BNL-105176-2014-TECH

Booster Technical Note No. 132;BNL-105176-2014-IR

SKEW QUADRUPOLE CORRECTIONS

S. Tepikian

October 1988

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

SKEW QUADRUPOLE CORRECTIONS

AD
BOOSTER TECHNICAL NOTE
NO. 132

S. TEPIKIAN
OCTOBER 10, 1988

ACCELERATOR DEVELOPMENT DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

SKEW QUADRUPOLE CORRECTORS

S. Tepikian

Abstract

Two sources of skew quadrupole errors are rotations of normal quadrupole fields and vertical displacements of sextupole fields which can include the eddy current sextupole fields in the dipole magnets at injection. These excite the coupling resonance $\nu_x - \nu_y = 0$ and the sum resonance $\nu_x + \nu_y = 9$ which can be crossed by the space charge tune spread. We correct these resonances with 24 skew quadrupole correctors organized in 4 families. We find for the worst case the strongest skew quadrupole corrector is 0.36% of the main quadrupole strength. Additionally, we find the skew quadrupoles impractical in controlling the beam shape by using coupling because of the space charge tune spread even at the top energy.

1. Introduction

All accelerators contain skew quadrupoles fields due to misalignments. These fields are due to rotational errors in normal quadrupoles, vertical displacements of normal sextupoles and other sources as well. The skew quadrupoles can excite two resonances which may be crossed due to tune spread. For the AGS-Booster the resonances are

$$\nu_x - \nu_y = 0, \quad \nu_x + \nu_y = 9$$

as shown in Fig. 1. Additionally, Fig. 1 shows the expected tune spread due to space charge[1] for protons at injection.

In the AGS-Booster, we included four sources of skew quadrupole errors:

(1) 0.3mrad rotations in the focusing and defocusing quadrupoles,

(2) 0.3mm vertical displacements of the chromaticity correcting sextupoles,

(3) 0.3mm vertical displacements of the dipoles with the eddy current sextupole fields at injection,

(4) 0.3mm displacements between the central axis of the dipoles and the central axis of the eddy current sextupoles,

where we assumed 3 standard deviations on a uniform distribution of errors. These errors excite the resonances stated above and must be corrected to reduce any possible beam loss.

The two resonances given above can be corrected using skew quadrupole correctors. There will be four per superperiod, two in the first full cell and two in the third full cell in the correction trim coil assembly as shown in Fig. 2.

The relation between the skew quadrupoles and resonance strengths are given in section 2. Section 3 gives the scheme to determine the strengths of the correctors. In section 4 we calculate the tune spread at top energy in the AGS-Booster in order to see if it is practical to change the shape of the beam (i.e. from flat to round or vice versa). Finally, a conclusion is given in section 5.

2. Theory

The motion of a particle in the beam of an accelerator can be described by the following Hamiltonian

$$H = \frac{p_x^{\,2}}{2} + \frac{p_y^{\,2}}{2} + \left[K(s) + \frac{1}{\rho^2(s)}\right] \frac{x^2}{2} + M(s)\; xy - K(s)\; \frac{y^2}{2}$$

where x and y are the particles position with respect to the equilibrium, p_x and p_y are the conjugate momenta, K(s) is the quadrupole strength, M(s) is the skew quadrupole strength, $\rho(s)$ is the radius of curvature and s is the independent variable (i.e. the time variable). We have assumed the particle is at the equilibrium momentum.

The resonance strengths can be found by applying two canonical transformations to the Hamiltonian H and then expanding the potential term in a fourier series. The first canonical transformation (which transforms the Hamiltonian to action-angle[2] variables) is given by the generating function shown below:

$$F(x,y,\psi_x,\psi_y,s) = -\frac{x^2}{2\beta_x(s)}\left[\tan\psi_x - \frac{\beta_x'(s)}{2}\right] - \frac{y^2}{2\beta_y(s)}\left[\tan\psi_y - \frac{\beta_y'(s)}{2}\right]$$

where ψ_x and ψ_y are the new angle variables, primes denote d/ds and the functions $\beta_x(s)$ and $\beta_y(s)$ are the solutions of the following equations

$$\frac{\beta_x(s)\beta_x''(s)}{2} - \frac{(\beta_x'(s))^2}{4} + \left(K(s) + \frac{1}{\rho^2(s)}\right)\beta_x^2(s) = 1$$

$$\frac{\beta_y(s)\beta_y''(s)}{2} - \frac{(\beta_y'(s))^2}{4} - K(s)\,\beta_y^2(s) = 1.$$

