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PROPOSAL TO ELIMINATE PARASITIC HARMONICS IN THE AGS RF LOW LEVEL BY MEANS OF A PHASE-LOCKED LOOP

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PROPOSAL TO ELIMINATE PARASITIC HARMONICS IN THE AGS RF LOW LEVEL BY MEANS OF A PHASE-LOCKED LOOP

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1. Introduction

The present rf low level of the AGS consists of a multiple heterodyne system. The output signal presents phase perturbations, when the swept frequency is equal to the 6th or the 7th harmonics of the 455 Kc/s oscillator. We try to avoid this fact by the control of the frequency and phase of the output signal of the first mixer. That is why we have studied the performances and the feasibility of a partly digital phase-locked loop (PLL).

2. Basic Operation

We consider only a part of the rf low level (Fig. 1); we keep the mixer and the 455 Kc/s oscillator, but the scheme is changed to get a PLL by addition of two buffers and limiters, a phase detector, a loop filter and a voltage controlled oscillator (VCO). The proposed block-diagram is on Fig. 2.

The loop acts like a translation loop, in which the offset frequency is the frequency F_o [1]. The buffers and limiters transform the sine waves at the frequency F_o into digital signals compatible with the used phase detector (usually TTL or ECL). The loop filter fixes the order of the loop with the feedback chain and is very important. The VCO must be designed to track only the sum of both frequencies, because the loop can be locked-in likewise for the difference of both frequencies.

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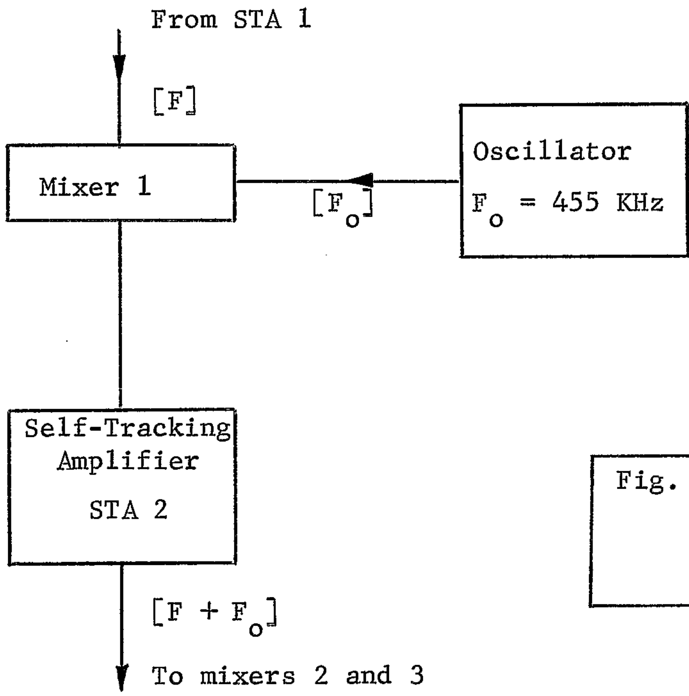

Fig. 1: Block diagram of a part of the present rf low level of the AGS.

