BNL-105066-2014-TECH

Booster Technical Note No. 19;BNL-105066-2014-IR

Coherent Instability in the Booster

S. Y. Lee

March 1986

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

COHERENT INSTABILITY IN THE BOOSTER

Booster Technical Note

No. 19

S. Y. LEE, J. M. WANG, W. F. ZHAO

MARCH 10, 1986

HIGH ENERGY FACILITIES
Brookhaven National Laboratory
Upton, N.Y. 11973

Coherent Instability in the Booster

S.Y. Lee, J.M. Wang and X.F. Zhao

Brookhaven National Laboratory

ABSTRACT

Longitudinal and transverse instability for the booster is evaluated. We found that the fast growth in the microwave region is not important. The slow growth of the head-tail mode can not be damped due to large space charge impedance at the low energy. Because of its small growth rate, the total growth is found to be maximum a few percent in the transverse phase space area.

1) Introduction

The coherent instability for a low energy booster is dominated by the space charge effect. To obtain the limitation of the booster performance and to evaluate the requirement of the correction elements in the booster, we shall concentrate in this paper to the collect effects. Section 2 discuss the single bunch phenomena and section 3 discuss the coupled bunch modes. The conclusion is given in section 4.

2) Single bunch collective effects.

In the single bunch collective effect we shall discuss the longitudinal motion as well as the transverse resistive wall instability. Table 1 list the relevant parameters in the booster performance for various ion species.

Table 1 Booster Parameters in the coherent effect

part.species	P	D	C	S	Cu	I	Au
A	1	2	12	32	63	127	197
<Q>	1	1	6	14	21	29	33
N(10**9)	3000	100	22	6.7	4.7	3.2	2.2
beta(inj)	0.5662	0.1768	0.1262	0.1002	0.0782	0.0595	0.0463
gamma(inj)	1.21319	1.01600	1.00805	1.00505	1.00307	1.00177	1.00107
eps(10-6pi)	50	50	50	50	50	50	50
epsn	34.3456	8.98148	6.36085	5.03534	3.92201	2.98028	2.31748
eta	-0.6360	-0.9253	-0.9406	-0.9465	-0.9504	-0.9530	-0.9544
space charge							
dnu(F/B)	0.07681	0.02235	0.05929	0.05901	0.07813	0.08727	0.08289
@injection							
Trev(10^-6s)	1.18875	3.80696	5.33336	6.71727	8.60704	11.3121	14.5371
N(10^9)/mus		3.28	0.5125	0.125	0.06875	0.0375	0.03125
Tfill(mus)	100	30.4878	42.9268	53.6	68.3636	85.3333	70.4
<N>fill		8.00843	8.04872	7.97942	7.94275	7.54353	4.84275
frev*2*pi	5285532	1650445	1178089	935376.	730004.	555438.	432215.
rf scenario							
	V=450 Volts Phi=0. degree and h=3						
area-evs/amu	0.11763	0.06310	0.06234	0.05805	0.05051	0.04172	0.03570
f(syn)*2pi	3247.04	3026.27	3063.24	2878.64	2520.38	2090.22	1792.21
Theta^	0.30710	0.30710	0.30710	0.30710	0.30710	0.30710	0.30710
deltaE	107409.	35986.1	152261.	300171.	401347.	508491.	525129.
dE/E(^-3)	0.09509	0.01902	0.01351	0.01002	0.00682	0.00429	0.00286
dp/p(^-3)	0.29663	0.60854	0.84889	0.99848	1.11553	1.21262	1.33420
Resist Wall	0.85714	0.03	0.035	0.00021	769230	377	
coastingbeam							
U=	-39790.	-3615.9	-6846.2	-5409.5	-5590.0	-4750.5	-3510.9
V=	422.964	4.70375	5.28709	2.93800	2.08495	1.17289	0.59420
Total growth	1.04320	1.00014	1.00022	1.00015	1.00014	1.00010	1.00004

2a) Microwave instability fast growth

The fast growth microcwave instability can be estimated by the Keil-Schnell formula. The beam paticles are injected and extracted below transition energy. Table 2 lists the impedance limit of the booster, where

$$|Z_{\parallel}/n| < \frac{2\pi|\eta|EA}{qeI_p}\left(\frac{\sigma_p}{p}\right)^2$$

$$|Zt/n| < \frac{4\sqrt{2\pi}\,|\eta|EA}{qeI_p\bar{\beta}}\left(\frac{\sigma_p}{p}\right)$$

Table 2. Microwave instability limit

	part.spices	P	D	C	S	Cu	I	Au
Z/n	long(Ohm)	120.	24000	2.9E6	2.E7	9.E7	4.E8	2.E9
Zt	trans(MOhm/m)	97.	9.4E4	8.4E5	5.E6	2.E7	8.E7	3.E8

The total impedance of the booster ring can be calculated by using programs such as URMEL and TBCI of T. Weiland of DESY. K.Y. Ng of FNAL calculated the impedance of the booster ring to obtain Z/n=3.3 Ohms for the longitudinal motion and Z/n=.02MOhms for the transverse motion. We therefore conclude that the fast growth microwave instability will not be important in the booster.

