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EVALUATION OF THE CHROMATICITY SEXTUPOLES FOR THE AGS BOOSTER

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HIGH ENERGY FACILITIES

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ABSTRACT

In this note the comparison has made between chosen set (1, 2, 4, 7) of chromaticity sextupoles and full set (1, 2, 3, 4, 5, 6, 7, 8) of sextupoles which would be ideal for Booster.

Mathematical model representing Booster lattice is based on certain approximations for eddy current sextupoles and for chromaticity sextupoles. In this note we also evaluate how good are these approximations.

INTRODUCTION

The AGS Booster is designed to accelerate protons from 200 MeV of injection energy. At that energy the typical energy spread could be $\Delta E/E=0.00064$ which corresponds to momentum spread $\Delta p/p=0.002$ which in turn can shift the tune by order of $\Delta\nu=0.01$. This tune shift due to momentum spread is known as a natural chromaticity.

The best way to accommodate chromaticity corrections for the Booster would be implementation of sextupoles near to each quadrupole as it is shown in Fig. 1.

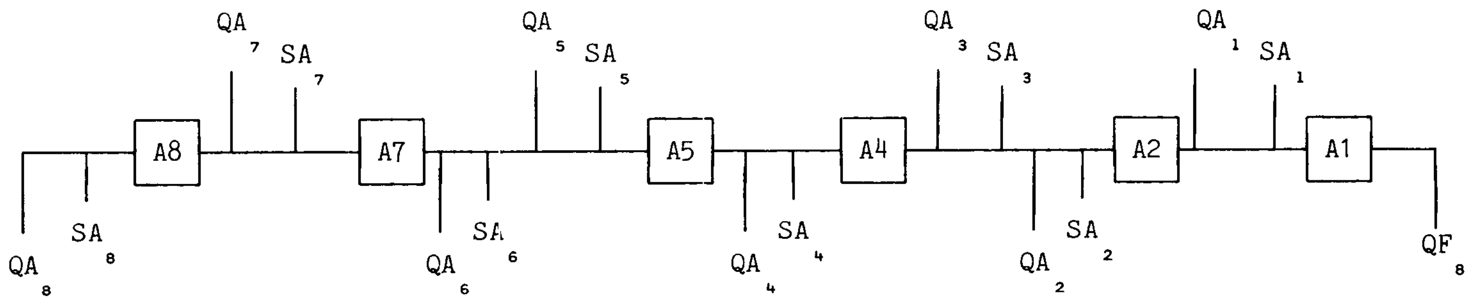

Fig. 1. Schematic Diagram of the Booster superperiod:

A_i - dipoles, i = 1,2,4,5,7,8;
QA_i - quads , i = 1,2,3,4,5,6,7,8;
SA_i - sext's , i = 1,2,3,4,5,6,7,8 is full set or i = 1,2,4,7 is chosen set.

However, due to needs for the space required for injection, acceleration, ejection, and abortion the set of (1,2,4,7) sextupoles was adopted for chromaticity correction. We'll refer to that set as a chosen set while (1,2,3,4,5,6,7,8) as a full set.

This note is a continuation of [Ref. 1,2] where evaluation of chosen set was initiated. In following sections we answer two questions:

- how good the chosen set compare to full set of sextupoles?
- how good a computational model for evaluation of chosen set?

As a criteria for comparison we use a linear chromaticities

$$\xi_x = d\nu_x/d\delta, \quad \xi_y = d\nu_y/d\delta, \quad \delta = \Delta p/p \tag{1}$$

and linear amplitude dependence of tune

$$\alpha_{xx} = \partial\nu_x/\partial\varepsilon_x, \quad \alpha_{xy} = \partial\nu_x/\partial\varepsilon_y = \partial\nu_y/\partial\varepsilon_x = \alpha_{yx}, \quad \alpha_{yy} = \partial\nu_y/\partial\varepsilon_y \tag{2}$$

all determined by Taylor expansion up to linear terms:

$$\nu_x(\delta) = \nu_x(o) + \xi_x\cdot\delta, \quad \nu_y(\delta) = \nu_y\ (o) + \xi_y\cdot\delta, \tag{3}$$

$$\nu_x(\varepsilon_x,\varepsilon_y) = \nu_x(o,o) + \alpha_{xx}\cdot\varepsilon_x + \alpha_{xy}\cdot\varepsilon_y,$$

$$\nu_y(\varepsilon_x,\varepsilon_y) = \nu_y(o,o) + \alpha_{yx}\cdot\varepsilon_x + \alpha_{yy}\cdot\varepsilon_y. \tag{4}$$

