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HARMONIC CURRENTS IN AN ARRAY OF VARIABLE PHASED POWER SUPPLIES

M. Meth

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Collider Accelerator Department

Brookhaven National Laboratory

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M. METH and J. SANDBERG

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ALTERNATING GRADIENT SYNCHROTRON DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

HARMONIC CURRENTS IN AN ARRAY OF VARIABLE PHASED POWER SUPPLIES

The literature and discussion of an array of multi-phase power supplies generally assumes that the various power supplies are operated in phase (ie. each supply is at maximum voltage and firing angles are equal). In this case the generated harmonic currents completely reinforce, which is not true at the BNL Accelerator installations. Power supplies are independently adjusted to arbitrary voltages by varying the firing angle of the supplies. The phase of the line currents reflect the firing angles of the power supplies. Since the harmonic components of the line currents are typically not in phase, they can either reinforce or cancel. This study shows that in the general case where firing angles are dissimilar, the percent harmonics are always reduced from those generated when identical firing angles are assumed.

Recent measurements of the harmonic composition of line currents at the 480-volt level and the 13.8 kV level at BNL seem to verify the independent phasing of the various supplies. The arithmetic addition of the various harmonics on the 480-volt line was significantly greater than the composite harmonics on the 13.8kV feeder, including corrections for the voltage transformation ratio and commutation overlap.

The elements of this phenomenon can be readily explained by examining two-independent power supplies energized from a common feed, see Figure 1. Each power supply is a delta, 6-phase star. Power supply 1 is full on; supply 2 is delayed in phase 30° (on a 60-hz scale) and has an output of .866 of its maximum voltage. The analysis is on a per unit basis using the rated DC voltages and current as base values. Figure 2A is the idealized line current for power supply 1. The line current for power supply 2 is the same as for 1 but delayed 30° (60 Hz scale). The composite line current is given in Figure 2C. It is obvious that the waveform of Figure 2A is richer in harmonics than is the waveform of Figure 2C. Idealized waveforms neglect the rounding of the edges due to commutation overlap. For each of the waveforms analyzed the origin is selected to maintain odd-function symmetry. For example, in Figure 2C the origin has been translated 15°. For a development of how the parameter of Figure 1 are derived, see Appendix 1.

The harmonic analysis of the current waveforms of Figure 2A and Figure 2C is summarized in Table I. (See Appendix 2) Note the substantial reduction in the 5th and 7th harmonic components (factor of 4) and the reduction in the total harmonic distortion (factor of 2) by the addition of these two waveforms.

The addition of two current waveforms of equal magnitude with a phase difference of 36° (relative rectified voltage of value 1.0 and 0.81) will cancel the 5th harmonic distortion and reduce the 7th harmonic distortion by a factor of 1.6. The 5th harmonic component will be reduced from this phenomenon for all phase difference that are less than 72° (rectified voltages of 1.0 and 0.31).

As another example of the reduction of harmonic distortion due to the addition of line currents is the case of 45° phase back. One supply is full-on and the other is phased back 45°. The rectified voltages are 1.0 and 0.707, the two rectified currents are equal. The harmonic analysis for this case is included and is summarized in Table II. Note that the 5th, 11th, and 13th harmonic distortion is reduced by a factor of 2.6; where as the 7th harmonic distortion remains the same. Table

II summarizes the performance of two 6-pulse supplies as a function of phase – back (α) of one of the two supplies.

Harmonic reduction and or cancellation does not affect the power flow between the line and the supply. Power flow requires that the current and voltage be of the same frequency and in phase for real power to flow or in quadrature for reactive power to flow. The integral involving the product of current and voltage of different frequencies, including harmonic relationships, is zero and does not contribute to power flow and to power balance.

The analysis can be performed either in the time domain or in the frequency domain. Time domain analysis involves developing the Fourier Series of the composite waveform. Frequency domain analysis involves linear superposition of the phase – shifted harmonic terms of the individual supplies (See Appendix 2).