2b) Slow growth collective effects

The coherent growth due to the resistive wall is related to the transverse coupling impedance, Zt

$$Z_t = -\frac{i}{e\beta I_0\Delta}\int\langle F_y\rangle dz = iRZ_0\left[\frac{1}{\beta^2\gamma^2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)-(1+i)\frac{\delta}{b^3}\right]$$

The coefficients for the dispersion relation, U+(1+i)V, is given by

$$U+(1+i)V = \frac{Nc r_0}{2\pi a^2 Q\beta\gamma}\left[-\frac{1-\kappa^2}{\gamma^2}+(1+i)\beta^2\kappa^2\frac{\mu\delta}{b}\right]$$

We shall solve the dispersion relation,

$$1 = \left[U+(1+i)V\right]\int\frac{f(p)dp}{\omega-(n-Q)\Omega}$$

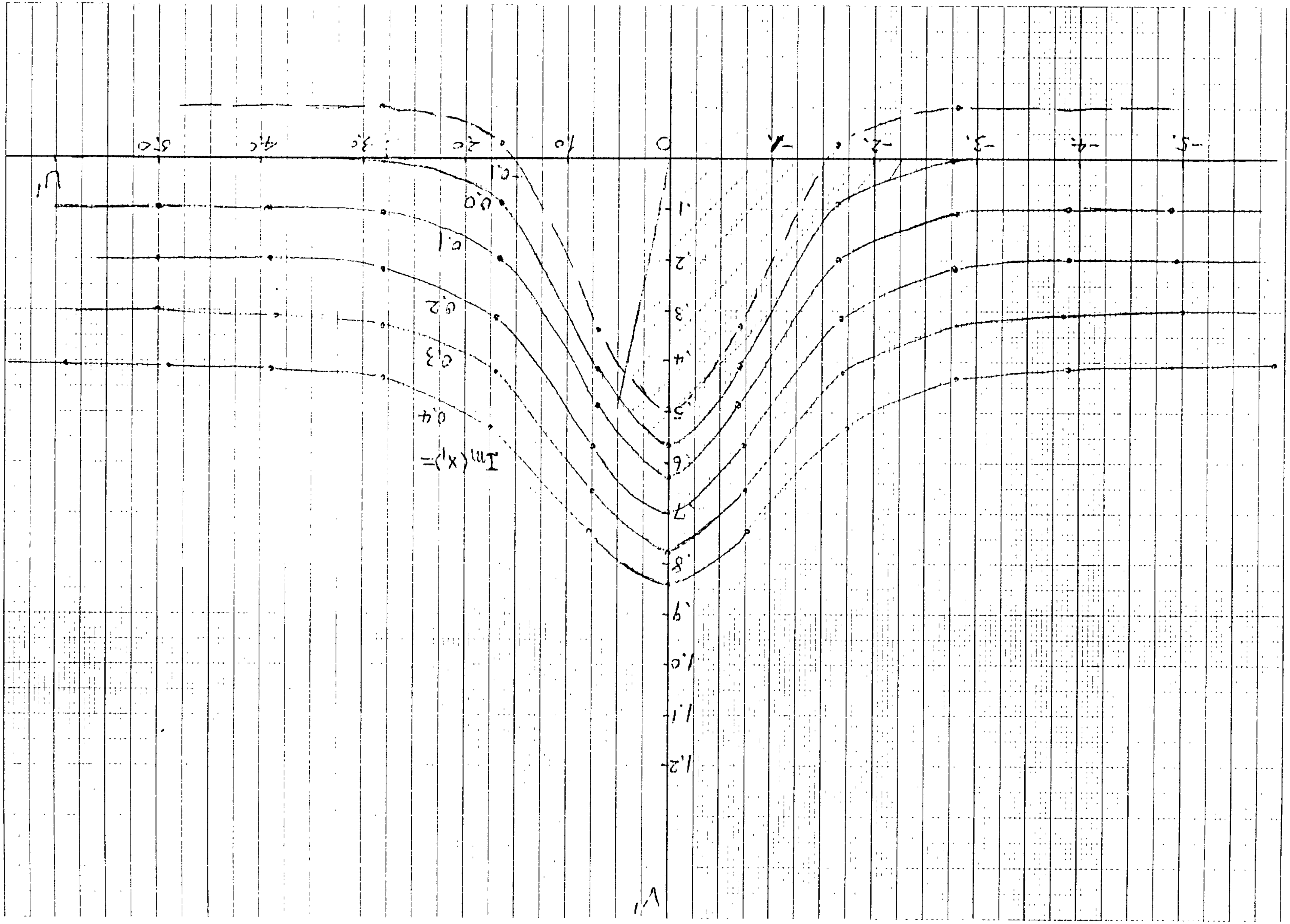
by using the Gaussian distribution in the momentum distribution. The result is given in Fig. 1, where we have used the normalized unit, i.e.

