BNL-105171-2014-TECH

Booster Technical Note No. 127;BNL-105171-2014-IR

Effect of resonances on the space charge limit

G. Parzen

August 1988

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

DISCLAIMER

EFFECT OF RESONANCES ON THE SPACE CHARGE LIMIT

AD
BOOSTER TECHNICAL NOTE
NO. 127

G. PARZEN

AUGUST 29, 1988

ACCELERTOR DEVELOPMENT DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

EFFECT OF RESONANCES ON THE SPACE CHARGE LIMIT

G. PARZEN

INTRODUCTION

One may distinguish between two kinds of space charge limit:

1) The intrinsic limit, which is the space charge limit in the absence of magnetic field errors. This limit is due to the forces generated by the beam itself.

2) The resonance limit, which is the space charge limit due to the presence of random field errors, that can generate nearby resonances.

In the past, the resonance limit has received more attention. The intrinsic limit is difficult to compute, and the mechanism involved is difficult to understand. However, studies done with the simulation program described below indicate that the experimentally observed space charge limit is fairly close to the computed intrinsic limit. Three accelerators have been studied, the AGS, the PS Booster and the Fermilab Booster. In each case, the computed intrinsic limit was found to be about a factor of 2 higher than the beam limit achieved in each accelerator.

In this presentation, the results found when random magnetic errors are present will be given.

The simulation study suggests a different model for the space charge limit, than that given by the resonance limit approach. It suggests that the intrinsic limit provides an upper bound for the space charge limit which is not far from what is actually achieved by accelerators. The resonances present, if strong enough, can prevent the accelerator from achieving the intrinsic limit.

CONCLUSIONS FOR THE RESONANCE LIMIT

The intrinsic space charge limit appears to play a dominant role in determining the space charge limit.

The presence of random field errors which can drive resonances does not produce much beam growth when the beam intensity is below the intrinsic space limit by about a factor of two. This assumes that the random field errors are of a magnitude that may reasonably be expected to exist in an accelerator.

When the beam intensity gets close to the intrinsic space charge limit, more than about 50% of the intrinsic space charge limit, then the random field errors may produce enough growth to lower the space charge limit below the intrinsic limit.

The space charge limit in the AGS Booster appears to be less sensitive to the presence of random field errors than the space charge limit in the AGS. The difference in sensitivity is about a factor of three.

Tolerances have been found on the size of random field errors in order to avoid appreciable reduction of the space charge limit below that given by the intrinsic limit.

For beam intensities well below the intrinsic limit, more than about a factor of 2 below, the space charge forces were found to have a stabilizing effect and to reduce the growth due to resonances.

REVIEW OF THE SIMULATION PROGRAM AND OF INTRINSIC LIMIT RESULTS

Space charge effects are an important limitation on the performance of low energy accelerators like the AGS Booster.

Simulation Program

Using a tracking program, space charge forces are entered as a kick at each element of the lattice; kick $\approx E_x, E_y$ and the length of the element.

To compute the electric field of the beam E_x, E_y, the beam shape in x, y space is assumed to be gaussian with the two parameters σ_x, σ_y which are different at each element in the lattice.

The parameters σ_x, σ_y are determined by tracking a sample of about 16 - 24 particles. The growth of this sample determines the growth in σ_x, σ_y.

Particle Sample

Sample may have 16 particles, 4 groups of 4. Each group of 4 starts at same ϵ_x, ϵ_y but different x, x′, y, y′. One group of 4 starts at ϵ_x, ϵ_y at edge of beam. Other groups start at smaller ϵ_x, ϵ_y. Groups can have $\Delta p/p \neq 0$.

Initial State of the Beam

The beam is assumed to exist in the accelerator in the shape it would have in the absence of space charge forces. This beam shape depends on the injection procedure. The space charge forces are then turned on, and the subsequent growth of the beam is studied.

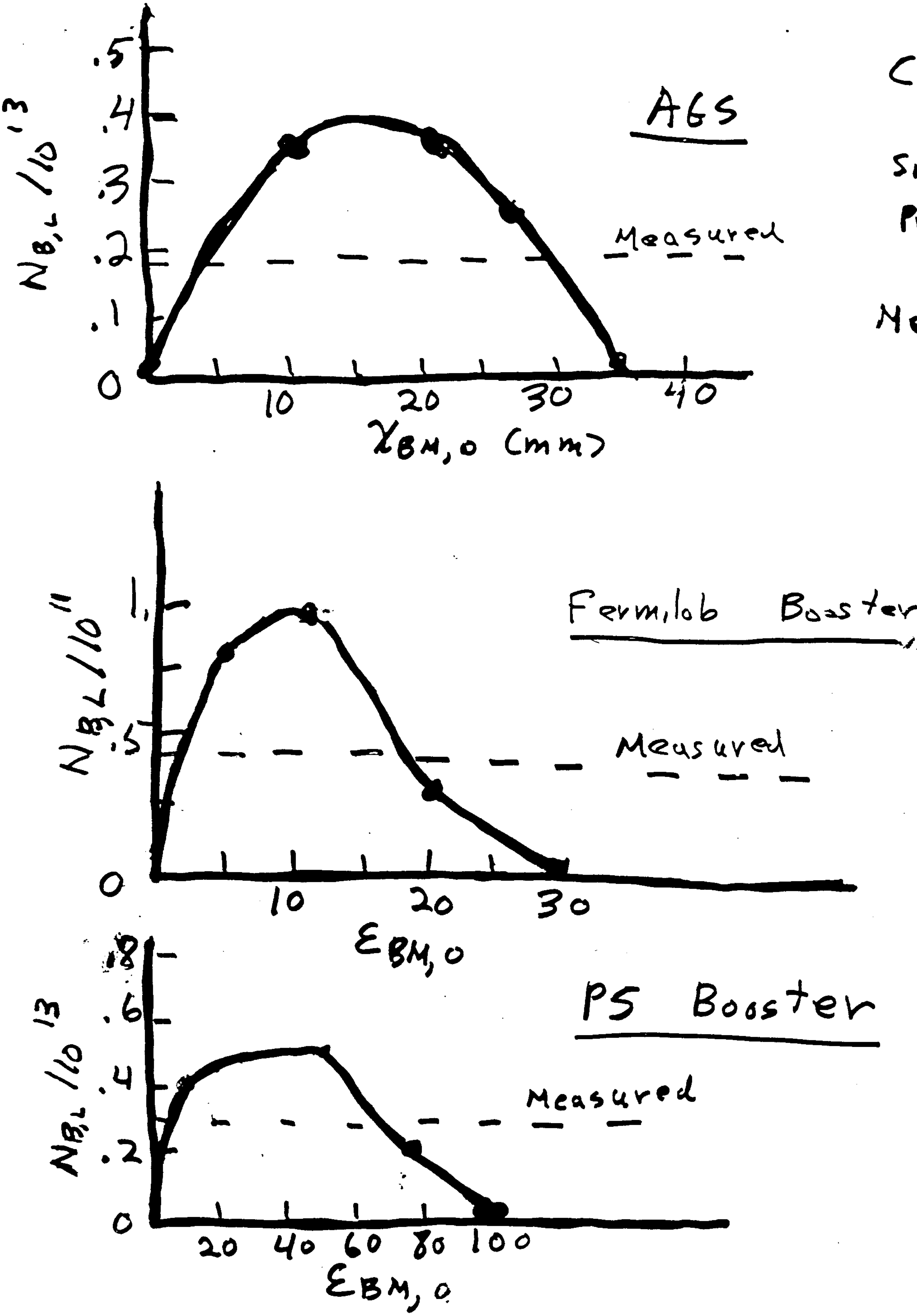
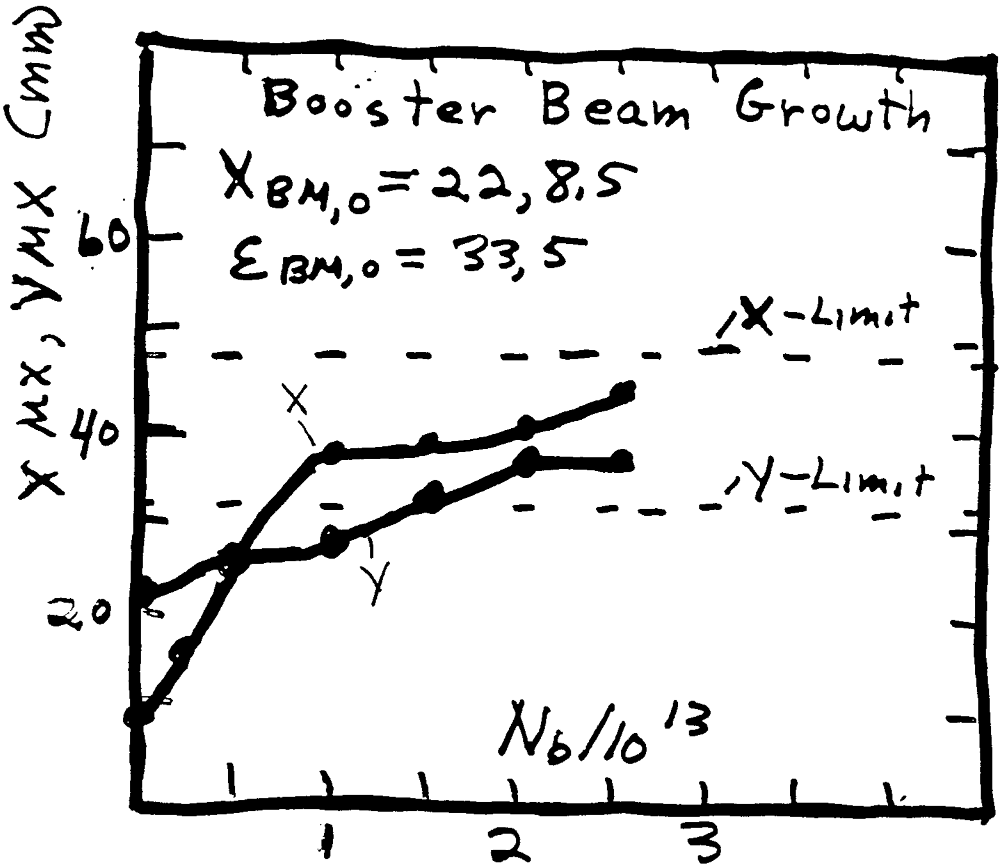
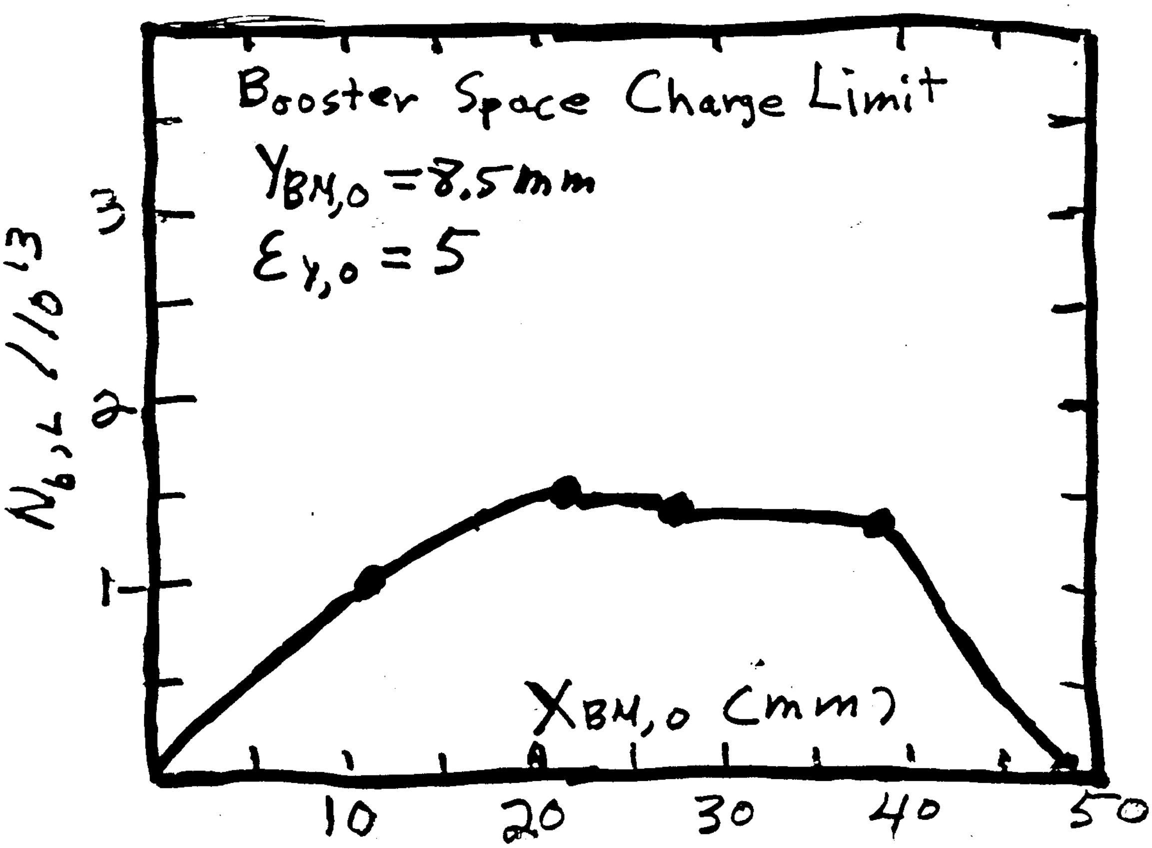


