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# DECAY OF FLUCTUATION STATES IN STOCHASTIC COOLING

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AGS DIVISION TECHNICAL NOTE

No. 122

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I. Introduction

Stochastic cooling<sup>1</sup> operates by sensing (and subsequently partially correcting) a statistical fluctuation in a finite sample of particles. It is pointed out in this note that the mean life of such a statistical fluctuation is so short as to make the physical realization of stochastic cooling impractical.

II. Background

Suppose the beam sample,  $N$ , be divided into two parts with  $N_1$  and  $N_2$  particles on the positive and negative side, respectively, of the mean beam position. The sensed average position is

$$\bar{y} = \frac{N_1 - N_2}{N_1 + N_2} \cong \frac{N_1 - N_2}{N} s = \frac{n}{N} s$$

where  $s$  is a measure of the total beam size and we set  $n \ll N$  so that we can take the denominator,  $N$ , to be a (quasi) constant.

If  $N_1$  and  $N_2$  are normally distributed, independent variables  $N_1 - N_2$  is also normally distributed with mean zero and variance  $N$ . The probability density distribution function of  $n$  is

$$p(n) = \frac{1}{(2\pi N)^{1/2}} \exp\left(-\frac{n^2}{2N}\right),$$

and transforming to the variable,  $\bar{y}$ ,

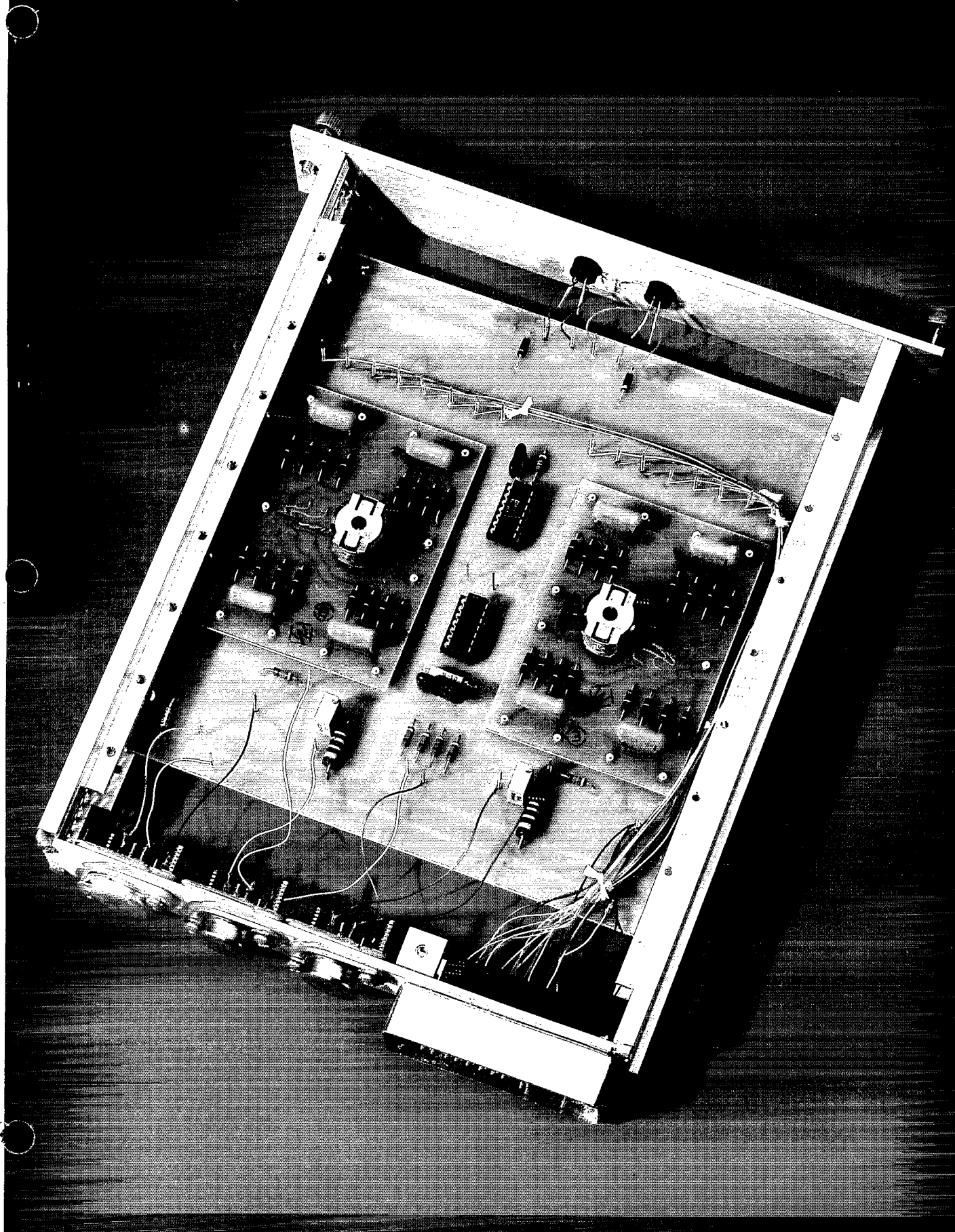


Fig. 2 Bias Supply in NTM package

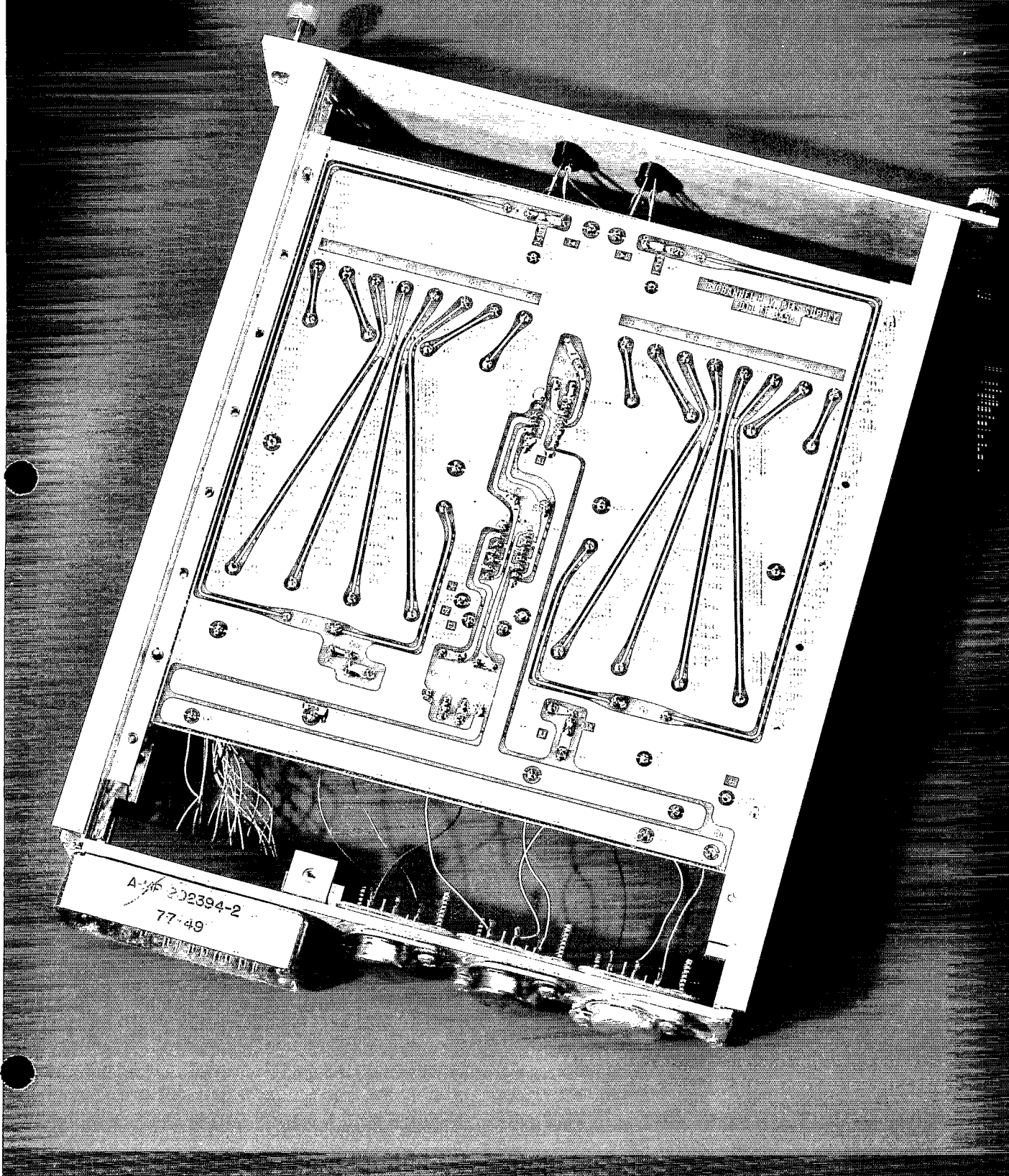


Fig. 3 Bias Supply Circuit Board

$$p(\bar{y}) = \frac{1}{s} \left( \frac{N}{2\pi} \right)^{\frac{1}{2}} \exp \left( - \frac{\bar{y}^2 N}{2s^2} \right)$$

with

$$\langle \bar{y}^2 \rangle = s^2 / N$$

i.e.

$$s^2 = \sigma^2 \cong \frac{\sum_{i=1}^{N_T} y_i^2}{N_T} \cong \frac{\sum_{i=1}^N y_i^2}{N}$$

$N_T$  is the total universe out of which  $N$  is a sample ( $N_T \gg N$ ).

In a real beam we do not have a static situation such as drawing balls from an urn and the statistical fluctuation,  $N \neq \langle N \rangle$ , which produces the detected  $\bar{y}$  will persist for only some period of time, i.e. there are probability after effects.<sup>2</sup> The mean life of the fluctuation,  $\bar{y}$ , is the same as the mean life of the fluctuation,  $N$ .<sup>3</sup>

### III. Calculation of Mean Life

We seek the probability that a particle will escape from the sample,  $N$ , which is within the azimuthal length from  $-\frac{L}{2}$  to  $+\frac{L}{2}$ . In one revolution a particle will lag or lead a particle with central momentum  $p$  by

