
\text { none } & 0.6 & 0.000463
\end{array}
$$

Note that these results were obtained using estimated length and angle errors which gave, on the actual survey, a large chi-square. If the large chi-square is due to an underestimate of the errors (and not a mistake), then the estimated errors should be increased by a factor of $9.4 / 3=1.8$, and the Monte Carlo estimates will all increase by the same factor.

## bS. ERrors in coordinates

The position of each monument is determined by the survey to within some error ellipse. By symmetry (ignoring the complication due to the use of the temporary monument in the real world), the error ellipses should have their axes along the radial and azimuthal directions, and should be the same for each monument. Of more interest are the errors of one monument relative to another, taking correlations into account. These can be determined from the Monte Carlo data, but there are many possible ways to do it, depending on just which correlations are chosen to be removed and which ones to stay. We want to see the effect of "bumpiness" in the monument figure, but not the effect of things like translations or rotations of the whole figure, which do not affect the physics of the AGS. The results here are from one choice, which is a compromise between trying to understand what is physically significant, and what was computationally feasible.

Figures $B 6$ - $B 8$ show the radial and azimuthal errors of each point, relative to monument LA. For each of the three figures, 250 data sets were generated with both length and angle errors, length errors only, or angle Errors only, Each of the data sets was closed, translated, and rotated, as described for the actual survey data, to give a set of monument coordinates. What is plotted is the rms errors, relative to the first monument, projected into the radial and azimuthal directions:

$$
\begin{align*}
& d x_{i}=\left(x_{i}-x_{i}^{0}\right)-\left(x_{1}-x_{i}^{0}\right)  \tag{BRa}\\
& d y_{i}=\left(y_{i}-y_{i}^{0}\right)-\left(y_{1}-y_{i}^{0}\right)  \tag{BIb}\\
& d r_{i}=d x_{i} \cos \theta_{i}+d y_{i} \sin \theta_{i}  \tag{BSa}\\
& d Q_{i}=d y_{i} \cos \theta_{i}-d x_{i} \sin \theta_{i}  \tag{BS}\\
& d r_{i}(r m s)=\sqrt{\sum d r_{i}^{2} / N}  \tag{BAa}\\
& d Q_{i}(r m s)=\sqrt{\sum d a_{i}^{2} / N} \tag{GAb}
\end{align*}
$$

where ( $x_{i}^{0}, y_{i}^{0}$ ) is the ideal location of monument $i$, and $\theta_{i}$ is the azimuthal position of monument $i$ (multiples of 15 degrees).

The error ellipses for these relative errors do not necessarily have their axes along the radial and azimuthal directions. The correlations which give the axis directions could be calculated from the Monte Carlo data if needed, but it is not done here.

The affect of the variation in average radius has not beer removed from the figures, although perhaps it should be, because a change in average radius will not cause orbit distortions. The amount of average radius change, dR, is shown below. Also shown, in the last two columns, are the rms radial and azimuthal errors, relative to the center of the figure (not relative to
monument L.A) .

| sig-len | sig-ang | df (rms) | dr (rins) | da (rms) |
| :--- | :--- | :--- | :--- | :--- |
| 0.014 in | 0.6 sec | 0.0115 | 0.0154 | 0.0158 |
| 0.014 | none | 0.0115 | 0.0134 | 0.0150 |
| none | 0.6 | 0 | 0.0074 | 0.0049 |

## B4. EFFECT OF SINGLE MEASUREMENT ERRORS

It is not intuitively obvious how a single measuring error - either a statistical error or a blunder - propagates through the closure calculations and affects the monuments. Figures Bq and B 10 show the monument displacements caused by a single length error (.014 inch) or angle error (0.6 sec), respectively. Figure Ell shows the fourier components caused by this distortion. Figure B12 shows the deviations of the real monuments from their ideal locations (with a very different magnification) in case anyone would like to guess where there might be blunders in the survey data.

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## Figure captions

1. Radial deviations, present survey and 1962 survey.
2. Present radial deviations, compared with 1962 deviations multiplied by 4.
3. $X$ and $y$ deviations of present survey and old surveys. The present points are plotted using $d x$ and $d y$, and because of the size of the deviations, and the curvature of the polar coordinate system, do not accurately represent dr and da. See Figure B12 for another picture.

