

Coherent Instability in the Booster

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COHERENT INSTABILITY IN THE BOOSTER

Booster Technical Note

No. 19

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ABSTRACT

Longitudinal and transverse instability for the booster is evaluated. We found that the fast growth in the microwave region is not important. The slow growth of the head-tail mode can not be damped due to large space charge impedance at the low energy. Because of its small growth rate, the total growth is found to be maximum a few percent in the transverse phase space area.

1) Introduction

The coherent instability for a low energy booster is dominated by the space charge effect. To obtain the limitation of the booster performance and to evaluate the requirement of the correction elements in the booster, we shall concentrate in this paper to the collect effects. Section 2 discuss the single bunch phenomena and section 3 discuss the coupled bunch modes. The conclusion is given in section 4.

2) Single bunch collective effects.

In the single bunch collective effect we shall discuss the longitudinal motion as well as the transverse resistive wall instability. Table 1 list the relevant parameters in the booster performance for various ion species.

Table 1 Booster Parameters in the coherent effect

part. species	P	D	C	S	Cu	I	Au
A	1	2	12	32	63	127	197
<Q>	1	1	6	14	21	29	33
N(10**9)	3000	100	22	6.7	4.7	3.2	2.2
beta(inj)	0.5662	0.1768	0.1262	0.1002	0.0782	0.0595	0.0463
gamma(inj)	1.21319	1.01600	1.00805	1.00505	1.00307	1.00177	1.00107
eps(10-6pi)	50	50	50	50	50	50	50
epsn	34.3456	8.98148	6.36085	5.03534	3.92201	2.98028	2.31748
eta	-0.6360	-0.9253	-0.9406	-0.9465	-0.9504	-0.9530	-0.9544
space charge dnu(F/B)	0.07681	0.02235	0.05929	0.05901	0.07813	0.08727	0.08289
@injection							
Trev(10^-6s)	1.18875	3.80696	5.33336	6.71727	8.60704	11.3121	14.5371
N(10^9)/mus		3.28	0.5125	0.125	0.06875	0.0375	0.03125
Tfill(mus)	100	30.4878	42.9268	53.6	68.3636	85.3333	70.4
<N>fill		8.00843	8.04872	7.97942	7.94275	7.54353	4.84275
frev*2*pi	5285532	1650445	1178089	935376.	730004.	555438.	432215.
rf scenario	V=450 Volts Phi=0. degree and h=3						
area-evs/amu	0.11763	0.06310	0.06234	0.05805	0.05051	0.04172	0.03570
f(syn)*2pi	3247.04	3026.27	3063.24	2878.64	2520.38	2090.22	1792.21
Theta^	0.30710	0.30710	0.30710	0.30710	0.30710	0.30710	0.30710
deltaE	107409.	35986.1	152261.	300171.	401347.	508491.	525129.
dE/E(^-3)	0.09509	0.01902	0.01351	0.01002	0.00682	0.00429	0.00286
dp/p(^-3)	0.29663	0.60854	0.84889	0.99848	1.11553	1.21262	1.33420
Resist Wall coastingbeam	0.85714	0.03	0.035	0.00021	769230	377	
U=	-39790.	-3615.9	-6846.2	-5409.5	-5590.0	-4750.5	-3510.9
V=	422.964	4.70375	5.28709	2.93800	2.08495	1.17289	0.59420
Total growth	1.04320	1.00014	1.00022	1.00015	1.00014	1.00010	1.00004

2a) Microwave instability fast growth

The fast growth microwave instability can be estimated by the Keil-Schnell formula. The beam particles are injected and extracted below transition energy. Table 2 lists the impedance limit of the booster, where

$$|Z_{||}/n| < \frac{2\pi |\eta| EA}{9e I_p} \left(\frac{\sigma_p}{P}\right)^2$$

$$|Z_t/n| < \frac{4\sqrt{\pi} |\eta| EA}{9e I_p \beta} \left(\frac{\sigma_p}{P}\right)^{\frac{1}{2}}$$