$$U'+(1+i)V' = \left[U+(1+i)V\right]/\Delta S'$$

with

$$\Delta S = |(n-Q_0)\eta+Q'|\,\Omega_0\frac{\Delta p}{p}$$

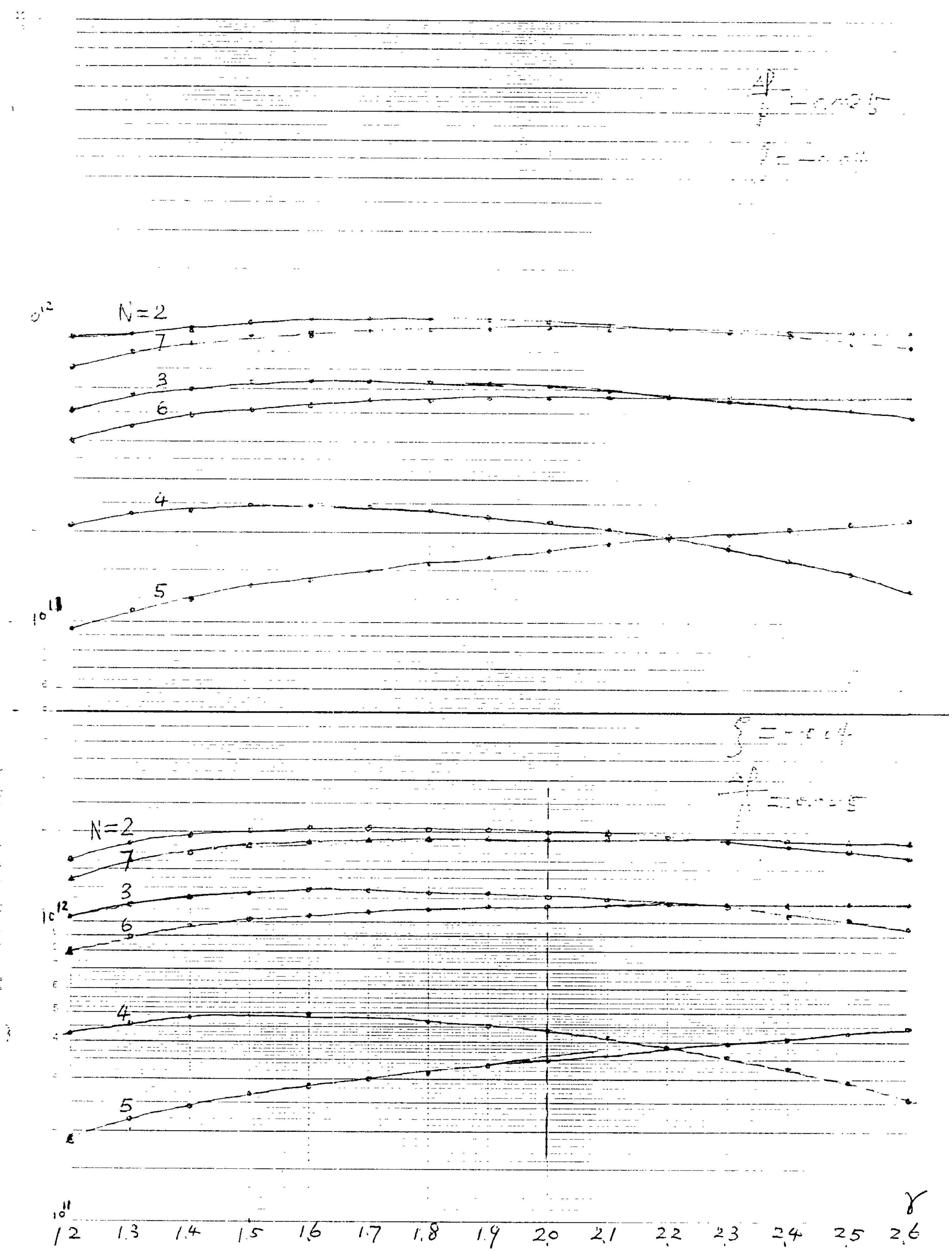
461510



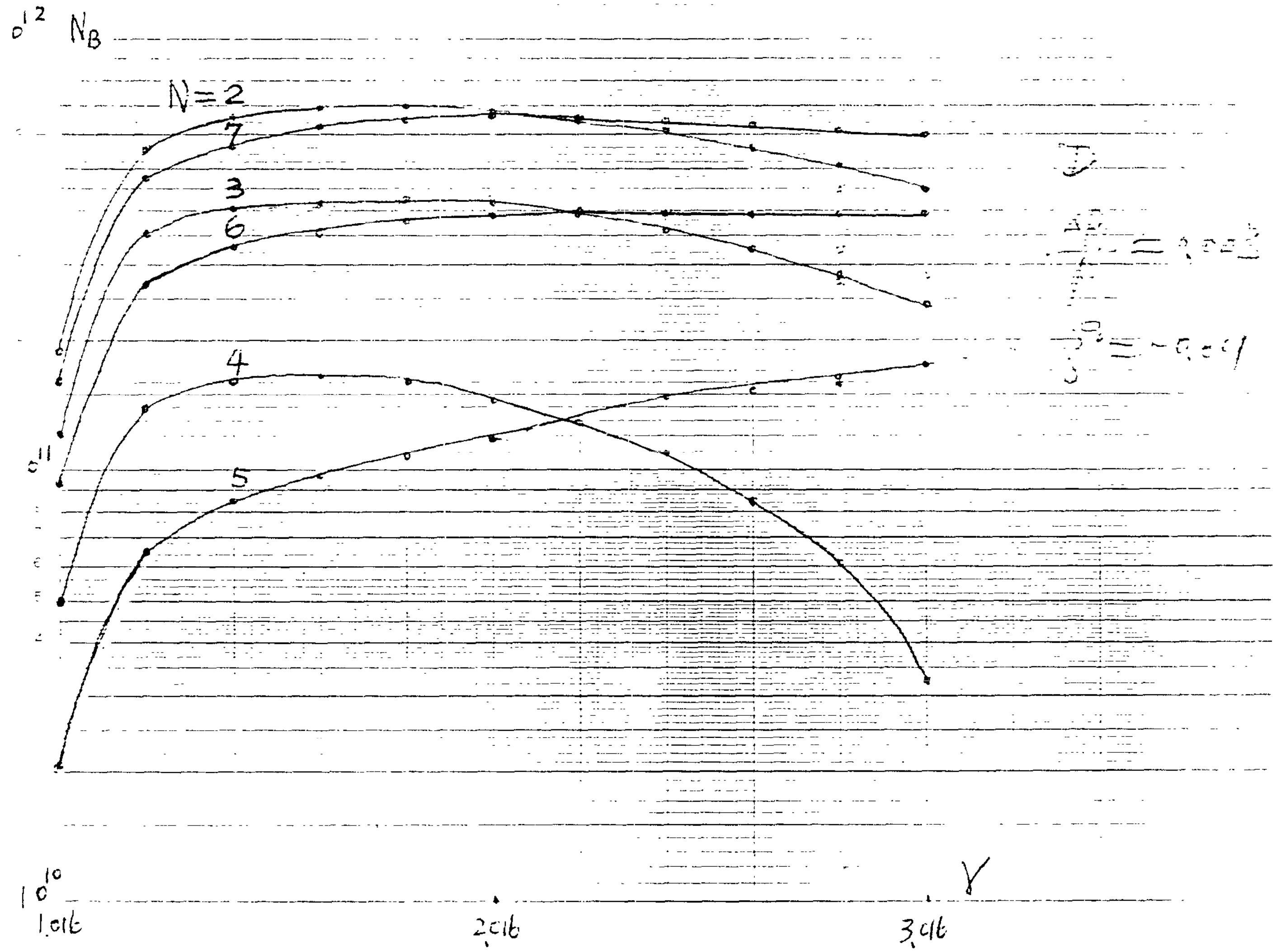
We observe that when the real part,U (corresponding to imaging part of the impedance) becomes very large. The landau damping becomes ineffective. The growth rate equals the resistive wall contribution. Table 1 lists the V component. The total growth of the transverse phase space is given by

$$G = \exp \left[\int \mathrm{Im}\,\omega \, dt \Big|_{\mathrm{Im}\,\omega > 0} \right]$$

