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EDDY CURRENTS IN BOOSTER VACUUM CHAMBERS

J. Claus

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Collider Accelerator Department

Brookhaven National Laboratory

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Brookhaven National Laboratory

Upton, N.Y. 11973

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(J. Claus)
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Abstract

The eddy currents in the vacuumchambers of the booster magnets and some of their consequences are discussed. These effects appear to be large enough to merit special attention. Possible ways to minimize some of them are described.

Introduction

The variation with time of the magnetic fields in cycling synchrotrons induce eddycurrents in the metallic parts of their magnets. These eddycurrents can delay and distort the guiding field. The effect is usually most pronounced at injection, when the ratio B/B tends to be high. This is particularly so in fast cycling machines with metallic vacuumchambers, in part because the eddycurrents in the vacuumchamber walls represent sources of field close to the particle trajectories. We investigate this effect for the booster proposed by BNL and find that it is sufficiently large to merit special attention. It is studied first separately for the dipoles and the quadrupoles. One conclusion is that the time delays in the dipoles could be rather larger than they are in the quadrupoles. Then some potential consequences of such differences in timedelays are discussed. It is found that they may induce time dependent variation of the focussing properties that can be large enough to require compensation. The effect of the eddycurrents on the fieldshape in the dipoles is also considered, separately for a constant B and for a B that varies sinusoidally with time, as a consequence of, e.g., ripple in the powersupply. For the former the well known contribution to the sextupolefield is recovered, the latter appears to induce very complex behaviour of the fieldshape with time.

1. Dipoles

The dipoles have windowframe-type yokes with airgaps of width w_p = 10" and height g = 3 1/4". A stainless steel vacuumchamber of somewhat complex cross-section is placed within these gaps. The gapfield B is essentially uniform over the extent of the chamber in the absence of eddy currents. However, the field must vary periodically as function of time and eddy-currents are generated in consequence. These currents delay and distort the net field relative to the intended one. They, and the fields they generate, are proportional in magnitude to the rate of change of the main field and therefore constant (in time) if the rate of change is constant. We make use of this constancy in estimating the magnitude of this eddy-current effect as function of the rate of change of the mainfield. It is also convenient to replace the actual vacuumchamber by an imaginary one of rectangular cross-section. We give this imaginary chamber a width w = 7", a height h = 2 3/4" and a wallthickness δ yet to be determined. The eddy-current density varies linearly with the distance x to the centerlines of top- and bottom covers and is zero on those centerlines according to:

$$j_z = \sigma E_z = -\sigma \dot{B} x \tag{1.1}$$

where σ represents the electrical conductivity of the material and $\dot{B}$ the rate of change of the mainfield. This current distibution, together with the currents in the sidewalls, gives rise to an eddy current induced field according to:

$$\Delta\tilde{B}(x) = -\mu_0 \frac{\sigma \dot{B}\delta}{g} \frac{w}{2}\left[\frac{w}{2}\left(1-\left(\frac{2x}{w}\right)^2\right)+h\right] \tag{1.2}$$

On the centerline $x = 0$, so that there:

$$\Delta\tilde{B}(0) = -\mu_0 \frac{\sigma \dot{B}\delta}{g} \frac{w}{2}\left(\frac{w}{2} + h\right) \qquad (=0.000322\ \dot{B}) \tag{1.3}$$

One may say that the dipole field is delayed, relative to the main field by:

$$\Delta t = \frac{\Delta\tilde{B}(0)}{\delta B/\delta t} = -\mu_0 \frac{\sigma\delta}{g} \frac{w}{2}\left(\frac{w}{2} + h\right) \qquad (=0.322\ \text{msec}) \tag{1.4}$$

It is also distorted by a sextupole field of strength:

$$\Delta\tilde{B}_s(x) = \mu_0 \frac{\sigma \dot{B}\delta}{g} \frac{2}{w} x^2 \qquad (=0.257\dot{B}x^2\ \text{T}) \tag{1.5}$$

