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MULTIPOLE COMPONENTS FROM THE EDDY CURRENT CORRECTION COILS

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Abstract

Eddy current correction coils on the vacuum chamber of the AGS Booster can induce non-negligible multipoles. These multipoles may greatly reduce the dynamical aperture of the AGS Booster.

I. Introduction

For fast cycling synchrotrons with large $\dot{B}/B$, it is known to have the problem of eddy current multipoles. Recently, the multipoles due to various shapes of vacuum chambers have been derived[2]. It is worthwhile to note that the sextupole term dominate. These eddy current sextupole causes some concern on the performance of the low energy high intensity machine.

Due to these concerns, the AGS Booster is designed with correct coils on the vacuum chamber shown in Fig. 1 to correct the sextupole component. In this note, I would like to address the question of multipoles due to these correction coils.

II. Multipole Expansion

Following Beth, the magnetic field due to the correct carrying wire is given by[1]

$$H = \frac{I}{2\pi(Z-Z_o)} \qquad (1)$$

where $Z = x + iy$ and $H = H_y + iH_x$, Z_o is the location of the coil. Figure 1 shows the geometry of the current carrying wires, where b = 1.375", a = 3.25", g= 3.25" and c = 0.95". The wires are proposed to locate at ($\pm x_1$, $\pm y_1$) and ($\pm x_2$, $\pm y_2$) where x and y satisfies $y = \pm mx \pm b$ equation for the vacuum chamber and $x_1 = 1.25$" and $x_2 = 2x_1$.

Equation 1 leads to the magnetic field, measured at a point (x,0) as

$$H_y(x) = \frac{I}{\pi} \sum_{m=1}^{2} m \left(\frac{x-x_m}{(x-x_m)^2 + y_m^2} \qquad \frac{x+x_m}{(x+x_m)^2 + y_m^2} \right) \qquad (1)$$

Due to the symmetry of the vacuum chamber, we obtain only H_y component. Similarly, the magnetic field due to the nearby image currents has to be added to Eq. (1) as,

$$H_y(x) = \frac{I}{\pi} \sum_{m=1}^{2} m \left[\frac{x-x_m}{(x-x_m)^2 + (y_m)^2} - \frac{x+x_m}{(x+x_m)^2 + (y_m)^2} \right.$$

$$+ \sum_{m=1}^{2} m \left[\frac{x-x_m}{(x-x_m)^2 + (g-y_m)^2} - \frac{x+x_m}{(x+x_m)^2 + (g-y_m)^2} \right]$$

$$\left. + \sum_{m=1}^{2} m \left[\frac{x-x_m}{(x-x_m)^2 + (g+y_m)^2} - \frac{x+x_m}{(x+x_m)^2 + (g+y_m)^2} \right] \right]$$

where the last two terms arises from the nearby image current contribution. The multipole coefficient can then be obtained by

$$b_n = \frac{1}{B} \cdot \frac{\mu_o}{n!} \left. \frac{\partial^n H}{\partial x^n} \right|_{x = 0}$$

These coefficients are given by

$$\Delta b_o = \frac{\Delta B_o}{B} = \frac{\mu_o I}{\pi B} \sum_{k=-1}^{1} \sum_{m=1}^{2} \frac{-2mx_m}{D}$$

$$b_2 = \frac{\mu_o I}{\pi B} \sum_{k=1}^{1} \sum_{m=1}^{2} mx_m \left(\frac{-2}{D^2} + \frac{8\ (y_m + kg)^2}{D^3} \right)$$

$$b_4 = \frac{\mu_o I}{\pi B} \sum_{k=-1}^{1} \sum_{m=1}^{2} mx_m \left(\frac{-2}{D^3} - \frac{8\ (y_m + kg)^2}{D^4} + \frac{32x_i^2\ (y_m + kg)^2}{D^5} \right)$$

$$b_6 = \frac{\mu_o I}{\pi B} \sum_{k=1}^{1} \sum_{m=1}^{2} mx_m \left(\frac{-2}{D^4} + \frac{48(y_m+kg)^2}{D^5} - \frac{60(y_m+kg)^4}{D^6} + \frac{128(y_m+kg)^6}{D^7} \right)$$

$$b_8 = \frac{\mu_o I}{\pi B} \sum_{k=1}^{1} \sum_{m=-1}^{2} mx_m \left(\frac{-2}{D^5} + \frac{80(y_m+kg)^2}{D^6} - \frac{480(y_m+kg)^4}{D^7} + \frac{896(y_m+kg)^6}{D^8} - \frac{512(y_m+kg)^8}{D^9} \right)$$

with $D \equiv x_m^2 + (y_m + kg)^2$

to be substituted into the above equations. The multipole coefficients are measured in [meter^{-n}].