The actions, J_x and J_y, are related to x and y as follows

$$x = \sqrt{2J_x\beta_x(s)}\,\cos\psi_x$$

$$y = \sqrt{2J_y\beta_y(s)}\,\cos\psi_y.$$

Note, the emittance of the beam is related to the action by $2J_x = \varepsilon_x/\pi$ and $2J_y = \varepsilon_y/\pi$. If there are no skew quadrupoles present (i.e. M(s) = 0) then the emittances (and the actions) are invariants of the motion[3].

Using the generating function, F, the new Hamiltonian becomes

$$H_1 = \frac{J_x}{\beta_x(s)} + \frac{J_y}{\beta_y(s)} + M(s)\sqrt{2J_x\beta_x(s)}\,\sqrt{2J_y\beta_y(s)}\,\cos\psi_x\,\cos\psi_y$$

The next transformation will eliminate the time dependence in the first two terms in the Hamiltonian H_1. This transformation is given by the following generating function

$$G(I_x, I_y, \psi_x, \psi_y, s) = I_x[\psi_x - \mu_x(s)] + I_y[\psi_y - \mu_y(s)]$$

where I_x and I_y are the new action variables and

$$\mu_x(s) = \int_0^s \frac{dt}{\beta_x(t)} - \frac{2\pi}{C} \nu_x s$$

$$\mu_y(s) = \int_0^s \frac{dt}{\beta_y(t)} - \frac{2\pi}{C} \nu_y s$$

and

$$\nu_x = \frac{1}{2\pi} \int_0^C \frac{dt}{\beta_x(t)} ,$$

$$\nu_y = \frac{1}{2\pi} \int_0^C \frac{dt}{\beta_y(t)}$$

with C being the circumference of the accelerator. Note, $\mu_x(s)$ and $\mu_y(s)$ are periodic functions of s with a period of C/P where P is the periodicity of the lattice.

The new action angle variables are related to the old as follows

$$I_x = J_x$$

$$I_y = J_y$$

$$\xi_x = \psi_x - \mu_x(s)$$

$$\xi_y = \psi_y - \mu_y(s)$$

where ξ_x and ξ_y are the new angle variables.

The transformation of Hamiltonian H_1 with the generating function G leads to the following Hamiltonian

$$H_2 = \frac{2\pi}{C}\left[\nu_x I_x + \nu_y I_y + 2[2I_x]^{1/2}\,[2I_y]^{1/2}\mathrm{Im}\left(\sum_{n=-\infty}^{\infty} e^{-i2\pi ns/C}\right.\right.$$

$$\left.\left.\left(A_n\, e^{i(\xi_x-\xi_y)} + B_n\, e^{i(\xi_x+\xi_y)}\right)\right)\right]$$

where

$$A_n = \frac{i}{4\pi}\int_0^C M(t)\beta_x^{1/2}(t)\beta_y^{1/2}(t)e^{i[\mu_x(t)-\mu_y(t)+2\pi nt/C]}\,dt$$

$$B_n = \frac{i}{4\pi}\int_0^C M(t)\beta_x^{1/2}(t)\beta_y^{1/2}(t)e^{i[\mu_x(t)+\mu_y(t)+2\pi nt/C]}\,dt$$

The terms A_n and B_n give the resonance strengths for the following resonances

$$\nu_x - \nu_y = n, \quad \nu_x + \nu_y = n$$

respectively. In the next section we will minimize the resonance strengths for the resonances given in the introduction.

3. Strategy

The AGS-Booster consist of six superperiods and each superperiod contains eight correction trim coil assemblies. Four of these assemblies will contain the skew quadrupole correctors. Using these trim coils we want to generate a resonances that cancels those resonances excited by the skew quadrupole errors. Thus, in an ideal accelerator with no skew quadrupole errors, we want to find the strengths of the skew quadrupole correctors to excite any given imperfection resonance strength.