Fig. 1
Comparison of Simulation Program Results with Measured Results

Fig. 2


Booster Beam Growth
$X_{BM,0}$ = 22, 8.5
$\varepsilon_{BM,0}$ = 33, 5
X-Limit
Y-Limit
X
Y
$N_b/10^{13}$
1
2
3
60
40
20
XMX, YMX (mm)


Booster Space Charge Limit
$Y_{BM,0}$ = 8.5 mm
$\varepsilon_{Y,0}$ = 5
$X_{BM,0}$ (mm)
10
20
30
40
50
1
2
3
$N_{b,L}/10^{13}$

HOW STUDIES OF RESONANCE EFFECTS WERE DONE

In order to drive the resonances, random error field multipoles, a_k, b_k were introduced in each magnet in the lattice. The random error field in each magnet due to each multipole is given by:

$$B_y = B_o\, b_k\, x^k,$$

$$B_x = B_o\, a_k\, x^k$$

where B_o is the main dipole field. In these studies the a_k, b_k in each magnet have the same rms values, which are different for different k. In each magnet, the a_k, b_k are different and are randomly chosen.

The beam dimensions are first allowed to grow due to space charge forces in the absence of random error field multipoles. When the beam reaches its final dimensions, the random a_k, b_k are then introduced and the particle motion is studied assuming that the beam dimensions are fixed. This procedure is good for determining the onset of appreciable beam growth due to the random error a_k, b_k.

It is possible to let the beam grow due to the effects of the random a_k, b_k. The algorithm that has so far been used to compute the growth in the beam dimensions from the growth of the particles in the sample studied could be more in error in this situation.

It is also easier to understand and interpret the effects seen when the beam dimensions are held fixed.

Two procedures were used to study the effect of the resonances due to the random a_k, b_k.

1) The Sample Procedure: In this procedure sample of about 16 particles, distributed over the beam, is tracked while in the presence of the resonances due to the random a_k, b_k, and in the presence of the space charge forces of the beam. The non-linear space charge forces produce a strong dence of the particle ν value on its betatron oscillation amplitude. The beam growth is found from the growth in the sample.

2) The Test Group Procedure: In this procedure, a test group of about 4 particles each having the same starting emittances $\epsilon_x = \epsilon_y$ is tracked, and the particle growth is observed. This is repeated for various values of the starting emittance $\epsilon_x = \epsilon_y$.

The second procedure, the test group procedures, allows one to explore the growth of particles whose ν values are close to resonances. It requires more effort and time than the sample procedure. The sample procedure may overlook the growth of some particles which may be close to a narrow resonance and which are not part of the sample.

The studies done indicate that both procedures usually give similar results. The resonance like effects appear to be broad enough not to be entirely missed by the sample procedure.

RESULTS FOR THE AGS

As a first step in studying the effects of resonances, the ν values reached by the particles in the beam were computed. The space charge forces shift the ν values, and this shift depends on the size of the betatron oscillations.

To compute the ν values, the beam dimensions are first allowed to grow due to space charge forces in the absence of random error field multipoles. When the beam reaches its final dimensions, the beam dimensions are fixed and a single particle is tracked. The motion of this single particle is Fourier analyzed to find its ν value, and the starting emittance is varied to find the dependence of the ν value on the emittance.

The next figure shows the ν values reached by one particle when its emittance is varied, and $N_b = .2 \times 10^{13}$ protons/bunch. $\epsilon_x = \epsilon_y$ and $x_0 = (\beta_x \epsilon_x)^{1/2}$, $\beta_x = 22$m. For $N_b = .2 \times 10^{13}$/bunch, the beam dimensions grow to $\pm$ 29 x $\pm$ 26 mm. Particles with $\epsilon_x = \epsilon_y$ and a starting $x_0 \geq 18$ mm may be considered to be outside the beam since a particle with $X_0 > 18$ mm will go outside the limits of $\pm$ 29 x $\pm$ 26 mm. The particles inside the beam can reach the second order $\nu = 7.5$ resonance lines, and the third order $\nu = 7.66667$ and $\nu = 7.3333$ resonance lines. $\Delta\nu_x = 1.28$, $\Delta\nu_y = 1.43$ at $\epsilon_x = 0$.

AGS, BEAM GROWTH DUE TO RANDOM a_1, b_1 (Sample Procedure Comments on Fig.4)

One sees that the effect of the random a_1, b_1 depends on how close N_b is to the intrinsic limit, $N_b = .38 \times 10^{13}$/bunch.

At $N_b = .2 \times 10^{13}$, which is 54% of the intrinsic limit, one may estimate a tolerance on the random a_1, b_1, for ho beam growth, of about $a_1 = b_1 = 5 \times 10^{-6}$/cm rms. there is little information about the random a_1, b_1 present n the AGS. $a_1 \simeq b_1 \simeq 5 \times 10^{-6}$ may not be far from the amount actually present in the AGS.

One might conclude that for the AGS, for N_b above $.2 \times 10^{13}$, or 54% of the intrinsic limit, one may have beam growth due to the random a_1, b_1.