The current harmonic distortion from multi-phase power supplies follows the 1/n rule (n = harmonic numbers: 5, 7, 11, 13...) at the lowest voltage level, generally 480 volts at BNL. At distribution and transmission voltage levels (13.8 kV and 69 kV) there is no obvious rule to predicate the value of harmonic currents knowing the currents of the individual power supplies. However, it is obvious from the examples cited that the composite harmonic distortion is significantly less than the arithmetic sum of the individual supplies.

As a second example consider three independent power supplies powered by a common power line. One supply is full on; the other two are phased back; One by 18° and the other by 36°. Relative rectified voltages are 1.0, 0.951 and 0.809. The load currents of the three supplies are equal, normalized value of 1.0. Figure 3B depicts the composite line current. The harmonic analysis is summarized in Table III and is compared to the composite line current resulting from three independent supplies that are each operated full on.

The line current for the three phased back supplies has considerably less harmonic distortion than the line current for two phased back supplies. In fact, as the number of supplies with random phase back increase, the harmonic distortion on the feeder decreases.

Two conclusions can be drawn from this study: first, harmonic currents at the 13.8 kV and 69 kV levels must be measured and not calculated from measurements taken at the 480 volt level. Second, measurements of 480 volt harmonic currents made by outside consultants in prior years is not particularly useful in determining the labs harmonic load. This loading is required for the design of power line filters. The employment of the arithmetic sum for the composite current could result in substantial over rating the components of the filter.

PARAMETERS	Waveform of FIG. 2A	Waveform of FIG. 2C
Fundamental Components, rms	0.7797	1.506
5th Harmonic rms 5th Harmonic %	0.1559 20%	0.0807 5.35%
7th Harmonic rms 7th Harmonic %	0.1114 14.3%	0.0576 3.82%
11th Harmonic rms 11th Harmonic %	0.0709 9.1%	0.1369 9.1%
13th Harmonic rms 13th Harmonic %	0.0599 7.7%	0.1159 7.7%
RMS Value	0.8165	1.5275
Total Harmonic Distortion rms Total Harmonic Distortion %	0.2424 31.1%	0.2554 16.96%

Table I
Harmonic Analysis of Current Waveforms

PARAMETERS	PHASE BACK						
	0°	30°	36°	45°	60°	72°	90°
Rectified Voltages	1.0, 1.0	1.0, 0.867	1.0, 0.809	1.0, 0.707	1.0, 0.5	1.0, 0.31	1.0, 0
Displacement Power Factor	1.0	0.966	0.951	0.924	0.866	0.809	0.707
Fundamental Current rms	1.559	1.507	1.483	1.441	1.350	1.262	1.103
5th Harmonic rms	0.312	0.0806	0	0.119	0.270	0.312	0.2205
Distortion	20%	5.35%	0	8.3%	20%	24.7%	20%
7th Harmonic rms	0.223	0.0576	0.131	0.206	0.193	0.069	0.1575
Distortion	14.3%	3.82%	8.83%	14.3%	14.3%	5.45%	14.3%
11th Harmonic rms	0.142	0.137	0.135	0.054	0.123	0.115	0.1002
Distortion	9.1%	9.1%	9.1%	3.76%	9.1%	9.1%	9.1%
13th Harmonic rms	0.120	0.116	0.0705	0.046	0.104	0.036	0.0848
Distortion	7.7%	7.7%	4.75%	3.2%	7.7%	2.9%	7.7%
RMS Value	1.6330	1.5275	1.506	1.472	1.414	1.3165	1.1547
Total Harmonic Distortion rms	0.486	0.246	0.2596	0.300	0.421	0.375	0.3416
Total Harmonic Distortion %	31.1%	16.2%	17.5%	20.8%	31.1%	29.7%	31.1%