$$\ell = \eta c \frac{\Delta p}{p}$$

if its momentum is different from the central momentum by  $\Delta p$  ( $\eta = \frac{-2}{\gamma_T} - \frac{-2}{\gamma}$ ).

We assume the sample to be uniformly distributed from  $-\frac{L}{2}$  to  $+\frac{L}{2}$ . The probability of loss (in the positive  $\ell$  direction) is equal to the probability of being in  $\delta x$  times the probability that  $\ell > \frac{L}{2} - x$  integrated over the range of  $x$  which can contribute. The probability that  $\ell > \frac{L}{2} - x$  is

$$\int_{\frac{L}{2}-x}^{\frac{\ell_m}{2}} d\ell p(\ell) \quad (1)$$

Where  $p(\ell)$  is the probability density distribution of  $\ell$  and  $\ell_m/2$  is the maxi-

minimum (positive) value of  $l$ . The probability of loss is then

$$P = \int_{x_0}^{L/2} \frac{dx}{L} \int_{L/2-x}^{\frac{l_m}{2}} dl p(l)$$

Where  $x_0 = -L/2$  for  $l_m/L \geq 2$  and  $x_0 = \frac{1}{2}(L-l_m)$  for  $l_m/L \leq 2$ ; i.e. we include in the integral only those particles which can be lost. If the distribution  $p(l) = 1/l_m$  (uniform) Eq. 1 is equal to