That this field appears as a pure sextupole is a consequence of the shape assumed for the vacuum chamber, it will be more complex in practice according to the actual distribution of metal in the chamber cross-section. The location of the chamber in the gap will affect the field shape also. Because of the symmetry the dipole-type multipoles (i.e. multipoles of order 2(2n-1)) will dominate. In the numerical examples we used σ = 10 mho/m for the stainless steel of the vacuumchamber and δ = 1.5 mm for its wallthickness. In proton operation the guide field is to swing from 0.156T to 0.411T and back in 0.1 sec thus the smallest $\dot{B}$ possible is 5.1 T/sec. At injection the corresponding field offset ΔB is then ΔB = -1.64 mT, that is, 1.05%.

The situation is not so simple if dB/dt varies with time, because the field is pulsed with an approximately triangular pulse with a fundamental frequency of up to 10Hz or due to ripple in the main powersupply, for example. There are three effects:

1. the field within the vacuumchamber is shifted in phase relative to the current in the coils, the phase shift depending on frequency;
2. the fieldshape is perturbed, the degree of perturbation depends on frequency;
3. because the impedance of the magnet is a complicated function of frequency, the relation between the voltage ripple of the powersupply and the current ripple in the magnet is also complicated.

It is convenient to subdivide the space in the gap in three regions: the two regions under the poles to the left and right vertical walls of the vacuumchamber, the third one the space enclosed by that chamber. The vertical walls act as localized conductors of relatively low resistance. Together they may be regarded as a separate loop in which considerable current is induced. This loop is the dominant source of phase shifts and delays. The top and bottom covers of the chamber can be seen as extended sheets of relatively high resistivity and are the principal source of field distortions. In estimating this effect we assume that the magnet is driven by a sinusoidal current of predetermined frequency. Using standard eddy current formalism we calculate first the field distribution inside the vacuumchamber in terms of the B at the innersurfaces of the vertical chamberwalls. From this we obtain the total flux embraced by the loop formed by those walls. Then we calculate the current induced in that loop, using the flux just obtained and the resistance of the loop. Since the B on the inner surface of the walls is determined by both the magnet current and the loop current we can now calculate the magnet current that is necessary to obtain that B. Finally we calculate the total flux passing through the gap and use it to determine the terminal voltage of the magnet as well as its impedance. We disregard any effects due to the distributed capacitances and stray inductances in the magnet coils since they are not pertinent for this problem. We also disregard the currents in the laminations of the yoke, although these might very well be of consequence, and the leakage inductance of the loop formed by the vertical walls of the vacuumchamber. This leakage occurs if the height of the walls is less than the gapheight and/or the reluctance of the iron yoke is not zero. Using the conventional complex notation for time dependent variables we obtain for B(x) within the chamber as function of horizontal position x relative to the chamber axis:

$$\underline{B}(x) = \underline{B}\left(\frac{w}{2}\right) \frac{\cosh((\alpha + i\beta)x)}{\cosh((\alpha + i\beta)w/2)} \tag{1.6}$$

For all practical purposes $\alpha \approx \beta \approx (\omega\mu_0\sigma\delta/g)^{1/2} = 0.3788$ where, as before,

	σ	$= 10^6$ mho/m,	electrical conductivity of SS
	δ	$= 1.5$ mm,	thickness of top and bottom covers
	g	$= 3\,1/4''$,	gap height
	w	$= 7''$,	width of top or bottom covers
and where	nu	$= \omega/2\pi$,	frequency

Using this, we obtain via

$$\underline{B}(x) = \underline{B}_w \sqrt{\frac{\cosh(2\alpha x) + \cos(2\beta x)}{\cosh(\alpha w) + \cos(\beta w)}}\ \exp(i(\varnothing_1 - \varnothing_2)) \qquad (1.7)$$

where

$$\tan(\varnothing_1) = \tanh(\alpha x).\tan(\beta x)$$
$$\tan(\varnothing_2) = \tanh(\alpha w/2).\tan(\beta w/2)$$

and where $\underline{B}_w = \underline{B}(w/2)$ is the field along the inner surfaces of the vertical chamberwalls.