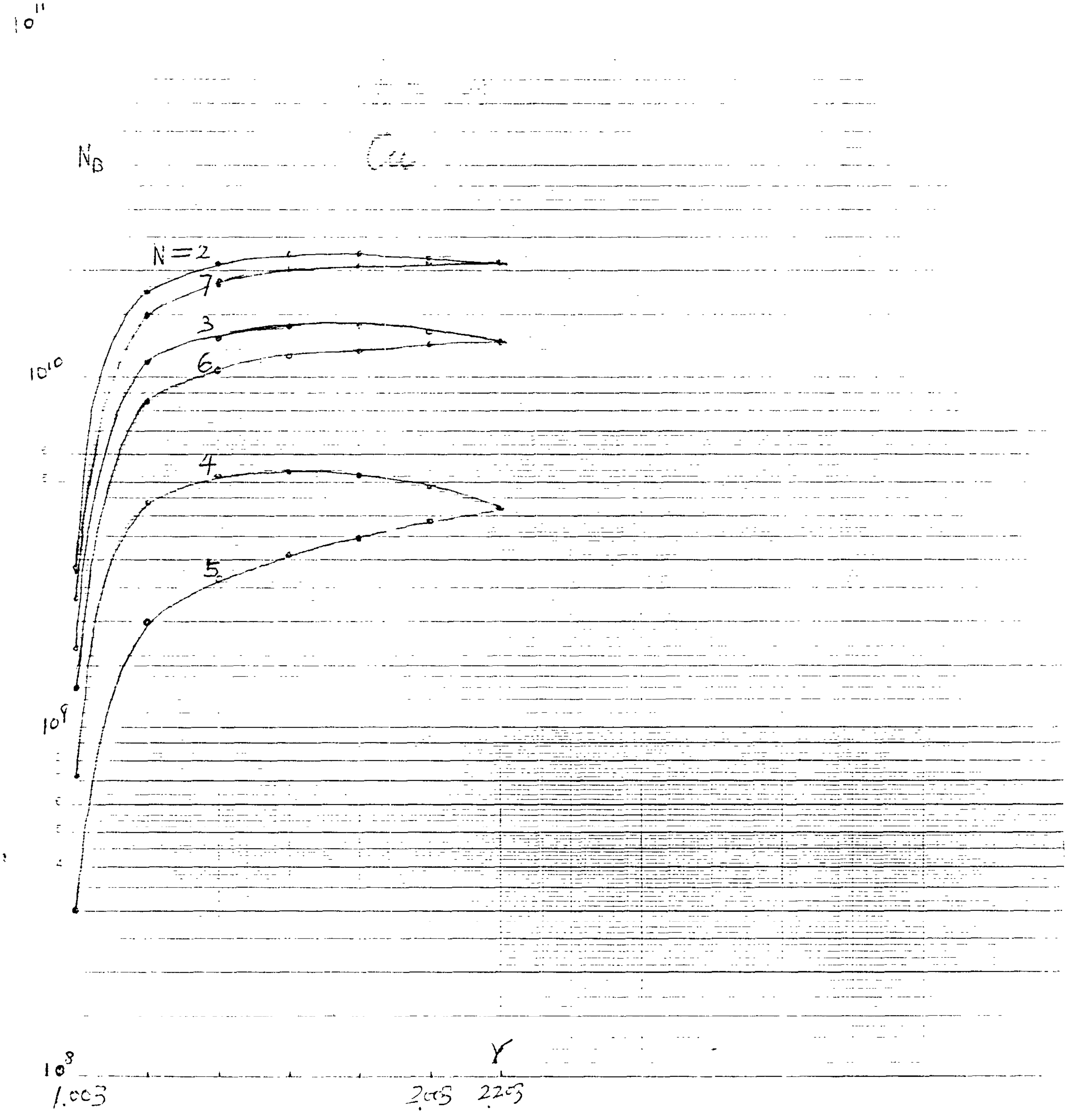
where we have integrated the growth rate with the storage time in the booster. Figs.2-6 show the instability limit as a function of the beam energy. At the low energy, the space charge impedance is large, we shall encounter instability irrespective of the Landau damping width. The Landau damping width, which induces mixing, shall be important in the dissipatioin of the growth into the phase space area. These damping width can be adjusted by the chromatic sextupoles. Since the total growth is of the order of a few percents. The transverse instability may not be important. Because of the slow growth, a feed back system shall still be considered to cure the head-tail mode.