Fig. 3

AGS Tune Spread

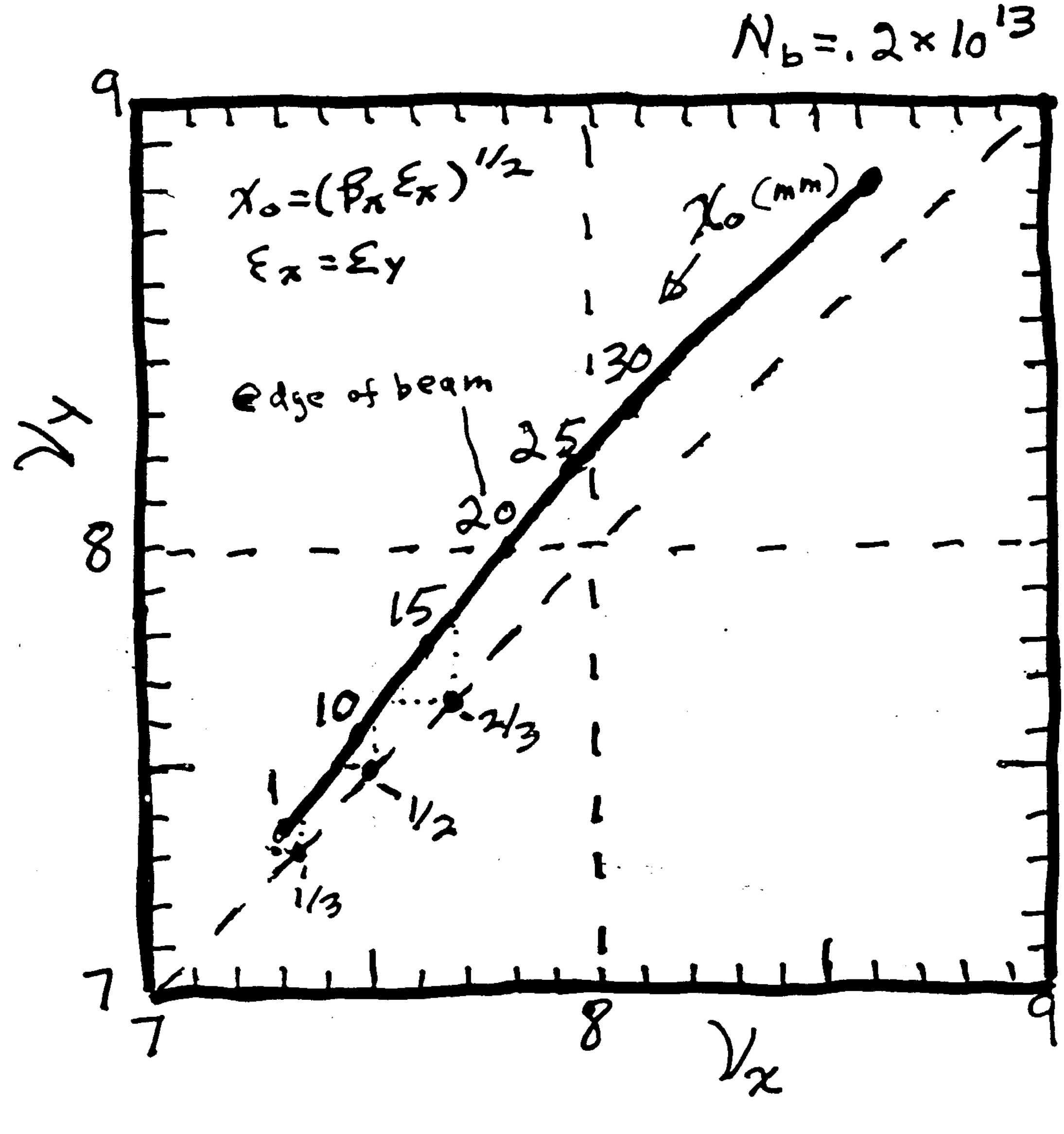
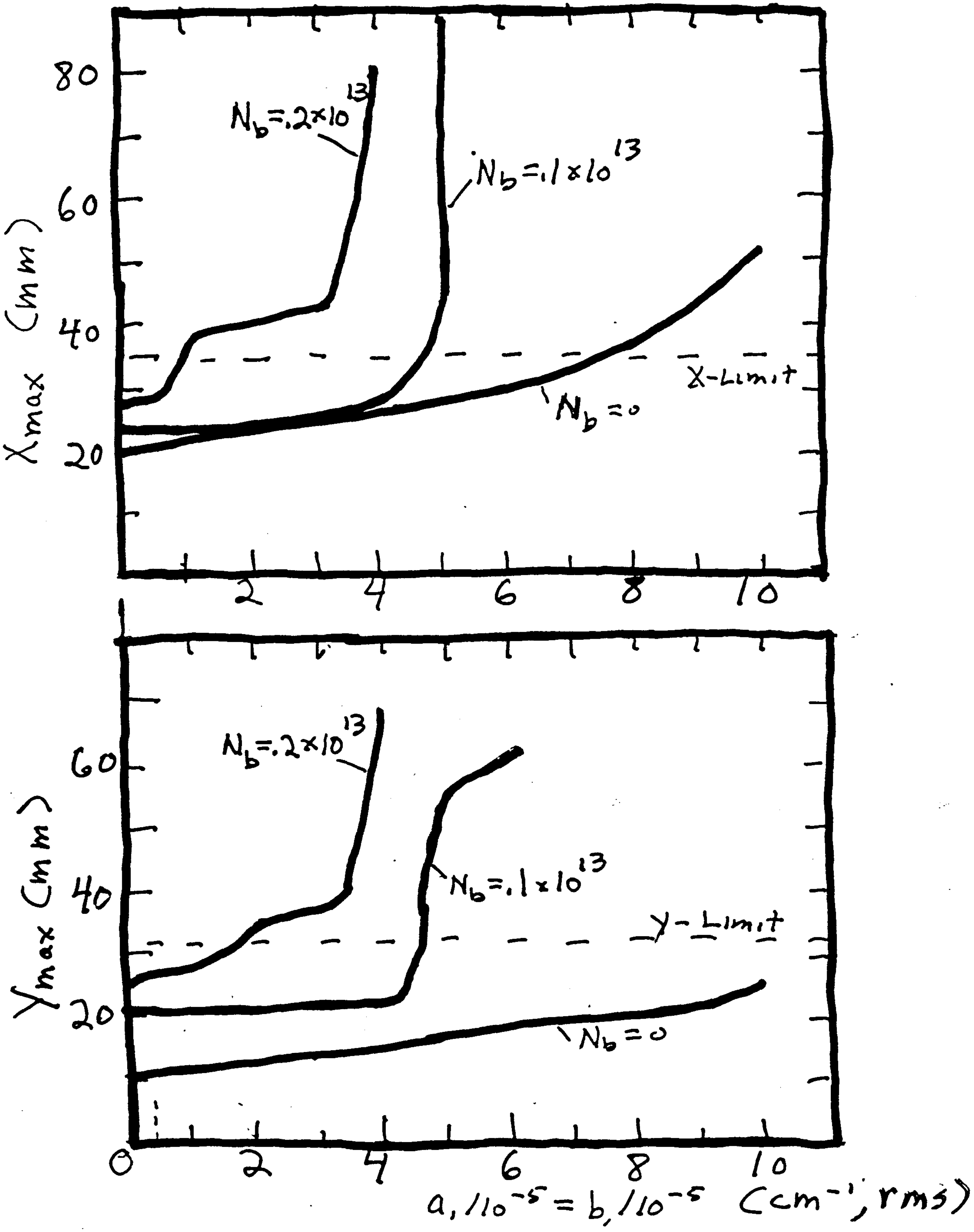


Fig. 4

AGS, – Random a_1, b_1 – Sample Procedure



AGS, BEAM GROWTH DUE TO RANDOM a_1, b_1. (Test Group Procedure Comments on Fig. 5)

Using the test group procedure one can explore the effects o the $\nu = 7.5$ resonance generated by the random errors. The starting emittance $\epsilon_x = \epsilon_y = X_o^2/\beta_x$ can be varied to make the test group pass through the resonance.

Beam growth is shown as a function of $x_o = (\beta_x \epsilon_x)^{1/2}$, for 3 values of the random a_1, b_1, $a_1 = b_1 = 0, .7 \times 10^{-5}$/cm, 2×10^{-5}/cm rms. The growth observed is about the same as found by the sample procedure for $N_b = .2 \times 10^{13}$/bunch.

One does see resonance like peaks, but the peaks are fairly broad and only very roughly correlated with the location of the resonances.