To cancel the sextupole component from the eddy current (vacuum chamber and the image current), we would set

$$b_2 = - b_2^E = - \frac{\mu_o \sigma h}{\pi} \frac{\dot{B}}{B} G_2 \simeq - 0.785\ m^{-2}$$

where $\mu_o = 4\pi \times 10^{-7}$ H/m, σ conductivity of the vacuum chamber, h the chamber thickness and G_2 is the geometric factor depending on the slope of the vacuum chamber.[2] The value of -0.785 m^{-2} was obtained specific for the Booster vacuum chamber and acceleration rate.[3]

The current for the correction coil should be ramped proportional $\dot{B}$, not $\dot{B}/B$.

To correct the sextupole, we excite I/B so that $b_2 + b_2^E = 0$. The resulting I/B is shown on Table 1 for the coil x_1 [in inch] location (the other coil is located at $x_2 = 2x_1$). The highlighted line at $x_1 = 1.25$" corresponds to the proposed coil location.

Table 1 multipole coefficient b_n [$meter^{-n}$] is tabulated vs the coil location x_1 [inch]. The presently proposed coil location is at $x_1 = 1.25$".

x1	b0	b2	b4	b6	b8	I/B	b2/(I/B)
0.20	4.428E-04	-0.785	7.84E+02	-6.05E+05	3.62E+08	-16.833	0.04664
0.25	4.762E-04	-0.785	6.83E+02	-4.11E+05	1.42E+08	-14.837	0.05291
0.30	5.202E-04	-0.785	5.75E+02	-2.43E+05	6.49E+06	-13.919	0.05640
0.40	6.435E-04	-0.785	3.74E+02	-3.61E+04	-5.32E+07	-13.912	0.05642
0.50	8.190E-04	-0.785	2.21E+02	3.01E+04	-3.39E+07	-15.477	0.05072
0.60	1.052E-03	-0.785	1.21E+02	4.87E+04	-5.77E+07	-18.243	0.04303
0.70	1.345E-03	-0.785	5.24E+01	7.64E+04	-9.67E+07	-22.122	0.03548
0.75	1.517E-03	-0.785	2.30E+01	9.58E+04	-1.06E+08	-24.492	0.03205
0.80	1.706E-03	-0.785	-5.71E+00	1.16E+05	-1.05E+08	-27.172	0.02889
0.90	2.146E-03	-0.785	-6.52E+01	1.50E+05	-7.55E+07	-33.593	0.02337
1.00	2.686E-03	-0.785	-1.29E+02	1.64E+05	-2.66E+07	-41.748	0.01880
1.10	3.357E-03	-0.785	-1.94E+02	1.56E+05	2.09E+07	-52.194	0.01504
1.20	4.205E-03	-0.785	-2.59E+02	1.29E+05	5.49E+07	-65.736	0.01194
1.25	4.715E-03	-0.785	-2.90E+02	1.11E+05	6.56E+07	-74.014	0.01061
1.30	5.294E-03	-0.785	-3.20E+02	9.11E+04	7.22E+07	-83.538	0.00940
1.40	6.718E-03	-0.785	-3.78E+02	4.87E+04	7.51E+07	-107.303	0.00732
1.50	8.616E-03	-0.785	-4.34E+02	6.02E+03	6.82E+07	-139.613	0.00562
1.60	1.121E-02	-0.785	-4.90E+02	-3.48E+04	5.56E+07	-184.569	0.00425

Since the b_n coefficients are in dimension of [$meter^{-n}$]. The beam dimension is only ± 2 inches. The coefficients do not give much insight into the problem. Next section, we shall analyze the effect of these higher multipoles. We, however, note that the b_6 due to the correction coils and its image current is two order larger than that of eddy current, while b_4 is one order larger.

III. <u>Effect of Multipoles on the Beam Dynamics</u>

We know that the multipole gives a kick to the beam according to

$$\Delta x'_M = \frac{b_n\, x^n}{\rho}\, \ell$$

where x, x′ are the phase space coordinate, ℓ is the length of the magnet, ρ is the radius of curvature. For the AGS Booster $\ell = 2.4$ m, $\rho = 13.75$ m. Table 2 shows the resulting kicks, $\Delta x'$, due to those multipoles. Note that at x = 2" the b_4, b_6, and b_8 multipole gives about -0.34 mr, 0.33 mr and 0.51 mr, respectively for the proposed configuration [coil at 1.25 inch location]. Fig. 2 shows $\Delta x'$ for b_4, b_6 and b_8 at x = 1.5" and x = 2" for various coil locations [at x_1].