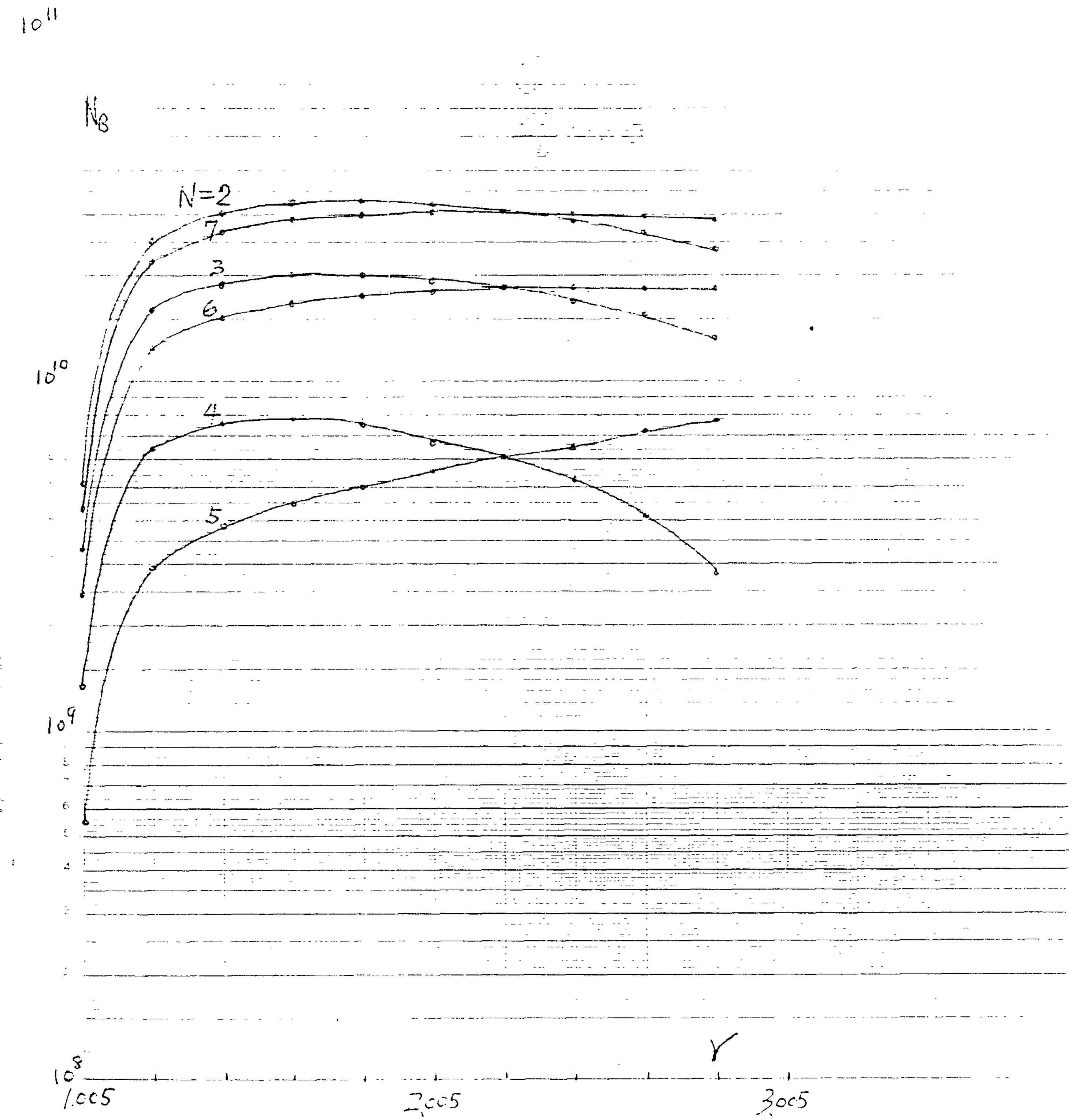
N=2
7
3
6
4
5
10^{11}
N=2
7
3
10^{12}
6
4
5
10^{11}
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6
γ



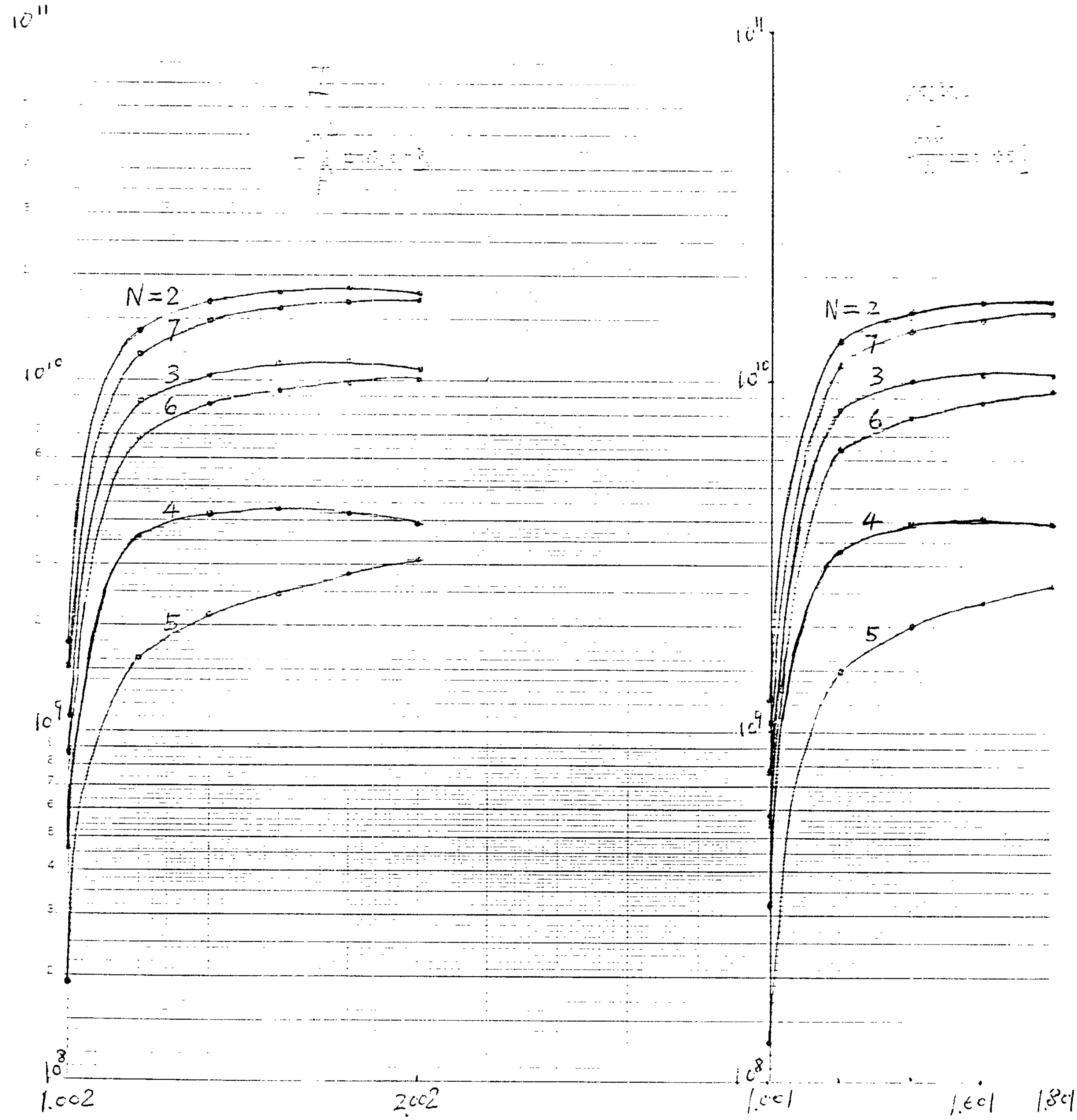
10^{12} N_B
N=2
7
3
6
4
5
10^{11}
10^{10}
1.0E
2.0E
3.0E
γ