We are concerned with the two resonances listed in the introduction. The corresponding strengths are A_0 and B_9. Since these numbers are complex, there are four conditions to be satisfied. The integrals in the previous section relating the skew quadrupole strengths to the resonance strengths can be converted to sums by using the thin lens approximation as

$$A_n = \frac{i}{4\pi} \sum_{p=1}^{6} \sum_{q=1}^{4} M_{pq} \beta_x^{1/2}(t_{pq}) \beta_y^{1/2}(t_{pq}) e^{i[\mu_x(t_{pq}) - \mu_y(t_{pq}) + 2\pi n t_{pq}/C]}$$

$$B_n = \frac{i}{4\pi} \sum_{p=1}^{6} \sum_{q=1}^{4} M_{pq} \beta_x^{1/2}(t_{pq}) \beta_y^{1/2}(t_{pq}) e^{i[\mu_x(t_{pq}) + \mu_y(t_{pq}) + 2\pi n t_{pq}/C]}$$

where the outer sum is over the superperiods, M_{pq} are the integrated skew quadrupole strength of the correctors and t_{pq} are the positions of the center of the correctors.

Due to the periodicity of the β and μ functions the above sums can be simplified. First, we define

$$t_q = t_{(p=1)q}$$

then

$$t_{pq} = t_q + (p-1)C/6$$

for p = 1, 2, 3, ... 6 and q = 1, 2, 3 and 4. Defining the coefficients f_p so that

$$M_{pq} = f_p M_q$$

defines four families of skew quadrupole correctors. The sums can now be written as

$$A_n = \frac{i}{4\pi} \left[\sum_{p=1}^{6} f_p e^{i\pi n(p-1)/3} \right] \sum_{q=1}^{4} M_q \beta_x^{1/2}(t_q) \beta_y^{1/2}(t_q) e^{i[\mu_x(t_q) - \mu_y(t_q) + 2\pi n t_q/C]}$$

$$B_n = \frac{i}{4\pi} \left[\sum_{p=1}^{6} f_p e^{i\pi n(p-1)/3} \right] \sum_{q=1}^{4} M_q \beta_x^{1/2}(t_q) \beta_y^{1/2}(t_q) e^{i[\mu_x(t_q) + \mu_y(t_q) + 2\pi n t_q/C]}$$

Choosing $f_p = \cos[\pi n(p-1)/3]$ for n = 0 and for n = 9 leads to the greatest contribution to each of the two resonances separately. Furthermore, the two resonances of f_p for n = 0 and for n = 9 are orthogonal (i.e. f_p

of $n = 0$ doesn't contribute to B_9 and $n=9$ doesn't contribute to A_0).

Now we can solve for the unknowns M_q for a given choice of A_0 and B_9. Due to the orthogonality between $n = 0$ and $n = 9$ we have eight unknowns (from two independent sets of M_q) with four conditions. These additional unknowns are handeled by the following scheme:

$$M_1 = M_3 \text{ and } M_2 = M_4$$

A listing of a program that uses the above strategy for solving for the strengths of the skew quadrupole correctors is given in the appendix. Using this program we have calculated the strengths for various different random seeds for the skew quadrupole errors given in the introduction. We have assumed the eddy current sextupole of $B'' = .24\ T/m^2$ in the dipoles.[4] In each case we used a uniform distribution of errors. The table below shows the random number seed versus the maximum strength of the corrector

SEED	INTEGRATED STRENGTH m^{-1}	% OF MAIN QUAD. STRENGTH
51351	0.0001717	0.31%
8795	0.0001716	0.31%
85003	0.0001210	0.22%
6221	0.0001239	0.22%
900863	0.0001991	0.36%
5819	0.0001350	0.24%

where we assumed a trim coil 10 cm long.

4. Coupling

The coupling resonance $\nu_x - \nu_y = 0$ that is excited by the skew quadrupoles can couple the transverse emittances (i.e. $\varepsilon_x + \varepsilon_y$=constant). Thus, if ε_x is increased then ε_y must decrease and vice versa. Can this phenomena be used to transform the beam from a flat to a round or vice versa in the AGS-Booster?

