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SIX DIMENSIONAL TRACKING SIMULATION FOR H- INJECTION

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OCTOBER 25, 1989

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Technical Note

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SIX DIMENSIONAL TRACKING SIMULATION for H^- INJECTION IN AGS BOOSTER

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October 25, 1989

Abstract

The effect of Coulomb multipole scattering on the beam emittance is studied with the 6 dimensional injection program ARCHSIM by A. Thiessen.[1] Since the energy loss due to Bhabha scattering is small, the six dimensional simulation can be approximated into transverse 4 dimensional and longitudinal 2 dimensional tracking. The emittance growth agrees well with the multiple scattering theory. The beam loss occurs mainly in the adiabatic capture process in the longitudinal phase space.

I. Introduction

During the H^- injection process, the incident beam passes through the foil many times. The multiple Coulomb scattering between the incident particle and the stripper gives rise to emittance growth or beam loss. To control the emittance growth, one may need orbit bump manipulation in minimizing the effect of multiple scattering.

Besides the multiple Coulomb scattering, the particle in the low energy synchrotron would experience strong space charge force. Tracking with space charge has been extensively studied previously.[2,3] We thus shall not repeat the calculation. This report concentrates on the emittance growth due to the multiple Coulomb scattering.

II. Basic Formula

The Coulomb scattering differential cross-section for p or a target Z_T is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Z_T e^2}{pv}\right)^2 \frac{1}{(\theta^2 + \theta^2_{min})^2} \tag{1}$$

for $\theta < \theta_{max}$, where

$$\theta_{min} = \frac{\hbar}{pa} \simeq \frac{Z_T^{1/3} m_e c}{192p}$$

$$\theta_{max} = \frac{\hbar}{pR} = \frac{275 m_e c}{A^{1/3}\ p}$$

Thus, the total scattering cross-section is

$$\sigma_T = \int \frac{d\sigma}{d\Omega}\ d\Omega = \pi \left(\frac{2Z_T e^2}{pv}\right)^2 \frac{1}{\theta^2_{min}} \tag{2}$$

For 200 MeV proton on the carbon foil we found that

$\theta_{min} \simeq 7.5 \times 10^{-6}$ rad
$\sigma_T \simeq 1.26 \times 10^{6}$ barns
$\theta_{max} \simeq 9.54 \times 10^{-2}$ rad

The mean square angle for the Coulomb scattering becomes

$$<\theta^2> = 4\theta^2_{min}\ \ell n\ (210\ Z^{-1/3}) \tag{3}$$

for a stripper of thickness t, the number of collision per transit is given by

$$n = \sigma_T N_A\ \frac{\rho t}{A} \tag{4}$$

where $N_A = 6.02 \times 10^{23}$ atoms/mole, ρ and A are the density and atomic weight respectively. In the following, we assume $\rho t = 100\ \mu g/cm^2$ carbon foil. We obtain n = 6.3 collisions/transits.

Since the multiple scattering is a random process, the final distribution will be a Gaussian with the angular divergence proportional to number of collisions as

$$< \Phi^2 > = N \quad n.<\theta^2>, \tag{5}$$

where N is the number of turns that particles passes through the foil. Since the emittance is given by

$$\epsilon = \frac{x^2 + (\alpha x + \beta x')^2}{\beta} \tag{6}$$

The emittance growth becomes (see appendix)

$$\Delta\epsilon = 2\beta\ x < \Phi^2> = 2\beta\ n <\theta^2> N \tag{7}$$

Thus, the emittance growth is proportional to N and the β- amplitude at the stripper location. For the AGS Booster, we expect $\Delta\epsilon_y \simeq 0.1\ N\ \pi$mm mrad.

III. Numerical Simulation

A six dimensional simulation program was written by Thiessen et al.[1] to simulate the multiturn injection. The process includes energy loss mechanism due to Bhabha scattering and the Coulomb multiple scattering. The machine parameter at the stripper location is:

βx = 9.8939 m
αx = 1.4535
βy = 5.4126 m
αy = 0.9565
x_p = 2.5147 m
x_p' = 0.3903
Q_x = 4.82
Q_y = 4.83
γ_T = 4.8812

The injection beam has emittance $\epsilon = 6\pi$ mm mrad from the 200 MeV LINAC. The pulse length is about 100 μs. Thus, the number of turns in the injection process is about 80. In our simulation, we choose 2000 particles divided into N turns injections.

