# SKEW QUADRUPOLE CORRECTIONS 

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## Abstract

Two sources of skew quadrupole errors are rotations of normal quadrupole fields and vertical displacements of sextupole fields which can include the eddy current sextupole fields in the dipole magnets at injection. These excite the coupling resonance $\nu_{x}-v_{y}=0$ and the sum resonance $v_{x}+v_{y}=9$ which can be crossed by the space charge tune spread. We correct these resonances with 24 skew quadrupole correctors organized in 4 families. We find for the worst case the strongest skew quadrupole corrector is $0.36 \%$ of the main quadrupole strength. Additionally, we find the skew quadrupoles impractical in controlling the beam shape by using coupling because of the space charge tune spread even at the top energy.

## 1. Introduction

All accelerators contain skew quadrupoles fields due to misalignments. These fields are due to rotational errors in normal quadrupoles, vertical displacements of normal sextupoles and other sources as well. The skew quadrupoles can excite two resonances which may be crossed due to tune spread. For the AGS-Booster the resonances are

$$
v_{x}-v_{y}=0, \quad v_{x}+v_{y}=9
$$

as shown in Fig. 1. Additionally, Fig. 1 shows the expected tune spread due to space charge ${ }^{1}$ for protons at injection.

In the AGS-Booster, we included four sources of skew quadrupole errors:
(1) 0.3 mrad rotations in the focusing and defocusing quadrupoles,
(2) 0.3 mm vertical displacements of the chromaticity correcting sextupoles,
(3) 0.3 mm vertical displacements of the dipoles with the eddy current sextupole fields at injection,
(4) 0.3 mm displacements between the central axis of the dipoles and the central axis of the eddy current sextupoles,
where we assumed 3 standard deviations on a uniform distribution of errors. These errors excite the resonances stated above and must be corrected to reduce any possible beam loss.

The two resonances given above can be corrected using skew quadrupole correctors. There will be four per superperiod, two in the first full cell and two in the third full cell in the correction trim coil assembly as shown in Fig. 2.

The relation between the skew quadrupoles and resonance strengths are given in section 2. Section 3 gives the scheme to determine the strengths of the correctors. In section 4 we calculate the tune spread at top energy in the AGS-Booster in order to see if it is practical to change the shape of the beam (i.e. from flat to round or vice versa). Finally, a conclusion is given in section 5 .

## 2. Theory

The motion of a particle in the beam of an accelerator can be described by the following Hamiltonian

$$
H=\frac{p_{x}^{2}}{2}+\frac{p_{y}^{2}}{2}+\left[K(s)+\frac{1}{\rho^{2}(s)}\right] \frac{x^{2}}{2}+M(s) x y-K(s) \frac{y^{2}}{2}
$$

where $x$ and $y$ are the particles position with respect to the equilibrium, $p_{x}$ and $p_{y}$ are the conjugate momenta, $K(s)$ is the quadrupole strength, $M(s)$ is the skew quadrupole strength, $\rho(s)$ is the radius of curvature and $s$ is the independent variable (i.e. the time variable). We have assumed the particle is at the equilibrium momentum.

The resonance strengths can be found by applying two canonical transformations to the Hamiltonian $H$ and then expanding the potential term in a fourier series. The first canonical transformation (which transforms the Hamiltonian to action-angle ${ }^{2}$ variables) is given by the generating function shown below:

$$
F\left(x, y, \psi_{x}, \psi_{y}, s\right)=-\frac{x^{2}}{2 \beta_{x}(s)}\left[\tan \psi_{x}-\frac{\beta_{x}^{\prime}(s)}{2}\right]-\frac{y^{2}}{2 \beta_{y}(s)}\left[\tan \psi_{y}-\frac{\beta_{y}^{\prime}(s)}{2}\right]
$$

where $\psi_{x}$ and $\psi_{y}$ are the new angle variables, primes denote $d / d s$ and the functions $\beta_{x}(s)$ and $\beta_{y}(s)$ are the solutions of the following equations

$$
\begin{gathered}
\frac{\beta_{x}(s) \beta_{x}^{\prime \prime}(s)}{2}-\frac{\left(\beta_{x}^{\prime}(s)\right)^{2}}{4}+\left[K(s)+\frac{1}{\rho^{2}(s)}\right] \beta_{x}^{2}(s)=1 \\
\frac{\beta_{y}(s) \beta_{y}^{\prime \prime}(s)}{2}-\frac{\left(\beta_{y}^{\prime}(s)\right)^{2}}{4}-K(s) \beta_{y}^{2}(s)=1 .
\end{gathered}
$$