AGS, Beam Growth – Random a_1, b_1 – Test group Procedure

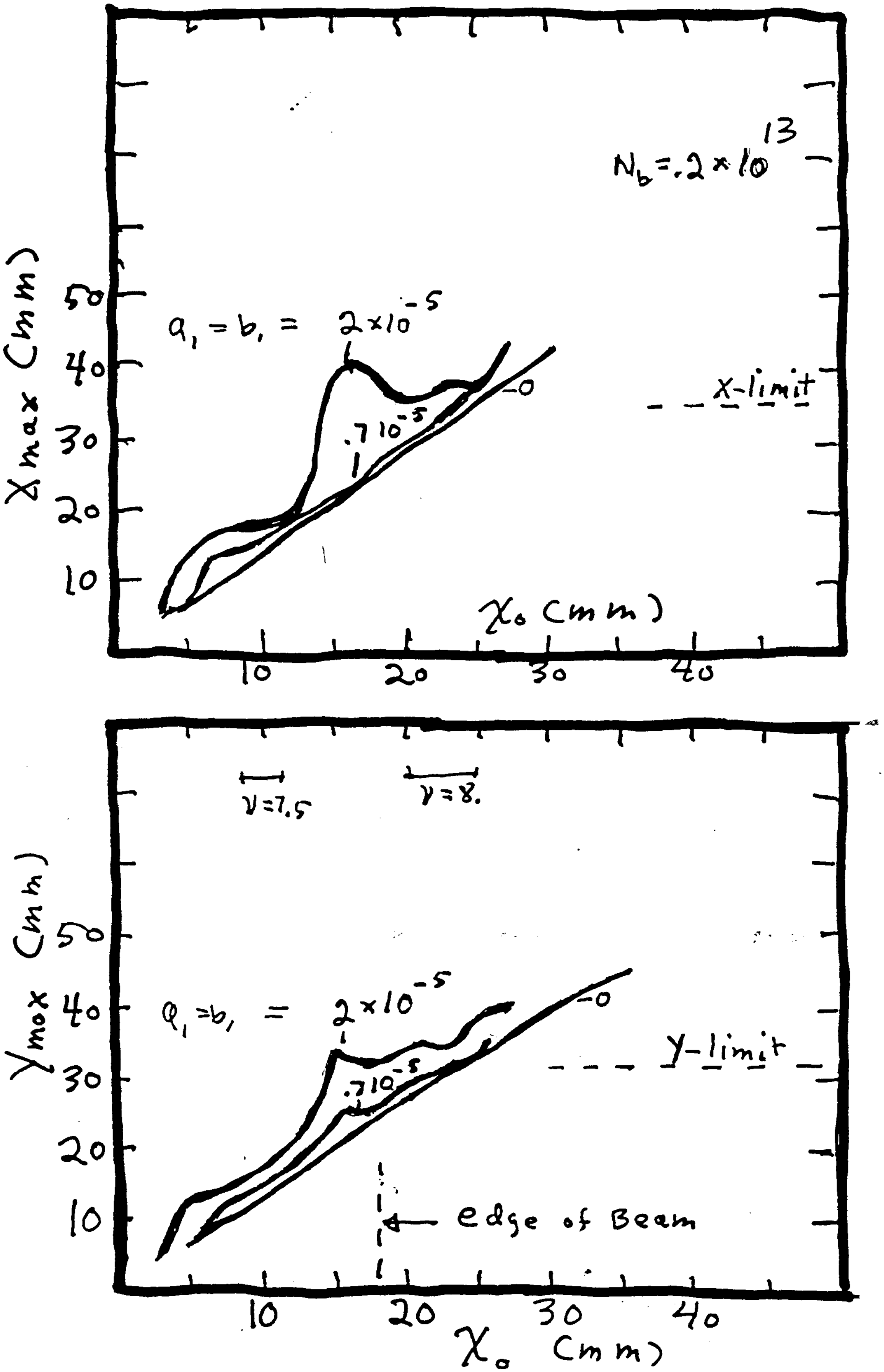


Fig. 5.

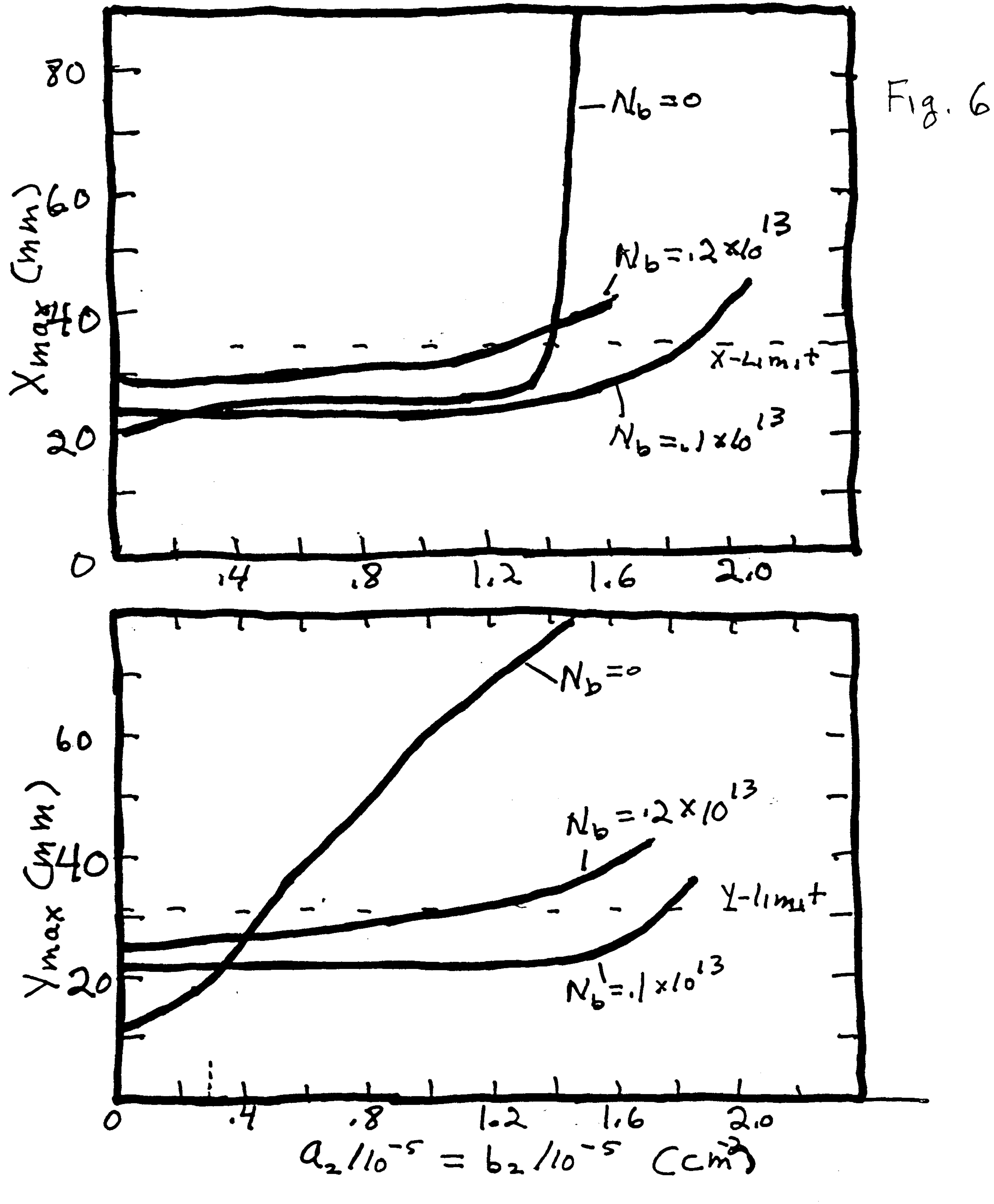
AGS, BEAM GROWTH DUE TO RANDOM a_2, b_2 (Sample procedure Comments on Fig 6)

One striking result is the large growth seen with $N_b = 0$, which can be understood by noting that the operating ν values are $\nu_x = 8.6$, $\nu_y = 8.8$ which is very close to the third order sum resonance $2\nu_x + \nu_y = 26$.

The first effect of adding space charge forces is to reduce the beam growth by introducing a strong dependence of the ν value on the amplitude of the betatron oscillation. The space charge forces only start to produce appreciable beam growth because of the random a_2, b_2 when N_b get close to the intrinsic limit. $N_b = .38 \times 10^{13}$/bunch.

At $N_b = .2 \times 10^{13}$, 54% of the intrinsic limit, one may roughly estimate a tolerance on the random a_2, b_2 for no beam growth of about $a_2 = b_2 = 3 \times 10^{-6}/\text{cm}^2$ rms.