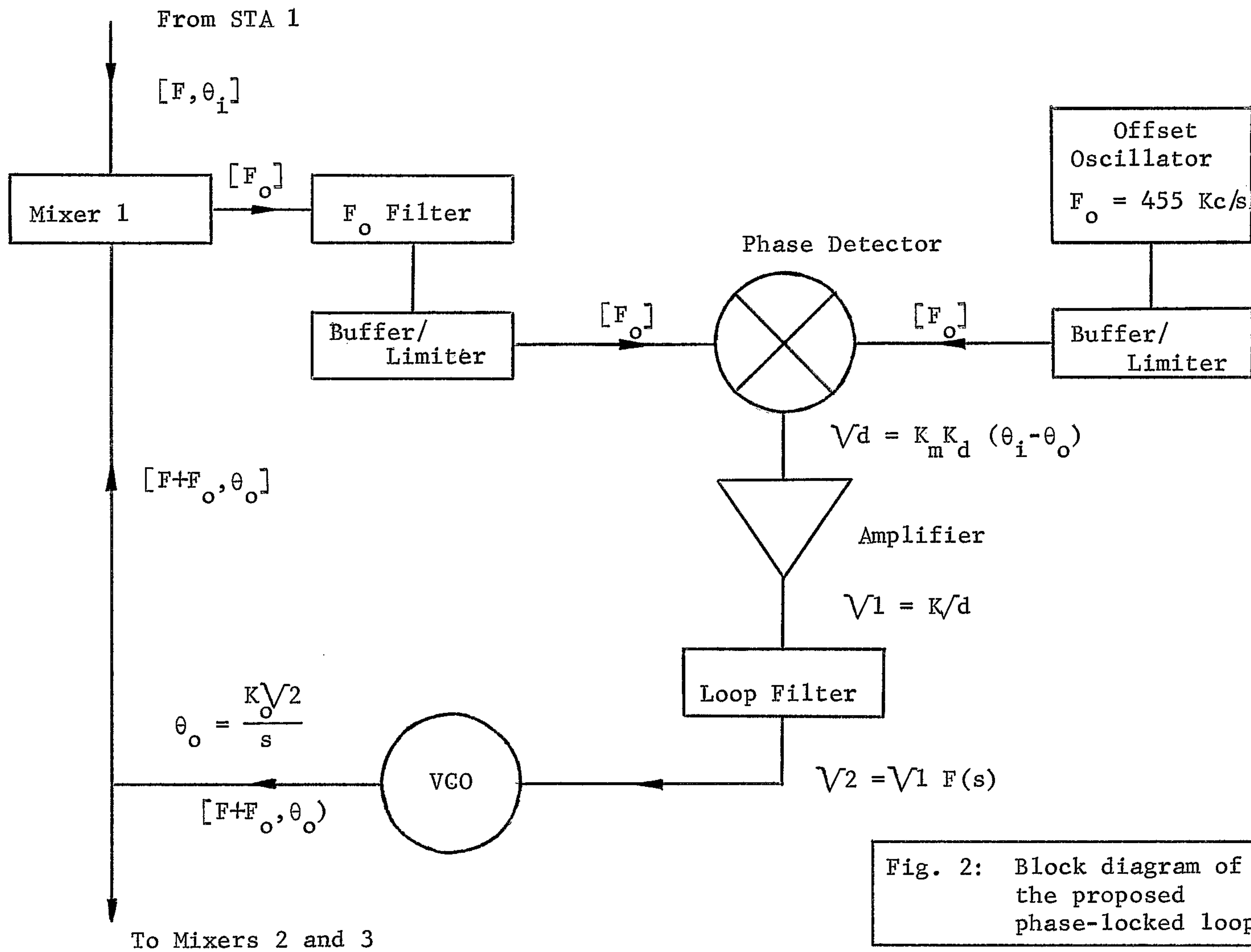

Fig. 2: Block diagram of the proposed phase-locked loop.

3. Loop Fundamentals

The basic transfer functions of each element are shown in Fig. 2.

3.1 Mixer 1 and Phase Detector and Offset Oscillator

It was shown[2] that the transfer function of this whole is:

$$V_d = K_m K_d \sin(\theta_i - \theta_o)$$

or

$$\boxed{V_d = K_m K_d (\theta_i - \theta_o)}$$

with the linear approximation, because $(\theta_i - \theta_o)$ remains small.

3.2 Loop Filter

$$\boxed{V_2 = V_d\, F(s)}$$

F(s) is the function, which allows to fix the order of the loop. F(s) will be ascertained later.

3.3 VCO

The transfer function of the VCO can be given, either versus the frequency shift: $\Delta\omega = K_o V_2$, or versus the phase shift:

$$\boxed{\theta_o = \frac{K_o V_2}{s}}$$

In these transfer function, K_m, K_d and K_o are the gain constants of the mixer, the phase detector and the VCO, while s is the laplace complex variable.

3.4 Closed-Loop Transfer Function

This transfer function will be:

$$P(s) = \frac{F_{output}}{F_{input}} = 1 + \frac{F_o}{F}$$

An other expression of P(s), according to the phase, will be:

$$P(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_o K_m K_d F(s)}{s}\left[1 - \frac{\theta_o(s)}{\theta_i(s)}\right] \text{ or } \boxed{P(s) = \frac{K_o K_m K_d F(s)}{S + K_o K_m K_d F(s)}}$$

3.5 Open-Loop Transfer Function

This transfer function, written according to the phase, will be:

$$\boxed{Q(s) = \frac{\theta_o}{\theta_i - \theta_o} = \frac{K_o K_m K_d F(s)}{s}}$$

4. Choice of the Loop Order

The order of the loop refers to the highest degree of the polynomial expression: $1 + Q(s) = 0$.

For some reasons not expanded here, which are bound to the theory of the servo systems, we will choose a second-order loop. The loop filter will be an active filter (Fig. 3) with:

$$\boxed{F_1(s) = \frac{A(1 + sCR_2)}{1 - sCR_2 + (1-A)\ sCR_1} \# \frac{1 + sCR_2}{sCR_1}} \quad \text{for large A.}$$

To adjust the gain of the loop we will add in the feedback chain an amplifier with a K gain and we will get:

$$\boxed{F(s) = K\ F_1(s)}$$

5. Response of the Loop for a Linearly Changing Input Frequency

We must consider the response of the loop. When the input frequency F changes; at injection and during the first 70 milliseconds, the rate is almost linear and maximum; its numerical value is:

$$\left[\Delta\dot{F}\right]_M = 34\ \text{Mc/s}^2 \quad .$$

This rate decreases then and becomes almost null at the end of the acceleration (Fig. 4).