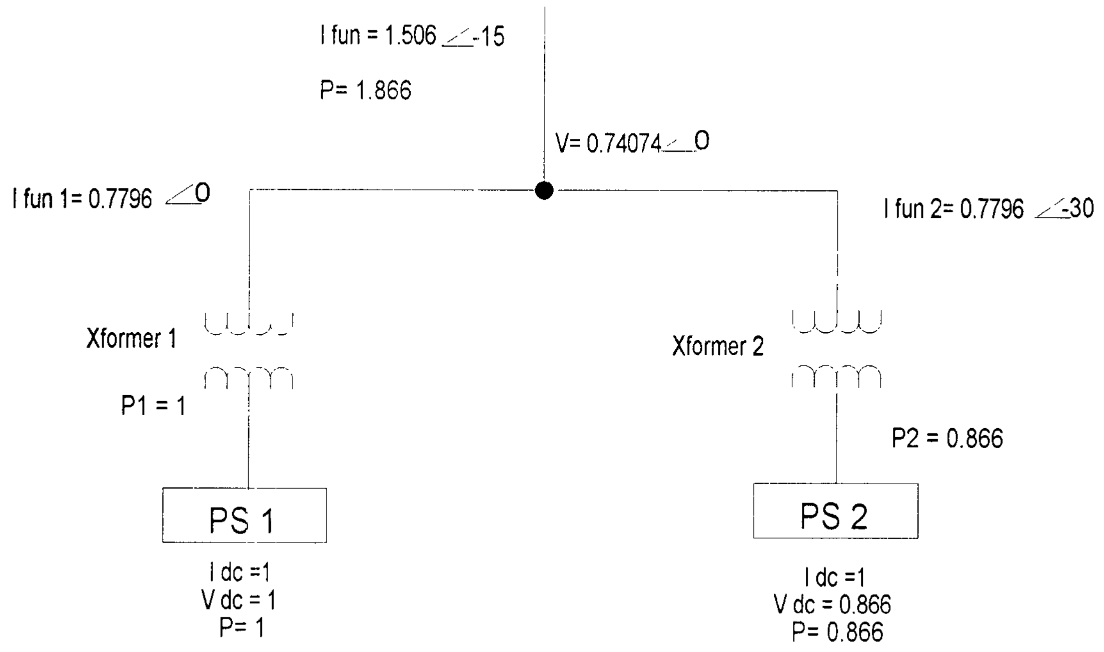
Table II

Performance of two 6-pulse supplies as a function of phase-back (α) of second supply. First supply is not phased-back.

PARAMETER	PHASE BACK	
	0 °, 0°, 0°	0°, 18°, 36°
Rectified Voltage	1.0, 1.0, 1.0	1.0, 0.951, 0.809
Displacement Power Factor	1.0	0.951
Fundamental Current rms	2.339	2.263
5th Harmonic rms Distortion	0.468 20%	0.156 6.9%
7th Harmonic rms Distortion	0.335 14.3%	0.02 0.9%
11th Harmonic rms Distortion	0.213 9.1%	0.064 2.75%
13th Harmonic rms Distortion	0.180 7.7%	0.011 0.5%
17th Harmonic rms Distortion	0.138 5.9%	0.100 4.4%
19th Harmonic rms Distortion	0.123 5.3%	0.119 5.3%
RMS Value	2.449	2.2804
Total Harmonic Distortion rms Total Harmonic Distortion %	0.727 31.1%	0.281 12.43%