Expression (1.6) shows that the field is modified in a complex manner: both its amplitude and its phase depend upon the transverse position x. On the axis we have:

$$\underline{B}_0 = \frac{\underline{B}_w}{(\cosh(\alpha w) + \cos(\beta w))^{1/2}} \cos(\varnothing_2) \qquad (1.8)$$

while at times $\omega t = n\pi/2$:

$$\underline{B}(x) = \underline{B}_w \sqrt{\frac{\cosh(2\alpha x) + \cos(2\beta x)}{\cosh(\alpha w) + \cos(\beta w)}}\ \cos(\varnothing_1 - \varnothing_2) \qquad (1.9)$$

Integrating (1.5) we find for the flux that passes between the vertical chamberwalls per meter azimuthal length:

$$\underline{\Phi} = A\ .\ \underline{B}_w \qquad (1.10)$$

where

$$A = \frac{2}{\alpha + i\beta}\ \frac{\sinh((\alpha + i\beta).w/2)}{\cosh((\alpha + i\beta).w/2)}$$

and for the electric field that is generated in those walls:

$$E = -0.5\ d\Phi/dt = -i\ 0.5\ \omega\ \underline{\Phi} \qquad (1.11)$$

The current in the walls is then:

$$I_w = E\ \sigma\ \delta\ h \qquad (1.12)$$

where $h < g$ is their height. The current I_c in the excitation coils can now be determined via the relation:

$$B_w = \frac{\mu_0}{g}\ (I_c + I_w) \qquad (1.13)$$

resulting in

$$I_c = \frac{g}{\mu_0} B_w (1 + i\tfrac{1}{2}\omega\sigma\delta h \frac{\mu_0}{g} A) \qquad (1.14)$$

With the relation $B_w(I_c)$ known, one can calculate the magnet impedance per unit length dZ/dl. For this one needs the total flux that links the coil. We take for the effective fieldwidth the polewidth w_p plus a gapheight g. The two regions outside the vacuumchamber have then a combined width $w_s = w_p + g - w$. The flux in these regions is only driven by the coil current I_c. Adding it to the flux that links the vacuumchamber we find for the flux that links the coil:

$$Psi = \frac{\mu_0}{g} I_c \left(w_s + \frac{A}{1 + \tfrac{1}{2}\omega\ \sigma\delta h \frac{\mu_0}{g} A} \right) \qquad (1.15)$$

The magnet impedance per unit length dZ/dl is then:

$$dZ/dl = i\omega \frac{\mu_0}{g} \left(w_s + \frac{A}{1 + \tfrac{1}{2}\omega\ \sigma\delta h \frac{\mu_0}{g} A} \right) \qquad (1.16)$$

It should be noted that our expressions are normalized to an excitation coil of one turn, I_c has to be changed to nI_c, and Z to $n^2 Z$, if it has n turns. It is clear, that we disregarded the ohmic resistance of the coil in determining Z. We may now calculate the magnetic field within the vacuumchamber as function of the coil current I_c or as function of the magnet terminal voltage $U = Z.I_c$. The analytical expressions become cumbersome and are not particularly enlightening. We do not reproduce them here, and give the results of a numerical evaluation of B_0/U, B_0/I and Z as functions of frequency instead. They are summarized in figs 1 - 3, which show the behaviour of amplitude and phase of each function. The phase is given as an angle in the case of $Z = U/I_c$ and as a timedelay $t = \varnothing/\omega$ in the case of B_0/I and of B_0/U.