Another basis for comparison between different sextupoles sets was a particle tracking. All calculations of chromaticities and tune amplitude dependence were performed on CDC-7600 computer using SYNCH program, while MAD program was used on VAX computer for tracking.

The nominal tune was $\nu_x(o) = 4.82$, $\nu_y(o) = 4.83$ for all calculations. To provide that tune we ran SYNCH program, fitting the strength K1 of focusing QF and defocusing QD quadrupoles to the given tune. The results are in Table I.

TABLE I.

Quads	The length [L] = M	The field [B] = T	Mag. rigid [Bp] = T-M	The strength [K1 = B1/Bp] = M^{-2}
QF	0.50375	0.156325	2.14962	1.19994
QD	0.251875	0.156325	2.14962	-1.23693

Another important parameter for calculations is the strength of eddy current sextupoles. There are different notations and units using in magnet and lattice studies. Following [Ref. 3] we'll derive one relationship for the field expansion coefficients.

Usually the field of the magnet is represented in magnet design studies as

$$B_y = B_o + B_o \sum_{n=1}^{\infty} b_n \frac{x}{r_o}^n , \qquad (5)$$

where r_o is normalization radius.

In lattice design studies the same field is represented by

$$B_y = B_o + \sum_{n=1}^{\infty} B_n . \frac{x^n}{n!} \quad . \qquad (6)$$

Equating (5) to (6) one can find correspondence between B_n and b_n:

$$B_n = B_o \frac{n!}{r_o^n} b_n \quad . \qquad (7)$$

In particularly for the strength of eddy current sextupoles one can find using b_2 and r_o from [Ref. 4] and normalizing to the magnetic rigidity $B_o\rho$:

$$K2 = \frac{B_2}{B_o\rho} = \frac{b_2 . 2!}{\rho . r_o^2} = \frac{.78 . 10^{-4} . 2!}{13.75 \ (0.01)^2} = 0.11345 m^{-3} \quad . \qquad (8)$$

All the following calculations were performed with that values of parameters K1 and K2.

APPROXIMATION FOR EDDY CURRENT SEXTUPOLES

AND FOR CHROMATICITY SEXTUPOLES

There are two type of sextupoles in the Booster. First are eddy current sextupoles superimposed in each bending magnet of the length 2.4 m. The second are chromaticity sextupoles each of the length of 0.1 m. In lattice model both types are represented by point sextupoles.

To approximate eddy current sextupoles 3 schemes were tested with 3,5, and 9 point sextupoles equidistantly inserted into dipole as it is shown in Fig. 2. In each scheme integrated strength of point sextupoles is equal to integrated strength 2.4 x K2 of eddy current sextupole.

To approximate chromaticity sextupoles also 3 schemes were tested with drift and point sextupole as it is shown in Fig. 3.

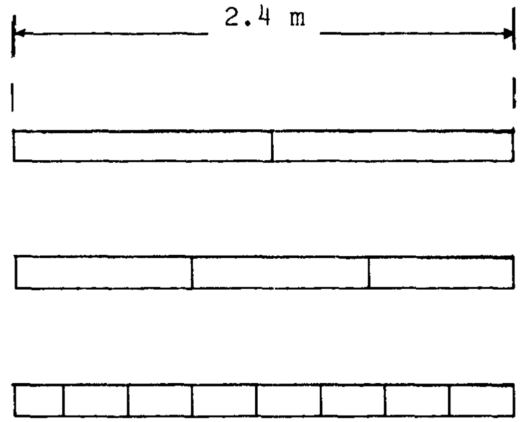

Fig. 2. 3,5 & 9 point sextupoles inserted in bending magnet.