If we assume that the frequency F changes as $\Delta\dot{F}\ t$, the input phase, which is the integral of the frequency, changes as: $\theta_i(t) = \frac{1}{2}\ \Delta\dot{F}\ t^2$ and the laplace transform of $\theta_i(t)$ is:

$$\boxed{\theta_i(s) = \frac{\Delta\dot{F}}{s^3}}$$

We must now study three kinds of behavior of the phase-locked loop: the tracking, the acquisition and the hold-in performance.

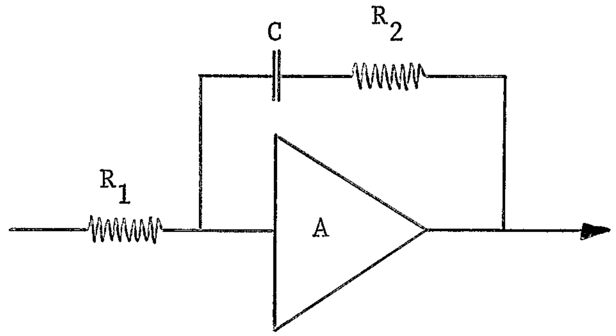

Fig. 3: Active Filter

$\tau_1 = CR_1 \qquad \tau_2 = CR_2$

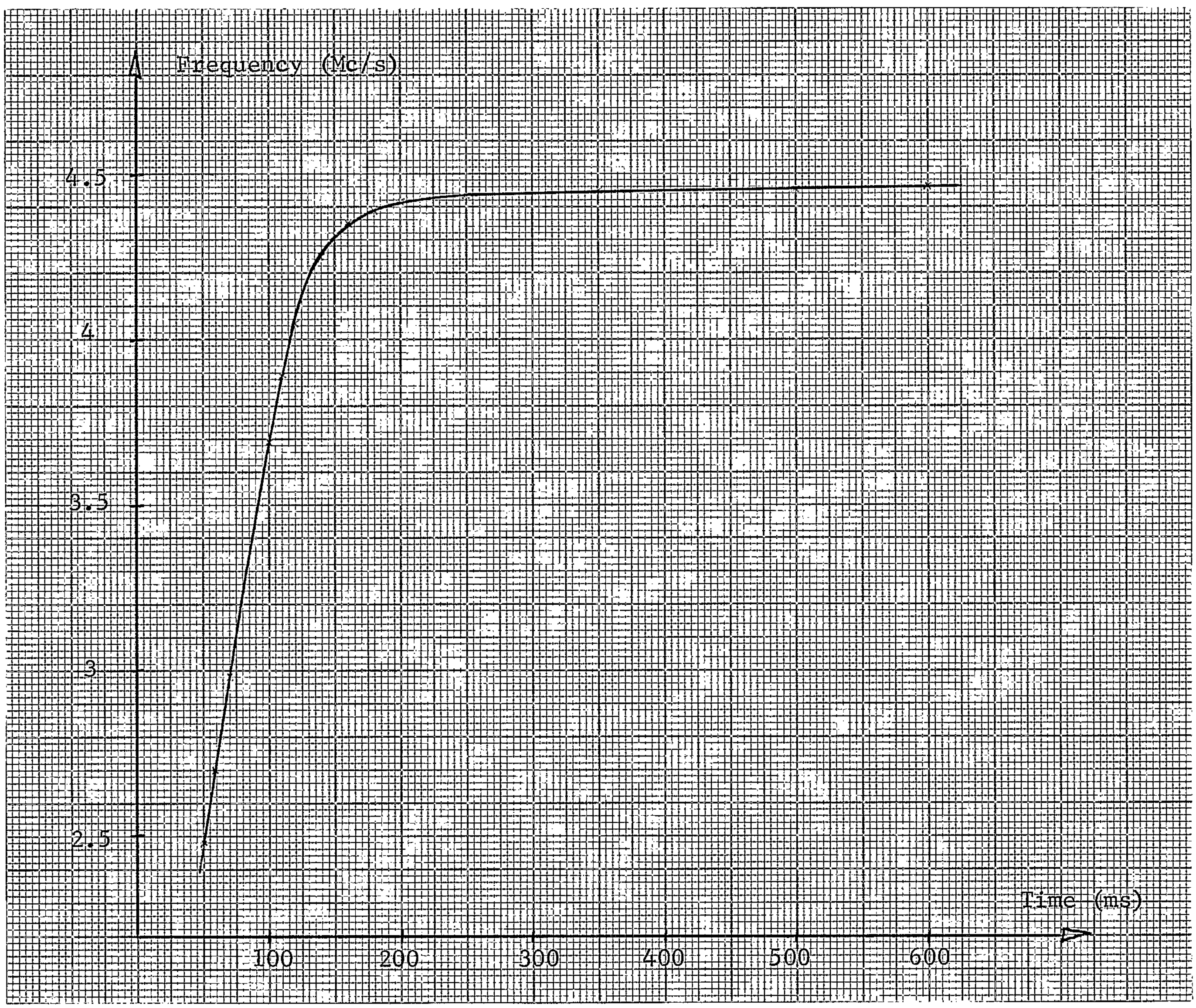

Fig. 4: AGS Accelerating Frequency vs. Time.