Table 2. Microwave instability limit

part.spices	P	D	C	S	Cu	I	Au
Z/n long(Ohm)	120.	24000	2.9E6	2.E7	9.E7	4.E8	2.E9
Zt trans(MOhm/m)	97.	9.4E4	8.4E5	5.E6	2.E7	8.E7	3.E8

The total impedance of the booster ring can be calculated by using programs such as URMEL and TBCI of T. Weiland of DESY. K.Y. Ng of FNAL calculated the impedance of the booster ring to obtain $Z/n=3.3$ Ohms for the longitudinal motion and $Z/n=.02$ MOhms for the transverse motion. We therefore conclude that the fast growth microwave instability will not be important in the booster.

2b) Slow growth collective effects

The coherent growth due to the resistive wall is related to the transverse coupling impedance, Z_t

$$Z_t = -\frac{i}{e\beta I_0 \Delta} \int \langle F_y \rangle dz = i R Z_0 \left[\frac{1}{\beta^2 \gamma^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) - (1+i) \frac{\delta}{b^3} \right]$$

The coefficients for the dispersion relation, $U+(1+i)V$, is given by

$$U+(1+i)V = \frac{Nc r_0}{2\pi a^2 Q \beta \gamma} \left[-\frac{1-\kappa^2}{\gamma^2} + (1+i) \beta^2 \kappa^2 \frac{\mu \delta}{b} \right]$$

We shall solve the dispersion relation,

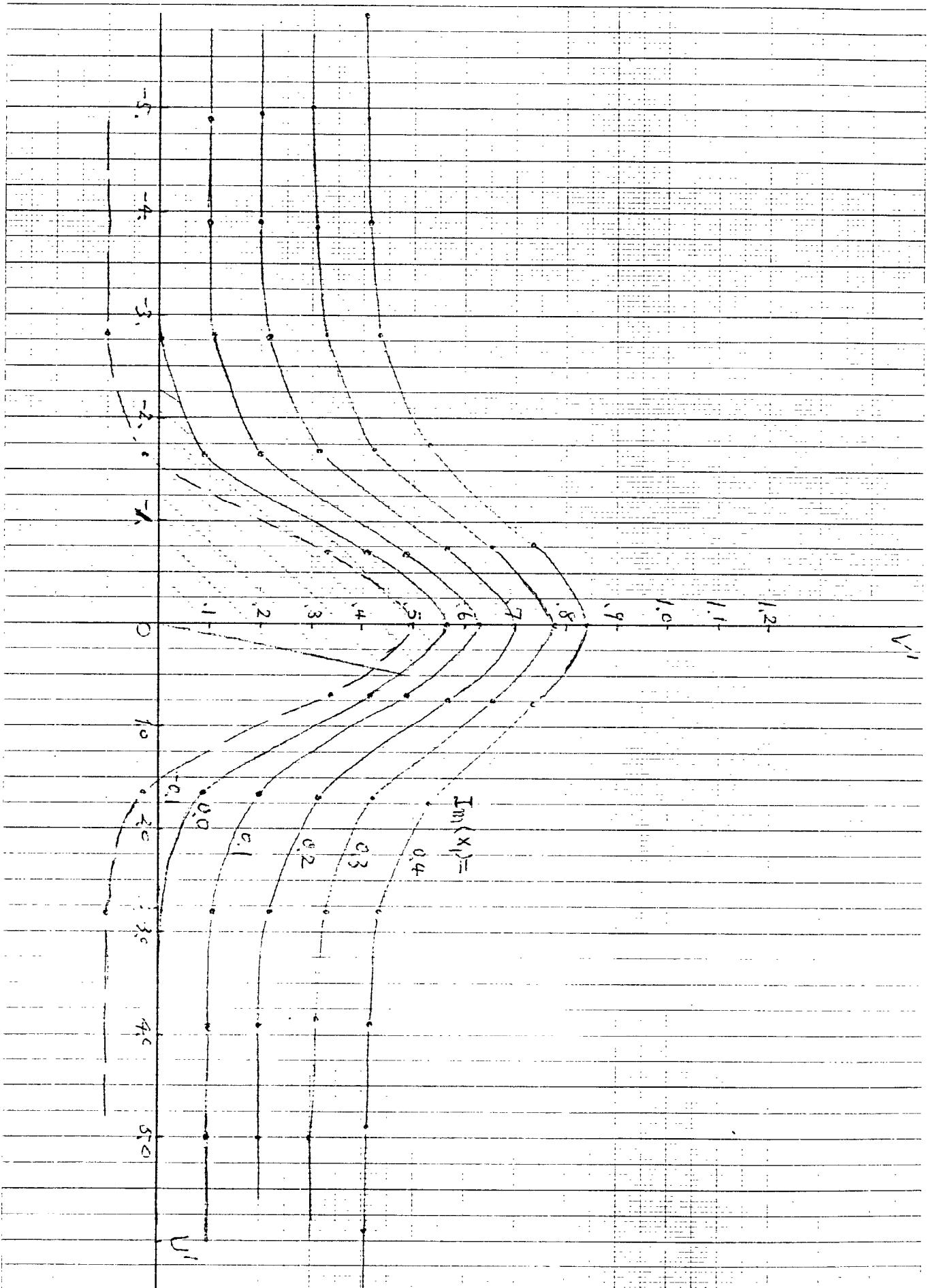
$$1 = [U + (1+i)V] \int \frac{f(p) dp}{\omega - (n - Q)\Omega}$$