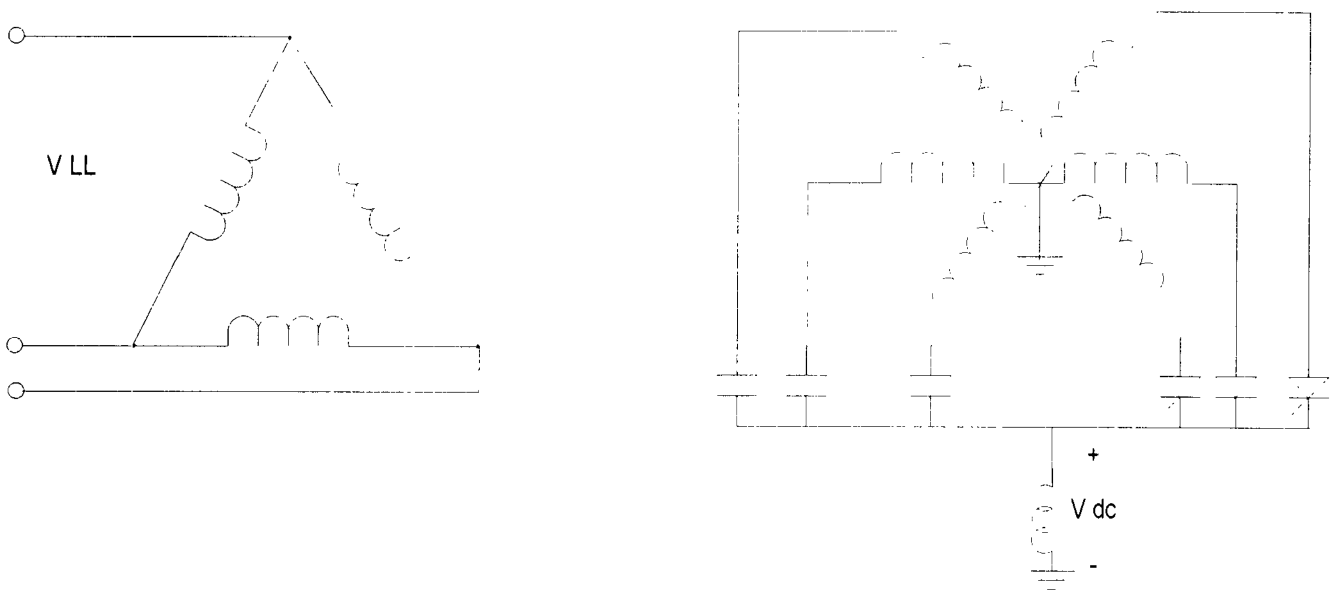
Table III

Performance of three 6-pulse supplies.