5.1 Tracking

If we suppose that our second-order loop is locked-in, we know that we make two errors: a steady-state acceleration error θ_a and a velocity error θ_v, that we can estimate:

5.1.1 Acceleration Error θ_a

It is shown[1] that the acceleration error θ_a is:

$$\boxed{\theta_a = \frac{2\pi\ \dot{\Delta F}}{F_n^2}}$$

where $\dot{\Delta F}$ is the rate of change of the input frequency and F_n is the "natural frequency" of the loop.

If we want to get an error smaller than 5°, we can determine the minimum natural frequency:

$$F_n^2 \geq \frac{2\pi\ [\dot{\Delta F}]M}{\theta_a}$$

$$\boxed{F_n \geq 49.5 \text{ Kc/s}}$$

Figure 5 plots the variation of θ_a versus time.

The other parameter, which defines the loop, is the damping factor ξ; we choose the value $\xi = \frac{\sqrt{2}}{2}$.

F_n and ξ are connected with the loop parameters K_o, K_m, K_d K and F(s) =

$$F(s) = \frac{1 + SCR_2}{SCR_1}$$

by the expressions:

$$F_n = \sqrt{\frac{K_o K_m K_d K}{CR_1}} \quad \text{and} \quad \xi = \frac{CR_2}{2}\sqrt{\frac{K_o K_m K_d K}{CR_1}}$$

5.1.2 Velocity error θ_v

At the acceleration error θ_a, we must add the velocity error θ_v[3], which increases with the time according to:

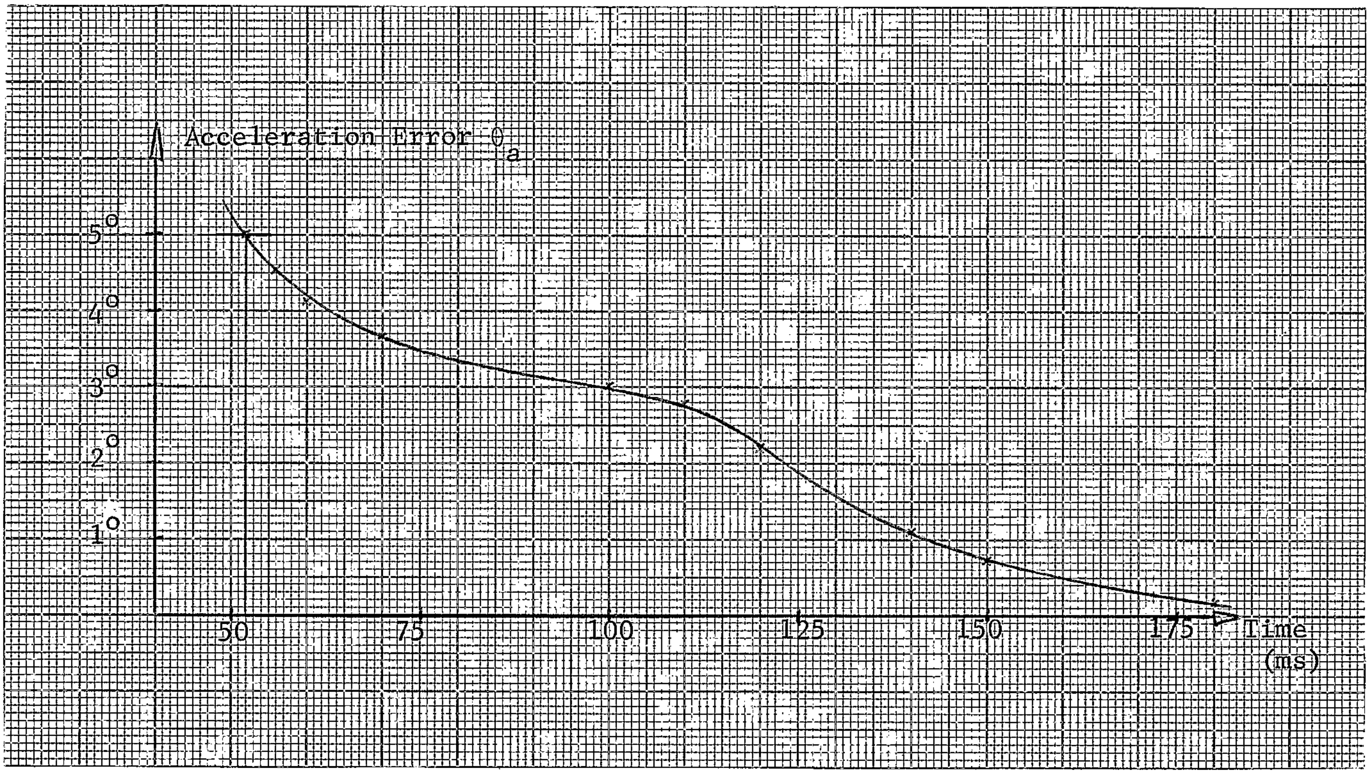

Fig. 5: Acceleration Error θ_a vs. Time in Tracking of the RF Signal from the Pick-up Electrode.