To investigate this consider the tune split of the AGS-Booster which is $\Delta\nu = \nu_y - \nu_x = 0.01$. With this tune split, the coupling will cause the beam to oscillate from flat to round to flat etc. about every 25 revolutions. This number may vary due to nonlinearities that can effect the tunes and thus, the tune split.

Different particles in the beam may have different tune splits due to tune spread. In order for all the particles to be coupled coherently, then we get the following requirement on the tune spread

$$\delta\nu_x,\ \delta\nu_y << 0.005.$$

The major cause of tune spread in the AGS-Booster is due to space charge effects. The space charge tune spread is scaled with beam energy as follows

$$\gamma^2\beta\ \delta\nu_{SP} = \text{constant}.$$

Parzen[1] has calculated the space charge tune spread for protons at 200Mev in the AGS-Booster giving

$$\delta\nu_x = -.52, \qquad \delta\nu_y = -.58$$

From the scaling of space charge tune spread we find

$$\delta\nu_x = -.070, \qquad \delta\nu_y = -.077$$

for protons at 1.5Gev.

Hence, all the particles will not coherently transform from a flat beam to a round beam at the top energy of the AGS-Booster.

5. Conclusion

A particle in the AGS-Booster can cross the coupling resonance $\nu_x-\nu_y=0$ and the sum resonance $\nu_x+\nu_y=9$, which is excited by the skew quadrupole errors, due to the large space charge tune spread at injection. We have formulated a strategy for correcting these resonances using half of the correction trim coil assemblies available in the Booster for skew quadrupole correctors. Using the arrangement of the correctors described in section 3, we have been able to correct the two resonances. For the errors given in the introduction, the largest skew quadrupole corrector is found to be 0.36% of the main quadrupole's field strength.

Additionally, the four errors listed in the introduction give about equal contribution to the resonance strengths and thus, to the maximum strength of the corrector required. With this, we can easily scale the results if the actual errors are different than those assumed in section 1.

References

1. G. Parzen, Booster Tech Note #127
2. H. Goldstein, Classical Mechanics, (Addison Wesley, 1965)
3. E. D. Courant and H. S. Snyder, Annals of Physics 3, 1-48 (1958)
4. G. Morgan and S. Kahn, Booster Tech Note #4
5. W. H. Press, et al, Numerical Recipes (Cambridge University Press, 1986)