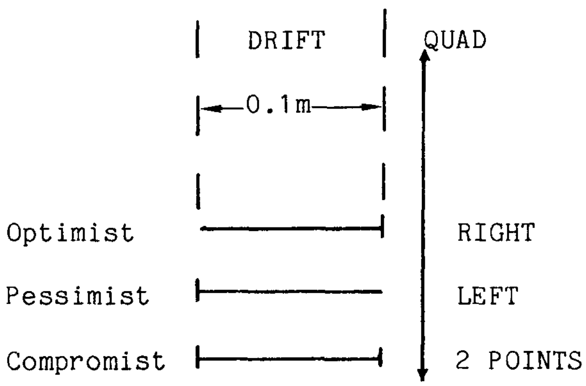

Fig. 3. Right, left and 2 point schemes approximating chromaticity sextupoles.

By obvious reason inherited from quadrupoles all the chromaticity sextupoles fall into families of focusing (horizontally) SF and defocusing (horizontally) SD. For each approximation scheme the results of testing are presented in the Table II.

TABLE II

Column Number	1	2	3	4	5	6
Eddy Current Scheme	3 points	3 points	5 points	9 points	3 points	$\left\lvert\frac{Col.4-Col.1}{Col.4}\right\rvert$ x100%
Chromatic Scheme	RIGHT	2 points	2 points	2 points	LEFT	
SF[K2]	.112091	.065171	.071355	.072903	.150190	23.1
SD[K2]	-.815308	-.415251	-.415543	-.415613	-.846626	0.02
Integrated Strength	K2	2xK2	2xK2	2xK2	K2	
$\delta = \Delta p/p$	-.002(0).002	-.002(0).002	-.002(0).002	-.002(0).002	-.002(0).002	-.002(0).002
ξ_x	-.113(0).109	-.121(0).118	-.126(0).122	-.127(0).123	-.133(0).129	11.0(0)11.4
ξ_y	.167(0)-.168	.178(0)-.180	.183(0)-.185	.183(0)-.186	.190(0)-.192	.09(0).09
ν_x	4.821(4.82)4.819	4.821(4.82)4.819	4.821(4.82)4.819	4.821(4.82)4.819	4.821(4.82)4.819	0(0)0
α_{xx}	14.6(14.2)13.8	17.4(17.0)16.5	18.3(17.9)17.4	18.6(18.1)17.6	20.9(20.4)19.9	21.5(21.5)21.6
α_{xy}	-4.95(-4.38)-3.89	-7.30(-6.67)-6.12	-8.52(-7.86)-7.29	-8.83(-8.17)-7.58	-9.32(-8.61)-7.99	43.9(46.4)48.7
ν_y	4.83(4.83)4.83	4.83(4.83)4.83	4.83(4.83)4.83	4.83(4.83)4.83	4.83(4.83)4.83	0(0)0
α_{yx}	-4.95(-4.38)-3.89	-7.30(-6.67)-6.12	-8.52(-7.86)-7.29	-8.83(-8.17)-7.58	-9.32(-8.61)-7.99	43.9(46.4)48.7
α_{yy}	94.5(94.9)95.3	96.1(96.6)97.1	98.2(98.6)99.1	98.7(99.2)99.7	99.9(100)101.	.04(.04).04

One can see from this table that all numbers in each line increase in absolute value monotonically from Col. 1 to Col. 5 justifying the names for RIGHT scheme as optimistic and for LEFT scheme as pessimistic. No wonder that optimistic scheme provides less values of chromaticity ξ and amplitude coifficient α. This is because the RIGHT scheme point sextupole is closer to the corresponding quadrupole than it would be in LEFT scheme.

The last column in Table II provides the relative errors in percent, the errors we can expect using optimistic scheme of Column 1 instead of more accurate scheme of Column 4. Assuming that these errors are acceptable we'll proceed to the next section using optimistic scheme throughout all further calculations.