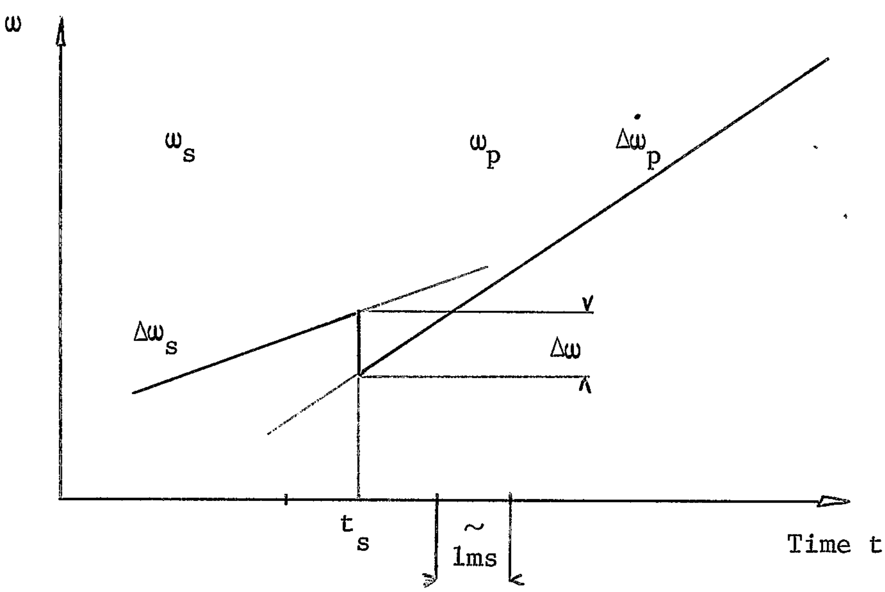

Fig. 6: Frequency Jump at the Switching Time.

$$\frac{\theta_\nu}{t} = \lim_{t\to\infty} \frac{d\theta_e(t)}{dt} = \lim_{s\to o} s \; [s\;\theta_e(s)]$$

$$= \lim_{s\to o} \frac{s^3\;\theta_i(s)}{s + K_o K_m K_d K F(s)} = \lim_{s\to o} \frac{\Delta\dot{\omega}}{s + K_o K_m K_d K F(s)}$$

$$\frac{\theta_\nu}{t} = \frac{\Delta\dot{\omega}}{K_o K_m K_d K F(o)}$$

Let us call the product $K_o K_m K_d K F(o)$ the velocity constant K_ν. If the loop filter is an active filter, F(o) is very large. The accumulated phase error after an elapsed time t is:

$$\theta_\nu = \frac{\Delta\dot{\omega} t}{K_\nu} = \frac{2\pi\;\Delta\dot{F} t}{K_\nu} \quad .$$

In fact, θ_ν is the sum of elementary velocity errors $d\theta_\nu$, because the rate of change of the input frequency varies into the cycle

$$\theta_\nu = \int_{t_1}^{t_2} \frac{2\pi}{K_\nu} \times \frac{dF}{dt} \times dt = \int_{F_1}^{F_2} \frac{2\pi}{K_\nu} \times dF$$

$$\boxed{\theta_\nu = \frac{2\pi}{K_\nu}\,(F_2 - F_1)}$$

We thus get the very simple result, that the velocity error is independent on the shape of variation of the frequency, but depends only on the difference between the frequency at injection and the maximum rf frequency. In practice, we will see later, that θ_ν can be neglected, because it is much smaller than θ_a.

5.2 Switching From the Starting Oscillator to the P.U.E. Signal

We want to get a fast acquisition of the PLL at start, when the beam is injected and the rate of change of the frequency is the largest. In fact, the PLL will be in lock on the more slowly swept frequency of the starting oscillator ($\Delta\dot{F}_s \simeq 18.5$ Mc/s^2). But when the switching between the signal from the starting oscillator and the signal from the pick-up electrode is performed, both frequencies are not exactly the same and it is obtaining a frequency jump (Fig. 6); the step amplitude is approximately from 14 to 20 KHz.

