

MULTIPLE SCATTERING IN THIN LAYERS

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In calculating the multiple scattering of charged particles it is common practice to assume that the distribution is gaussian and that the mean square angle of that distribution is given by the standard "handbook" formula. In this note I point out that this method, when applied to multiple scattering in thin layers, can be grossly in error, and that a "recipe" originally due to N. Bohr and applied by Bethe and Ashkin gives a good approximation to the mean square angle of the gaussian portion of the scattering distribution.

In passing through matter charged particles undergo a number of individual small angle coulomb scatters, (θ_i). These individual scatters combine statistically to give a net angle (Θ), with respect to the original direction, of "multiple" scattering. Fermi¹ shows that

$$\cos \Theta = \prod_{i=1}^{i=n} \cos \theta_i \quad (1)$$

for n individual scatters and that in the approximation that each $\theta_i \ll 1$

$$[\Theta^2] = \sum_{i=1}^n [\theta_i^2] \quad (2)$$

(square brackets indicate expectation values).

1. Nuclear Physics. Compiled by Orear, Rosenfeld and Schluter; Univ. of Chicago Press, 1949.

Note that Eq. (1) is exact and Eq. (2) is subject only to the small angle approximation. No restrictions are made on the probability distributions of the individual θ_i 's. If the individual θ_i distributions are the same and independent of each other we may write

$$[\theta^2] = \sum [\theta_i^2] = n[\theta^2] \quad , \quad (3)$$

so that the net mean square scattering angle is proportional to some suitable average over the initial scattering distribution. If n becomes very large the Central Limit Theorem of statistics tells us that the distribution of θ will approach the normal, or gaussian, distribution subject only to all of the $[\theta_i^2]$ being finite. There will always be a tail to the gaussian approximation which is due to single scattering but if n is large its effect is small.

The individual scatters are essentially given by the Rutherford distribution, $f(\theta)$, and

$$[\theta^2] = \int_{\theta_{\min}}^{\theta_{\max}} \theta^2 f(\theta) d\theta \approx \ln \frac{\theta_{\max}}{\theta_{\min}} \quad .$$

θ_{\min} is determined by the screening effect of atomic electrons and for relativistic particles is given by

$$\theta_{\min} \cong \frac{\lambda}{a} \quad ,$$

where λ is the de Broglie wavelength of the incident particle and a is the radius of the atom. For relativistic particles θ_{\max} is set by the finite size of the nucleus and

$$\theta_{\max} \cong \lambda/R \quad ,$$

where R is the radius of the nucleus. Rossi and Greisen² using these values for θ_{\min} and θ_{\max} and some other simplifying assumptions arrive at the oft quoted "handbook" formula³

$$[\theta^2] = \left(\frac{21.2}{pv}\right)^2 \frac{X}{X_0} \quad (4)$$

where pv is in units of MeV and X/X_0 is the number of radiation lengths traversed.

For thin layers, even though n is large enough that the central portion of the θ distribution is closely gaussian, Eq. (4) can grossly over estimate the mean square of that gaussian because of the contribution of single scattering events which are much larger than $[\theta^2]^{1/2}$. Rossi⁴ quotes the range of validity of Eq. (4) as

$$\frac{X}{X_0} \gtrsim 46 A^{-2/3}$$

or
$$t \gtrsim 6.7 \left(\frac{137}{Z}\right)^2 A^{1/3} \text{ gm/cm}^2$$

The scattering distribution for thinner layers can be obtained by numerical solution of the exact (small angle approximation) integro-differential diffusion equation, e.g. Snyder and Scott⁵, Moliere⁶ or by Monte Carlo methods. Sternheimer⁷ has pointed out that the effect of single scattering can be approximated by the superposition of two or more gaussians.