Figure 1 shows the final beam emittance vs N, the number of turns in the injection process. The upper curve shows the final emittance vs N in multiple Coulomb scattering. The emittance increases very fast in the first few turns of the injection process. This is due mainly to the broad distribution of the Coulomb scattering in Eq.(1). After N=5, or Nn$\cong$ 30 interactions, the emittance grows linearly with N. Figure 1 indicates that $\Delta\epsilon \simeq 0.1\ N\ \pi$ mm mrad. This is in reasonable agreement with the estimate of Eq. (7).

The lower part of Fig. 1 shows the time saving feature of the ARCHSIM program, where the θ_{min} is multiplied by a bugger factor $f_B = 2.5$. This bugger factor decreases the total cross-section σ_T by 6.25 times. Since

$$n \propto \sigma_T \propto \theta_{min}^{-2}\ f_B^{-2}$$

$$<\theta^2> \propto \theta_{min}^2\ f_B^2,$$

Equation (7) should be independent of the bugger factor f_B. The resulting emittance growth in the lower curve of Figure 1 shows clearly similar behaviour as that of $f_B = 1$ in the upper curve, $\Delta\epsilon \simeq 0.07N\ \pi$mmmrad.

Because of the small number of collisions, the distribution function $\rho(x)$ shown in Figure 2 is far from Gaussian. The tail part is larger than the Gaussian. The total emittance of the beam shown in Fig. 1 (irrespective the distribution) is, however, small in comparison with the available dynamical aperture.

The beam loss of about 5% arises mainly from the adiabatic capture in the longitudinal motion. To eliminate the beam loss, one should chop and paint the RF bucket properly.[4-7]

IV. Conclusion

The numerical simulation indicates that the emittance grows very fast in the first few turns around the accelerator. After 10 passages through the foil, the emittance growth becomes linear and follows well with the multiple scattering theory. The energy loss due to Bhabha scattering is small at 200 MeV injection.

The total injection efficiency is about 95% in the present $\dot{B} = 0$ scenario. To eliminate beam loss, phase space painting with chopped beam is necessary.

In the present study, we do not change the orbit bump to avoid multiple Coulomb scattering. The orbit bump program can be used to paint the transverse phase space to minimize the effect of space charge. Since the emittance growth is small in the multiple scattering, the most important issue rests on the space charge effect, which has been extensively studied in the previous tracking calculations.[2,3]

References

1. H. A. Thiessen, AHF Tech. Note 89-001, April 20, 1989.
2. G. Parzen, AGS Booster Tech. Note #108, "Space Charge Effects in the AGS Booster", 2/1/88.
3. J. Wei, S. Y. Lee and A. G. Ruggiero, AGS Booster Tech. Note # 115, "R.F. Capture of the AGS Booster", 4/8/88.
4. E. Gianfelice, H. Schoenauer, EHF-87-36.
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<u>Appendix</u> Emittance Growth in the Multiple Coulomb Scattering

Let us assume that the beam profile can be represented by a Gaussian distribution, i.e.

$$\rho\ (x,x') = \frac{1}{\pi\epsilon_0}\ e^{-\frac{x^2+\ (\alpha x+\beta x')^2}{\beta\epsilon_0}} \tag{A/1}$$

where ϵ_0 is the emittance of the beam.

The passage of the beam through the foil can be represented by the kick angle θ of a Coulomb scattering, i.e.,

$$x' \rightarrow x' + \theta \tag{A.2}$$

with a Gaussean distribution function,

$$f\ (\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta}\ e^{-\theta^2/2\sigma_\theta^2} \tag{A.3}$$

The final distribution function after a passage through the foil becomes

$$\tilde{p}\ (x,x') = \frac{1}{\pi\epsilon_0}\ \frac{1}{\sqrt{2\pi}\sigma_\theta} \int_{-\infty}^{\infty} e^{-\frac{x^2+\ (\alpha x+\beta x'+\beta\theta)^2}{\beta\epsilon_0}}\ e^{-\frac{\theta^2}{2\sigma_\theta^2}}\, d\theta$$

$$= \frac{1}{\pi\epsilon_0}\ \frac{\tilde{\sigma}_\theta}{\sigma_\theta}\ e^{-\frac{x^2+\ (\alpha x+\beta x')\left(1-\frac{2\tilde{\sigma}_\theta^2 f}{\epsilon_0}\right)}{\beta\epsilon_0}} \tag{A.4}$$

where

$$\tilde{\sigma}_\theta^{-2} = \sigma_\theta^{-2} + 2\ \frac{\beta}{\epsilon_0} \simeq \sigma_\theta^{-2} \tag{A.5}$$