by using the Gaussian distribution in the momentum distribution. The result is given in Fig. 1, where we have used the normalized unit, i.e.

$$U' + (1+i)V' = [U + (1+i)V] / \Delta S'$$

with

$$\Delta S = |(n - Q_0) \eta + Q' | \Omega_0 \frac{4p}{P}$$



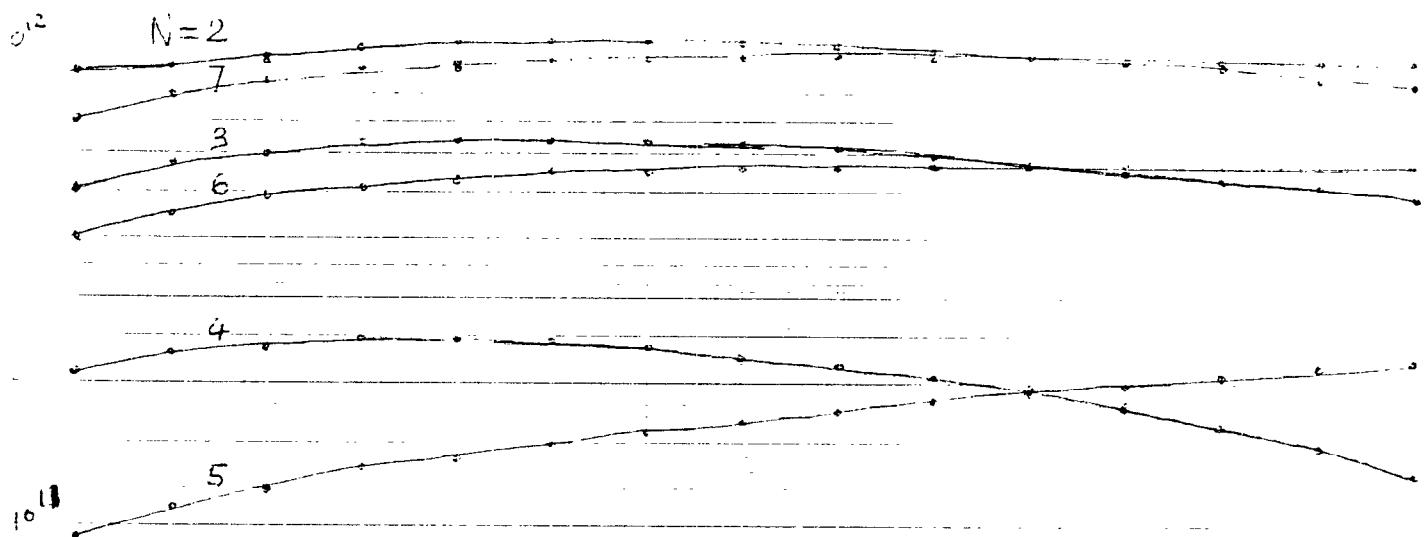
We observe that when the real part, U (corresponding to imaging part of the impedance) becomes very large. The Landau damping becomes ineffective. The growth rate equals the resistive wall contribution. Table 1 lists the V component. The total growth of the transverse phase space is given by

