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RF Beam Loading in the Booster

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INTRODUCTION:

Maximum beam loading will occur when accelerating protons, where intensities of $1.5\text{-}3 \times 10^{13}$ particles per pulse are anticipated. We shall consider briefly four aspects of the bunched beam rf cavity interaction problem. These are static beam loading, transient beam loading, detuning and waveform distortion.

I. <u>Static Beam Loading</u>:

We assume the equivalent circuit shown in Figure 1 is valid. There are two current sources I_g and I_b with R_g the generator impedance (real) and R_c representing the cavity losses. One can add in parallel a beam resistanceR_b = $V_c/I_b \sin \Phi_b$ and reactance $X_b = V_c/ I_b \cos \Phi_b$. Here I_b is the rf comparent of the beam current with $I_b = F\, \bar{I}_b$ where $\bar{I}_b$ is the DC beam current and $0 \leq F \leq 2$ depending upon the bunching factor. We have the additional relations

$$Z = \mathrm{Re}\, \cos\Theta e^{-j\Theta};\; Y = G \sec\Theta e^{j\Theta} = G + jB$$

$$R_e = \frac{R_g\, R_c}{R_g + R_c}\;;\; R_o = \sqrt{L/Cj\omega_r} = 1/\sqrt{LC\, j}\; Q_o = R_c/\omega_r L =$$

$$\omega_r C R_c \gg 1 \quad Q_l = R_e/\omega_r L \quad \bar{I}_b \sin \Phi_s = I_b \frac{\sin\Phi_b}{2}$$

$$\tan \Theta = (\omega/\omega_r - \omega_r \omega)\, Q_l;\; \Delta\omega = (\omega - \omega_r) \ll \omega_r$$

Now we consider the case where the cavity is detuned so that $B = -1/X_b = -I_b \cos \Phi_b/V_c$ which requires that

$$\tan\Theta = -I_b \cos\Phi_b/Vc$$

Then looking into the cavity from the generator one sees a purely resistive load of R_c in parallel with R_b. Hence the generator current will be in phase with the total cavity voltage V_c. If there are no control loops operating i.e. tuning AGS, phase, or radial then stability of the coherent motions of the beam bunch is lost when

$$I_b \sin \Phi_b = V_c/R_e = I_{go} = 2\bar{I}_b \sin \Phi_s$$

or equivalently when $R_b = R_e$ i.e. the power delivered to the beam is the same as that dissipated in the cavity and generator in parallel. One can write the total generator current as $I_g = I_{go} + I_b \sin\Phi_b$ so that for $\Phi_s = 30°$, F=2, $I_g = I_b = 2\, I_{go}$ at the stability limit. However, if the control loops are properly designed then stability can be obtained for $I_b/I_{go} \approx 3$. Therefore, we shall adopt as a design criteria the requirement that $R_e = R_b$ and, of course, that the cavity is detuned according to the relation given above.

If we assume 10^{13}/bunch in the Booster then $\bar{I}_{bmax} = 6.25$ Amp at 1 GeV and since the bunches will never be less than ≈ 150° long an F=1.67 is reasonable. 11is gives an $I_b = 10.4$ Amp so that for $V_c = 8$Kv and $\Phi_s = 30°$ $\Phi_b = 36.8°$ and, we obtain an $R_b = 1.28$K Ω/gap. Here we assume a total gap voltage of 32 Kv at 1 GeV and a maximum $\dot{B}$ of 5.7T/sec. Now measurements on the present AGS cavities at 8 Kv and 3.9 MHz indicate an R_c =10.67 KΩ/ gap so that an R_g of 1.45 KΩ/gap would be required. The total power delivered to the beam is $\bar{I}_B V_c \sin\Phi_s = 100$ KW or 25 KW/gap. The power in the cavity is ≈ 3 KW/gap or 12 KW total. Note that because the form factor F is not two Φ_b the phase of the rf component of the beam current is not equal to Φ_s the stable phase angle. However, this does not affect our requirement that the power delivered to the beam equal that lost in Re.

For reasons given below if the present AGS cavity of four gaps is to be used in the Booster it is desirable to increase the individual gap capacity by at least a factor of two. If this can be accomplished by increasing the ferrite bias current then since the μQ product is essentially constant R_c will not change and the necessary R_g would be the same. An additional requirement of 10Kv/gap at injection where $f_{rf} = 2.5$ MHz is also desirable at high currents for reasons outlined later. This would mean about 5KW loss per gap but since V_c is grater and $\sin\Phi_b$ is much smaller initially, the total power requirements would not increase.