The actions, $J_{x}$ and $J_{y}$, are related to $x$ and $y$ as follows

$$
\begin{aligned}
& x=\sqrt{2 J_{x} \beta_{x}(s)} \cos \psi_{x} \\
& y=\sqrt{2 J_{y} \beta_{y}(s)} \cos \psi_{y}
\end{aligned}
$$

Note, the emittance of the beam is related to the action by $2 J_{x}=\varepsilon_{x} / \pi$ and $2 J_{y}=\varepsilon_{y} / \pi$. If there are no skew quadrupoles present (i.e. $M(s)=0$ ) then the emittances (and the actions) are invariants of the motion ${ }^{3}$.

Using the generating function, $F$, the new Hamiltonian becomes

$$
H_{1}=\frac{J_{x}}{\beta_{x}(s)}+\frac{J_{y}}{\beta_{y}(s)}+M(s) \sqrt{2 J_{x} \beta_{x}(s)} \sqrt{2 J_{y} \beta_{y}(s)} \cos \psi_{x} \cos \psi_{y}
$$

The next transformation will eliminate the time dependence in the first
two terms in the Hamiltonian $H_{1}$. This transformation is given by the following generating function

$$
G\left(I_{x}, I_{y}, \psi_{x}, \psi_{y}, s\right)=I_{x}\left[\psi_{x}-\mu_{x}(s)\right]+I_{y}\left[\psi_{y}-\mu_{y}(s)\right]
$$

where $I_{x}$ and $I_{y}$ are the new action variables and

$$
\begin{aligned}
& \mu_{x}(s)=\int_{0}^{s} \frac{d t}{\beta_{x}(t)}-\frac{2 \pi}{C} v_{x} s \\
& \mu_{y}(s)=\int_{0}^{s} \frac{d t}{\beta_{y}(t)}-\frac{2 \pi}{C} v_{y} s
\end{aligned}
$$

and

$$
\begin{aligned}
& v_{x}=\frac{1}{2 \pi} \int_{0}^{c} \frac{d t}{\beta_{x}(t)}, \\
& v_{y}=\frac{1}{2 \pi} \int_{0}^{c} \frac{d t}{\beta_{y}(t)}
\end{aligned}
$$

with $C$ being the circumference of the accelerator. Note, $\mu_{x}(s)$ and $\mu_{y}(s)$ are periodic functions of $s$ with a period of $C / P$ where $P$ is the periodicity of the lattice.

The new action angle variables are related to the old as follows

$$
\begin{gathered}
I_{x}=J_{x} \\
I_{y}=J_{y} \\
\xi_{x}=\psi_{x}-\mu_{x}(s) \\
\xi_{y}=\psi_{y}-\mu_{y}(s)
\end{gathered}
$$

where $\xi_{x}$ and $\xi_{y}$ are the new angle variables.
The transformation of Hamiltonian $H_{1}$ with the generating function $G$ leads to the following Hamiltonian

$$
\begin{gathered}
H_{2}=\frac{2 \pi}{C}\left[v_{x} I_{x}+v_{y} I_{y}+2\left[2 I_{x}\right]^{1 / 2}\left[2 I_{y}\right]^{1 / 2} I m\left[\sum_{n=-\infty}^{\infty} e^{-i 2 \pi n s / C}\right.\right. \\
\left.\left.\left(A_{n} e^{i\left(\xi_{x}-\xi_{y}\right)}+B_{n} e^{i\left(\xi_{x}+\xi_{y}\right)}\right)\right]\right]
\end{gathered}
$$

where

$$
\begin{aligned}
& A_{n}=\frac{i}{4 \pi} \int_{0}^{C} M(t) \beta_{x}^{1 / 2}(t) \beta_{y}^{1 / 2}(t) e^{i\left[\mu_{x}(t)-\mu_{y}(t)+2 \pi n t / C\right]} d t \\
& B_{n}=\frac{i}{4 \pi} \int_{0}^{C} M(t) \beta_{x}^{1 / 2}(t) \beta_{y}^{1 / 2}(t) e^{i\left[\mu_{x}(t)+\mu_{y}(t)+2 \pi n t / C\right]} d t
\end{aligned}
$$