AGS, Random a_2, b_2 - Sample Procedure



AGS, BEAM GROWTH DUE TO RANDOM a_2, b_2 (Test Group Procedure Comments on Fig. 7)

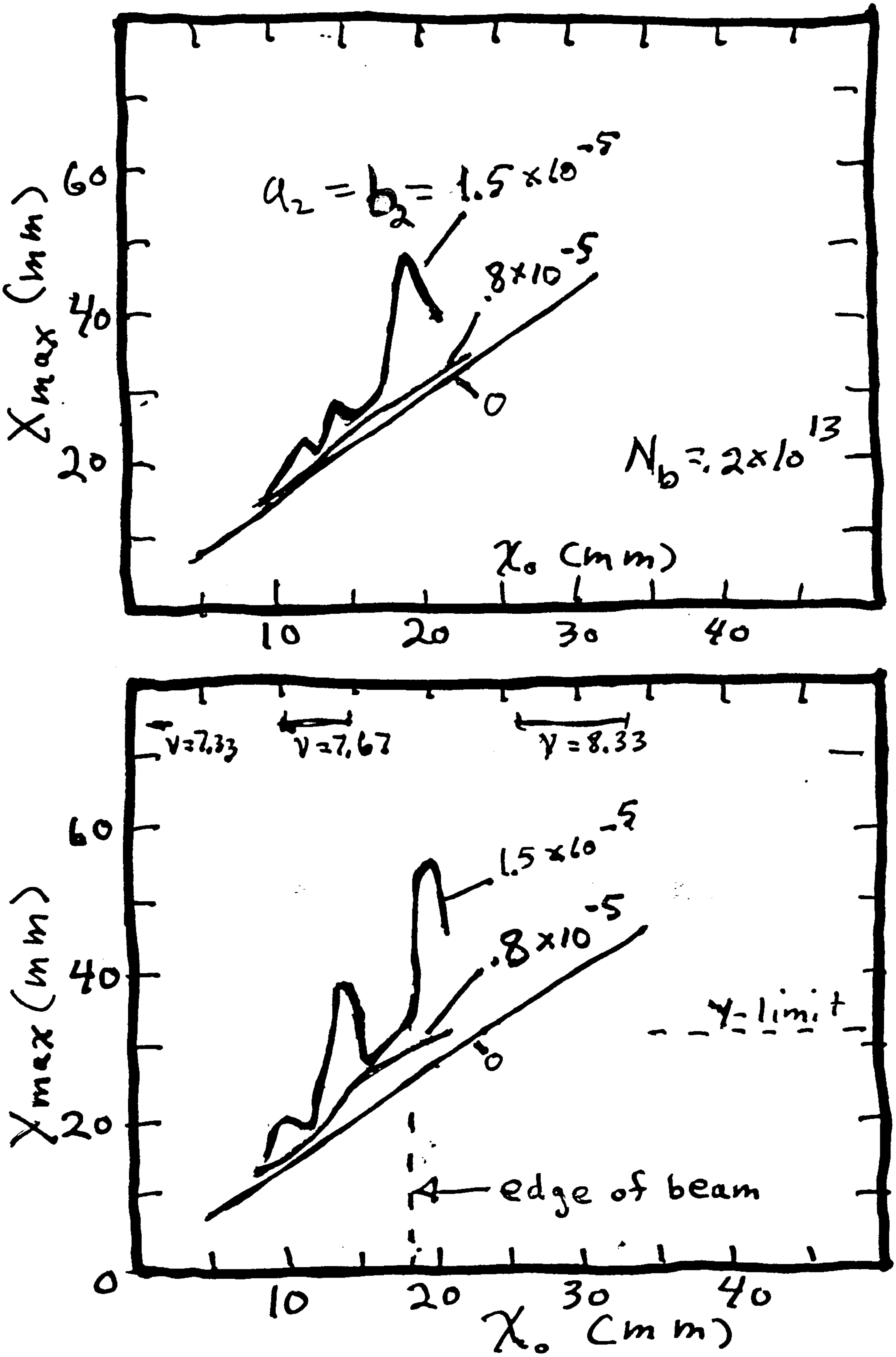
Using the test group procedure one can explore the effects the $\nu = 7.33333$ and $\nu = 7.66667$ third order resonances.

Beam growth is shown as a function of $x_0 = (\beta_x \epsilon_x)^{1/2}$ for 3 values of the random a_2, b_2, $a_2 = b_2 = 0, .8 \times 10^{-5}, 1.5 \times 10^{-5}/cm^2$ rms.

Resonance like peaks are observed, which are only roughly correlated with the location of the resonances. No sign was seen of the $\nu = 7.3333$ resonance. The growth found agrees fairly well with that found by the sample procedure.

AGS Beam Growth - Random a_2, b_2 - Test Group Procedure

Fig. 7



RESULTS FOR THE BOOSTER

The following figures give the results found for the Booster.

BOOSTER TUNE SPREAD (Comments on Fig. 8)

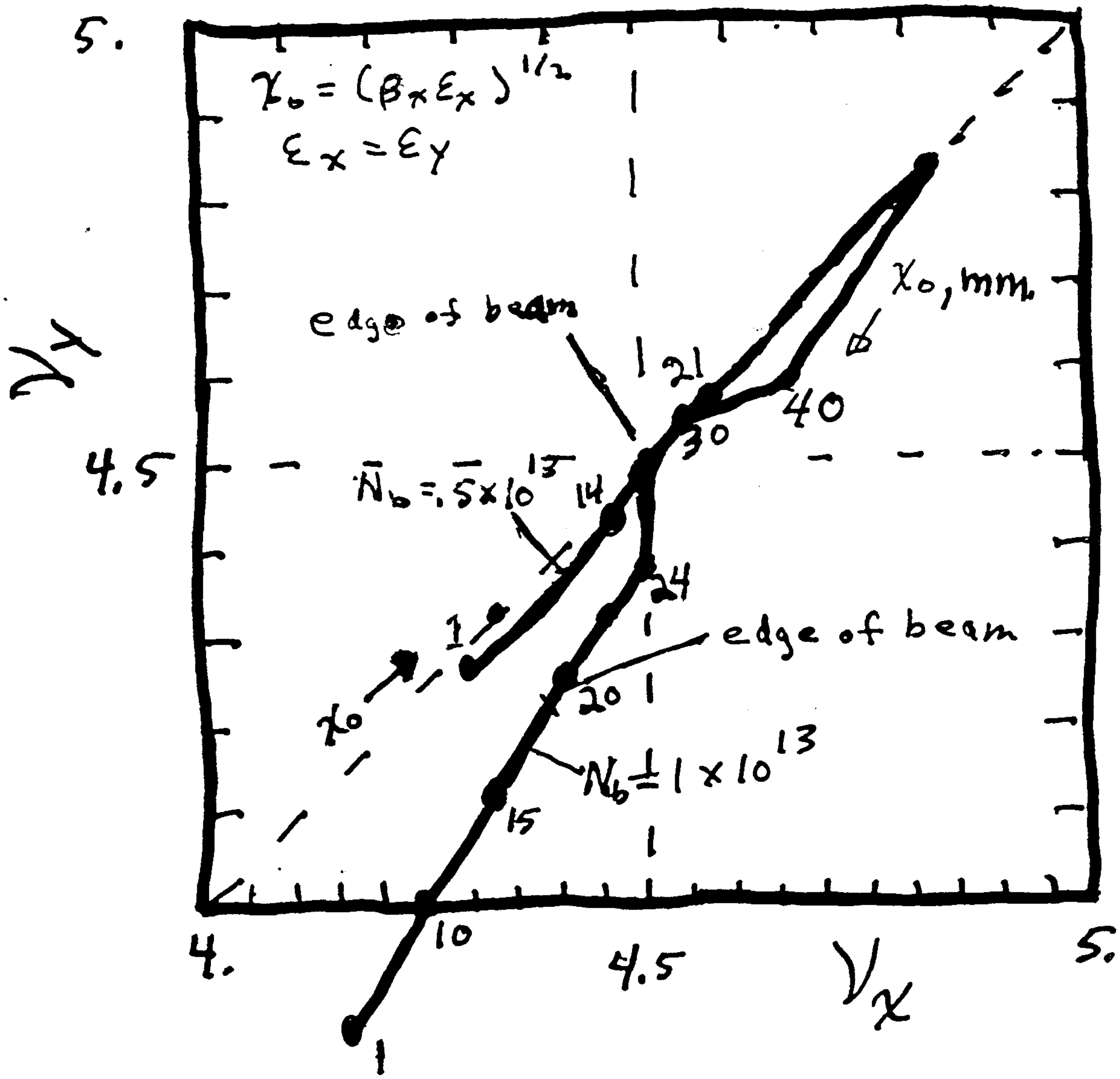
$\epsilon_x = \epsilon_y$, $x_o = (\beta_x \epsilon_x)^{1/2}$, $\beta_x = 13.8$m. For $N_b = 1 \times 10^{13}$/bunch, the beam dimensions grow to ± 39 x 27 mm particles with $\epsilon_x = \epsilon_y$. A starting $x_o \geq 20$ mm may be considered to be outside the beam since a particle with $x_o = 20$ mm, $\epsilon_x = \epsilon_y$, will go outside the limits of ± 39 x ± 27 mm. For $N_b = 1$, the particles in side the beam can reach the second order resonance $\nu_y = 4$ and the third order $\nu = 4.333$ resonance lines.

For $N_b = .5 \times 10^{13}$/bunch, the beam dimensions grow to ± 28 x ± 26 mm when no random a_k, b_k are present. Particles with $x_o \geq 18$ mm, $\epsilon_x = \epsilon_y$ can be considered to be outside the beam. The resonances that can be reached are the $\nu = 4.333$ and $\nu = 4.5$ resonance lines.

For $N_b = .5 \times 10^{13}$/bunch, $\Delta\nu_x = -.52$, $\Delta\nu_y = -.58$ for $\epsilon_x = \epsilon_y = 0$, $x_o = 0$.