A1. Radial deviations for various closure methods, relative to ideal positions.

A2. Radial deviations for various closure methods, relative to least squares fit.

A3. Simple traverse, showing geometry for least squares fit.
E1. Distribution of weighted sum-of-squares of "measured" minus "adjusted" values (i.e., "chi-square") for 250 Monte Carlo runs. These plots use bar charts to simulate histograms: the number ticks should be interpreted as the left edge of the bin.

E2. Distribution of weighted sum-af-squares of "measured" minus "true" values.
E3. Distribution of weighted sum-of-squares of "adjusted" minus "true" values.
B4. Fourier amplitudes of Monte Carlo data. (The 12th harmonic should be half that shown).

E5. Distribution of amplitudes of 9th harmonic.
B6. Radial and azimuthal errors (rms) relative to monument LA, from Monte Carlo data with length and angle errors.

B7. Radial and azimuthal errors (rms) relative to monument LA, from Monte Carlo data with length errors only.

Bg. Radial and azimuthal errors (rms) relative to monument LA; from Monte Carlo data with angle errors only.

B9. Monument displacements for a single . 014 inch length measurement error.
B10. Monument displacements for a single 0.6 second angle measurement error.
B11. Harmonics generated by the displacements of Figures B 9 and B10.
B12. Displacements of "actual" monuments from ideal locations. Note that the scale of the displacements is much different from Figures 89 and B10.

TABLE 1. Monument survey data with least-squares fit for coordinates. The data and coordinates in this table use 'temporary' monument $\mathrm{fg}^{\prime}$ '.