Appendix

Below is a listing of the program that was used to calculate the skew quadrupole corrector strengths. The program is written in FORTRAN 77 and was tested on the VAX. The input consists of two files. The first file includes the number of correctors per superperiod; the number of superperiods; position, betatron functions and the phase advance for each corrector in the first superperiod; the betatron tunes and the circumference. The second file consist of the real and imaginary parts of A_0 and B_9. After the program has run the array skquad(*) contains the strengths of all the correctors.

```
      program correctors
c
c      This program calculates the coefficients for each correcting
c      skew quadrupole to determine the resonance strengths the correctors
c      drive.
c
c      We use the thin lens approximation.
c
      implicit real*8 (a-h,o-z)
c
c        bx      = the beta function (x direction)
c        by      = the beta function (y direction)
c        cct     = the trim coil package contribution to resonance
c        cfac    = a transformation factor for the resonance strength
c        circ    = the machine circumference
c        cstr    = the real part of the resonance strength to be
c                  excited
c        csup    = the real part of the superperiod contribution
c                  to the resonance
c        maxsize = maximum size for skew quadrupoles per superperiod
c        mskq    = maximum size of skew quadrupoles array
c        mux     = the phase advance (x direction, 2*pi included)
c        muy     = the phase advance (y direction, 2*pi included)
c        nct     = the number of trim coil packages per superperiod
c        nres    = the number of resonances
c        nsup    = the number of superperiods
c        nsupmax = maximum superperiod
c        sct     = the trim coil package contribution to resonance
```

```
c        sfac    = a transformation factor for the resonance strength
c        snorm   = the coefficient for a given skew quadrupole from one
c                  superperiod to the next
c        sstr    = the imaginary part of the resonance strength to be
c                  excited
c        ssup    = the imaginary part of the superperiod contribution
c                  to the resonance
c        suml    = position of the center of the trim coil package
c        skquad  = skew quadrupole array
c        vx      = tune in x direction
c        vy      = tune in y direction
c        wx      = !
c        wy      = !  a resonance => wx*vx + wy*vy = wp
c        wp      = !
c
      parameter (maxsize=100, nres=2, nsupmax=60, mskq=maxsize*nsupmax,
     1           pi=3.141592653589793d0, twopi=6.2831853071795865d0)
      real*8 mux,muy
      complex*16 ccoef
      character smtrm*4,stype*4
      dimension suml(maxsize),bx(maxsize),mux(maxsize),by(maxsize),
     1          muy(maxsize),wx(nres),wy(nres),wp(nres),csup(nres,
     2          nsupmax),ssup(nres,nsupmax),cct(nres,maxsize),sct(nres,
     3          maxsize),cstr(nres),sstr(nres),snorm(nres,nsupmax),