2. Rev. Mod. Phys. 13, 240 (1941).
 3. The origin (and usefulness) of the multiplier $(1 + \epsilon)$ in the LBL Particle Properties review is obscure.
 4. High Energy Particle, Prentice Hall (1952).
 5. Phys. Rev. 76, 220 (1949).
 6. Zeits. f. Naturforschung 3a, 78 (1948).
 7. Rev. Sci. Instrum. 25, 1070 (1954).

N. Bohr⁸ has emphasized the statistical similarity of the single scattering contribution in thin layers to the contribution of large energy losses in thin layers (now commonly called the Landau effect). In the latter, calculating the mean energy loss in a manner analogous to the Rossi-Greisen method gives a value larger than the most probable energy loss. The distribution of energy loss is approximately gaussian around the most probable value. Bohr points out that if the upper limit of the averaging integral is taken to be that energy loss (angle θ_1) for which there is, on the average, only one energy loss (scattering angle) larger than it throughout the layer a good gaussian approximation is arrived at.

The use of a gaussian approximation in multiple scattering is particularly useful if one is interested in easily calculating e.g. the FWHM or $1/e$ value of the distribution function. Bethe and Ashkin⁹ using Bohr's prescription, $\theta_{\max} = \theta_1$, arrive at

$$\theta_1^2 = 0.157 Z (Z + 1) \frac{t}{A(pv)^2} ,$$

and
$$[\theta^2] = \theta_1^2 \ln (1.13 \times 10^4 Z^{4/3} A^{-1} t) , \quad (5)$$

for $v \cong c$, where pv is in MeV and t is in grams/cm^2 . The argument of the logarithm, $\theta_1^2/\theta_{\min}^2$, in Eq. (5) is of the order of the average number of scatters so that the region of validity of Eq. (5) is $\theta_1^2 \gg \theta_{\min}^2$. Sternheimer¹⁰ has pointed out that Eq. (5) agrees (prediction of $1/e$ value of the distribution)

8. Danske Viden. Sels. Math.-Fys. Medd. 18 (1940-8), also Phil. Mag. (6), 30, 581 (1915). I commend these two papers to anyone interested in a concise and lucid exposition of the physical principles involved.

9. "Exp. Nuc. Phys." (E. Segre, Ed.) Vol. I, Wiley, 1953.

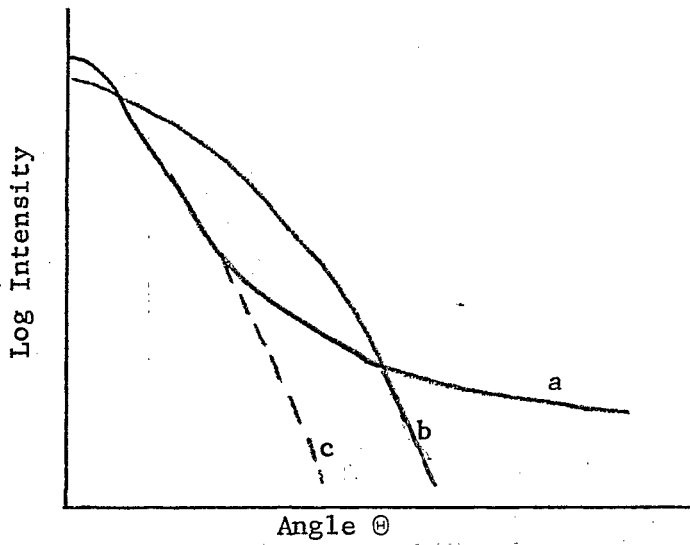
10. "Methods of Exp. Phys." (Yuan, Wu, Eds.) Vol. 5, Part A, Acad. Press, 1961.

with the rather meager data as well as the more sophisticated theories and that Eq. (4) is a gross over estimate.

Figure 1 sketches the relation of the two gaussian approximations using Eq. (4) and Eq. (5).

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a is the actual multiple scattering distribution with single scattering tail; b is the gaussian approximation using Eq. (4); c is the gaussian approximation using Eq. (5).

Fig. 1. Multiple scattering in thin layers.