$$G = \exp \left[\int \text{Im } \omega \, dt \Big|_{\text{Im } \omega > 0} \right]$$

where we have integrated the growth rate with the storage time in the booster. Figs. 2-6 show the instability limit as a function of the beam energy. At the low energy, the space charge impedance is large, we shall encounter instability irrespective of the Landau damping width. The Landau damping width, which induces mixing, shall be important in the dissipation of the growth into the phase space area. These damping width can be adjusted by the chromatic sextupoles. Since the total growth is of the order of a few percents. The transverse instability may not be important. Because of the slow growth, a feed back system shall still be considered to cure the head-tail mode.

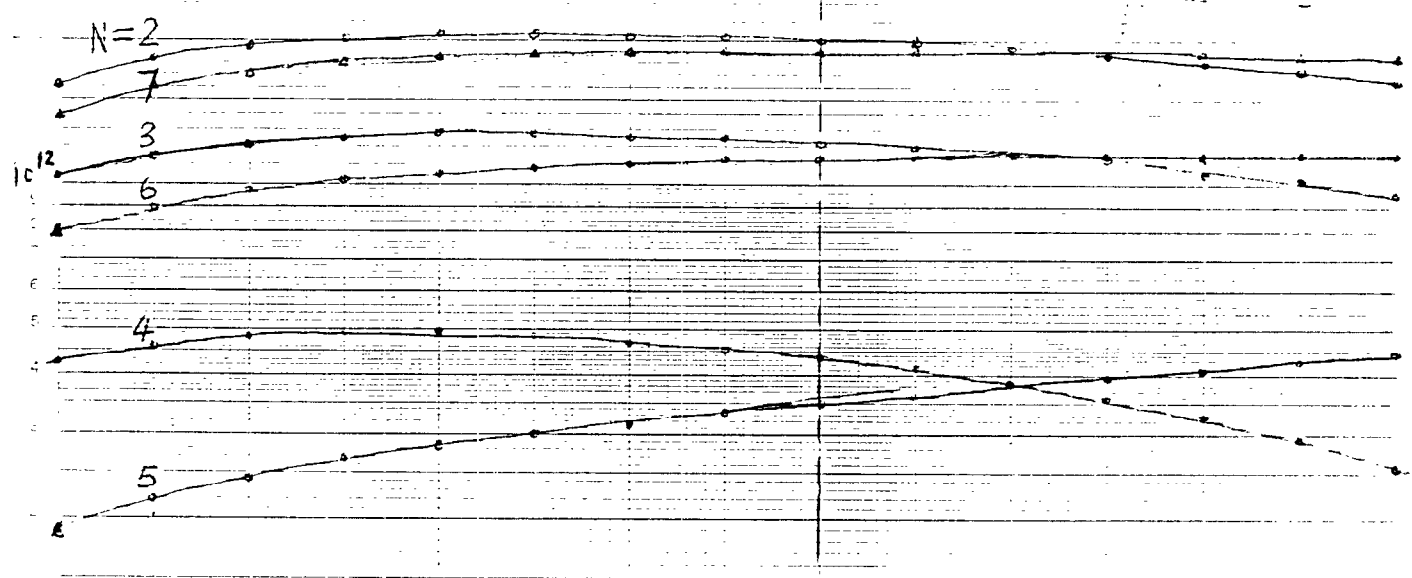
$$\frac{4}{5} = -0.25$$

$$\frac{1}{2} = -0.5$$



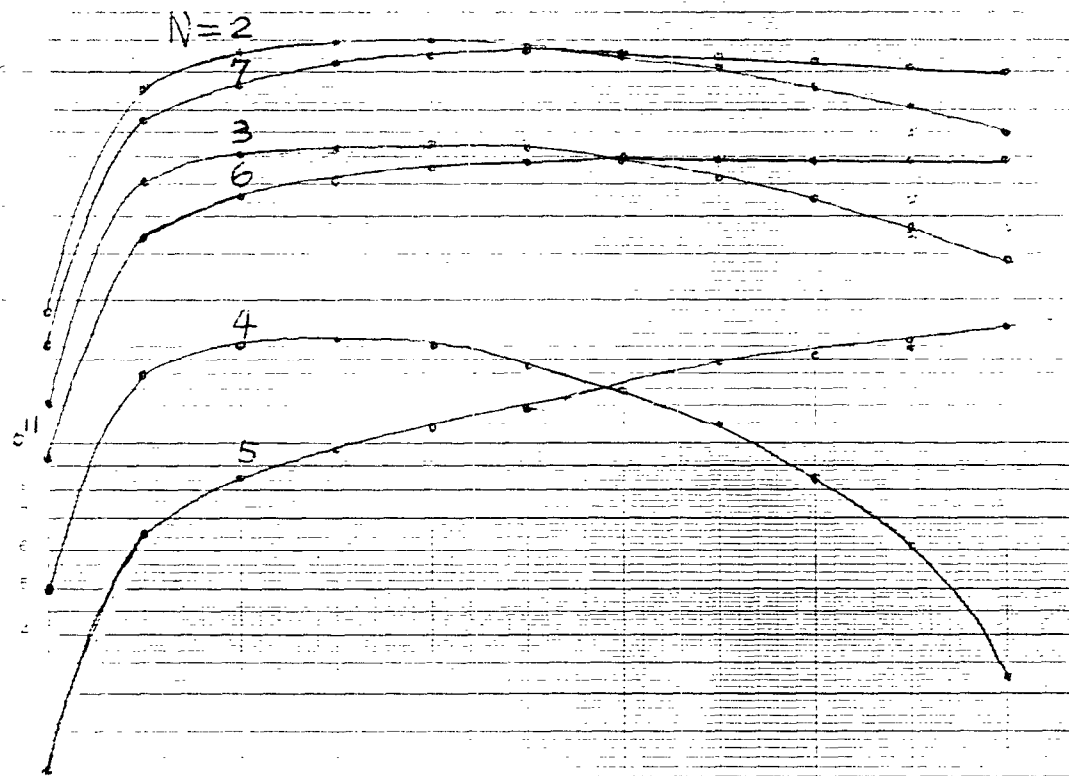
$$\frac{1}{2} = -0.5$$

$$\frac{4}{5} = -0.25$$



12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 γ

$10^{12} N_B$



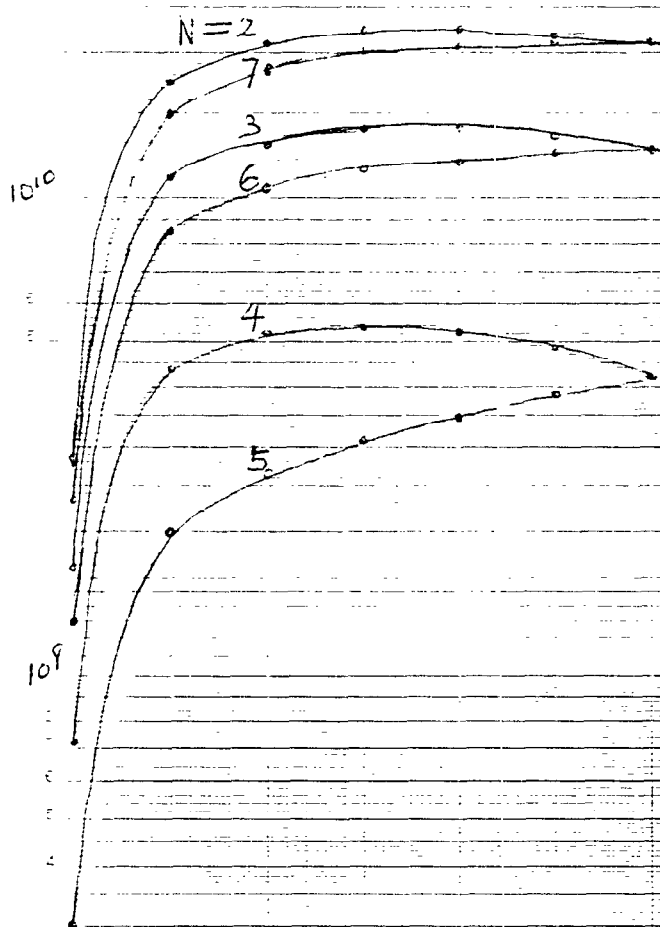
$\frac{dN}{dt} = 0$
 $\frac{dN}{dt} = -0.01$