Two Independent Power Supplies 30 degrees apart

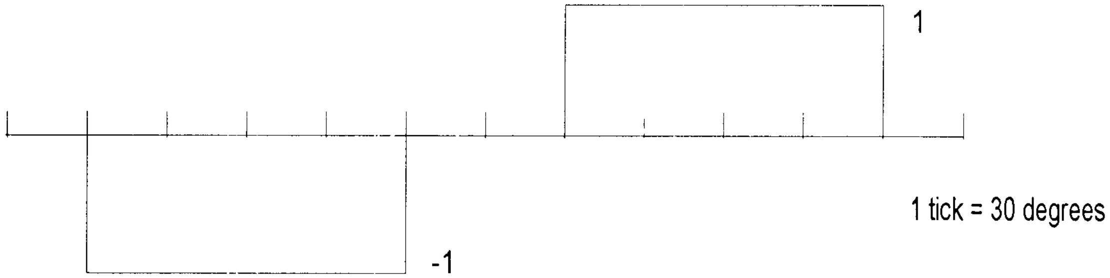
Figure 1 A



Power Supply; Delta, 6-Phase Star

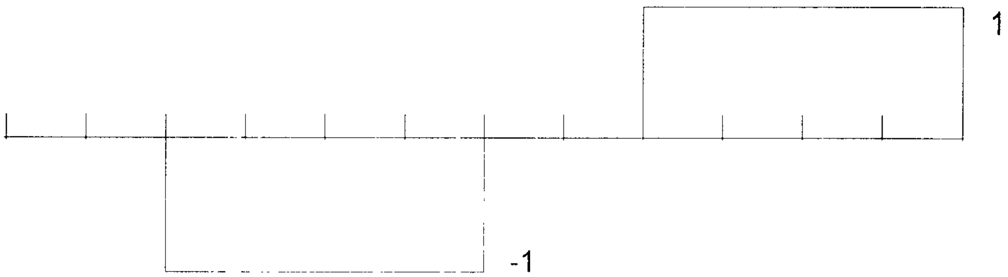
Figure 1 B

Figure 1



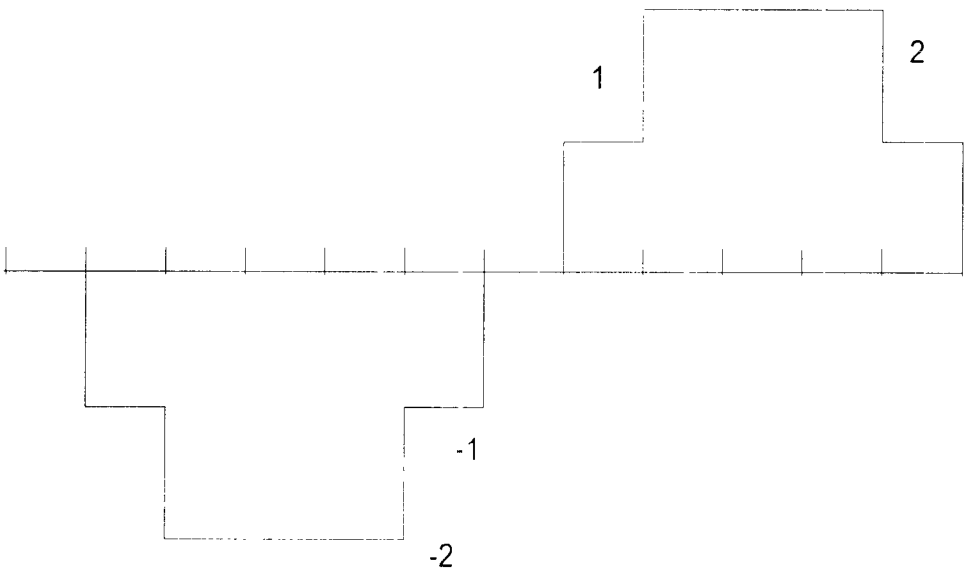
Line Current for Power Supply 1

Figure 2 A



Line Current for Power Supply 2

Figure 2 B



Composite Line Current for Power Supplies 1 & 2

Figure 2 C

Figure 2

1 tick = 6 degrees

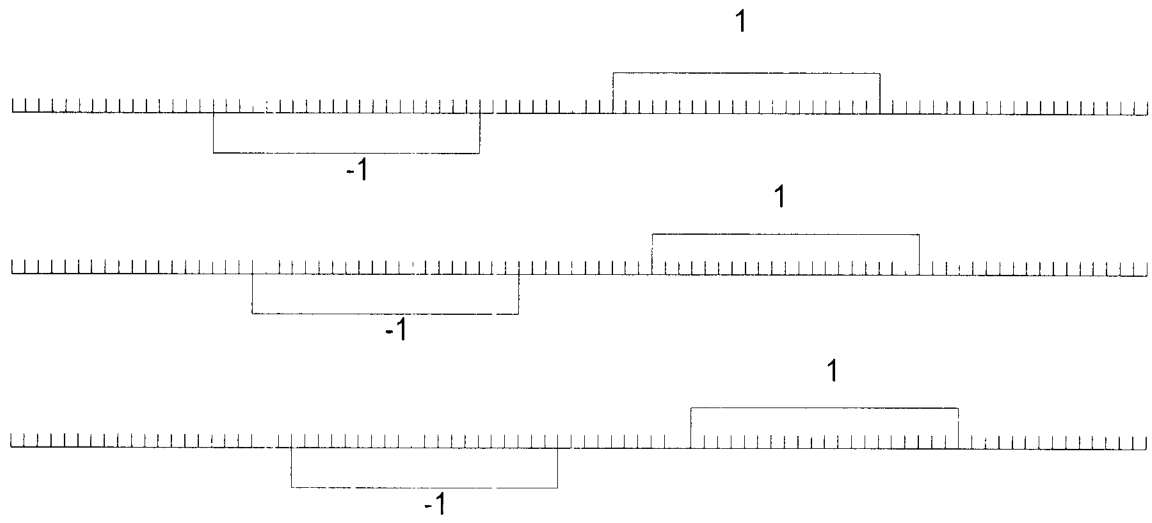


Figure 3 A

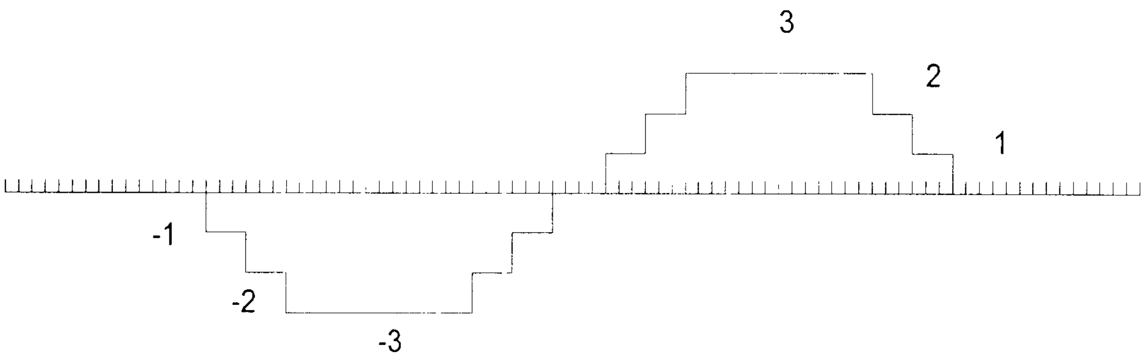


Figure 3 B

Composite Line Current for ThreePower Supplies

Phase Back of 0, 18, & 36 degrees.

Figure 3

Appendix 1

Power Balance/Fundamental Current

For a six pulse bridge connection:

$$V_{dc} = \sqrt{2}\ V_{LL}\ (\tfrac{6}{\pi})\ \mathrm{Sin}\ (\tfrac{\pi}{6})\ \mathrm{Cos}\ \alpha$$

$$V_{dc} = 1.35\ V_{LL}\ \mathrm{Cos}\ \alpha$$

Where α is the phase-back angle.

$$P_{dc} = I_{dc}\ V_{dc} = \sqrt{3}\ V_{LL}\ I_{fun}\ \mathrm{Cos}\ \theta$$

Where I_{fun} is the fundamental component of the AC line current.

θ is the angle between I_{fun} and V_{LL}. In this case $\alpha = \theta$

$$V_1 = V_2 = \tfrac{1}{135} = 0.7407$$

$$P_{ac} = P_{dc}$$

$$P_1 = P_{dc} = 1 \qquad\qquad P_2 = V_{dc}\ I_{dc} = 0.866$$

$$Pin = P_1 + P_2 = 1.866$$

$$I_{fun1} = \frac{P1}{\sqrt{3}\, V_{LL}\ Cos\,\theta} = \frac{1}{\sqrt{3}\ (0.7407)\ 1} = 0.7797$$