|  |  |  |  | Measured |  | Adjusted |  |  | Residuals |  | Coordinates |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mron | , | I de |  | sec | angle | length | angle | length | - angle(sec) | lengt | \| X | $Y$ |
| 11 | la | \| 15 | 0 |  | 15.002417 |  |  | 1348 | 0.193 | -0.000 | -4989.331 | 1336 |
| ; 2 | a | \| 14 | 59 | 49.6 | 99 | 348.370 | 14.997 | 348 | 0.100 | 0.0017 | -5165.298 | -0.012 |
| 13 | ab | \| 15 | 0 |  | 5.001472 | 348.4 | 15.001474 | 1348.404 | 0.009 | 0.0036 | -4989.33 | 1336.8 |
| 14 | $b$ | \| 15 | 0 |  | . 00108 | 8. | 15.001062 | 1348.427 | -0.074 | 0.0052 | -4473.3 | 2582 |
| 15 | be | : 14 | 59 | 57.6 | 14.999889 | 48.42 | 14.999848 | 1348.430 | -0.144 | 0.006 | $-3652.423$ | 3652.378 |
| 16 | c | 1 14 | 59 | 56.3 | 14.998972 | 1348.378 | 14,998918 | 1348.385 | -0.194 | 0.0074 | -2582.62 | 4473 |
| 17 | cd | 114 | 59 | 49. | 14.997111 | 1348.401 | 14.9970 | 1348.408 | -0.223 | 0.0078 | -1336.880 | 496 |
| 18 | $d$ | 115 | 0 | 10.5 | 15.002917 | 1348.399 | 15.002853 | 1348.406 | -0.227 | 0.0076 | -0.016 | -5165.308 |
| 9 | de | \| 14 | 59 | 53.7 | 14.998250 | 1348.345 | 14.988 | 1348.352 | -0.207 | 0.0069 | 1336. 05 | 981 |
| 110 | e | ! 15 | 0 | 19.3 | 15.00536 | 48.458 | 15.005 | 1348.463 | -0.164 | 0.0058 | 2582.587 | 4473. |
| ! 11 | ef | : 14 | 59 | 41.4 | 14.994833 | 1348.452 | 14.994805 | 1348.455 | -0.101 | 0.0042 | : 3652.347 | -3652. 402 |
| 1 12 | f | 1 12 | 11 | 14.3 | 12.187306 | 1036.920 | 12.1872 | 1036.930 | -0.022 | 0.01 | 4473.2 | 258 |
| : 13 | $\mathrm{fg}^{\prime}$ | 1 16 | 42 | 57.7 | 16.716028 | 1664.079 | 16.716040 | 1664.081 | 0.045 | 0.0021 | 4916.647 | 1645.26 |
| 114 | g | :16 | 5 | 56.8 | 16.097111 | 1348.392 | 16.099155 | 1348.390 | 0.160 | -0.0017 | 5165.442 | 0.1 |
| 15 | gh | 115 | 0 | 39.2 | 15.010889 | 1348.317 | 15.010958 | 1348.302 | 0.252 | -0.0143 | \| 4989.426 | 1336.9 |
| 116 | h | 114 | 58 | 54.4 | 14.981778 | 1348.444 | 14.981 | 1348.438 | 0.335 | -0.0052 | - 4473.199 | 2582.5 |
| 1 17 | hi | 1 15 | 0 | 21.9 | 15.006083 | 1348.438 | 15.0061 | 1348.431 | - 0.404 | -0.0066 | : 3652.443 | 3652.40 |
| 118 | i | : 15 | 0 |  | 15.001111 | 1348.406 | 15.001237 | 1348.398 : | 0.454 | -0.0074 | : 2582.664 | 4473 |
| 19 | $\mathrm{i}_{\mathrm{j}}$ | : 15 | 0 |  | 15.001250 | 1348.422 | : 15.001384 | 1348.414 | - 0.483 | -0.0078 | 1336.898 | 4989.27 |
| 120 | j | 114 | 59 | 46.1 | 14.996139 | 1348.383 | 14.99627 | 1346.375 | 0.487 | -0.0076 | 0.012 | 5165 |
| : 21 | jk | 115 | 0 | 13.6 | 15.003776 | 1348.450 | 15.003907 | 1348.442 : | ; 0.467 | -0.0069 | -1336.831 | 4989.2 |
| 122 | k | : 14 | 59 | 42.6 | 14.995167 | 1348.457 | : 14.975284 | 1348.450 : | : 0.424 | -0.005日 | :-2582.607 | 4473.1 |
| 123 | kl | 115 | 0 |  | 15,000806 | 1348.382 | : 15.000905 | 1348.377 | - 0.361 | -0.0042 | : -3652.437 | 3652 |
| 124 | 1 | 115 | 0 |  | 15.000278 | 1348.400 | 15.000356 | 1348.397 | - 0.283 | -0.0024 | : -4473.302 | 2582. |
| (1) | (la) |  |  |  |  |  | \| |  | - |  | 1-4989,331 | 1336 |


| Closure errors: Angle: | Raw | Fitted |  | Sum of squares: |
| :---: | :---: | :---: | :---: | :---: |
|  | -3.1 | $0.0 E+00$ | seconds | 1.9E+00 1.0E-03 |
|  | -0.209 | -1.1E-07 | inchas | Heighted: |
|  | 0.017 | -1.4E-06 | inches | 5.4008274 .039403 |
|  |  |  |  | Total Chisquare: |
| Data to locate permanent monument fq: |  |  |  | 9.440231 |
| Distance fro fg to g: | 1348.392 |  |  |  |
| Angle fog-ght (turning angle) | 15.008611 |  |  |  |
|  |  |  |  |  |

TABLE 2. Monument coordinates ( $X-Y$ and polar) and deviations fron ideal coordinates. This tahle uses permanent monument fg.