$$\frac{1}{l_m} \left( \frac{l_m}{2} - \frac{L}{2} + x \right)$$

and we get (multiplying by 2 to include loss in the negative  $l$  direction)

$$P = \frac{l_m}{4L} \quad \text{for } \frac{l_m}{L} \leq 2$$

$$= 1 - \frac{L}{l_m} \quad \text{for } \frac{l_m}{L} \geq 2$$

A physically more realistic  $p(l)$  might be a parabolic one, i.e.

$$p(l) = \frac{3}{2} \frac{1}{l_m} - \frac{6}{l_m^3} l^2$$

and, as above, we get

$$P = \frac{3}{16} \frac{l_m}{L} \quad \text{for } \frac{l_m}{L} \leq 2$$

$$= 1 - \frac{3}{2} \frac{L}{l_m} + \left( \frac{L}{l_m} \right)^3 \quad \text{for } \frac{l_m}{L} \geq 2$$

and we see that  $P$  is not very sensitive to the exact form of  $p(l)$ .

It can be shown<sup>4</sup> that the mean life of a state of fluctuation,  $N$ , for continuous observation is given by

$$T_N = 1/(N + \langle N \rangle) P_0$$

where  $P(\delta t) = P_0 \delta t + O(\delta t^2)$ . We see that the solutions of P for  $\frac{\ell_m}{L} \leq 2$  satisfy this so we have  $(N \cong \langle N \rangle)$

$$T_N \cong \left( \frac{1}{2} \langle N \rangle \frac{\ell_m}{L} \right)^{-1}$$

Substituting  $\langle N \rangle = N_T L / C$  and  $\ell_m = 2\eta \frac{\Delta p_m}{p} C$  we get

$$T_N \cong \left( N_T \eta \frac{\Delta p_m}{p} \right)^{-1} \text{ revolutions .}$$

For  $10^{10}$  antiprotons stored in the ISA at 30 GeV with  $\Delta p/p = \pm 10\%$  we get

$$T \cong 10^{-5} \text{ revolutions} \cong 10^{-10} \text{ sec}$$

or  $\cong 3$  cm of azimuth.

There is another mechanism which contributes to the decay of the state of fluctuation, N. As above, assume the sampled beam to be divided into  $N_1$  and  $N_2$  particles on either side of the mean position. Then, due to betatron oscillation, in one revolution any particle will cross from region 1 to region 2 (or vice-versa)  $\nu$  times, and the mean life of the state  $N_1$  is

$$\left( \frac{\langle N_1 \rangle}{\langle N_1 \rangle + \langle N_2 \rangle / \nu} \right)^{-1}$$

$$T_{N_1} \cong (2\nu N_1)^{-1} \cong T_{N_2}$$

so that the mean life of the state, N, using  $\langle N_1 \rangle \cong \langle N_2 \rangle = 1/2 N$ , is

$$T_N \cong (2\nu N)^{-1} \text{ revolutions .}$$

Substituting  $N = 10^5$ ,  $\nu \cong 25$  we get

$$T \cong 1/50 \times 10^{-5} \text{ revolutions .}$$



References

1. L.W. Smith, AGS Tech Note No. 120, and references therein.
2. S. Chandrasekhar, Stochastic Problems in Physics and Astronomy, Rev. Mod. Phys. 15, 1 (1943).
3. The mean life  $T_N$  is given by

$$\frac{1}{T_N} = \frac{1}{T_{N_1}} + \frac{1}{T_{N_2}}$$

or with m smaller slices

$$\frac{1}{T_N} = \sum_{i=1}^m \frac{1}{T_i}$$

4. Reference 2, p.53.

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