Table 2 The multipole kicks $\Delta x'$ at 1.5" and 2" are tabulated vs the coil location x_1 [inch].

x1	Δx' B4@1.5"	Δx ' b 6@1.5"	Δx' b8@1.5"	Δx' B4@2"	Δx' B6@2"	Δx' B8@2"
0.20	2.88E-04	-3.23E-04	2.80E-04	9.11E-04	-1.81E-03	2.80E-03
0.25	2.51E-04	-2.19E-04	1.10E-04	7.94E-04	-1.23E-03	1.10E-03
0.30	2.12E-04	-1.30E-04	5.03E-06	6.69E-04	-7.30E-04	5.02E-05
0.40	1.37E-04	-1.93E-05	-4.12E-05	4.34E-04	-1.08E-04	-4.12E-04
0.50	8.14E-05	1.60E-05	-2.63E-05	2.57E-04	9.02E-05	-2.63E-04
0.60	4.45E-05	2.60E-05	-4.47E-05	1.41E-04	1.46E-04	-4.47E-04
0.70	1.93E-05	4.08E-05	-7.49E-05	6.09E-05	2.29E-04	-7.48E-04
0.75	8.47E-06	5.12E-05	-8.22E-05	2.68E-05	2.87E-04	-8.21E-04
0.80	-2.10E-06	6.20E-05	-8.14E-05	-6.63E-06	3.49E-04	-8.13E-04
0.90	-2.40E-05	8.01E-05	-5.85E-05	-7.58E-05	4.50E-04	-5.85E-04
1.00	-4.74E-05	8.76E-05	-2.06E-05	-1.50E-04	4.92E-04	-2.06E-04
1.10	-7.15E-05	8.30E-05	1.62E-05	-2.26E-04	4.67E-04	1.62E-04
1.20	-9.51E-05	6.88E-05	4.25E-05	-3.01E-04	3.86E-04	4.25E-05
1.25	-1.07E-04	5.92E-05	5.08E-05	-3.37E-04	3.33E-04	5.08E-04
1.30	-1.18E-04	4.86E-05	5.59E-05	-3.72E-04	2.73E-04	5.59E-04
1.40	-1.39E-04	2.60E-05	5.82E-05	-4.39E-04	1.46E-04	5.81E-04
1.50	-1.59E-04	3.21E-06	5.28E-05	-5.04E-04	1.81E-05	5.28E-04
1.60	-1.80E-04	-1.86E-05	4.31E-05	-5.70E-04	-1.04E-04	4.30E-04

In order to measure the importance of these nonlinear kicks, we should compare the corresponding kicks on the orbit due to the quadrupoles, i.e.

$$\Delta x_Q' = \frac{x}{f} = \frac{x}{\frac{B\rho}{B' \ell_Q}} \simeq \frac{x}{3.52}$$

where f is the focal length of the quadrupoles.

Thus, to measure the importance of the multipole kick is to find the ratio of $\Delta x_M'/\Delta x_Q'$. Table 3 shows the ratio at x = 1.5" and 2" for b_4, b_6 and b_8. We observe that effect is rather important for b_4, b_6 and b_8 at x = 2 inches. The multipole kick is about 3.5% of the quadrupole kick. The tracking calculation should be performed to test the stability of the beam at this condition.

Table 3 Ratio of the Multipole Kick to that of the Corresponding Quadrupole Kicks