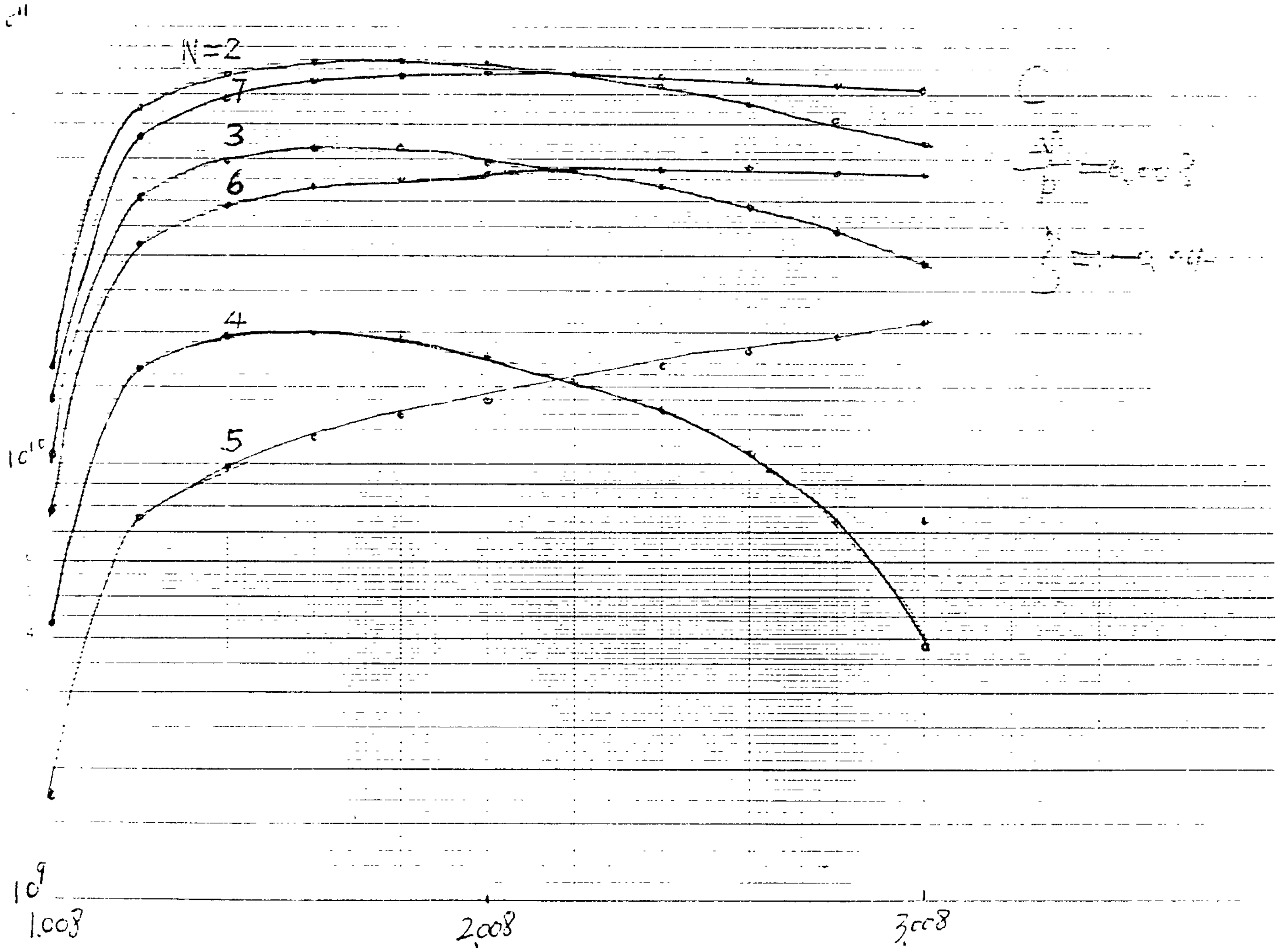
10^11
N_B
Cu
N=2
7
3
6
10^10
4
5
10^9
10^8
Y
1.003
2.005
2.205