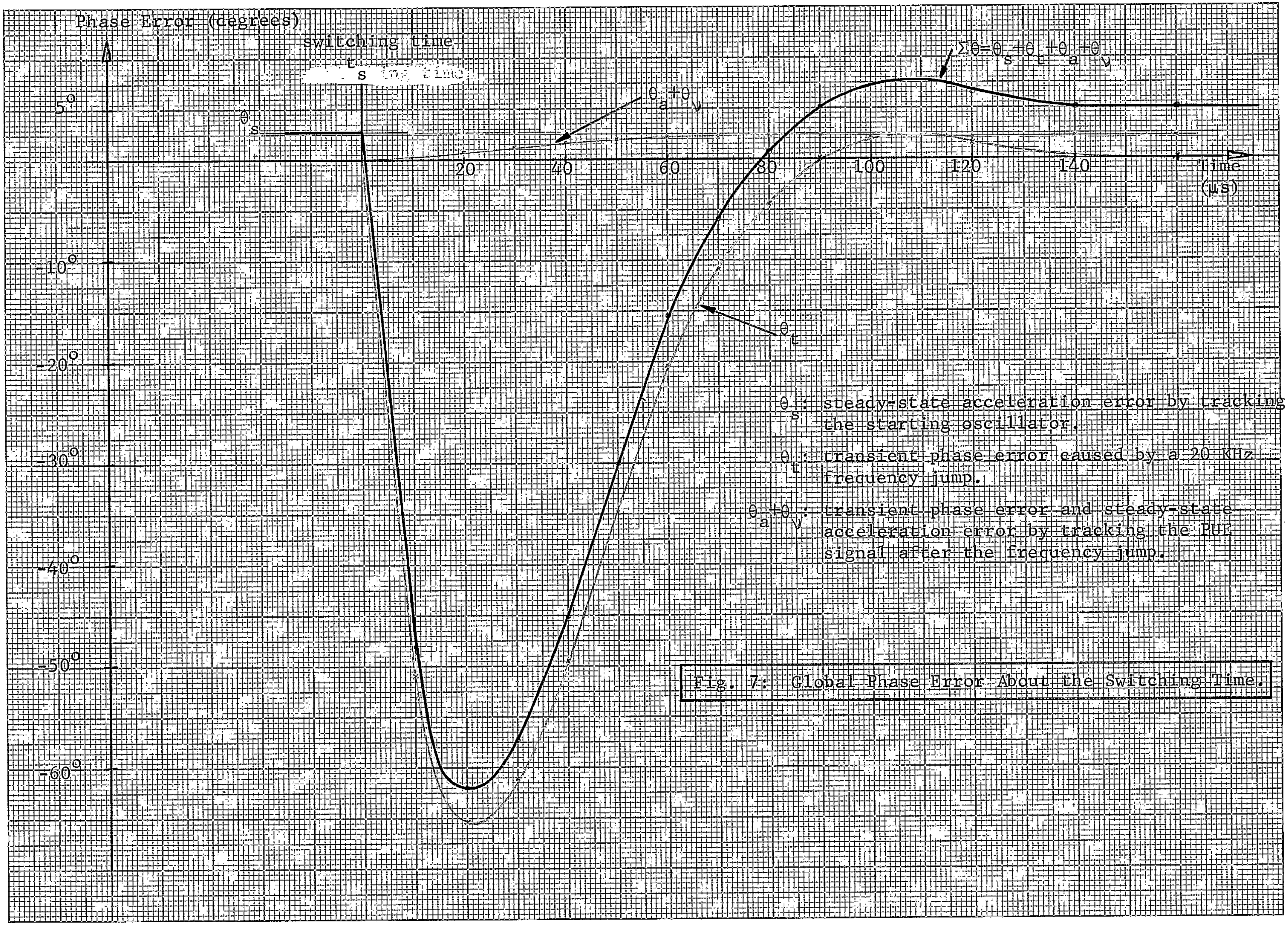

Fig. 7: Global Phase Error About the Switching Time.

We can presume that the PLL is locked-in by the starting oscillator. The steady-state acceleration error θ_s is constant and equal to

$$\theta_s = 2\pi \frac{\Delta\dot{F}_s}{F_n^2} = 0.0475 \text{ rd} = 2.72^\circ \quad .$$

The velocity error will be very small and can be neglected.

Just as the switching happens, the phase error θ_c is the sum of three phase errors:

- The phase error θ_s constant.
- The transient phase error, related to the frequency step, of which the amplitude is large enough, but the length is small (90 μs). There is no steady-state error resulting from this frequency step.
- The phase error θ_p, related to the rate of change of the frequency after the switching.

The shape of the global phase error just at the switching is plotted on Fig. 7. For this drawing the normalized curves plotted by Hoffman[4] have been used. The transient phase error can be corrected by a phase shifter.

5.3 <u>Acquisition</u>

We must initially lock-in the PLL on the starting oscillator and this acquisition may be achieved by some means:

a) The loop will lock up without shipping cycles, if the frequency difference between input and VCO is less than the lock-in frequency ΔF_L:

$$\Delta F_L \approx 2\,\xi\,F_n = F_n\sqrt{2}$$

$$\underline{\Delta F_L \simeq 70 \text{ KHz.}}$$

The lock-up transient time is on the order of $1/F_n$:

$$\underline{t_L \simeq 20\ \mu\text{s.}}$$

b) The VCO frequency can slowly slide toward the input frequency if the difference frequency is less than the pull-in frequency ΔF_p. A good formula for loops with high gain is

$$\Delta F_p = \sqrt{2\,(2\,\xi\,F_n\,K_v - F_n^2)}$$

The time required for a loop to pull into lock for an initial frequency offset ΔF is approximately

$$T_p \approx \frac{\Delta F^2}{2\xi F_n^3} .$$

If it is wanted to get a pull-in time less than 1 ms, the maximum difference between input and VCO frequencies will be:

<u>$\Delta F = 414$ KHz</u>.

c) To acquire lock more fastly, it is possible to apply a sweep voltage to the VCO, in order that the VCO frequency sweeps into the input frequency. In practice, the probability of lock is 1, if the sweep rate $\dot{\Delta F}/F_n^2$ is less than $\frac{1}{2}$.