xl	B4@1.5"	b 6@1.5"	b8@1.5"	B4@2"	B6@2"	B8@2"
0.20	2.64E-02	-2.96E-02	2.57E-02	6.26E-02	-1.25E-01	1.93E-01
0.25	2.30E-02	-2.01E-02	1.01E-02	5.46E-02	-8.48E-02	7.53E-02
0.30	1.94E-02	-1.19E-02	4.61E-04	4.60E-02	-5.002E-02	3.45E-02
0.40	1.26E-02	-1.77E-03	-3.78E-03	2.99E-02	-7.46E-03	-2.83E-02
0.50	7.47E-03	1.47E-03	-2.41E-03	1.77E-02	6.20E-03	-1.81E-02
0.60	4.08E-03	2.39E-03	-4.10E-03	9.67E-03	1.01E-02	-3.07E-02
0.70	1.77E-03	3.74E-03	-6.87E-03	4.19E-03	1.58E-02	-5.15E-02
0.75	7.77E-04	4.69E-03	-7.53E-03	1.84E-03	1.98E-02	-5.64E-02
0.80	-1.92E-04	5.69E-03	-7.46E-03	-4.56E-04	2.40E-02	-5.59E-02
0.90	-2.20E-03	7.35E-03	-5.37E-03	-5.21E-03	3.10E-02	-4.02E-02
1.00	-4.34E-03	8.04E-03	-1.89E-03	-1.03E-02	3.39E-02	-1.42E-02
1.10	-6.55E-03	7.62E-03	1.49E-03	-1.55E-02	3.21E-02	1.11E-02
1.20	-8.72E-03	6.31E-03	3.90E-03	-2.07E-02	2.66E-02	2.92E-02
1.25	-9.77E-03	5.43E-03	4.66E-03	-2.32E-02	2.29E-02	3.49E-02
1.30	-1.08E-02	4.46E-03	5.13E-03	-2.56E-02	1.88E-02	3.84E-02
1.40	-1.27E-02	2.38E-03	5.34E-03	-3.02E-02	1.00E-02	4.00E-02
1.50	-1.46E-02	2.95E-04	4.84E-03	-3.47E-02	1.24E-03	3.63E-02
1.60	-1.80E-04	-1.86E-05	4.31E-05	-5.70E-04	-1.04E-04	4.30E-04

Fig. 3 show the ratio $\Delta x'_M/\Delta x'_Q$ at x = 1.5" and 2", respectively.

Figs. 1-4 show that there is no coil location which will minimize the multipole kicks for all multipoles.

IV. Conclusion

The eddy current correction coil proposed in the booster vacuum chamber seems to produce substantial amount of higher multipoles, which may affect the beam dynamics at the aperture of ± 2 inches. It is important to reconsider the beam dynamics issue before the correction coil is placed on the vacuum chamber. Our analysis indicates that the multipole cannot be minimized with respect to the coil location. It is doubtful that the eddy current correction coil will be useful to the booster performance. In my opinion, it is easier to deal with the sextupoles than the induced higher multipoles.

Finally, I have also calculated multipoles from the three coils b_2 correction scheme, i.e. correction coils are located at x_1, x_2 and x_3, such that $x_2 = 2x_1$, $x_3 = 3x_1$.
The effect of multipole kicks remains large. I have also taken into account the infinite set of images. The result agrees with the present analysis of nearby images to within 5%.

Let us apply the similar criteria to RHIC. The maximum beam size is expected to be no more than 1 inch. For RHIC, quadrupole focal length f = 10 m, $\rho \simeq 243$ m, length of dipole ℓ = 10 m, we obtain then

$$\Delta x'_M = b_n \ \ell/\rho \simeq 4 \times 10^{-6} \ [b_n \times 10^4]$$

$$\Delta x'_Q = \frac{0.025}{10} = 0.0025$$

where the multiple coefficient b_n is in the unit of $[\text{inch}]^{-n}$. The ratio of these phase space kicks becomes

$$\Delta x'_M/\Delta x'_Q = 1.6 \times 10^{-3} \; [bn \times 10^4]$$

For $b_n \times 10^4 \simeq 1$ [inch^{-n}], the ratio becomes 0.16%.

For $b_n \times 10^4 \simeq 10$, the ratio will be 1.6%, which is large and can affect the dynamical aperture.

We have also proposed a prescription for expressing the importance of the higher multipoles. The multipole coefficient should be expressed as:

b_n measured at the expected maximum beam size

For the booster, the maximum beam size is expected to be ± 2 inches. The b_4, b_6 and b_8 from the eddy current correction coils are -19.0×10^{-4}, 18.5×10^{-4} and 28.6×10^{-4} at 2 inches. The effect of these higher multipoles gives $\Delta x'$ of $b_n \cdot \ell/\rho$, which is - 0.34 mr, + 0.33 mr and 0.51 mr for b_4, b_6 and b_8, respectively. This effect is not negligible in comparison with the similar quantity of $\Delta x'_Q$ = 14.5 mr from each quadrupole.

For the booster, 2" is a realistic beam size, the ratio multipole to the quadrupole kick of 3.5% is indeed an important problem to be resolved.

