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DECAY OF FLUCTUATION STATES IN STOCHASTIC COOLING

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AGS DIVISION TECHNICAL NOTE

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I. Introduction

Stochastic cooling[1] operates by sensing (and subsequently partially correcting) a statistical fluctuation in a finite sample of particles. I point out in this note that the mean life of such a statistical fluctuation is so short as to make the physical realization of stochastic cooling impractical.

II. Background

Suppose the beam sample, N, be divided into two parts with N_1 and N_2 particles on the positive and negative side, respectively, of the mean beam position. The sensed average position is

$$\bar{y} = \frac{N_1 - N_2}{N_1 + N_2} \cong \frac{N_1 - N_2}{N} s = \frac{n}{N} s$$

where s is a measure of the total beam size and we set $n \ll N$ so that we can take the denominator, N, to be a (quasi) constant.

If N_1 and N_2 are normally distributed, independent variables $N_1 - N_2$ is also normally distributed with mean zero and variance N. The probability density distribution function of n is

$$p(n) = \frac{1}{(2\pi N)^{\frac{1}{2}}} \exp\left(-\frac{n^2}{2N}\right) ,$$

and transforming to the variable, $\bar{y}$,

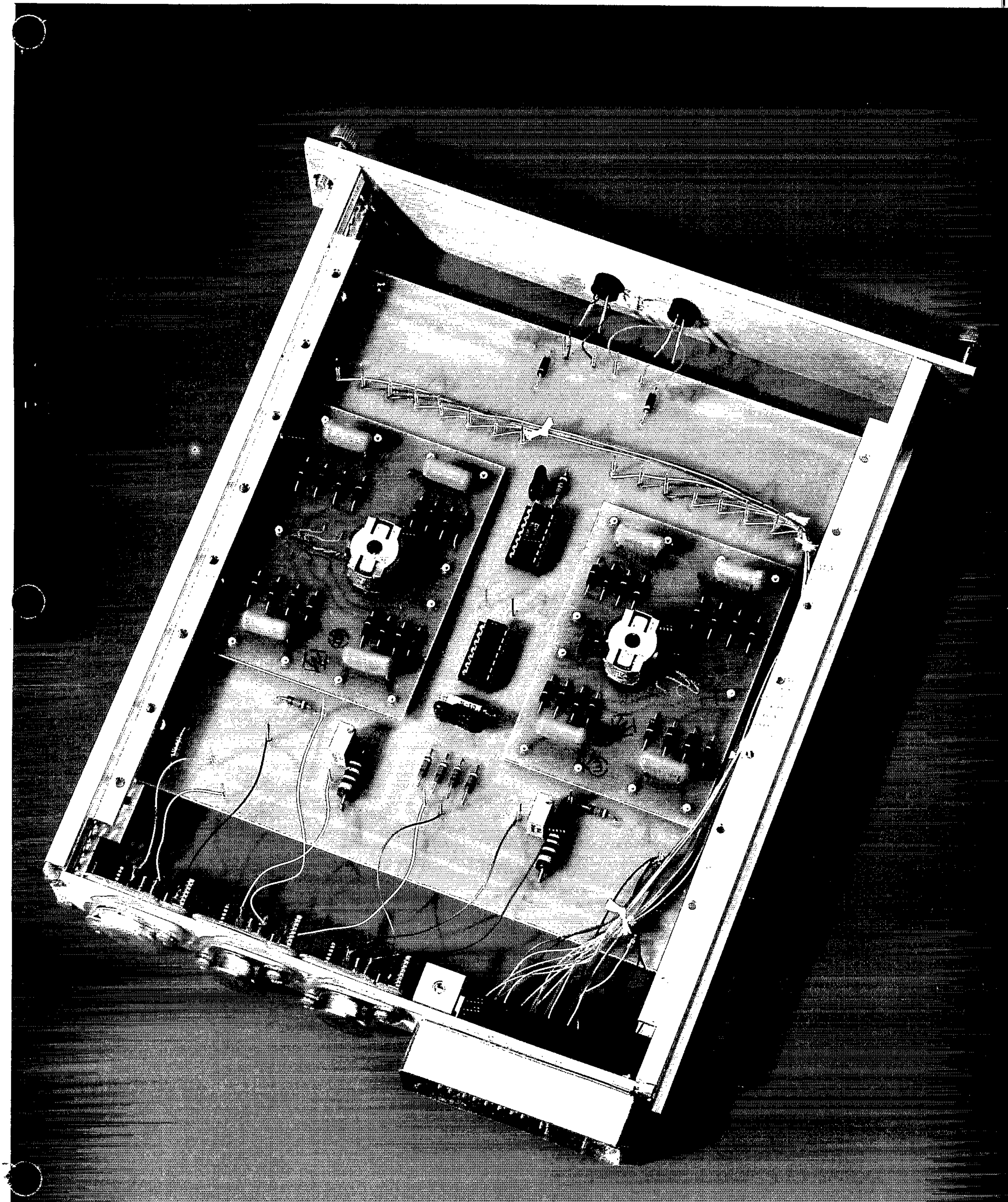

Fig. 2 Bias Supply in NIM package

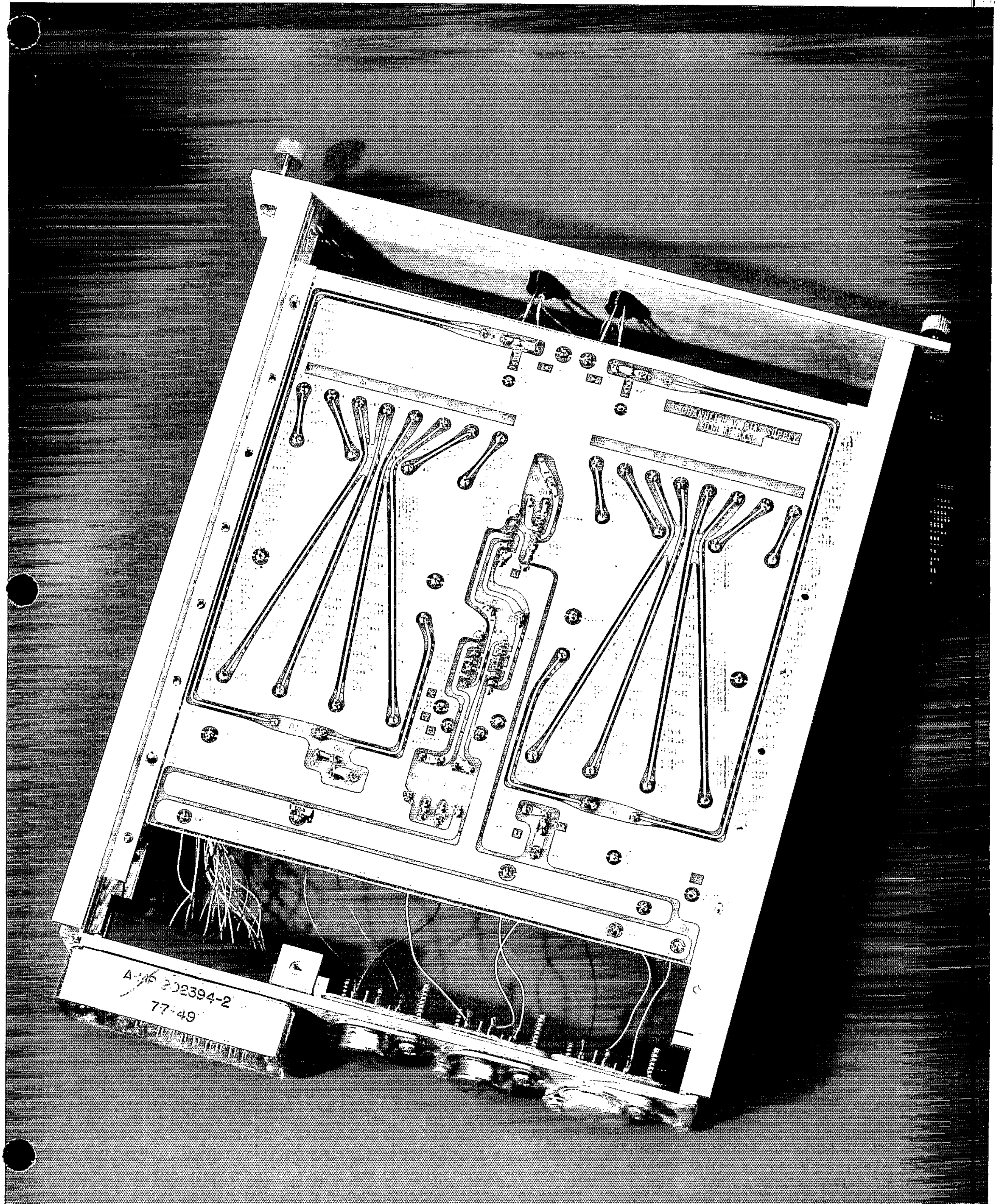

Fig. 3 Bias Supply Circuit Board

$$p(\bar{y}) = \frac{1}{s}\left(\frac{N}{2\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{\bar{y}^2 N}{2s^2}\right)$$

with

$$\langle \bar{y}^2 \rangle = s^2/N \quad ,$$

i.e.

$$s^2 = \sigma^2 \cong \frac{\sum^{N_T} y_i^2}{N_T} \cong \frac{\sum^{N} y_i^2}{N} \quad .$$

N_T is the total universe out of which N is a sample ($N_T \gg N$).

In a real beam we do not have a static situation such as drawing balls from an urn and the statistical fluctuation, $N \neq \langle N \rangle$, which produces the detected $\bar{y}$ will persist for only some period of time, i.e. there are probability after effects.[2] The mean life of the fluctuation, $\bar{y}$, is the same as the mean life of the fluctuation, N.[3]

III. Calculation of Mean Life

We seek the probability that a particle will escape from the sample, N, which is within the azimuthal length from $-\frac{L}{2}$ to $+\frac{L}{2}$. In one revolution a particle will lag or lead a particle with central momentum p by

$$\ell = \eta \, C \, \frac{\Delta p}{p}$$

if its momentum is different from the central momentum by Δp ($\eta = \gamma_T^{-2} - \gamma^{-2}$).