II. Transient Beam Loading:

Transient beam loading will occur in the Booster only at injection where during the capture process I_b the rf component of beam current will increase from zero to some initial valve in a time that will be too short for the turning loop to respond. In order to maintain the proper phase relation between V_c and I_b a feed forward compensation loop will be employed during the capture process. The signal from a sum pickup electrode will be filtered and the rf component of the beam current after amplification will be added in the proper phase with the cavity drive signal to cancel the beam induced signal at f_{rf}. This will require some additional reactive current from the generator until the turning loop has made the necessary correction of Θ. One can minimize the required reactive current transient by detuning the cavity by $\Theta/2$ prior to injection. The proper operation of this compensation requires a fast phase lock loop to be functioning at all times.

III. Detuning:

Below the transition energy Θ the detuning angle is negative so that $\omega_r > \omega_{rf}$ and for $Q_1 >> 1$ we can write

$$\frac{\Delta\omega}{\omega_r} = -\frac{I_b R_e \mathrm{Cos}\Phi_b}{2Q_1 C} = \frac{-I_b \mathrm{Cos}\Phi_b}{2\omega_r C V_c} = -\frac{I_b \mathrm{Cos}\Phi_b}{2I_{cir}} = \frac{I_b R_o \mathrm{Cos}\Phi_b}{2\ V_c}$$

We shall calculate $\Delta\omega$ for the present AGS cavity where C the gap capacity is $\approx$ $375\mu\mu f$. Again we use the values at 3.9MHz and assume $\bar{I}_b$=6.25 Amp. Then

$$\frac{\Delta\omega}{\omega_r} = \frac{-\ 10.4 \times .8}{16 \times .375 \times 2\pi \times 3.9} = -\ 5.68 \times 10^{-2}$$

or $\Delta\omega \cong -\ 5.68 \times 2\ \pi \times 3.9 \times 10^4 = -\ 2\ \pi\ 222 \times 10^3$

That is the cavity resonance is about 222Kc above f_{rf} at 1 GeV for $V_c = 4 \times 8$ Kv, $\Phi_s = 30°$ ($\Phi_b = 36.8°$). Since the rotation frequency f_o is 1.3 MHz here the principal effect of the detuning is to produce some impedance at ($f_{rf} + f_o$) or $4f_o$. In order to calculate the value of this impedance we use the expression

$$\tan\Theta' = (\omega/_{\omega_r} - \omega_r/\omega)\ Q_1$$

to find Θ with $\omega = \omega_{rf} + \omega_o$, $\omega_r = \omega_{rf} + 2\pi x222x10^3$ and then evaluate $Z = R_e \mathrm{Cos}\Theta'\ e^{-j\Theta}$. We find for $Q_1 = \omega C R_e = 11.75$

$$\tan\Theta' = \left(\frac{5.2}{4.122} - \frac{4.122}{5.2}\right)11.75 = 5.6 \text{ or } \Theta' = 79.9°$$

thus $Z\ (5.2\ \mathrm{MHz}) = 1.28 \times 10^3 \times .176\ (1.176 + .984j)\Omega$

$= (40 + 221j)\Omega$

It is the real part of this impedance that could drive the n=2 coupled bunch mode. The total real impedance for four gaps of 160Ω could be tolerated since in the event the bunches become unstable due to a lack of Landau damping (not likely due to the almost full bucket) a simple single channel feedback loop as used on the CERN Booster or the NSLS UV ring can be employed to suppress the oscillations. However, an increase in the gap capacity by a factor of at least two is desirable for other reasons so we calculate $\Delta\omega$ and Z for this case also. We assume C=750 μμf but the same R_e so that Q_1 = 23.5 and $\Delta\omega$ = 222 KC/2 = 111 KC. Then we find tan θ' = 12.34 or θ' = 85.37° and Z = (8.4 + 102j)Ω so that the total real impedance at 5.2 MHZ would be nearly 33Ω.

IV. Waveform Distortion:

Here we are concerned with the beam induced voltages at harmonics of the rf voltage i.e. at 6 f_o, 9f_o, etc. Since they will alter the shape of the buckets defined by V_{rf}. The important parameter here is Z (nf_o)/n where in our case n=6,9,12 etc. are relevant. For frequencies well above resonance Z_{cav} = $-j/\omega\, C_{gap}$ so that we have

$$Z_{cav}/n = -j/2\pi n^2 f_o C_g$$

and calculate the valves at injection (f_o=827Kc) and 1GeV(f_o=1.3MHZ) for the present AGS cavity with C_g=375μμf and n=6. We obtain $|Z_c/6|$=14.3Ω @ 4.96MHz and $|Z_c/6|$ = 9.1Ω at 7.8 MHz. Now the cavity impedance is not the only source of beam induced voltage. We must also consider the space charge term due to impedance presented to the beam by the vacuum chamber. This can be written as

$$\frac{Z_{sc}}{n} = -j\ g_o Z_o/2\beta\gamma^2$$

where Z_o = 370Ω and g_o = 1+ln (b/a) with b = vacuum chamber radius, a = beam radius. We shall assume g_o = 1.25 and find at 200 MeV

$$Z_{sc}/n = -j347\Omega$$

and at 1 GeV

$$Z_{sc}/n = -j62.3\Omega$$

Since we will have four accelerating gaps the impedance at 1 GeV due to the rf stations would be $-j36\Omega$ at n=6 and 57 Ω at injection. In order to reduce this impedance relative to Z_{sc}/n at these frequencies we propose that the gap capacity be increased by at least a factor of two so that at injection where the bucket is full the cavity contribution at $2f_{rf}$ will be less than 10% of Z_{sc}/n.