REFERENCES

1. R.A. Beth, J. Appl. Phys. 37, 2568 (1966).

2. S.Y. Lee, G. Morgan and S. Kahn, to be published.

3. G. Morgan, Booster Technical Note #4.

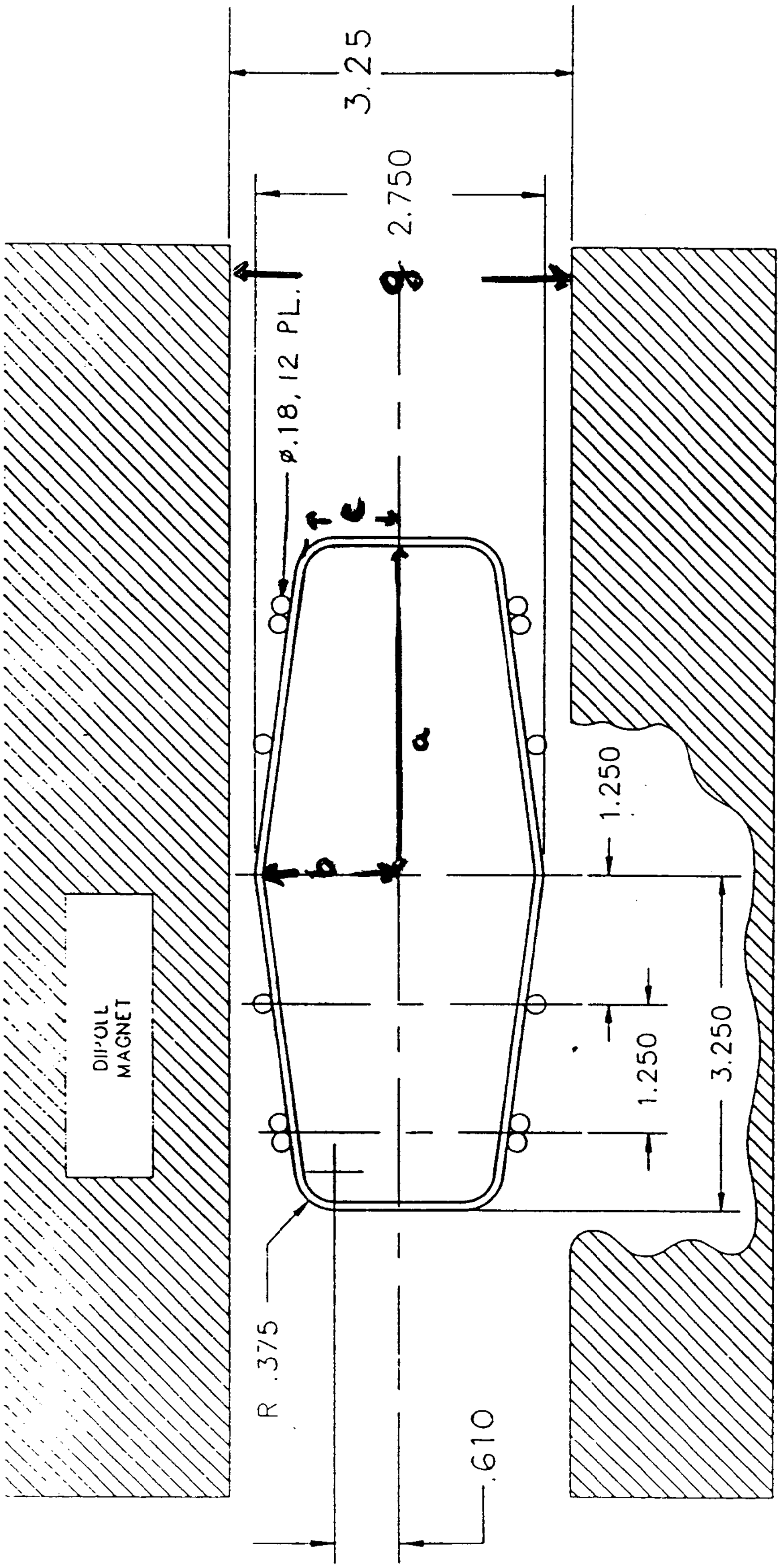
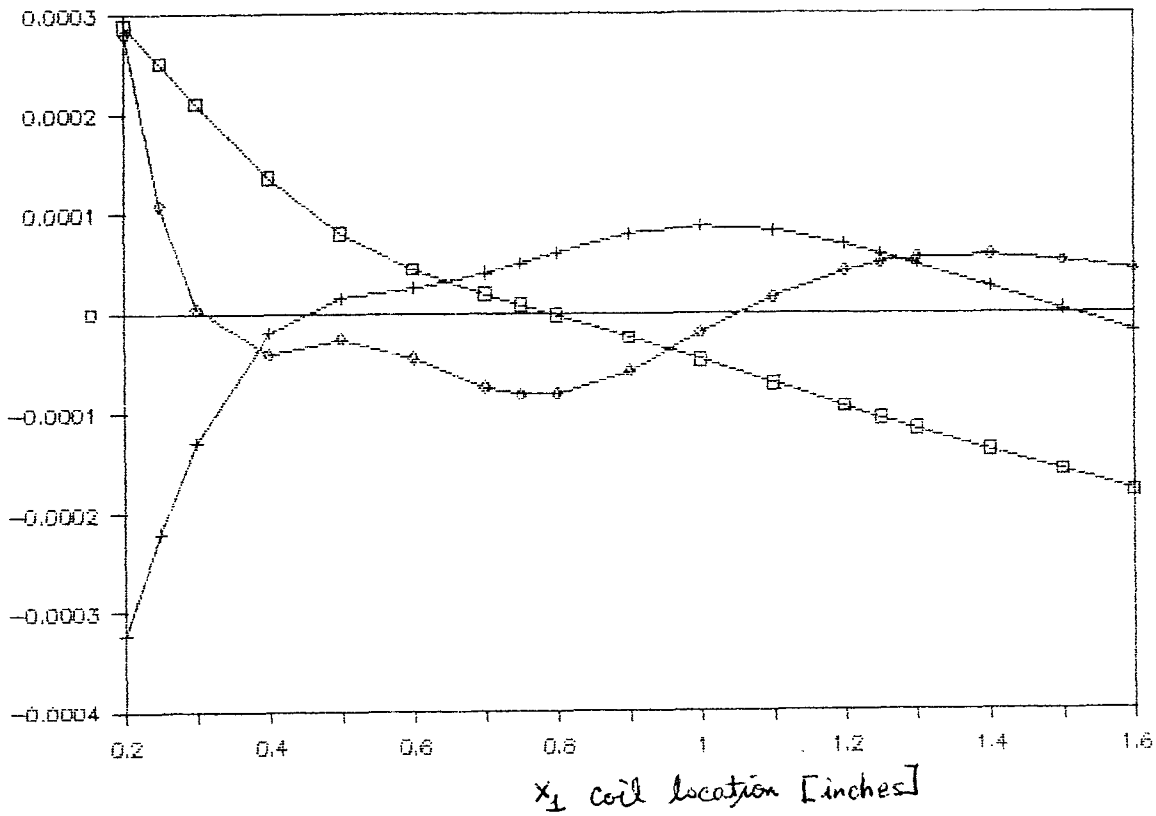