The terms $A_{n}$ and $B_{n}$ give the resonance strengths for the following resonances

$$
v_{\mathrm{x}}-v_{\mathrm{y}}=\mathrm{n}, \quad v_{\mathrm{x}}+v_{\mathrm{y}}=\mathrm{n}
$$

respectively. In the next section we will minimize the resonance strengths for the resonances given in the introduction.

## 3. Strategy

The AGS-Booster consist of six superperiods and each superperiod contains eight correction trim coil assemblies. Four of these assemblies will contain the skew quadrupole correctors. Using these trim coils we want to generate a resonances that cancels those resonances excited by the skew quadrupole errors. Thus, in an ideal accelerator with no skew quadrupole errors, we want to find the strengths of the skew quadrupole correctors to excite any given imperfection resonance strength.

We are concerned with the two resonances listed in the introduction. The corresponding strengths are $A_{0}$ and $B_{g}$. Since these numbers are complex, there are four conditions to be satisfied. The integrals in the previous section relating the skew quadrupole strengths to the resonance strengths can be converted to sums by using the thin lens approximation as

$$
\begin{aligned}
& A_{n}=\frac{i}{4 \pi} \sum_{p=1}^{6} \sum_{q=1}^{4} M_{p q} \beta_{x}^{1 / 2}\left(t_{p q}\right) \beta_{y}^{1 / 2}\left(t_{p q}\right) e^{i\left[\mu_{x}\left(t_{p q}\right)-\mu_{y}\left(t_{p q}\right)+2 \pi n t_{p q} / C\right]} \\
& B_{n}=\frac{i}{4 \pi} \sum_{p=1}^{6} \sum_{q=1}^{4} M_{p q} \beta_{x}^{1 / 2}\left(t_{p q}\right) \beta_{y}^{1 / 2}\left(t_{p q}\right) e^{i\left[\mu_{x}\left(t_{p q}\right)+\mu_{y}\left(t_{p q}\right)+2 \pi n t_{p q} / C\right]}
\end{aligned}
$$

where the outer sum is over the superperiods, $M_{p q}$ are the integrated skew quadrupole strength of the correctors and $t_{p q}$ are the positions of the center of the correctors.

Due to the periodicity of the $\beta$ and $\mu$ functions the above sums can be simplified. First, we define

$$
t_{q}=t_{(p=1) q}
$$

then

$$
\mathrm{t}_{\mathbf{p q}}=\mathrm{t}_{\mathbf{q}}+(\mathrm{p}-1) \mathrm{C} / 6
$$

for $p=1,2,3, \ldots 6$ and $q=1,2,3$ and 4 . Defining the coefficients $f_{p}$ so that

$$
M_{p q}=f_{p} M_{q}
$$

defines four families of skew quadrupole correctors. The sums can now be written as

$$
\begin{aligned}
& A_{n}=\frac{i}{4 \pi}\left[\sum_{p=1}^{6} f_{p} e^{i \pi n(p-1) / 3}\right] \sum_{q=1}^{4} M_{q} \beta_{x}^{1 / 2}\left(t_{q}\right) \beta_{y}^{1 / 2}\left(t_{q}\right) e^{i\left[\mu_{x}\left(t_{q}\right)-\mu_{y}\left(t_{q}\right)+2 \pi n t_{q} / C\right]} \\
& B_{n}=\frac{i}{4 \pi}\left[\sum_{p=1}^{6} f_{p} e^{i \pi n(p-1) / 3}\right] \sum_{q=1}^{4} M_{q} \beta_{x}^{1 / 2}\left(t_{q}\right) \beta_{y}^{1 / 2}\left(t_{q}\right) e^{i\left[\mu_{x}\left(t_{q}\right)+\mu_{y}\left(t_{q}\right)+2 \pi n t_{q} / C\right]}
\end{aligned}
$$