     4          sfac(nres),skquad(mskq),b(nres),a(nres,nres),cfac(nres),
     5          indx(nres)
c ----- the resonances
      data    wx/ 1.d0, 1.d0/,
     1        wy/ 1.d0,-1.d0/,
     2        wp/ 9.d0, 0.d0/,
     3    skquad/mskq*0.d0/
c ----- reading in the lattice parameters
      read(4,10) nct,nsup
   10 format(2i8)
      do 20 i=1,nct
          read(4,30) suml(i),bx(i),zmux,by(i),zmuy
          mux(i)=twopi*zmux
          muy(i)=twopi*zmuy
   30     format(5e16.9)
   20 continue
      read(4,30) circ,vx,vy
c ----- reading the resonance strengths
      read(5,*) (cstr(i),sstr(i),i=1,nres)
c ----- contribution to each resonance by superperiod
      part=twopi/dfloat(nsup)
      write(6,*)
      do 40 i=1,nres
          do 40 j=1,nsup
              arg=wp(i)*part*dfloat(j-1)
              csup(i,j)=dcos(arg)
              ssup(i,j)=dsin(arg)
   40 continue
      do 50 j=1,nsup
          write(6,60) j,(csup(i,j),ssup(i,j),i=1,nres)
   60     format(1x,i4,8f9.6)
   50 continue
c ----- contribution to each resonance from within each superperiod
      conv=twopi/circ
```

```
      do 70 i=1,nres
          do 70 j=1,nct
              ccoef=dcmplx(0.d0,dsqrt(bx(j)*by(j))/(4.d0*pi))*
     1              cdexp(dcmplx(0.d0,wx(i)*(mux(j)-conv*vx*suml(j))+
     2              wy(i)*(muy(j)-conv*vy*suml(j))+wp(i)*conv*suml(j)))
              cct(i,j)=dreal(ccoef)
              sct(i,j)=dimag(ccoef)
   70 continue
      write(6,*)
      do 80 j=1,nct
          write(6,60) j,(cct(i,j),sct(i,j),i=1,nres)
   80 continue
c
c       There are four skew quadrupoles per superperiod.  To correct
c       two resonances that are orthogonal, we have 4 conditions with
c       eight unknowns.  We set the first skew quadurupole equal to the
c       third and the second equal to the fourth.  The 4 unknowns are
c       then solved by two consecutive 2 x 2 sets of linear equations.
c
c ----- the resonance strengths are converted to deal with each superiod
c ----- separately
      do 90 i=1,nres
          cfac(i)=0.d0
          sfac(i)=0.d0
          do 90 j=1,nsup
              snorm(i,j)=csup(i,j)
              cfac(i)=cfac(i)+snorm(i,j)*csup(i,j)
              sfac(i)=sfac(i)+snorm(i,j)*ssup(i,j)
   90 continue
c ----- the linear equation to solve
c
c ----- the p=9 resonances
      den1=cfac(1)**2+sfac(1)**2
      b(1)=(cfac(1)*cstr(1)+sfac(1)*sstr(1))/den1
      b(2)=(cfac(1)*sstr(1)-sfac(1)*cstr(1))/den1
c #######
      a(1,1)=cct(1,1)+cct(1,3)
      a(2,1)=sct(1,1)+sct(1,3)
      a(1,2)=cct(1,2)+cct(1,4)
      a(2,2)=sct(1,2)+sct(1,4)
c #######
      call ludcmp(a,2,nres,indx,dsgn)
      call lubksb(a,2,nres,indx,b)
      do 100 i=1,nsup
          karg=4*(i-1)+1
          skquad(karg)=snorm(1,i)*b(1)
          skquad(karg+1)=snorm(1,i)*b(2)
          skquad(karg+2)=snorm(1,i)*b(1)
          skquad(karg+3)=snorm(1,i)*b(2)
  100 continue
c ----- for p=0 resonances
      den2=cfac(2)**2+sfac(2)**2
      b(1)=(cfac(2)*cstr(2)+sfac(2)*sstr(2))/den2
      b(2)=(cfac(2)*sstr(2)-sfac(2)*cstr(2))/den2
c #######
      a(1,1)=cct(2,1)+cct(2,3)
      a(2,1)=sct(2,1)+sct(2,3)