2. Quadrupoles

The quadrupoles have iron yokes with four expressed poles in a quadrupolar symmetry. We assume a circular cylindrical vacuumchamber of stainless steel with a diameter 2R of 8" and with a wallthickness δ still to be determined. We take it to be immersed in a truly quadrupolar field B such that

$$B_r(r,\Theta) = \dot{B}'.r.\sin(2\Theta), \qquad B_\Theta(r,\Theta) = \dot{B}'.r.\cos(2\Theta) \qquad (2.1)$$

where r and Θ represent position coordinates and $B' = B'(t)$ the fieldgradient. Because the field changes with time eddy currents are induced in the metallic vacuumchamber which generate an eddy current field. That field has again quadrupolar symmetry and a magnitude that is proportional with the rate of change of the main field. It follows that the perturbing field is constant in time if the rate of change of the main field is constant. Assuming that it is, we find for the eddy current distribution in the vacuumchamber:

$$j(\Theta) = \tfrac{1}{2}\sigma\dot{B}'R^2 \cos(2\Theta) \qquad (2.2)$$

and thus for for the induced quadrupole field:

$$\Delta\tilde{B}_r' = -\tfrac{1}{2}\mu_0\sigma\delta\dot{B}'R \sin(2\Theta), \qquad \Delta\tilde{B}_\Theta' = -\tfrac{1}{2}\mu_0\sigma\delta\dot{B}'R \cos(2\Theta) \qquad (2.3)$$

These last expressions assume that the reluctance beyond the outer surface of the vacuumchamber wall is negligeably small. That would require that the yoke consists of a block of high-μ_r material that fits tightly all around the vacuumchamber. In the actual quadrupoles such a tight fit exists only near the poletips. This does not affect the strength of the eddy current quadrupole very much but introduces 4(2n-1) poles. The shape and location of the vacuumchamber are also potential sources of distortion of the eddy current field. It is therefore important to have it properly centered and to give its shape at least quadrupolar symmetry if it can not be circular, this symmetry ensures that only 4(2n-1) poles are intro- duced and no others. One may say, in the same spirit as for the dipoles, that the eddy currents delay the actual quadrupole field relative to the expected one by

$$\Delta t = \Delta\tilde{B}'/\dot{B}' = \tfrac{1}{2}\mu_0\sigma\delta R \qquad (\approx 0.0957 \text{ msec}) \ (2.4)$$

if δ = 1.5 mm, as in the dipole. It follows that the offset of the quadrupole fields relative to their nominal value is:

$$\frac{\Delta\tilde{B}'}{B'} = \Delta t\frac{\dot{B}'}{B} = \Delta t\frac{\dot{B}'}{\dot{B}}\,\frac{\dot{B}}{B'} = \Delta t\frac{B'}{B}\,\frac{\dot{B}}{B'} = \Delta t\frac{\dot{B}}{B} = 3.12\ 10^{-3}$$

at the nominal instant of proton injection.

3. Consequences

The consequences of these eddy currents depend in part on the way the dipoles and the quadrupoles are excited. We consider only two of the many possible arrangements: in the first one all main excitation coils of all major magnets, dipoles and quadrupoles, are connected in series, and supplied from a single powersupply, in the other the dipoles and quadrupoles form two separate chains, each with its own supply. If transmissionline-like effects are neglected, all magnets see the same driving current in the first case, while the dipole and quadrupole currents are mutually semi-independent in the second (they are still coupled via the powergrid). We have seen that the eddy currents in the vacuum chamber of a magnet delay the magnetic field within it relative to the driving current. Eddy currents in the other metallic components, e.g., in the polepieces and in the yoke, contribute to this delay. If the magnets in a chain all have the same net delay, the whole field pattern, disregarding the multipoles due to eddy currents for the moment, is shifted by that amount, relative to the driving current and no error, apart from an easily compensated timing error, is generated. Different delays in individual magnets however, cause changes in field pattern that change with field level, i.e., with the energy of the synchronous particle. E.g., a difference in the delays of dipoles and quadrupoles causes a modulation of the betatron frequencies with synchronous energy if the closed orbit is kept fixed. The magnitude of the effect is easily calculated:

relative error in dipole field: $\frac{\Delta B}{B} = \frac{\dot{B}}{B}\Delta t_{dip}$

relative error in quadrupole field: $\frac{\Delta B'}{B'} = \frac{\dot{B}'}{B'}\Delta t_{quad}$

shift in tune: $\Delta\nu = \frac{N}{2\pi}\tan(\tfrac{1}{2}\Delta\psi)\left(\frac{\Delta B'}{B'} - \frac{\Delta B}{B}\right)$

$$= \frac{12}{\pi}\tan(36.2^\circ)\,\frac{\dot{B}}{B}\,(0.0957 - 0.322)10^{-3} \qquad (3.1)$$

where N is the number of cells and $\Delta\psi$ the phase advance per cell. The numerical example applies if the dipoles and quadrupoles are in series. From this, and from the rough numbers already given, we have at

proton injection ($B = 0.156$ T, $\dot{B} = 5.1$ T/sec): $\Delta\nu = -0.0206$
proton extraction ($B = 0.411$ T, $\dot{B} = 5.1$ T/sec): $\Delta\nu = -0.0078$

if the dipoles and quadrupoles are electrically in series without further corrections. Expression (3.1) shows that the effect increases with increasing dB/dt, with decreasing B and with increasing difference between delays ($\Delta t_{dip} - \Delta t_{quad}$), it would therefore be worst at the injection of the heaviest ions, for which B_{inj} is lowest and B_{ex} highest.

4. Remedies

There are three different cures for this problem. A very simple one is to make the difference in delays zero. This can be done by apropriate adjustment of the wall thicknesses of the vacuumchambers. It follows from expressions (1.3) and (2.4), that the delays due to the vacuumchambers in dipole and quadrupoles are equal if:

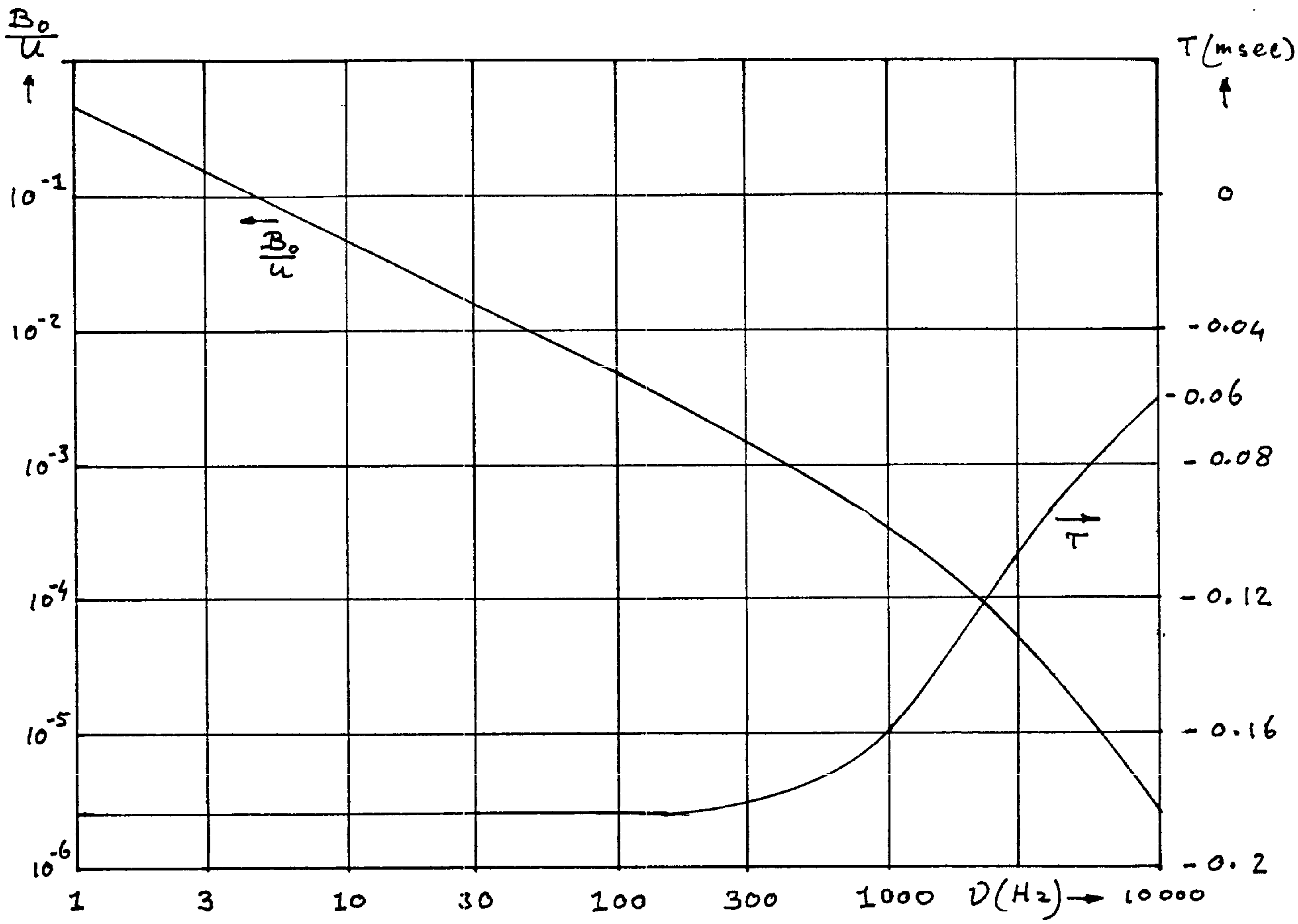
$$\frac{\delta_{quad}}{\delta_{dip}} = \frac{w}{gR} \left(\tfrac{1}{2}w + h\right) \qquad (\approx 3.36) \quad (4.1)$$