Figure 1. Three-turn loop on top and bottom of dipole vacuum chamber for local correction of eddy current effects.

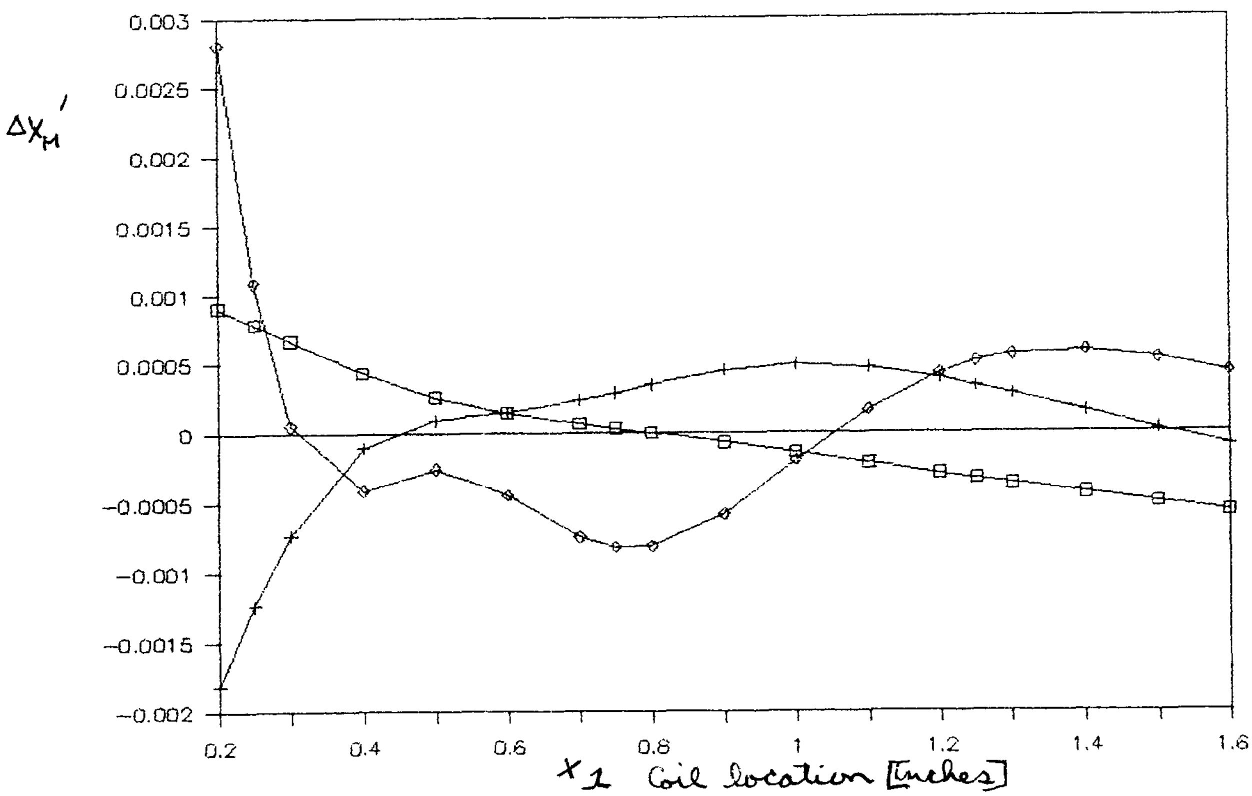
At $x = 1.5''$ location



□ b_4
+ b_6
▵ b_8
@ 1.5"

Fig. 2a

At 2 inch location



@ 2"