10^{10}
 1.016
 2.016
 3.016
 γ

10^{11}

N_B

Cu



10^3
 1.003

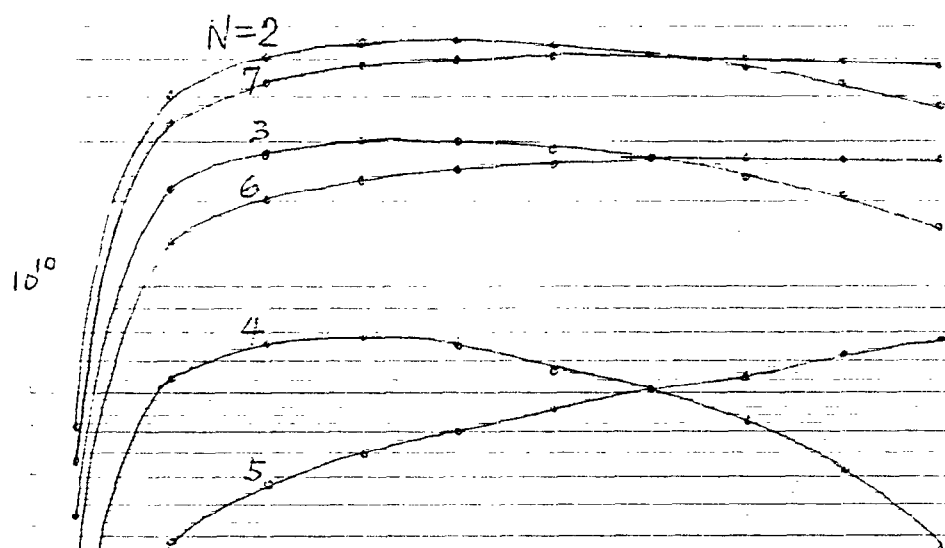
Y
 $2.003 \quad 2.203$

10
9
8
7
6
5
4

10^{11}

N_8

$$\frac{N}{L} = \frac{1}{2} \left(1 + \frac{1}{N} \right)$$



10^9

10^8

1.005

2.005

3.005

γ

10

9

8

7

6

5

4

3

2

1

10^{11}

9

8

7

6

5

4

3

2

1

10^{10}

9

8

7

6

5

4

3

2

1

10^9