COMPARISON OF TWO SETS OF CHROMATICITY SEXTUPOLES:
FULL SET AND CHOSEN SET

In Table III there are results of calculations of chromaticities and amplitude dependence for two sets of chromatic sextupoles. Calculations were performed using RIGHT scheme for chromatic sextupoles and 3 point scheme for eddy current sextupoles. Also x-coordinate of closed orbit x_{co} presented which refers to the end of Bocster superperiod. To complete a comparison of two sets a tracking calculations were done.

Three particles (protons) were chosen for tracking along the Booster lattice. Initial energy was 200 MeV and initial transfer momentum were zero for all particles. Initial coordinates are shown in Fig. 4. For all particles tracking was repeated three times for three values of momentum spread: δ = 0.002,0.0, + 0.002. Each time tracking was done for 300 revolutions with printing frequency after each five revolutions. All collected data were analyzed to choose maximal relative deviation according to formula

$$R_i = \max_n \frac{D_{in}}{L_i} \times 100\%$$

where i= 1, 2, 3 and L_i, D_{in} are distances shown in Fig. 4.

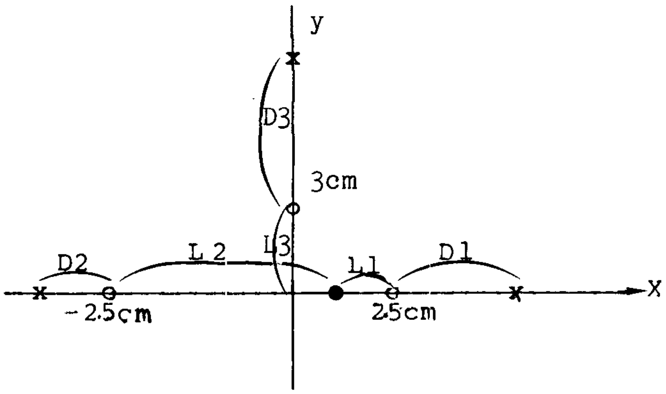

Fig. 4. Three particles inside of vacuum chamber.

0 - particle inital position
x - particle position after n-th turn
• - closed orbit position.

The resulting values R_i are kind of measure for the motion nonlinearlity. Those values in precent are shown in the last rows of the Table III.

TABLE III

Set Type	Chosen Set (1,2,4,7)	Full Set (1,2,3,4,5,6,7,8)
SF[K2]	.112091	.029395
SD[K2]	-.815308	-.480318
δ	-.002(0)+.002	-.002(0)+.002
ξ_x	-.113(0).109	-.037(0).036
ξ_y	.167(0)-.168	-.029(0).028
ν_x	4.8212(0)4.8190	4.8211(4.82)4.8189
α_{xx}	14.6(14.2)13.8	-1.16(-1.25)-1.34
α_{xy}	-4.95(-4.38)-3.89	-4.69(-4.80)-4.91
ν_y	4.8299(4.83)4.8299	4.8301(4.83)4.83
α_{yx}	-4.95(-4.38)-3.89	-4.69(-4.80)-4.91
α_{yy}	94.5(94.9)95.3	15.9(15.9)15.9
Xco	-.00106(0).0011	-.00107(0).00109
R1	6.5(1.5)0.0	8.5(3.8)0.0
R2	0.0(0.0)2.8	0.0(0.0)1.0
R3	0.0(0.1)0.0	0.0(0.0)0.1

One can see from this table that although the amplitude dependences α_{xx}, α_{yy} are much stronger for chosen set nevertheless the dynamical properties R_i are pretty similar and stable for both sets. And that's the beauty of chosen set.

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REFERENCES

1. X. F.Zhao, S. Tepikian, S. Y. Lee,
   Some Corrections for the ACS Booster, Accelerator Division Technical Note No.232, January 29, 1986, BNL AGS Department.
2. E. D. Courant and Z. Parsa,
   Chromaticity Correction for the AGS Booster with 1,2,4,7 Sextupole Configuration, Booster Technical Note No. 17, March 5, 1986, BNL HEF Department.
3. Ramesh Chandra Gupta, Private communication, 1986.
4. G. Morgan and S. Kahn,
   Calculation of Eddy Currents in the Beam Tube, Booster Technical Note No. 4, January, 1986, BNL.