It is possible to calculate the effects of the space charge induced voltage on the bucket area if the phase space distribution of the bunch is known. We shall do this for two different distributions, first assuming a constant density distribution (1) and second assuming a local elleptical energy distribution (2). In both cases one assumes that the bucket is full, which would be the case for protons at injection into the Booster. For the uniform distribution one calculates

$$\Lambda_{sc} = 4\pi h g_o E_o r_p N / R e V_c \gamma^2$$

where r_p is the classical proton radius (1.53×10^{-18}M), N the number of particles of charge e, h the harmonic number, R the machine radius and E_o the rest energy. If we assume $N = 3\times10^{13}$ and V_c=40 KV we obtain at 200 MeV $\Lambda_{sc} = 1$. Now in reference 1 there is plotted $(A-A_{sc}) / A$ vs Λ_{sc} for different valves of $\Gamma = \mathrm{SIN}\Phi_s$ where A is the bucket area due to V_c alone and A_{sc} the reduced bucket area. We find for $\Gamma = .2$ a area reduction of about 8% and for $\Gamma = 0$ a 5% reduction.

If we assume a local elliptical energy distribution so that the induced voltage has the same shape as the applied voltage (2) we can write

$$A = Ao \sqrt{K_t}$$

where

$$K_t = 1 + \frac{2\pi h \bar{I}_b \mathrm{Im}\ (Z/n)}{V_c\ f(\Phi_1,\Phi_2)}$$

for a sinusoidal applied voltage and

$$f(\Phi_1,\Phi_2) = \sin\Phi_2 - \sin\Phi_1 - \tfrac{1}{2}(\Phi_2-\Phi_1)(\cos\Phi_1+\cos\Phi_2)$$

with Φ_1,Φ_2 being the bucket limits.

For V_c = 40 KV, $\bar{I}_b$ = 3x1.32 Amp I_m (Z/n)= -347Ω and Γ = .2 we find A = .9 Ao where A_o is the area at zero beam current. If Γ = 0, A ≅.95A_o at 10 Kv/gap but if we assume 30 Kv for all the above cases we find the area reductions about 2.5-3% greater. Hence the bucket area effects even at $3x10^{13}$ are not large as long as we maintain large bucket areas and small stable phase angles at low energies.

At present the bunch areas in the AGS at $2x10^{13}$ in 12 bunches is 1 ev sec below the transition energy. Because of transverse instabilities and blow up of the area in crossing transition it is desirable to at least maintain this area for the bunches coming from the Booster: In order to do this it is necessary to have a V_c=4oKv at injection so that the stationary bucket area is greater than 1 ev sec. Then with an initial $\dot{B}$ ≈ .6T /sec and a Φ_s ≅ 2.50° the moving bucket area at maximum current would be ≈ 1 ev sec. If wish to maintain or increase slightly this bucket area during the acceleration cycle then it is desirable to maintain the 40 Kv gap voltage so that the $\dot{B}$ can be increased to the above mentioned value of 5.75T/sec. In this case the maximum power transferred to the beam remains at 100 KW for 10^{13}/bunch but the permissible R_e is increased to 1.2KΩ i.e. $(10/8)^2$ x 1.28 KΩ. Then since the cavity impedance at 3.9 MHz (assuming the present AGS configuration) decreases to 8KΩ/gap the generator impedance could be as high as 2.67KΩ/gap and still maintain the P_b=P_e criteria.

Conclusions:

In order to insure stable operation at high beam currents in the Booster it will be necessary to design an rf system that can deliver 100 KW to the beam at 40 KV total gap voltage and 10^{13} protons/bunch. This can be accomplished if the real impedance that the beam sees across the accelerating gaps at the rf frequency is equal to real part of the impedance at this frequency that the beam presents to the rf system i.e. $R_b=R_e$. If one of the present AGS cavities is to be used then it is desirable to increase the gap capacity by at least a factor of two i.e. $C_g \geqq 750$ μμf for four gaps or an equivalent value if more or less gaps are employed.

Reference

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2. A. Hofmann, F. Pedersen; IEEE Transactions on Nuclear Science Vol. NS-26 June 1979 #3 Pg. 3526