Note that the distribution (A.4) does not change the x distribution due to the multiple kick, i.e., the multiple kicks change only the x′ distribution. The Ellipse in the exponent of Eq. (A.4) is not a constant of motion but encompassed by an ellipse with emittance

$$\epsilon = \epsilon_0\ /\ \left(1-2\ \frac{\tilde{\sigma}_\theta^2\beta}{\epsilon_0}\right) \simeq \epsilon_0 + 2\ \tilde{\sigma}_\theta^2\beta \tag{A.6}$$

We thus obtain

$$\Delta\epsilon/\Delta N = 2\ \beta\ \tilde{\sigma}_\theta^2 \cong 2\ \beta\ n\ \langle\theta^2\rangle \tag{A.7}$$

Fig.1 Emittance Growth as a Function of Number of Turns of Injection ($\epsilon_{inj}=6\pi$)

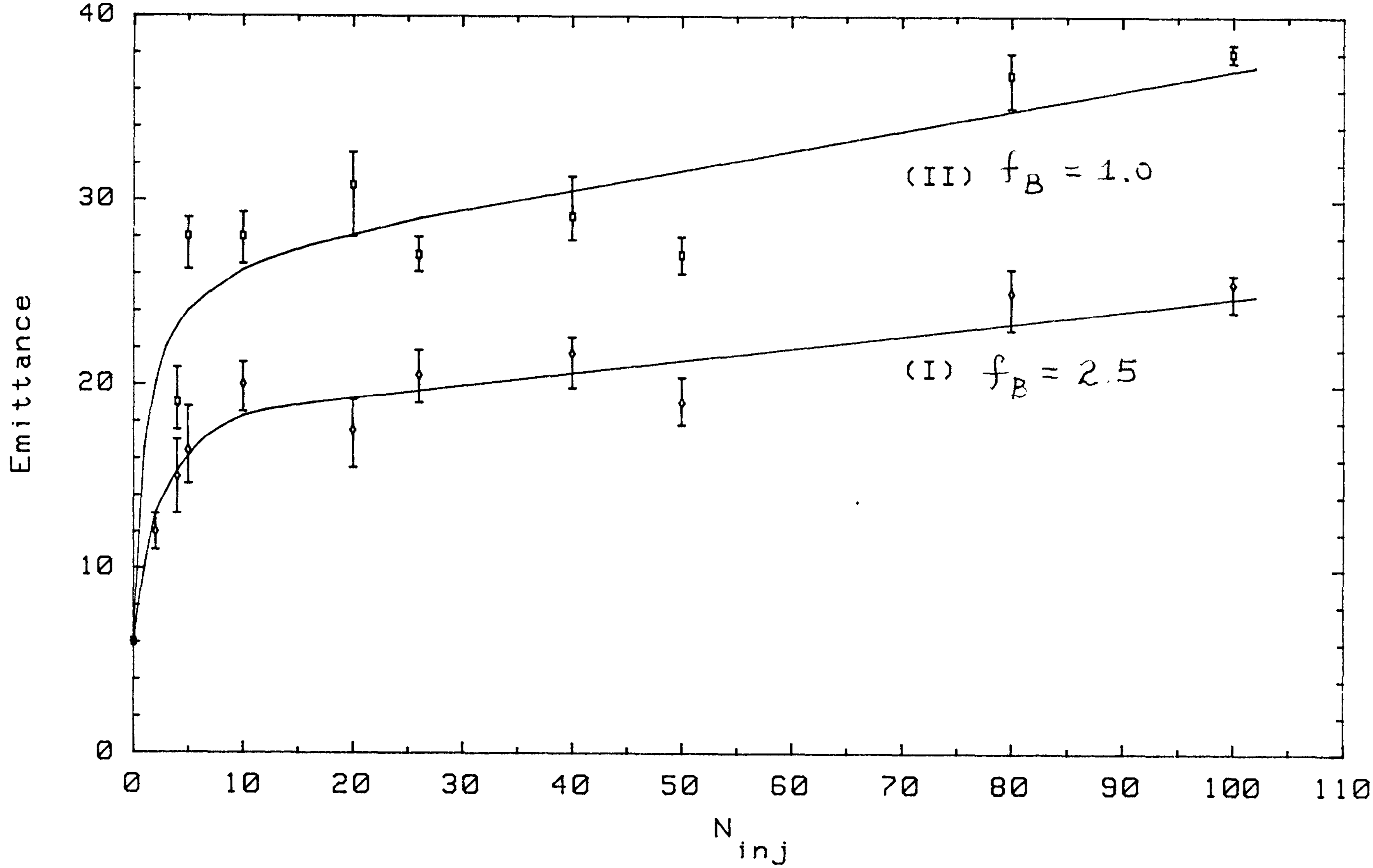
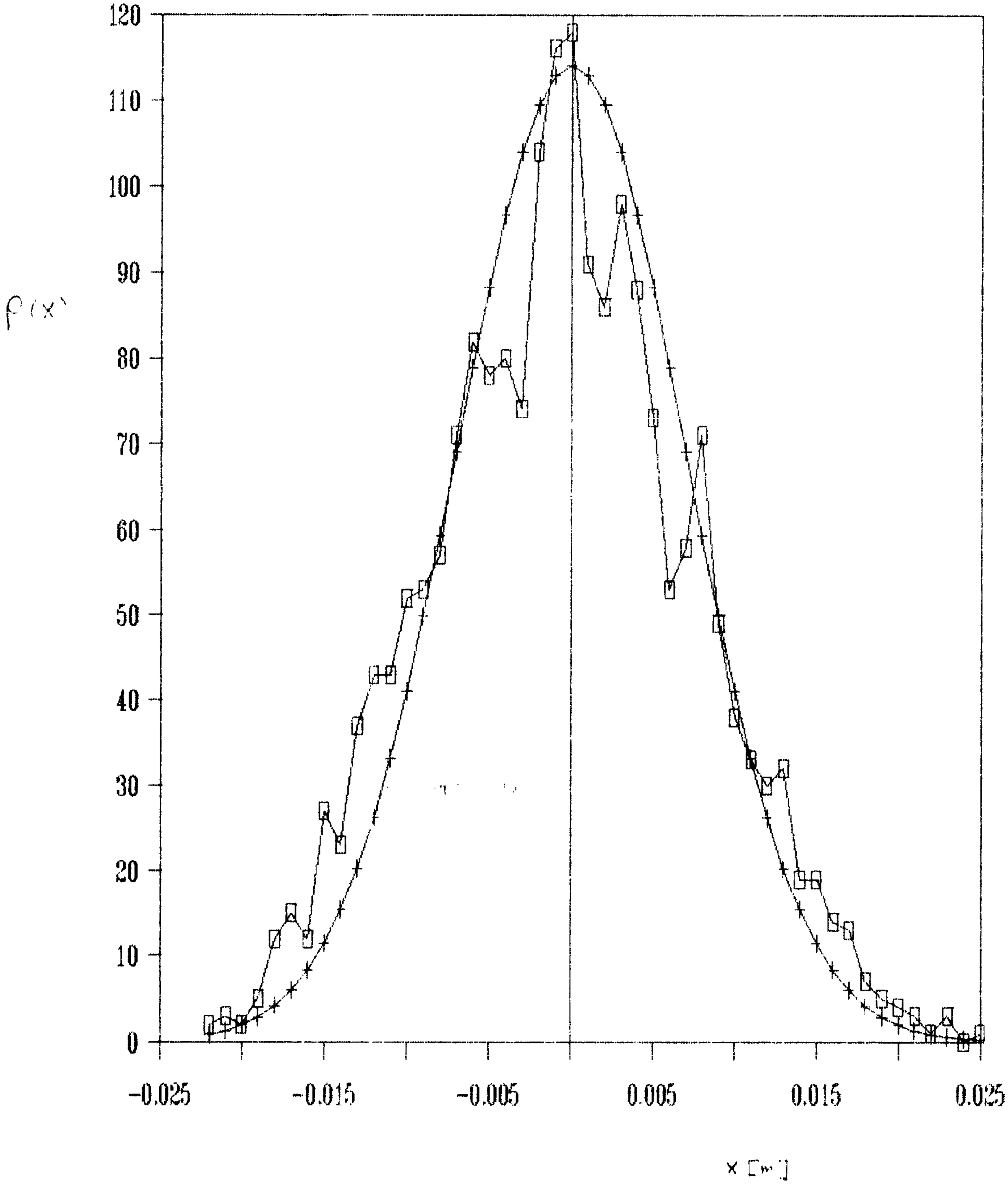


Figure 2



120
110
100
90
80
70
60
50
40
30
20
10
0
ρ(x)
-0.025
-0.015
-0.005
0.005
0.015
0.025
x [m]