We assume the sample to be uniformly distributed from $-\frac{L}{2}$ to $+\frac{L}{2}$. The probability of loss (in the positive ℓ direction) is equal to the probability of being in δx times the probability that $\ell > \frac{L}{2} - x$ integrated over the range of x which can contribute. The probability that $\ell > \frac{L}{2} - x$ is

$$\int_{L/2-x}^{\frac{\ell_m}{2}} d\ell \, p(\ell) \quad . \tag{1}$$

Where $p(\ell)$ is the probability density distribution of ℓ and $\ell_m/2$ is the maximum (positive) value of ℓ. The probability of loss is then

$$P = \int_{x_o}^{L/2} \frac{dx}{L} \int_{L/2-x}^{\frac{\ell_m}{2}} d\ell p(\ell) \quad .$$

Where $x_o = -L/2$ for $\ell_m/L \geq 2$ and $x_o = \frac{1}{2}(L-\ell_m)$ for $\ell_m/L \leq 2$, i.e. we include in the integral only those particles which can be lost. If the distribution $p(\ell) = 1/\ell_m$ (uniform) Eq. 1 is equal to

$$\frac{1}{\ell_m}\left(\frac{\ell_m}{2} - \frac{L}{2} + x\right)$$

and we get (multiplying by 2 to include loss in the negative ℓ direction)

$$P = \frac{\ell_m}{4L} \qquad \text{for} \quad \frac{\ell_m}{L} \leq 2$$

$$= 1 - \frac{L}{\ell_m} \qquad \text{for} \quad \frac{\ell_m}{L} \geq 2 \quad .$$

A physically more realistic $p(\ell)$ might be a parabolic one, i.e.

$$p(\ell) = \frac{3}{2}\frac{1}{\ell_m} - \frac{6}{\ell_m^3}\ell^2$$

and, as above, we get

$$P = \frac{3}{16}\frac{\ell_m}{L} \qquad \text{for} \quad \frac{\ell_m}{L} \leq 2$$

$$= 1 - \frac{3}{2}\frac{L}{\ell_m} + \left(\frac{L}{\ell_m}\right)^3 \qquad \text{for} \quad \frac{\ell_m}{L} \geq 2 \quad ,$$

and we see that P is not very sensitive to the exact form of $p(\ell)$.

It can be shown[4] that the mean life of a state of fluctuation, N, for continuous observation is given by

$$T_N = 1/(N+\langle N\rangle)P_o$$

where $P(\delta t) = P_o \delta t + O(\delta t^2)$. We see that the solutions of P for $\frac{\ell_m}{L} \leq 2$ satisfy this so we have $(N \cong \langle N \rangle)$

$$T_N \cong (\tfrac{1}{2}\langle N \rangle \frac{\ell_m}{L})^{-1} \quad .$$

Substituting $\langle N \rangle = N_T L/C$ and $\ell_m = 2\eta \frac{\Delta p_m}{p} C$ we get

$$T_N \cong (N_T \eta \frac{\Delta p_m}{p})^{-1} \text{ revolutions .}$$

For 10^{10} antiprotons stored in the ISA at 30 GeV with $\Delta p/p = \pm\ 10\%$ we get

$$T \cong 10^{-5} \text{ revolutions} \cong 10^{-10} \text{ sec}$$

or $\cong$ 3 cm of azimuth.

There is another mechanism which contributes to the decay of the state of fluctuation, N. As above, assume the sampled beam to be divided into N_1 and N_2 particles on either side of the mean position. Then, due to betatron oscillation, in one revolution any particle will cross from region 1 to region 2 (or vice-versa) ν times, and the mean life of the state N_1 is $(\langle N_1 \rangle \ll N_T/\nu)$

$$T_{N_1} \cong (2\nu N_1)^{-1} \cong T_{N_2}$$

so that the mean life of the state, N, using $\langle N_1 \rangle \cong \langle N_2 \rangle = 1/2\ N$, is

$$T_N \cong (2\nu N)^{-1} \text{ revolutions} \quad .$$

Substituting $N = 10^5$, $\nu \cong 25$ we get

$$T \cong 1/50 \times 10^{-5} \text{ revolutions.}$$

References

1. L.W. Smith, AGS Tech Note No. 120, and references therein.
2. S. Chandrasekhar, Stochastic Problems in Physics and Astronomy, Rev. Mod. Phys. 15, 1 (1943).
3. The mean life T_N is given by

$$\frac{1}{T_N} = \frac{1}{T_{N_1}} + \frac{1}{T_{N_2}}$$

or with m smaller slices

$$\frac{1}{T_N} = \sum_{i=1}^{m} \frac{1}{T_i} \quad .$$

4. Reference 2, p.53.

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