Fig. 2b

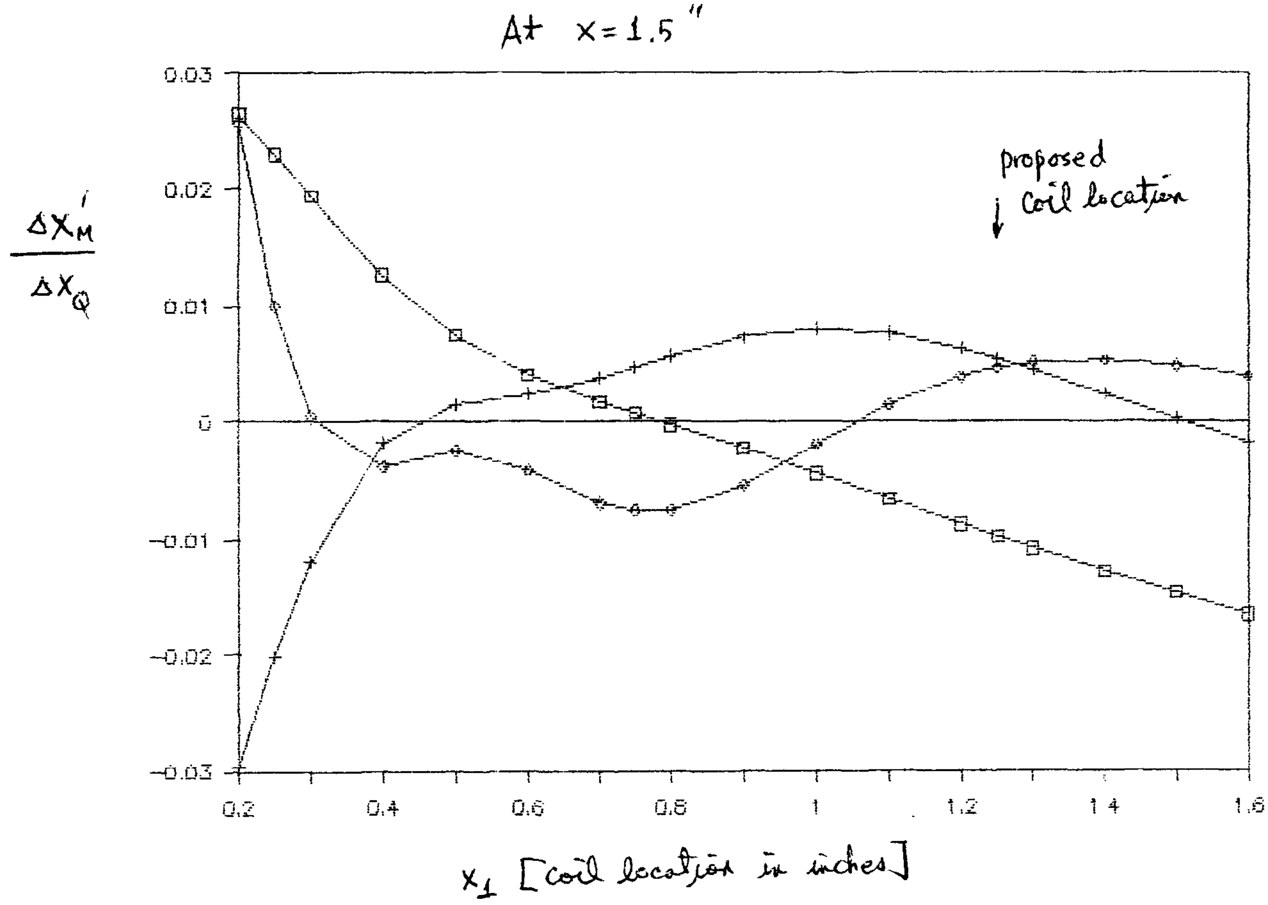

At x=1.5"
proposed
coil location
$\frac{\Delta X_M'}{\Delta X_Q}$
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
x_1 [coil location in inches]