      a(1,2)=cct(2,2)+cct(2,4)
```

```
      a(2,2)=sct(2,2)+sct(2,4)
c #######
      call ludcmp(a,2,nres,indx,dsgn)
      call lubksb(a,2,nres,indx,b)
      do 110 i=1,nsup
          karg=4*(i-1)+1
          skquad(karg)=skquad(karg)+snorm(2,i)*b(1)
          skquad(karg+1)=skquad(karg+1)+snorm(2,i)*b(2)
          skquad(karg+2)=skquad(karg+2)+snorm(2,i)*b(1)
          skquad(karg+3)=skquad(karg+3)+snorm(2,i)*b(2)
  110 continue
c ----- printing out the value of the skew quadrupoles
      do 120 i=1,nct*nsup
          write(7,130) skquad(i)
  130     format(4x,f19.15)
  120 continue
      end
c*******************************************************************
c       LU Decomposition of a linear system of equations
c*******************************************************************
c
c       W. H. Press, et al; 'Numerical Recipes',(Cambridge University
c           Press, 1986)
c
      SUBROUTINE LUDCMP(A,N,NP,INDX,D)
      implicit real*8 (a-h,o-z)
      PARAMETER (NMAX=100,TINY=1.0d-20)
      DIMENSION A(NP,NP),INDX(N),VV(NMAX)
      D=1.d0
      DO 12 I=1,N
        AAMAX=0.d0
        DO 11 J=1,N
          IF (dABS(A(I,J)).GT.AAMAX) AAMAX=dABS(A(I,J))
11      CONTINUE
        IF (AAMAX.EQ.0.d0) PAUSE 'Singular matrix.'
        VV(I)=1.d0/AAMAX
12    CONTINUE
      DO 19 J=1,N
        IF (J.GT.1) THEN
          DO 14 I=1,J-1
            SUM=A(I,J)
            IF (I.GT.1)THEN
              DO 13 K=1,I-1
                SUM=SUM-A(I,K)*A(K,J)
13            CONTINUE
              A(I,J)=SUM
            ENDIF
14        CONTINUE
        ENDIF
        AAMAX=0.d0
        DO 16 I=J,N
          SUM=A(I,J)
          IF (J.GT.1)THEN
            DO 15 K=1,J-1
              SUM=SUM-A(I,K)*A(K,J)
15          CONTINUE
            A(I,J)=SUM
          ENDIF
```

```
          DUM=VV(I)*dABS(SUM)
          IF (DUM.GE.AAMAX) THEN
            IMAX=I
            AAMAX=DUM
          ENDIF
16      CONTINUE
        IF (J.NE.IMAX)THEN
          DO 17 K=1,N
            DUM=A(IMAX,K)
            A(IMAX,K)=A(J,K)
            A(J,K)=DUM
17        CONTINUE
          D=-D
          VV(IMAX)=VV(J)
        ENDIF
        INDX(J)=IMAX
        IF(J.NE.N)THEN
          IF(A(J,J).EQ.0.)A(J,J)=TINY
          DUM=1.d0/A(J,J)
          DO 18 I=J+1,N
            A(I,J)=A(I,J)*DUM
18        CONTINUE
        ENDIF
19    CONTINUE
      IF(A(N,N).EQ.0.d0)A(N,N)=TINY
      RETURN
      END
C-------------------------------------------------------------------
      SUBROUTINE LUBKSB(A,N,NP,INDX,B)
      implicit real*8 (a-h,o-z)
      DIMENSION A(NP,NP),INDX(N),B(N)
      II=0
      DO 12 I=1,N
        LL=INDX(I)
        SUM=B(LL)
        B(LL)=B(I)
        IF (II.NE.0)THEN
          DO 11 J=II,I-1
            SUM=SUM-A(I,J)*B(J)
11        CONTINUE
        ELSE IF (SUM.NE.0.) THEN
          II=I
        ENDIF
        B(I)=SUM
12    CONTINUE
      DO 14 I=N,1,-1
        SUM=B(I)
        IF(I.LT.N)THEN
          DO 13 J=I+1,N
            SUM=SUM-A(I,J)*B(J)
13        CONTINUE
        ENDIF
        B(I)=SUM/A(I,I)
14    CONTINUE
      RETURN
      END
```