Choosing $f_{p}=\cos [\pi n(p-1) / 3]$ for $n=0$ and for $n=9$ leads to the greatest contribution to each of the two resonances separately. Furthermore, the two resonances of $f_{p}$ for $n=0$ and for $n=9$ are orthogonal (i.e. $f_{p}$
of $n=0$ doesn't contribute to $B_{9}$ and $n=9$ doesn't contribute to $A_{0}$ ).
Now we can solve for the unknowns $M_{q}$ for a given choice of $A_{0}$ and $B_{g}$. Due to the orthogonality between $n=0$ and $n=9$ we have eight unknowns (from two independent sets of $M_{q}$ ) with four conditions. These additional unknowns are handeled by the following scheme:

$$
M_{1}=M_{3} \text { and } M_{2}=M_{4}
$$

A listing of a program that uses the above strategy for solving for the strengths of the skew quadrupole correctors is given in the appendix. Using this program we have calculated the strengths for various different random seeds for the skew quadrupole errors given in the introduction. We have assumed the eddy current sextupole of $B^{\prime \prime}=.24 \mathrm{~T} / \mathrm{m}^{2}$ in the dipoles. ${ }^{4}$ In each case we used a uniform distribution of errors. The table below shows the random number seed versus the maximum strength of the corrector

| SEED | INTEGRATED STRENGTH <br> $\mathrm{m}^{-1}$ | $\%$ OF MAIN <br> QUAD. STRENGTH |
| ---: | :---: | :---: |
| 51351 | 0.0001717 | $0.31 \%$ |
| 8795 | 0.0001716 | $0.31 \%$ |
| 85003 | 0.0001210 | $0.22 \%$ |
| 6221 | 0.0001239 | $0.22 \%$ |
| 5819 | 0.0001991 | $0.36 \%$ |

where we assumed a trim coil 10 cm long.

## 4. Coupling

The coupling resonance $v_{x}-v_{y}=0$ that is excited by the skew quadrupoles can couple the transverse emittances (i.e. $\varepsilon_{x}+\varepsilon_{y}=\operatorname{constant).~Thus,~if~} \varepsilon_{x}$ is increased then $\varepsilon_{y}$ must decrease and vice versa. Can this phenomena be used to transform the beam from a flat to a round or vice versa in the AGS-Booster?

To investigate this consider the tune split of the AGS-Booster which is $\Delta \nu=\nu_{y}-\nu_{\mathrm{x}}=0.01$. With this tune split, the coupling will cause the beam to oscillate from flat to round to flat etc. about every 25 revolutions. This number may vary due to nonlinearities that can effect the tunes and thus, the tune split.

Different particles in the beam may have different tune splits due to tune spread. In order for all the particles to be coupled coherently, then we get the following requirement on the tune spread

$$
\delta v_{\mathrm{x}}, \delta v_{\mathrm{y}} \ll 0.005
$$

The major cause of tune spread in the AGS-Booster is due to space charge effects. The space charge tune spread is scaled with beam energy as follows

$$
\gamma^{2} \beta \delta v_{S P}=\text { constant. }
$$

Parzen ${ }^{1}$ has calculated the space charge tune spread for protons at 200 Mev in the AGS-Booster giving

$$
\delta v_{x}=-.52, \quad \delta v_{y}=-.58
$$

From the scaling of space charge tune spread we find

$$
\delta v_{\mathrm{x}}=-.070, \quad \delta v_{\mathrm{y}}=-.077
$$

for protons at 1.5Gev.
Hence, all the particles will not coherently transform from a flat beam to a round beam at the top energy of the AGS-Booster.

## 5. Conclusion

A particle in the AGS-Booster can cross the coupling resonance $v_{x}-v_{y}=0$ and the sum resonance $v_{x}+v_{y}=9$, which is excited by the skew quadrupole errors, due to the large space charge tune spread at injection. We have formulated a strategy for correcting these resonances using half of the correction trim coil assemblies available in the Booster for skew quadrupole correctors. Using the arrangement of the correctors described in section 3 , we have been able to correct the two resonances. For the errors given in the introduction, the largest skew quadrupole corrector is found to be $0.36 \%$ of the main quadrupole's field strength.