One expects that delay differences due to the differences in structure of the dipole and quadrupole yokes, if of any consequence, can be controled in a similar manner, e.g., by adjustment of the resistivity and/or thickness of the iron laminations. It is evident from the examples above that there is no need for high accuracy. It may be seen from (4.1), that the condition for equal delays is independent from the value of dI/dt. This means that, in first approximation, the magnetic fields behave synchronously, even if dI/dt changes during the accelerating cycle. Only in first approximation, because the various eddycurrent circuits exhibit a multitude of timeconstants so that the effective fields do not necessarily remain matched while dI/dt changes from one value to another. One expects this to be a minor effect.

Another possibility is to restrict the magnitude of I/I. This implies in general, that dI/dt will have to be reduced when I is small, i.e., early in the accelerating cycle, using the magnet power supply as a properly programmed function generator. This can be done, though perhaps not without increasing its ripple component. The consequences of such ripple will have to be studied. A third one is to have the dipole chain and the quadrupole chain separately excited and to delay the quadrupole powersupply with respect to the dipole powersupply by the difference in dipole and quadrupole delay times. The ripple currents to dipolechain and quadrupolechain are likely to be rather different in amplitudes and phases in this arrangement because their impedances are different.

5. Conclusion

It is evident that this discussion just skims the surface of its subject, any real magnet can be expected to behave in a much more complex manner than the simple model used for our estimates. These estimates show however effects of sufficient magnitude to require further investigation. It is advisable to measure the impedance- and transfer func tions as functions of frequency of prototypes of dipole dipoles and quadrupoles before deciding on the arrangement of circuits, powersupplies, characteristics of passive/active ripple filters etc.



$\frac{B_o}{U}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
T (msec)
0
- 0.04
- 0.06
- 0.08
- 0.12
- 0.16
- 0.2
$\frac{B_o}{U}$
T
1
3
10
30
100
300
1000
ν (Hz)
10000

Fig 1: $B_o/U = f(\nu)$