$$I_{fun2} = \frac{P_2}{\sqrt{3}\, V_{LL}\ Cos\,\theta} = \frac{.866}{\sqrt{3}\ (0.7407)\ Cos\ 30} = 0.7797$$

$$I_{fun}\ in = I_{fun1} + I_{fun2} = 0.7797 + 0.7797\ \measuredangle{-30}$$

$$= 1.506\ \measuredangle{-15}$$

Appendix 2

Harmonic Calculations

Period = 2π $$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (an \, Cos\, n\omega t + bn \, Sin\, n\omega t)$$

Fourier Analysis of:

Fig. 2A

an = 0 (odd function)

$$bn = \frac{2}{\pi} \int_0^{\pi} f\ (wt)\ Sin\ \frac{n\pi wt}{\pi}\ dwt$$

$$bn = \frac{2}{\pi n} \left[- Cos\, n\, wt \right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{2}{\pi n} \left[- Cos\, n \frac{5\pi}{6} + Cos\, n \frac{\pi}{6} \right]$$

For rms values

$$bn = \frac{2}{\pi n \sqrt{2}} \left[- Cos \frac{5\pi n}{6} + Cos\, n \frac{\pi}{6} \right]$$

n	bn rms
1	0.7797
5	-0.1559

Figure 2C

$$bn = \frac{2}{\pi n} \left[\left[- Cos\, n\, wt \right]_{\frac{\pi}{12}}^{\frac{3\pi}{12}} + \left[- 2\, Cos\, n\, wt \right]_{\frac{3\pi}{12}}^{\frac{3\pi}{4}} + \left[- Cos\, n\, wt \right]_{\frac{3\pi}{4}}^{\frac{11\pi}{12}} \right]$$

$$bn = \frac{2}{\pi n}\left[- \text{Cos}\frac{\pi}{4}n + \text{Cos}\frac{\pi}{12}n - 2\,\text{Cos}\frac{3\pi}{4}n + 2\,\text{Cos}\frac{\pi}{4}n - \text{Cos}\frac{11\pi}{12}n + \text{Cos}\frac{3\pi}{4}n \right]$$