Additionally, the four errors listed in the introduction give about equal contribution to the resonance strengths and thus, to the maximum strength of the corrector required. With this, we can easily scale the results if the actual errors are different than those assumed in section 1.

## References

1. G. Parzen, Booster Tech Note \#127
2. H. Goldstein, Classical Mechanics, (Addison Wesley, 1965)
3. E. D. Courant and H. S. Snyder, Annals of Physics 3, 1-48 (1958)
4. G. Morgan and S. Kahn, Booster Tech Note \#4
5. W. H. Press, et al, Numerical Recipes (Cambridge University Press, 1986)

## Appendix

Below is a listing of the program that was used to calculate the skew quadrupole corrector strengths. The program is written in FORTRAN 77 and was tested on the VAX. The input consists of two files. The first file includes the number of correctors per superperiod; the number of superperiods; position, betatron functions and the phase advance for each corrector in the first superperiod; the betatron tunes and the circumference. The second file consist of the real and imaginary parts of $A_{0}$ and $B_{9}$. After the program has run the array skquad(*) contains the strengths of all the correctors.
program correctors

C c drive.
C
C C

```
c This program calculates the coefficients for each correcting c skew quadrupole to determine the resonance strengths the correctors
    This program calculates the coefficients for each correcting
    drive.
    We use the thin lens approximation.
implicit real*8 (a-h,o-z)
```