5.4 Hold-in Performance

This notion is important for a PLL with a narrow bandwidth, because the dynamic error $\theta_a = \dot{\Delta F}/F_n^2$, which becomes: $\sin\theta_a = \dot{\Delta F}/F_n^2$, for a phase detector with a sinusoidal characteristic and must thus remain less than 1, can be very small. Here, F_n is large and the rate of change of the input frequency $\dot{\Delta F}$ can become high; despite this fact, the loop will not fall out of lock.

6. Practical Achievement of the PLL

Agreeably to Fig. 2, we can define each part of the PLL.

6.1 Mixer

It is the same, that this used presently, or anyone equivalent (SPECTRAN s5.BM 7 or MCL ZLW 1H/SRA 1H).

6.2 F_o Filter

The bandwidth of this filter will be large enough to allow the acquisition of the frequency, but narrow enough to eliminate all parasitic bands. It can be an active or passive bandpass filter.

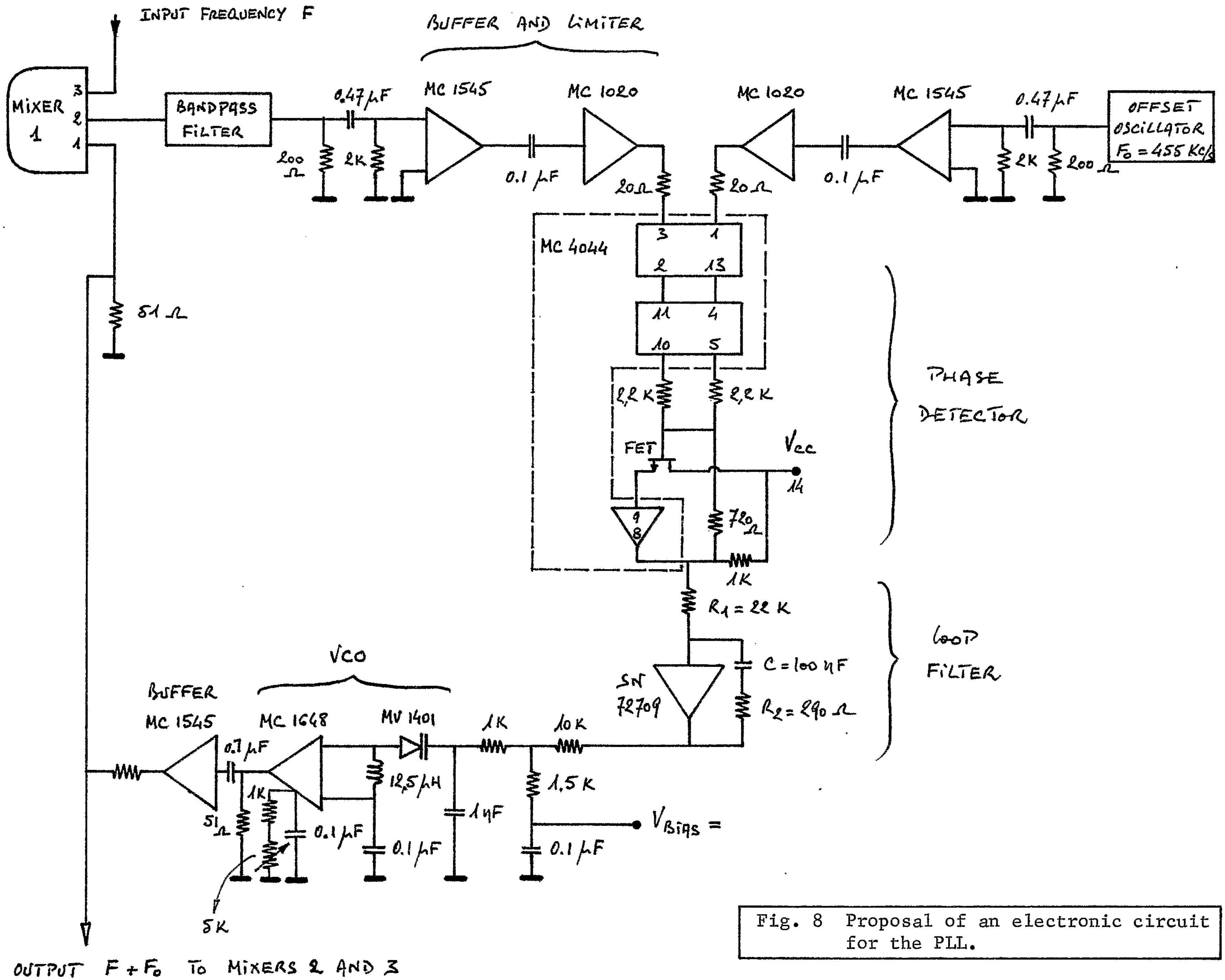

Fig. 8 Proposal of an electronic circuit for the PLL.