$$bn\ rms = \frac{2}{\pi n\sqrt{2}}\left[- \text{Cos}\ n\frac{\pi}{4}n + \text{Cos}\frac{\pi}{12}n - 2\,\text{Cos}\frac{3\pi}{4}n + 2\,\text{Cos}\frac{\pi}{4}n - \text{Cos}\frac{11\pi}{12}n + \text{Cos}\frac{3\pi}{4}n \right]$$

n	bn rms
1	1.506
5	.0807

Figure 3B

$$bn\ rms = \frac{2}{\pi n\sqrt{2}}\left[- \text{Cos}\ n\frac{\pi}{6} + \text{Cos}\ n\frac{\pi}{15} - 2\,\text{Cos}\ n\frac{4\pi}{15} + 2\,\text{Cos}\ n\frac{\pi}{6} - 3\,\text{Cos}\ n\frac{11\pi}{15} + 3\,\text{Cos}\ n\frac{4\pi}{15} - 2\,\text{Cos}\ n\frac{5\pi}{6} + 2\,\text{Cos}\ n\frac{11\pi}{15} - \text{Cos}\ n\frac{14\pi}{15} + \text{Cos}\ n\frac{5\pi}{6} \right]$$

n	bn rms
1	2.263
5	0.156

Phasor Analysis

Sumation of 2A + 2A $\sphericalangle 30$ (See Table I) corresponds to Fig. 2C

$I_1 = .7797 + .7797 \sphericalangle 30$

$I_1 = .7797 + .67524 + j\,0.38985$

$= 1.4549 + j\,0.38985$

$|I_1| = 1.506$

$I_5 = .1559 + .1559 \sphericalangle 150°$

$|I_5| = .0807$

Sumation of 2A + 2A $\sphericalangle 18°$ + 2A $\sphericalangle 36°$ (See Table I) corresponds to Fig. 3B

$I_1 = .7797 + .7797 \sphericalangle 18° + .7797 \sphericalangle 36°$

$= .7797 + .7415 + j\,0.24094 + .6308 + j\,0.4583$

$= 2.152 + j\,0.69924$

$|I_1| = 2.263$

$I_5 = .1559 + .1559 \sphericalangle 90 + .1559 \sphericalangle 180$

$|I_5| = .156$

Appendix 3

Harmonic Distortion

Rms Value of Figure 2A

$$I\ rms = \sqrt{\frac{2}{2\pi}\int_{o}^{\frac{2\pi}{3}} I^2 max\ dwt} = \sqrt{\frac{1}{\pi}\left(1^2\frac{2\pi}{3}\right)} = \sqrt{\frac{2}{3}} = 0.8165$$

Rms Value of Figure 2C

$$I\ rms = \sqrt{\frac{2}{2\pi}\left[2\int_{o}^{\frac{\pi}{6}} 1^2\ dwt + \int_{o}^{\frac{\pi}{2}} 2^2\ dwt\right.} = \sqrt{\frac{1}{\pi}\left(\frac{2\pi}{6}+\frac{4\pi}{2}\right)} = 1.5275$$

Rms Value of Figure 3B

$$I\ rms = \sqrt{\frac{2}{2\pi}\left[2\int_{o}^{\frac{\pi}{10}} 1^2\ dwt + 2\int_{o}^{\frac{\pi}{10}} 2^2\ dwt + \int_{o}^{\frac{7\pi}{15}} 3^2\ dwt\right.} = \sqrt{\frac{1}{\pi}\left(\frac{2\pi}{10}+\frac{8\pi}{10}+\frac{63\pi}{15}\right)} = 2.2804$$

Harmonic Distortion:

Id = Total Harmonic Distortion rms

$$I\ rms = \sqrt{I_{fun}^2 + I_D^2} \qquad \sqrt{\sum_{n=2}^{\infty} I_n^2} = I_D$$

$$I_D^2 = I_{rms}^2 - I_{fun}^2 \qquad \sqrt{\sum_{n=1}^{\infty} I_n^2} = I_{rms}$$

Figure 3B

$Id^2 = (2.2804)^2 - (2.263)^2 = 0.07883$

Id = 0.2807

$$I_{THD} = \frac{I_D \times 100}{I_{fun}} = \frac{0.2807 \times 100}{2.263} = 12.4\%$$