8

7

6

5

4

3

2

1

10^8

1.002

2.002

10^{11}

9

8

7

6

5

4

3

2

1

10^{10}

9

8

7

6

5

4

3

2

1

10^9

8

7

6

5

4

3

2

1

10^8

1.001

1.601

1.801

$N=2$

7

3

6

4

5

$N=2$

7

3

6

4

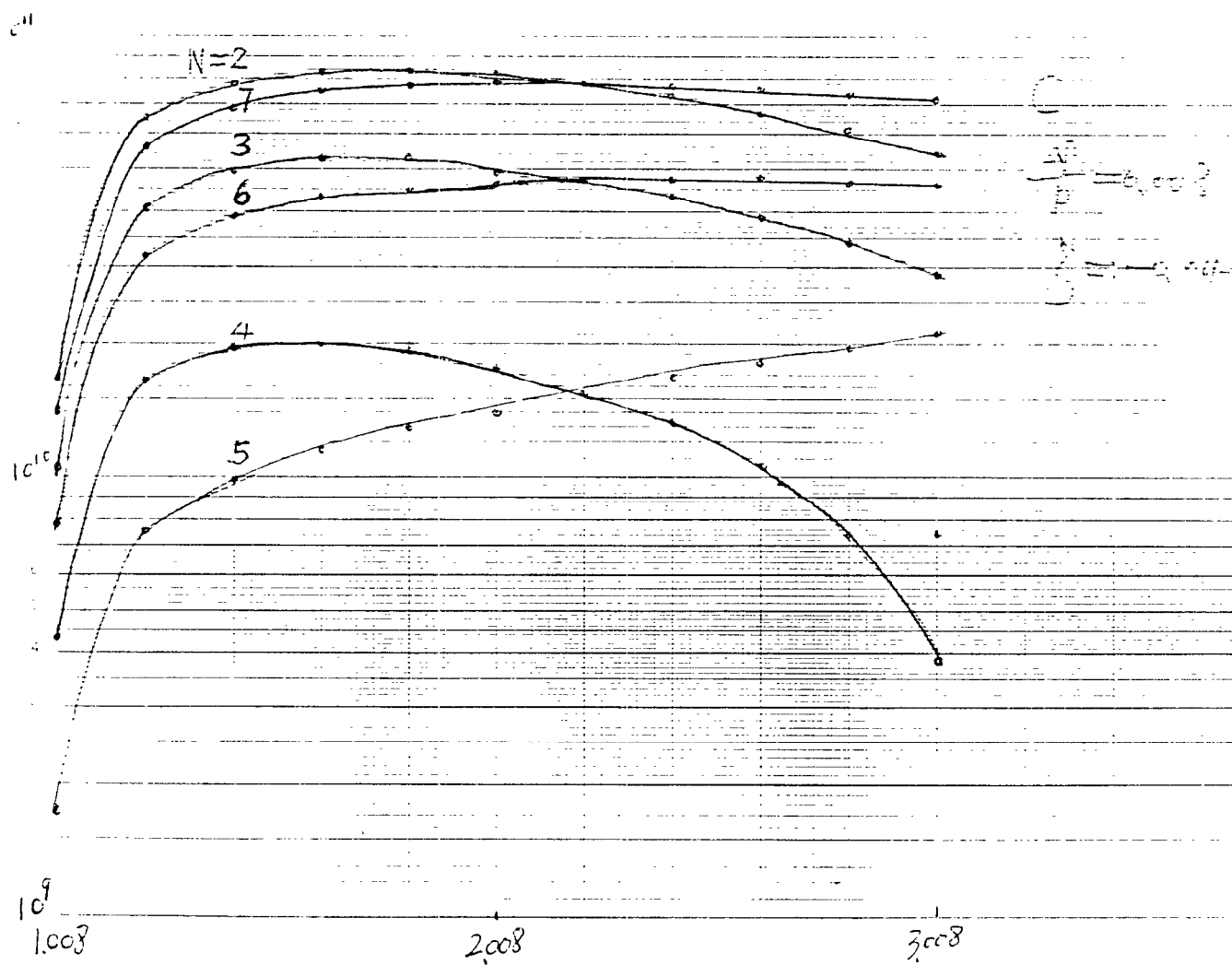
5

$\frac{1}{\lambda} = 0.1 \times 10^{-2}$

1.001

1.601

1.801



3) Couples bunch impedance in the rf detuning.