Fig. 8

Booster Tune Spread



BOOSTER, BEAM GROWTH DUE TO RANDOM a_1, b_1 (sample procedure)

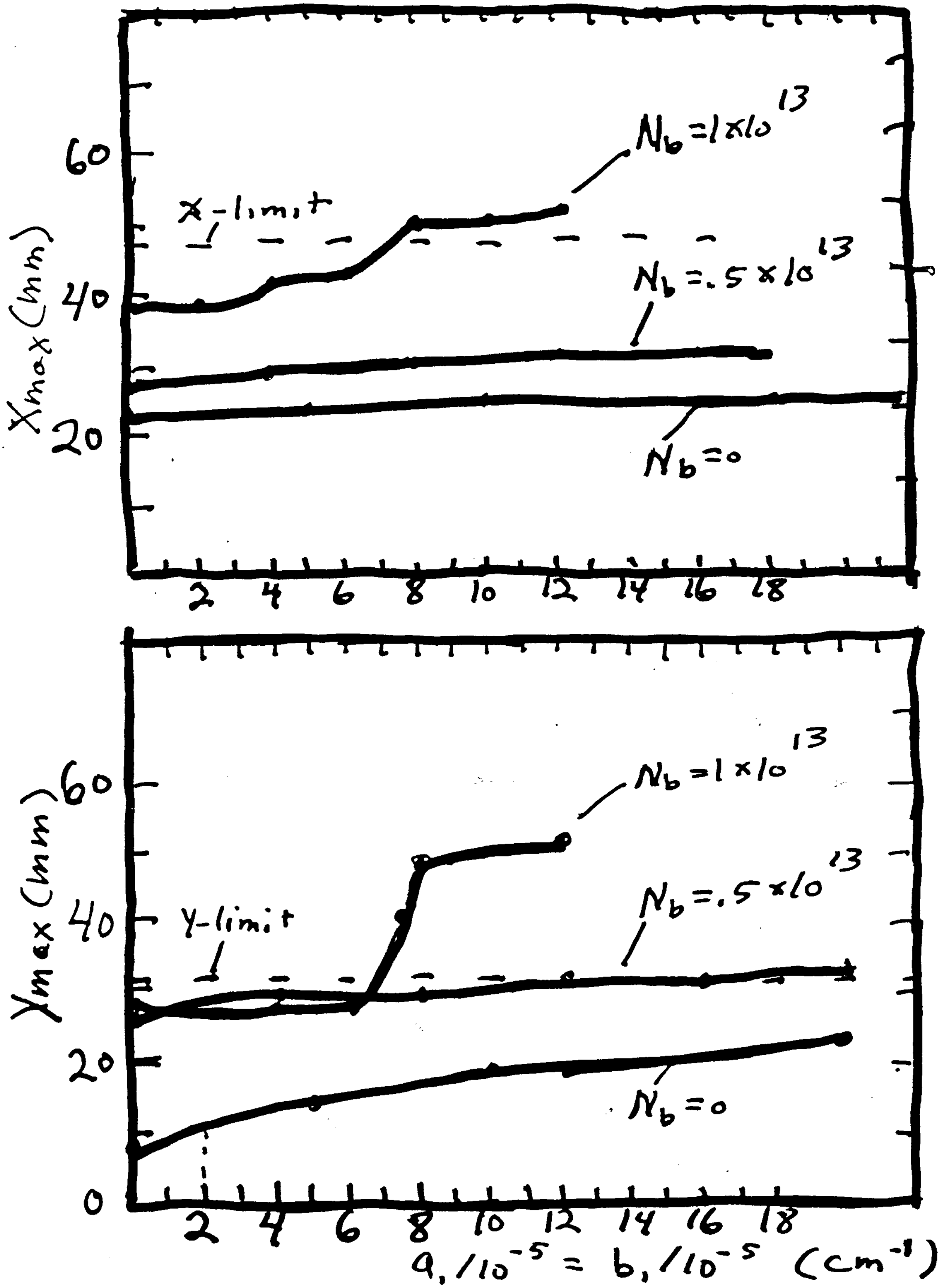
The effect of the random a_1, b_1 depends on how close N_b is to the intrinsic limit, N $N_b = 1.5 \times 10^{13}$/bunch.

At $N_b = 1 \times 10^{13}$, which is 70% of the intrinsic limit, one may estimate a tolerance on the random a_1, b_1 for no beam growth of about $a_1 = b_1 = 2 \times 10^{-5}$/cm rms. This is about 4 times larger than the AGS result.

It seems likely that the Booster may have little problem with the effects of the random a_1, b_1.

Booster, - Random a_1, b_1 - Sample procedure

Fig. 9.



BOOSTER, BEAM GROWTH DUE TO RANDOM a_1,b_1 (Test Group Procedure, Comments on Fig. 10)

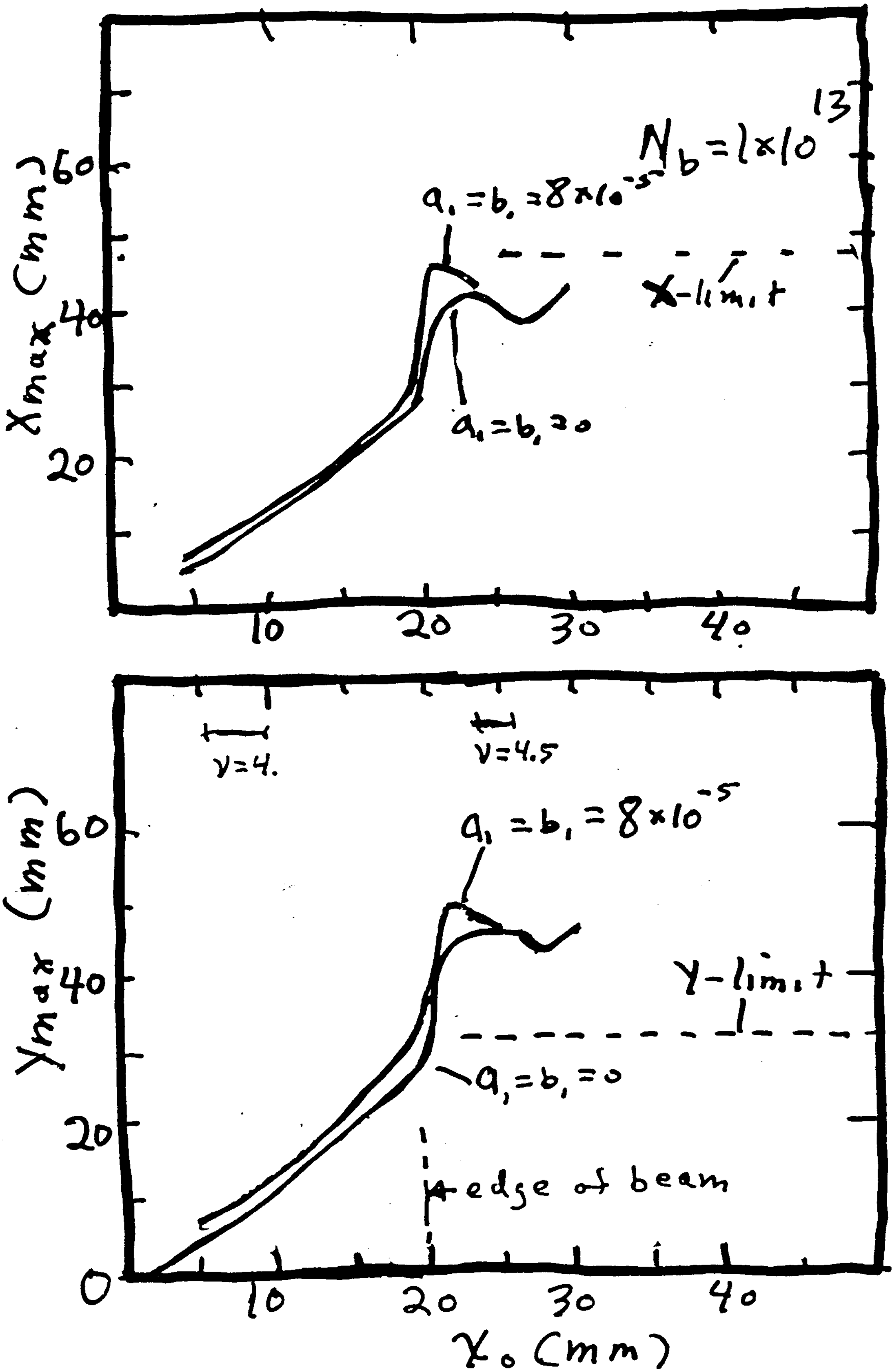
Beam growth is shown as a function of $x_o = (\beta_x \epsilon_x)^{1/2}$, for 2 values of the random a_1, b_1 $a_1 = b_1 = 0$, and $a_1 = b_1 = 8 \times 10^{-5}$/cm rms.

There is no sign of the $\nu = 4$ resonance. There is some indication of resonance like growth near $\nu = 4.5$. This growth is seen even when $a_1 = b_1 = 0$. One may note that $\nu = 4.5$ is a structure resonance for the Booster, as $\nu = 4.5 = 18/6$ and the Booster has a six fold periodicity.

With the test group procedure, there is difficulty in deciding whether the particles experiencing this growth are inside or outside the beam. The sample procedure indicates that these are inside the beam. In any case, they are certainly near the edge of the beam, and this may not produce an important loss of beam.

Booster - Beam Growth - Random a_1, b_1 - Test Group Procedure

Fig 10



BOOSTER, BEAM GROWTH DUE TO RANDOM a_2, b_2 (sample procedure. Comment on Fig. 11)

At $N_b = 1 \times 10^{13}$/bunch, 70% of the intrinsic limit, one may roughly estimate a tolerance on the random a_2, b_2 for n_o beam growth of about $a_2 = b_2 = 5 \times 10^{-6}/cm^2$ rms. This is about 2 times larger than the AGS result.