    bx \(\quad=\) the beta function ( \(x\) direction)
    by \(\quad=\) the beta function ( \(y\) direction)
    cct \(=\) the trim coil package contribution to resonance
    cfac \(=\) a transformation factor for the resonance strength
    circ \(=\) the machine circumference
    cstr \(=\) the real part of the resonance strength to be
        excited
    csup \(=\) the real part of the superperiod contribution
                to the resonance
    maxsize \(=\) maximum size for skew quadrupoles per superperiod
    mskq \(=\) maximum size of skew quadrupoles array
    mux \(\quad=\) the phase advance ( \(x\) direction, 2*pi included)
    muy \(\quad=\) the phase advance (y direction, 2*pi included)
    nct \(\quad=\) the number of trim coil packages per superperiod
    nres \(=\) the number of resonances
    nsup \(=\) the number of superperiods
    nsupmax \(=\) maximum superperiod
    sct \(=\) the trim coil package contribution to resonance
    c sfac = a transformation factor for the resonance strength
c snorm $=$ the coefficient for a given skew quadrupole from one
c superperiod to the next
c sstr $=$ the imaginary part of the resonance strength to be excited
c ssup $=$ the imaginary part of the superperiod contribution
c to the resonance
c suml = position of the center of the trim coil package
c skquad $=$ skew quadrupole array
$\mathrm{c} \quad \mathrm{vx} \quad=$ tune in x direction
$c \quad$ $\mathrm{cy} \quad=$ tune in $y$ direction
C WX = !
$c \quad$ wy $=$ ! a resonance $\Rightarrow w^{*} v x+w y^{*} v y=w p$
c wp =
c
parameter (maxsize=100, nres=2, nsupmax=60, mskq=maxsize*nsupmax,
1 $\mathrm{pi}=3.141592653589793 \mathrm{do}$, twopi=6.2831853071795865d0)
real*8 mux, muy
complex*16 ccoef
character smtrm*4, stype*4
dimension suml(maxsize), bx(maxsize), mux(maxsize), by(maxsize), muy(maxsize), wx(nres), wy (nres), wp(nres), csup(nres,
2 nsupmax), ssup(nres, nsupmax), cct(nres, maxsize), sct (nres, 3 maxsize), cstr(nres), sstr(nres), snorm(nres, nsupmax), 4 sfac(nres), skquad(mskq),b(nres), a(nres, nres), cfac(nres), 5 indx(nres)
c ----- the resonances
data $w x / 1 . d 0,1 . d 0 /$,
1 wy/ 1.do,-1.d0/,
2 wp/ 9.d0, o.dO/,
3 skquad/mskq*0.d0/
c ----- reading in the lattice parameters
read (4,10) nct, nsup
10 format (2i8)
do $20 \mathrm{i}=1$, nct
read(4,30) suml(i), bx(i), zmux, by(i), zmuy
mux(i)=twopi*zmux
muy(i)=twopi*zmuy
30 format(5e16.9)
20 continue
read(4,30) circ,vx, vy
c ----- reading the resonance strengths
read(5,*) (cstr(i), sstr(i), i=1, nres)
c ----- contribution to each resonance by superperiod
part=twopi/dfloat(nsup)
write(6, *)
do $40 \mathrm{i}=1$, nres
do $40 \mathrm{j}=1$, nsup
arg=wp(i)*part*dfloat(j-1)
$\operatorname{csup}(\mathbf{i}, j)=\operatorname{dcos}(\arg )$
$\operatorname{ssup}(i, j)=d \sin (\arg )$
40 continue
do $50 \mathrm{j}=1$, nsup
write (6,60) $j,(\operatorname{csup}(i, j), \operatorname{ssup}(i, j), i=1, \operatorname{nres})$
60 format( $1 \mathrm{x}, \mathrm{i} 4,8 \mathrm{f} 9.6$ )
50 continue
c ----- contribution to each resonance from within each superperiod conv=twopi/circ

```
    do 70 i=1,nres
        do 70 j=1,nct
            ccoef=dcmplx(0.d0, dsqrt(bx(j)*by(j))/(4.d0*pi))*
                        cdexp(dcmplx(0.dO,wx(i)*(mux(j)-conv*vx*suml(j))+
                        wy(i)*(muy(j)-conv*vy*suml (j))+wp(i)*conv*suml(j)))
            cct(i,j)=dreal(ccoef)
        sct(i,j)=dimag(ccoef)
    7 0 ~ c o n t i n u e
    write(6,*)
    do 80 j=1,nct
        write(6,60) j,(\operatorname{cct}(i,j),sct(i,j),i=1,nres)
80 continue
c
c There are four skew quadrupoles per superperiod. To correct
c two resonances that are orthogonal, we have 4 conditions with
c eight unknowns. We set the first skew quadurupole equal to the
c third and the second equal to the fourth. The 4 unknowns are
c then solved by two consecutive 2 x 2 sets of linear equations.
c ----- the resonance strengths are converted to deal with each superiod
c ----- separately
    do 90 i=1,nres
                cfac(i)=0.d0
                sfac(i)=0.d0
                do }90\textrm{j}=1\mathrm{ , nsup
                    snorm(i,j)=csup(i,j)
                cfac(i)=cfac(i)+snorm(i,j)*csup(i,j)