The booster shall be operated at $h=3$ harmonic mode, therefore there are possibly 3 coupled bunch mode in the booster. The eigenstate of the coupled bunch mode have the following coherent frequency shift,

$$\Delta\Omega = i \frac{q e \eta I_0 \omega_0^2}{2\pi E \omega_{s0}} \frac{1}{2^a (a-1)!} \hat{\theta}^{2a-2} Z_{eff} \quad ; \quad \hat{\theta} = \text{rms bunch length}$$

where I_0 is the total average current and the effective impedance is given by

$$Z_{eff} = \sum_{n=-\infty}^{\infty} (nh+s)^{2a-1} Z[(nh+s)\omega_0 + \Omega] e^{-\frac{1}{2}(nh+s)^2 \hat{\theta}^2} \quad ; \quad \Omega \approx a\omega_{s0}$$

$$\xrightarrow{h\hat{\theta} \gg 1} \frac{1}{Z} \cdot \frac{(a-1)!}{\hat{\theta}^{2a-1}}$$

Since the synchrotron frequency ω_s is a function of the synchrotron amplitude, the resulting frequency provides Landau damping for the coupled bunch motion, i.e.

$$\gamma = -i \frac{\eta \omega_0^3 h}{2\pi \beta c} a \left[\frac{1}{2^a a!} \right]^2 Z_{eff} \int_0^{\infty} dr \frac{r^{2a} \psi'_0(r)}{\Omega - a\omega_s(r)}$$

where we have integrated the synchrotron amplitude in the dispersion integral. Assuming that all the nonlinearity of the synchrotron frequency is provided by the natural rf voltage,

$$V(t) = \hat{V} \sin(h\omega_0 t)$$

we obtain the limiting impedance to be

$$|Z| \leq \frac{2\pi E \omega_{s0}}{q e \eta I_0 \omega_0^2} \frac{2^a a!}{\hat{\theta}^{2a-2}} 0.32 F_a' \frac{h^2}{16} \omega_{s0} (2.5 \hat{\theta})^2$$

Table 3 shows that limit impedance of the booster for the stability of the coupled bunch motion. The stability limit may be severe only in the case of proton beams, where $|Z_{eff}| < 6000 \text{ Ohm}$ and $|Z_{eff}| < 354 \text{ Ohm}$ is required to have stable coupled bunch motion for dipole mode and for quadrupole mode respectively.

Table 3 Impedance limit for the Coupled bunch mode instability

Coupled bunch instability							
Ion species	P	D	C	S	Cu	I	Au
dO \hbar megaS	1076.63	1003.43	1015.68	954.479	835.690	693.060	594.248
Zmax/factor	1151.22	1134339	2362859	6643300	1E+07	3E+07	9E+07
Zmax(a=1)	5825.34	5739921	1E+07	3E+07	7E+07	2E+08	4E+08
Zrw/Zmax	13.8020	0.02506	0.01424	0.00568	0.00321	0.00152	0.00063
Mode factors							
a	1	2	3	4	5	6	
Fa'	0.777	1.12	1.37	1.59	1.78	1.96	
total factor=	5.06014	309.345	24072.4	2369796	3E+08	4E+10	

4) Conclusion

In conclusion we have studied the possible coherent effects for the bunched beam in the booster. The transverse resistive wall instability can contribute at most a few percent of the phase space growth. The growth rate is much smaller than the synchrotron frequency. Thus these mode corresponds to the head tail instability, which can be controlled by the feed back system. We examine also the coupled bunch instability and calculate the preliminary effective impedance for the coupled bunch motion. The impedance shall be evaluated when the rf design is tested. Our conclusion is however that only the proton may be important to the coupled bunch motion.