Booster - Random a_2, b_2 → Sample-Procedure

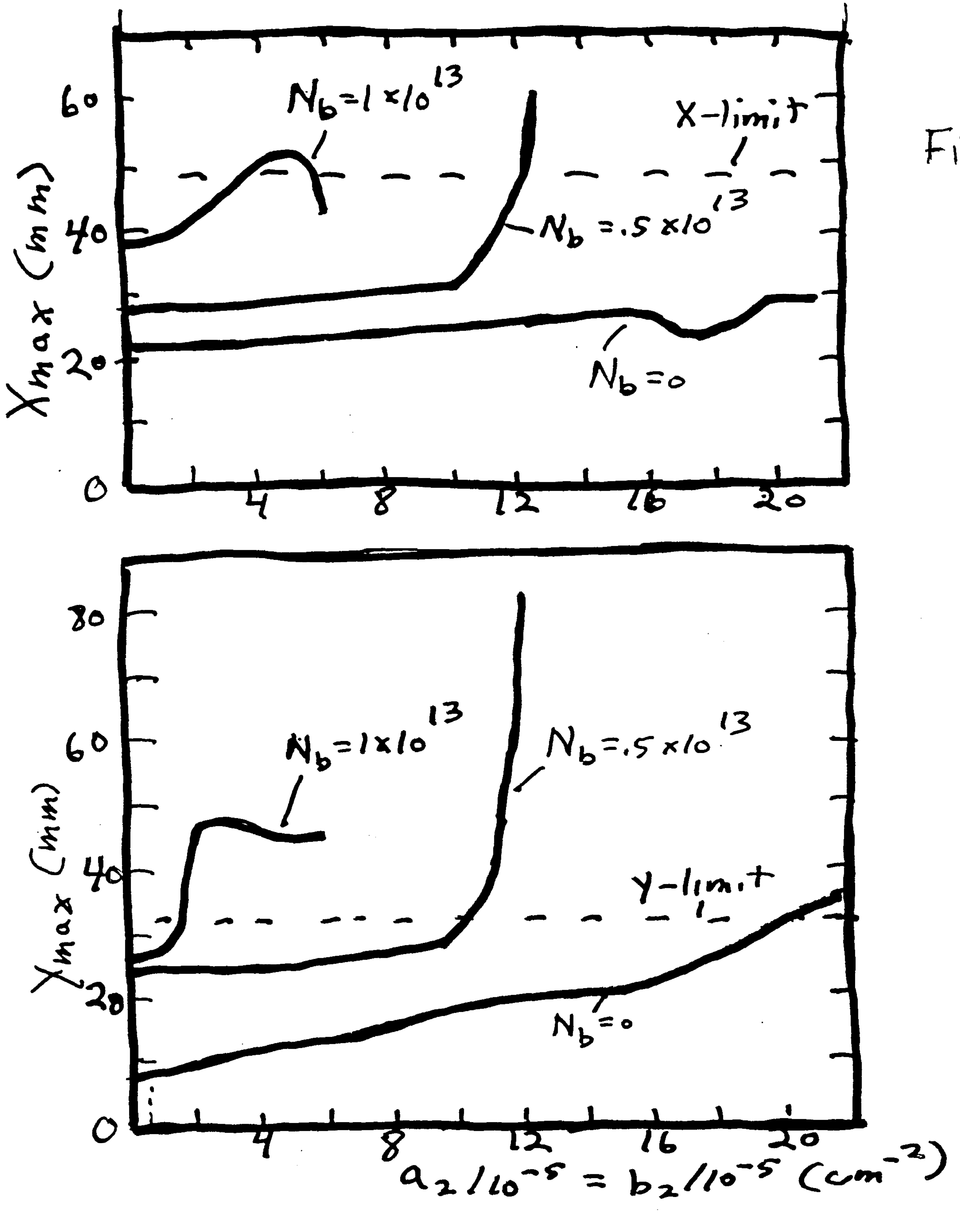


Fig. 11

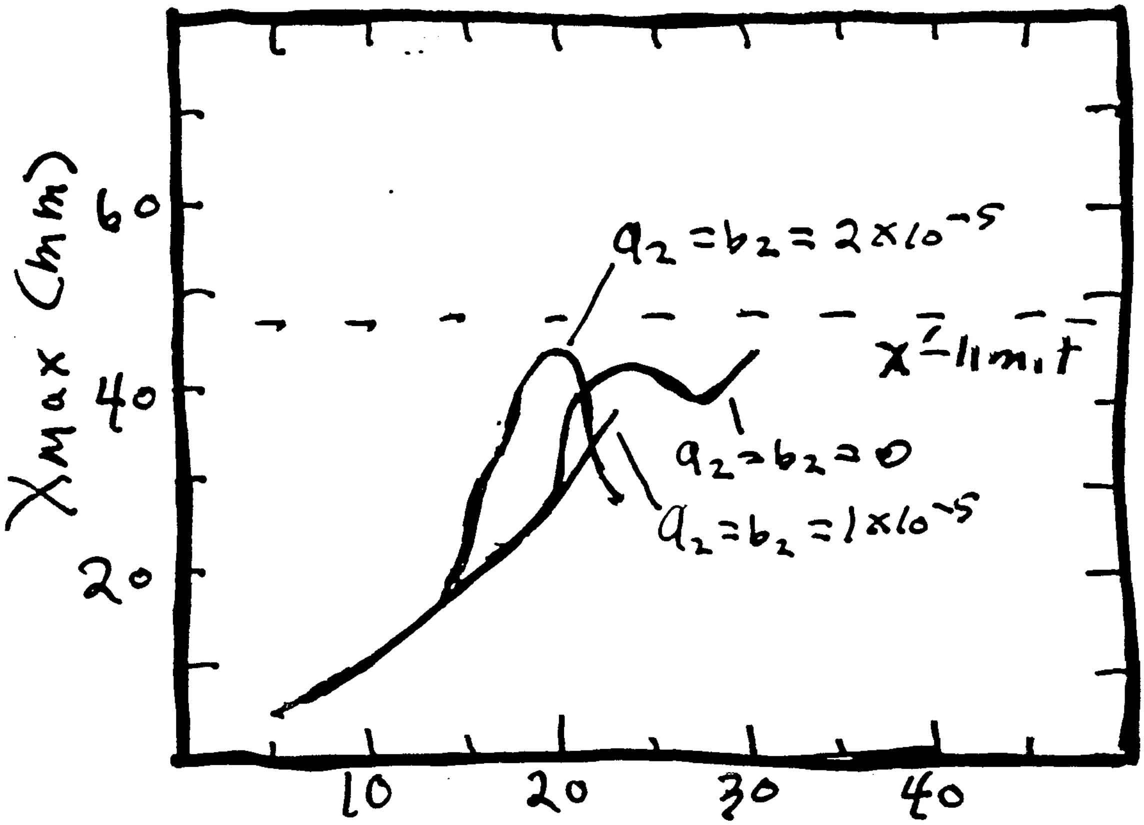
BOOSTER, BEAM GROWTH DUE TO RANDOM a_2, b_2, (Test Group Procedure. Comments on Fig. 12)

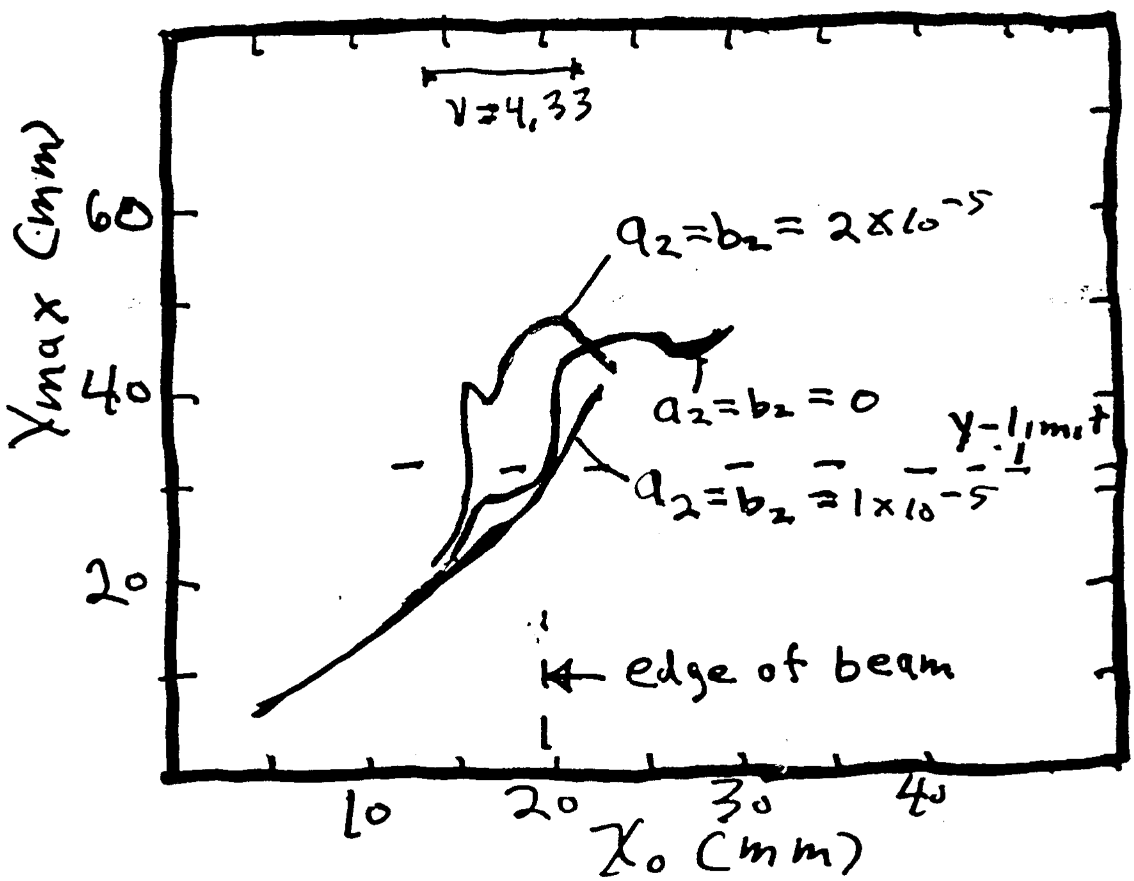
Beam growth shown as a function of $x_o = (\beta_x \epsilon_x)^{1/2}$, for 2 values of the random a_2, b_2, $a_2 = b_2 = 0$, $1 \times 10^{-5}/cm^2$ rms for $N_b = 1 \times 10^{13}$/bunch.