6.3 455 Kc/s Oscillator

The present oscillator can be kept.

6.4 Buffer and Limiter

These circuits transform the sinusoidal signals to digital MTTL inputs for the phase detector. We can use a broadband amplifier MC 1545, followed by a fast amplifier MC 1020, with a short rise time.

6.5 Phase Detector

For this frequency range, it is possible to use a MTTL MC 4044 phase detector; with two digital MTTL inputs, we obtain an output dc voltage of which the level depends on the phase difference.

6.6 Loop Filter

It can be achieved in two ways: either it is placed after the charge pump of the MC 4044 and uses the output amplifier of the MC 4044, or it is placed outside the MC 4044 and uses a distinct operational amplifier (SN 72709).

6.7 VCO

It is a MECL MC 1648 circuit with a resistor added between the AGC circuit and the ground, to get a sine-wave output. The output signal is used as input for the Mixer 1 on the one hand, for the Mixers 2 and 3 on the other hand.

A detailed picture of the PLL is on Fig. 8.

7. Numerical Calculus and Parameters for the PLL

7.1 Phase Detector Gain Constant K_d

The value of K_d for a phase-detector MC 4044 is usually: K'_d = 0.111 V/rd. The gain of the output amplifier is: $K_a = 2.2/0.72 \# 3$. The value of K_d is:

$$K_d = K'_d K_a = 0.34 \text{ V/rd.}$$

7.2 VCO Gain Constant K_o

An evaluation of K_o for a MC 1648 VCO used in the frequency range 2.95 - 4.95 MC/s is: K_o = 2 MHZ/V

$$K_o = 1.26.20^7 \text{ rd/v/s.}$$

7.3 Loop Filter Transfer Function F(s)

The transfer function of a second-order loop with an active filter (Fig. 3) is:

$$F(s) = \frac{A(sCR_2 + 1)}{sCR_2 + 1 + (1-A)\ sCR_1}$$

In practice, A is large but not infinite; when s comes to 0, we get $F(o) = A$; when s increases indefinitely, we get $F(\infty) = R_2/R_1$, if A remains constant to the high frequencies; else, $F(\infty)$ comes to 0.

For a SN 72709 operational amplifier, with the appropriate compensation, the closed-loop voltage gain is $A = 10^3$ down to 400 KHz and decreases beyond. The velocity constant or d-c loop gain: $K_v = K_o K_m K_d\ F(o)$ is, assuming to $K_m = 1$

$$K_v = 1.26 \times 10^7 \times 0.34 \times 10^3$$

$$\underline{K_v = 4.27 \times 10^9 \text{ Hz.}}$$

This value of K_v allows to evaluate the accumulated phase error θ_v previously indicated

$$\theta_v = \frac{2\pi}{K_v}(F_2 - F_1)\ .$$

$$\underline{\theta_v = 2.94 \text{ mrd} = 0.17^\circ}\ .$$

In the useful frequency range, it is possible to write

$$F(s) = \frac{1 + sCR_2}{sCR_1}$$

The loop is entirely defined by

- The loop gain: $K_L \doteq K_o K_d K_m = 4.27 \times 10^6$ Hz.
- The natural frequency $F_n = \sqrt{\dfrac{K_o K_d}{CR_1}} = 49.5 \times 10^3$ Hz.

• The damping factor $\xi = \frac{CR_2}{2}\sqrt{\frac{K_o K_d}{CR_1}} = \frac{\sqrt{2}}{2}$.

We also get: $C = 100$ nF, $R_1 = 22$ KΩ, $R_2 = 290$ Ω, and the time constants of the loop are:

$$\tau_1 = CR_1 = 2.22 \text{ msec.}$$

$$\tau_2 = CR_2 = 29\ \mu\text{sec.}$$

With the VCO's tank circuit, including the inductor L and the varactor MV 1401 or equivalent, the swept frequency range is the wanted range, if the input voltage of the varactor changes from 3 V to 5.5 V; the varactor capacitance decreases from 230 pF to 80 pF.

Let us notice that an additional inverter-amplifier with a gain equal to 1 may be necessary between the phase-detector and the VCO, to get a correct phase-relation in the loop.

8. Conclusions

It is theoretically possible to use a PLL to lock-in and track fastly changing frequencies like at the injection in a synchrotron. But the consequence is a broad bandwidth of the loop and, of course, a great noise sensitivity. If the signal is enough out of noise, the practical application of a PLL can be considered.

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