                sfac(i)=sfac(i)+snorm(i,j)*ssup(i,j)
    90 continue
c ----- the linear equation to solve
c
c ------ the p=9 resonances
    den1=cfac(1)**2+sfac(1)**2
    b(1)=(cfac(1)*cstr(1)+sfac(1)*sstr(1))/den1
    b(2)=(cfac(1)*sstr(1)-sfac(1)*\operatorname{cstr}(1))/den1
c #######
    a(1,1)=cct(1,1)+\operatorname{cct}(1,3)
    a(2,1)=sct (1, 1)+sct (1, 3)
    a(1,2)=cct (1,2)+\operatorname{cct}(1,4)
    a(2,2)=sct (1, 2)+sct (1,4)
c #######
    call ludcmp(a,2, nres,indx,dsgn)
    call lubksb(a,2,nres,indx,b)
    do 100 i=1,nsup
        karg=4*(i-1)+1
        skquad(karg)=snorm(1,i)*b(1)
        skquad(karg+1)=snorm(1,i)*b(2)
        skquad(karg+2)=snorm(1, i)*b(1)
        skquad(karg+3)=snorm(1,i)*b(2)
    100 continue
c ----- for p=0 resonances
    den2=cfac(2)**2+sfac(2)**2
    b(1)=(cfac(2)*cstr(2)+sfac(2)*sstr(2))/den2
    b(2)=(cfac(2)*sstr(2)-sfac(2)*cstr(2))/den2
c #######
    a(1, 1)=\operatorname{cct}(2,1)+\operatorname{cct}(2,3)
    a(2,1)=sct (2,1)+\operatorname{sct}(2,3)
    a(1,2)=\operatorname{cct}(2,2)+\operatorname{cct}(2,4)
```

```
    a(2,2)=sct (2, 2)+\operatorname{sct}(2,4)
c #######
    call ludcmp(a,2, nres,indx, dsgn)
    call lubksb(a,2, nres,indx,b)
    do }110\textrm{i}=1,\textrm{nsup
        karg=4*(i-1)+1
        skquad(karg)=skquad(karg)+snorm(2,i)*b(1)
        skquad(karg+1)=skquad(karg+1)+snorm(2,i)*b(2)
        skquad(karg+2)=skquad(karg+2)+snorm(2,i)*b(1)
        skquad(karg+3)=skquad(karg+3)+snorm(2,i)*b(2)
    110 continue
c ----- printing out the value of the skew quadrupoles
    do }120\textrm{i}=1\mathrm{ , nct*nsup
            write(7, 130) skquad(i)
    130 format(4x,f19.15)
    120 continue
    end
    c LU Decomposition of a linear system of equations
```



```
c
c W. H. Press, et al; 'Numerical Recipes',(Cambridge University
c Press, 1986)
c
    SUBROUTINE LUDCMP(A,N,NP, INDX, D)
    implicit real*8 (a-h,o-z)
    PARAMETER (NMAX=100,TINY=1.Od-20)
    DIMENSION A(NP,NP),INDX(N),VV(NMAX)
    D=1.d0
    DO 12 I=1,N
        AAMAX=0.dO
        DO 11 J=1,N
        IF (dABS(A(I, J)).GT. AAMAX) AAMAX= dABS(A(I,J))
        CONTINUE
        IF (AAMAX.EQ.O.dO) PAUSE 'Singular matrix.'
        VV(I)=1.dO/AAMAX
    CONTINUE
    DO }19\textrm{J}=1,\textrm{N
        IF (J.GT. 1) THEN
            DO 14 I=1,J-1
                SUM=A(I, J)
            IF (I.GT.1)THEN
                DO 13 K=1,I-1
                    SUM=SUM-A(I , K)*A(K, J)
                    CONTINUE
                    A(I , J )=SUM
                    ENDIF
        CONTINUE
        ENDIF
        AAMAX=0.dO
        DO 16 I=J,N
            SUM=A(I, J)
            IF (J.GT. 1)THEN
            DO 15 K=1,J-1
                SUM=SUM-A(I, K)*A(K,J)
            1 5
            CONTINUE
            A(I, J)=SUM
            ENDIF
```

```
        DUM=VV(I)*dABS(SUM)
        IF (DUM.GE. AAMAX) THEN
        IMAX=I
        AAMAX=DUM
        ENDIF
16 CONTINUE
    IF (J.NE. IMAX)THEN
        DO 17 K=1,N
            DUM=A( IMAX, K)
            A(IMAX,K)=A(J,K)
            A(J,K)=DUM
        CONTINUE
        D=-D
        VV(IMAX)=VV(J)
    ENDIF
    INDX(J)=IMAX
    IF(J.NE.N)THEN
    IF(A(J,J).EQ.O. )A(J, J)=TINY
    DUM=1.dO/A(J,J)
    DO 18 I=J+1,N
        A(I,J)=A(I,J)*DUM
            CONTINUE
        ENDIF
19 CONTINUE
    IF(A(N,N).EQ.O.dO)A(N,N)=TINY
    RETURN
    END
C-------------------------------------------------------------------------
    SUBROUTINE LUBKSB(A,N,NP, INDX, B)
    implicit real*8 (a-h,o-z)
    DIMENSION A(NP,NP),INDX(N),B(N)
    II=0
    DO 12 I=1,N
        LL=INDX(I)
        SUM=B(LL)
        B(LL)=B(I)
        IF (II.NE.O)THEN
            DO 11 J=II,I-1
                SUM=SUM-A(I,J)*B(J)
            CONTINUE
            ELSE IF (SUM.NE.O.) THEN
            II=I
        ENDIF
        B(I)=SUM
        CONTINUE
        DO 14 I=N, 1,-1
            SUM=B(I)
            IF(I.LT.N)THEN
            DO 13 J=I +1,N
                    SUM=SUM-A(I, J)*B(J)
            CONTINUE
            ENDIF
            B(I)=SUM/A(I,I)
            CONTINUE
            RETURN
            END
```



Fig. 1 Tune Diagram



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