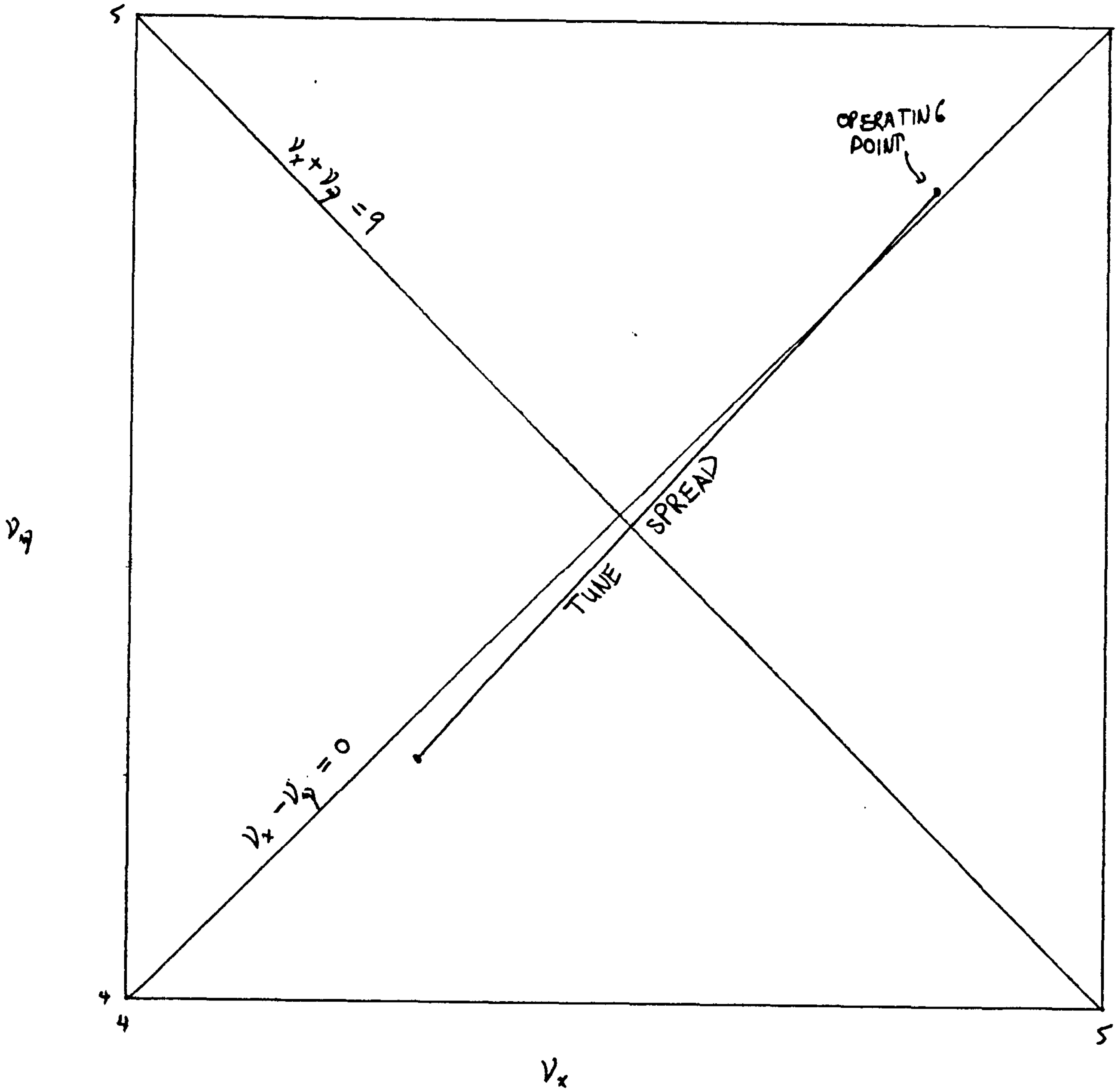

Fig. 1 Tune Diagram

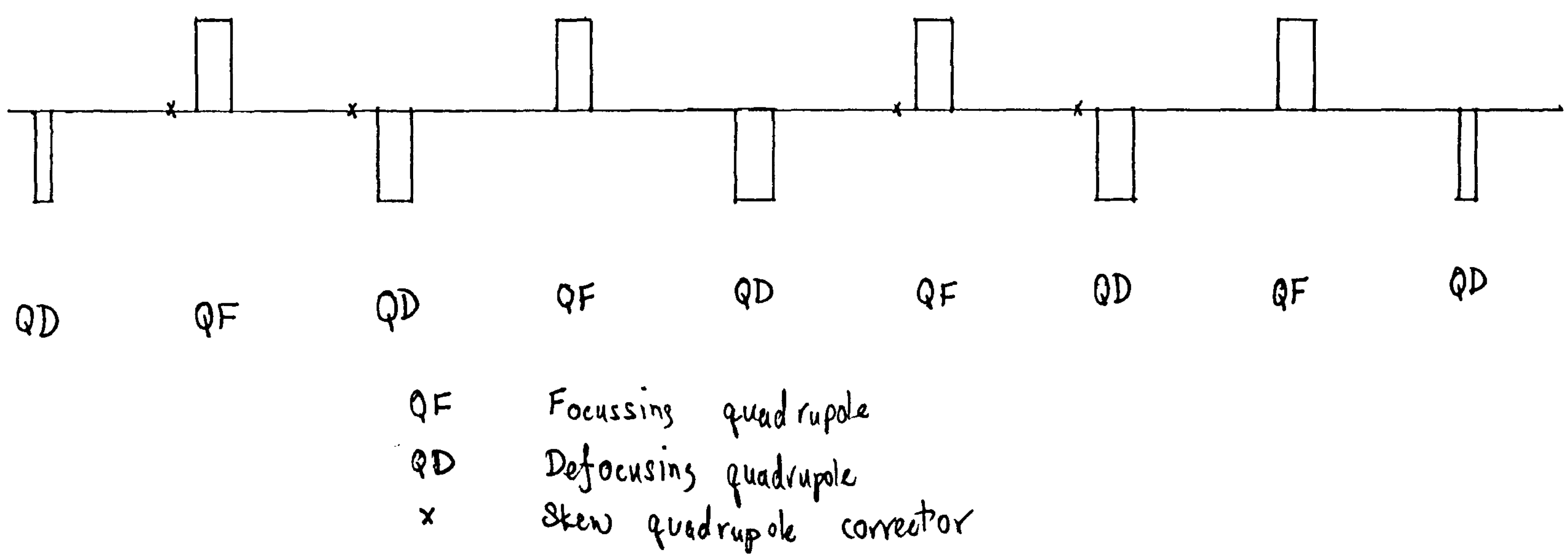

Fig. 2. Skew quadrupole corrector placement in AGS - Booster