10^{11}
N_B
N=2
7
3
6
10^{10}
4
5
10^9
10^8
Y
1.005
2.005
3.005



10^11
N=2
7
3
6
4
5
10^10
10^9
10^8
1.002
2002
1.001
1.601
1801



N=2
7
3
6
4
5
10^10
10^9
1.008
2.008
3.008

3) Couples bunch impedance in the rf detuning.

The booster shall be operated at h=3 harmonic mode, therefore there are possibly 3 coupled bunch mode in the booster. The eigenstate of the coupled bunch mode have the following coherent frequency shift,

$$\Delta\Omega = i \; \frac{q e \eta I_0 \omega_0^2}{2\pi E \omega_{s0}} \; \frac{1}{2^a (a-1)!} \; \hat{\theta}^{2a-2} \; Z_{eff} \qquad ; \; \hat{\theta} = \text{rms bunch length}$$

where I_0 is the total average current and the effective impedance is given by

$$Z_{eff} = \sum_{n=-\infty}^{\infty} (nh+s)^{2a-1} \; Z[(nh+s)\omega_0 + \Omega] \, e^{-\frac{1}{2}(nh+s)^2 \hat{\theta}^2} \qquad ; \; \Omega \simeq a\omega_{s0}$$

$$\xrightarrow[h\hat{\theta} \gtrsim 1]{} \quad \bar{Z} \cdot \frac{(a-1)!}{\hat{\theta}^{2a-1}}$$

Since the synchrotron frequency ω_s is a function of the synchrotron amplitude, the resulting frequency provides Landau damping for the coupled bunch motion, i.e.

$$1 = -i \; \frac{\eta \omega_0^3 h}{2\pi p \beta c} \; a \left[\frac{1}{2^a a!}\right]^2 Z_{eff} \int_0^\infty dr \; \frac{r^{2a} \psi_0'(r)}{\Omega - a\omega_s(r)}$$

where we have integrated the synchrotron amplitude in the dispersion integral. Assuming that all the nonlinearity of the synchrotron frequency is provided by the natural rf voltage,

$$V(t) = \hat{V} \sin(h\omega_0 t)$$

we obtain the limiting impedance to be

$$|Z| \leq \frac{2\pi E \omega_{s0}}{q e \eta I_0 \omega_0^2} \; \frac{2^a a!}{\hat{\theta}^{2a-2}} \; 0.32 \, F_a' \; \frac{h^2}{16} \omega_{s0} \, (2.5\hat{\theta})^2$$

Table 3 shows that limit impedance of the booster for the stability of the copled bunch motion. The stability limit may be severe only in the case of proton beams, where ¦Zeff¦<6000 Ohm and ¦Zeff¦<35[illegible]Ohm is required to have stable coupled bunch motion for dipole mode and for quadrupole mode respectively.

Table 3 Impedance limit for the Coupled bunch mode instability

Coupled bunch instability							
Ion species	P	D	C	S	Cu	I	Au
dOhmegaS	1076.63	1003.43	1015.68	954.479	835.690	693.060	594.248
Zmax/factor	1151.22	1134339	2362859	6643300	1E+07	3E+07	9E+07
Zmax(a=1)	5825.34	5739921	1E+07	3E+07	7E+07	2E+08	4E+08
Zrw/Zmax	13.8020	0.02506	0.01424	0.00568	0.00321	0.00152	0.00063

Mode factors						
a	1	2	3	4	5	6
Fa'	0.777	1.12	1.37	1.59	1.78	1.96
total factor=	5.06014	309.345	24072.4	2369796	3E+08	4E+10

4) Conclusion

In conclusion we have studied the possible coherent effects for the bunched beam in the booster. The transverse resistive wall instability can contribute at most a few percent of the phase space growth. The growth rate is much smaller than the synchrotron grequency. Thus these mode corresponds to the head tail instability,which can be controled by the feed back system. We examine also the coupled bunch instability and calculate the preliminary effective impedance for the coupled bunch motion. The impedance shall be evaluated when the rf design is tested. Our conclusion is however that only the proton may be important to the coupled bunch motion.