| $\begin{aligned} & \text { i } \\ & \text { imonument } \end{aligned}$ | Coordinates |  |  | Coord. | A Deviations ( $x-y$ ) |  | Dev. (polar) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ) | Y | radius | azinuth | dx | dy | dr | da |
| la 1 | -4989.331 | 1336.864 | 5165.330 | 165.000246 | 0.062 | -0.040 | -0.070 | 0.022 |
| 2 a 1 | : -5165.298 | -0.012 : | : 5165.298 | -179.999863 | : 0.102 | -0.012 | -0.102 | 0.012 |
| 3 ab | : -4989.331 | -1336.853 | : 5165.327 | 165.000370 | 0.062 | 0.051 ; | -0.073 | -0.033 |
| I | \| -4473.316 | -2582.615 | 5165.31 | 150.000531 | : 0.052 | 0.085 | -0.087 | -0.048 |
| br 1 | : -3652.423 | -3652.378: | : 5165.274 | 135.000354 | : 0.067 | 0.1121 | -0.126 | -0.032 |
| 6 c | : -2582. 625 | -4473. 230 : | : 5165.244 | -120.000045 | : 0.075 | 0.1371 | -0.156 | -0.004 |
| 7 cd | : -1336.880 | -4989.236 | 5165.242 | -105.000200 | : 0.024 | 0.158 | -0.158 | -0.018 |
| 18 d | - -0.016 | -5165.308 | : 5165.308 | -90.000181 | : -0.016 | 0.092 | -0.092 | -0.016 |
| 9 de | 1 1336.855 | -4989,308 | : 5165.305 | -75.000279 | -0.049 | 0.085 | -0.095 | -0.025 |
| : 10 | - 2582.567 | -4473. 358 | 5165.335 | -60.001029 | -0.113 | 0.010 | -0.065 | -0.093 |
| [11 ef \| | 3652.347 | -3652.402 | : 5165.238 | -45.000429 | $:-0.142$ | 0.0871 | -0.162 | -0.039 |
| ¢ 12 f ; | 4473.269 | $-2582.626$ | 5165.277 | -29,999836 | : -0.099 | 0.074 | -0.123 | 0.015 |
| 113 fg | 4999.257 | -1336.722 | 5165.221 | -14.998439 | : -0.137 | 0.182 | -0.179 | 0. |
| 114 g | 5165.442 | 0.1101 | : 5165.442 | 0.001225 | : 0.042 | 0.1101 | 0.042 | 0.110 |
| 1 15 gh : | 4999.426 | 1336.963 | : 5165.447 | 15.000542 | : 0.032 | 0.059 : | 0.047 | 0.04 |
| 116 h | 4473.199 | 2582.528 : | -5165.168 | 29.999277 | : -0.168 | -0.172 | -0.232 | -0.06 |
| 117 hi | 3652.443 | 3652.409 | - 5165.310 | 44.999729 | : -0.046 | -0.081 | -0.090 | -0.02 |
| 1建 i | 2582.664 | 4473.287 | : 5165.312 | 59.999895 | : -0.036 | -0.081 : | -0.088 | -0.00 |
| [19 ij | 1336.898 | 4989.276 : | : 5165.285 | 74.999727 | : -0.006 | -0.117 | -0.115 | -0.025 |
| 120 j i | 0.012 | 5165.225 | : 5165.225 | 89.999863 | : 0.012 | -0.175 | -0.175 | -0.012 |
| 121 jk : | 1-1336.831 | 4989.259 | : 5165.252 | 104.999608 | : 0.072 | -0.134 | -0.148 | -0.035 |
| 122 k \| | - 2582.607 | 4473.178 | : 5165.190 | 120.000163 | : 0.093 | -0.190 | -0.210 | 0.015 |
| 123 kl \| | 1-3652.437 | 3652.334 | : 5165.253 | 135.000807 | : 0.052 | -0.155 | -0.147 | 0.073 |
| 12411 | 1-4473.302 | 2582.613 | : 5165.299 | 150.000474 | : 0.066 | -0.087 | -0.101 | 0. |
| (1) (1a) | 1-4989.331 | 1336.864 : | - 5165.330 | 165.000246 |  | ; |  |  |
| laverages : | 10.000 | -0.000 | 1 5165.287 |  | 0.000 | -0.000 | -0.113 | 0.000 |

Figure 1
Monument Radial Position




Figure A1
RADIAL DEVIATIONS


Figure az RADIAL DEVIATIONS


Figure A3.

gure B1

> Sum-of-Squares Distribution
> measured - adjusted (chisquare)



0.025119
0.015849
0.010000
0.006310
0.003981
0.002512
0.002512
0.001585
0.001000
0.000631
0.000398
0.000251
0.000158
0.000100
length and angle

ıequnu
Figure B6

(u!) dodie sun

Figure B7
Monte Carlo Errors Relative to Monument LA


(u!) 101.10 sun


(seyou!) epnzllduv



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