Fig. 3a

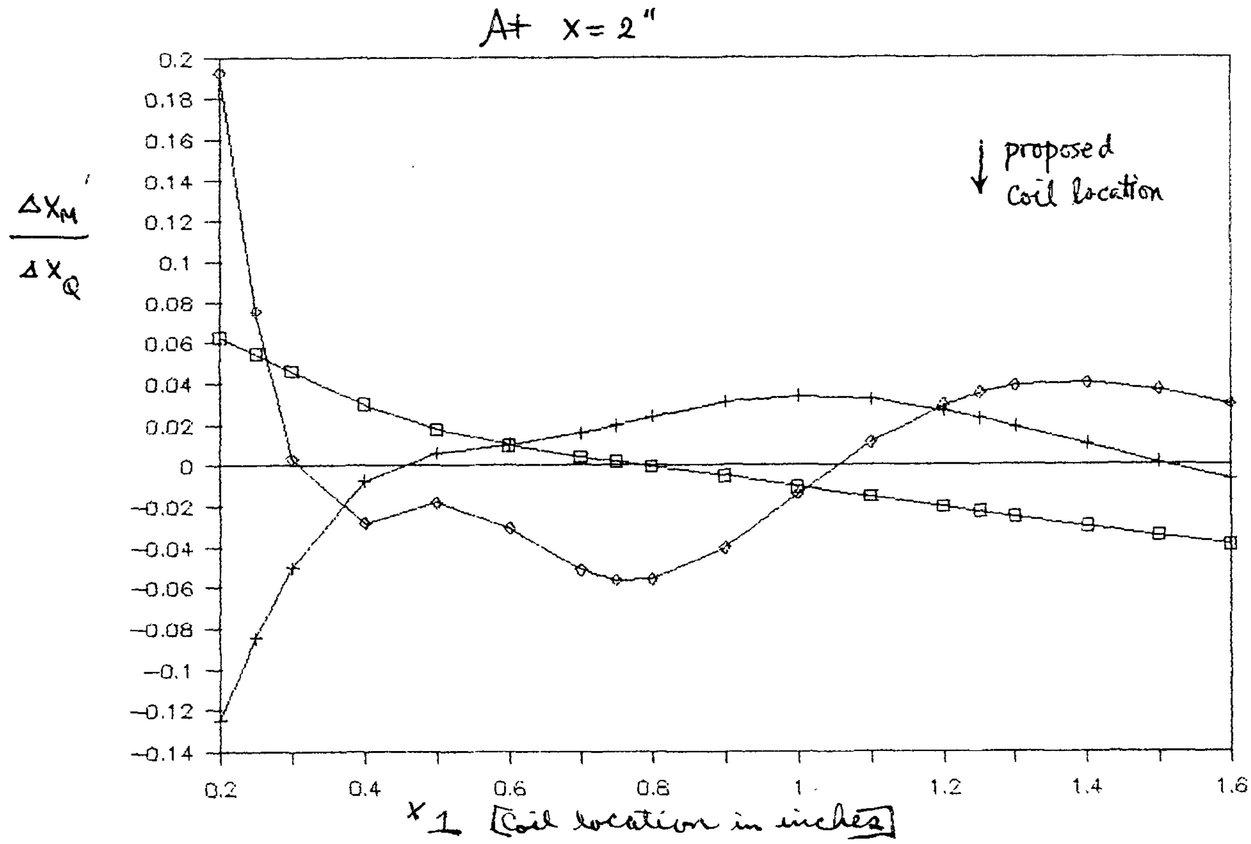

At x=2"
proposed
coil location
$\frac{\Delta X_M'}{\Delta X_Q}$
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
x_1 [coil location in inches]

Fig 3b