There is some indication of resonance like growth near $\nu = 4.3333$, which is the only set of third order resonance lines which are encountered by particles inside the beam. there is little growth for $a_2 = 1 \times 10^{-5}$, and considerably more growth for $a_2 = 2 \times 10^{-5}/cm^2$ rms when $N_b = 1 \times 10^{13}$/bunch.

Booster - Beamgrowth - Random a_2, b_2 - Test group Procedure

Fig. 12





APPENDIX A

TRANSIENT PHASE RESPONSE AT INJECTION

Consider a single bunch injected into a resonant cavity. The beam centroid is in quadrature with the unloaded cavity voltage. The beam current i_b is one-half of a sinusoid as indicated in Figure A-1.

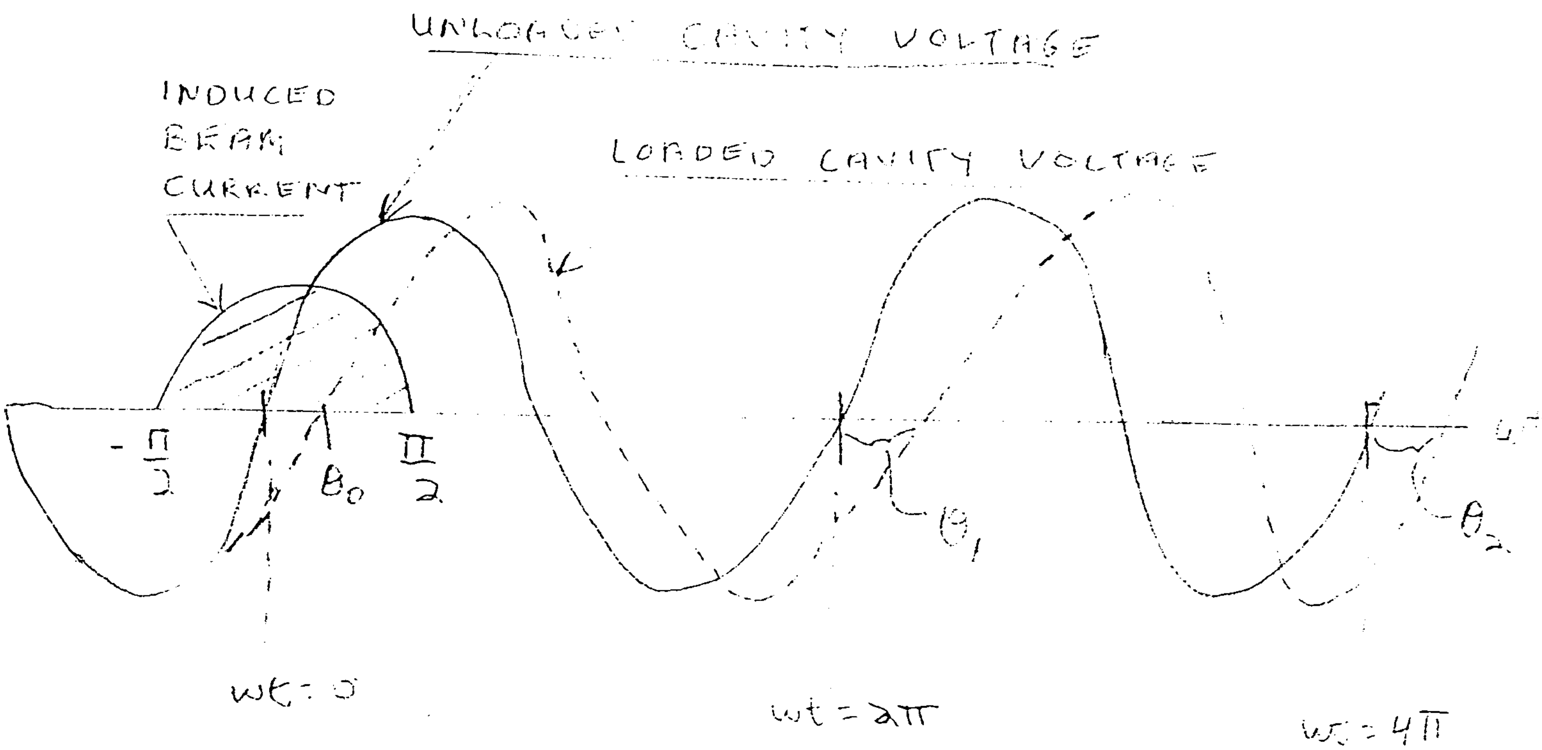


FIGURE A-1
INJECTION OF ONE BUNCH

The transient response for this single pulse is given by:

$$\nu(t) = v \sin wt - \frac{1}{2} Q_o \left\{ \frac{Q}{c} \right\} \left\{ 1 - e^{\frac{-wt + \pi/2}{2Q_o}} \cos wt. \right\} \qquad -\frac{\pi}{2} \leq wt \leq \frac{\pi}{2}$$

$$= V \sin wt - \frac{1}{2} Q_o \left\{ \frac{Q}{c} \right\} \left\{ e^{\left\{\frac{\pi}{4Q_o}\right\}} - e^{-\left\{\frac{\pi}{4\ Q_o}\right\}} \right\} e^{-\left\{\frac{wt}{2Q_o}\right\}} \cos wt \quad wt \geq \pi/2$$

where Q = charge per bunch
C = gap capacitance
V = peak gap voltage
Q_o = Q (quality factor) of the cavity

The transient loading causes a phase delay of the gap voltage that can be evaluated for each period following injection.

$$\theta_o = \frac{\frac{\pi}{8} \quad \frac{Q}{c}}{v - \frac{1}{4} \quad \frac{Q}{c}} = \frac{\pi}{8} \quad \frac{Q}{c} \quad \frac{1}{v}$$

$$\theta_1 = \frac{\pi}{4} \quad \frac{Q}{c} \quad \frac{1}{v} \quad e^{-\pi/Q_o}$$

$$\theta_N = \frac{\frac{\pi}{4} \quad \frac{Q}{c} \quad e^{(-\pi N Q_o)}}{v + \frac{\pi}{8} \quad \frac{1}{Q_o} \quad \frac{Q}{c} \quad e^{(-\pi N/Q_o)}}$$

which reduces to

$$\theta_N = \frac{\pi}{4} \quad \frac{Q}{c} \quad \frac{1}{v} \quad e^{(-\pi N/Q_o)}$$

for $V > \frac{1}{Q_o} \quad \frac{\pi}{8} \quad \frac{Q}{c}$

For a second bunch injected during the second RF cycle the net phase delay is approximated by

$$\Delta\theta_1 = \frac{1}{V} \left\{ \frac{Q}{C} \right\} \left\{ \frac{\pi}{8} + \frac{\pi}{4} \quad e^{(-\pi/Q_o)} \right\}$$

For the third bunch injected during the third RF cycle the net phase delay is approximated by

$$\Delta\theta_2 = \frac{1}{V} \left\{ \frac{Q}{C} \right\} \left\{ \frac{\pi}{8} + \frac{\pi}{4} \quad e^{-\pi/Q_o} \left[1 + e^{(-\pi/Q_o)} \right] \right\}$$

The phase shift encountered by the third bunch during the initial injection from the Booster into the AGS ring is a function of Q_o and is tabulated below.

Q_o	Phase Delay $\Delta\theta_2$	Energy Loss Per Gap ΔU
10	$1.39 \frac{1}{V} \left\{ \frac{Q}{C} \right\}$	$1.39 \left\{ \frac{Q}{C} \right\}$
20	$1.63 \frac{1}{V} \left\{ \frac{Q}{C} \right\}$	$1.63 \left\{ \frac{Q}{C} \right\}$
30	$1.83 \frac{1}{V} \left\{ \frac{Q}{C} \right\}$	$1.83 \left\{ \frac{Q}{C} \right\}$

The phase shift of the gap voltage introduces a deceleration of the beam. The energy loss per gap, ΔU, is given by the product of $\Delta\theta_2$ and V which is also tabulated as a function of Q_0.

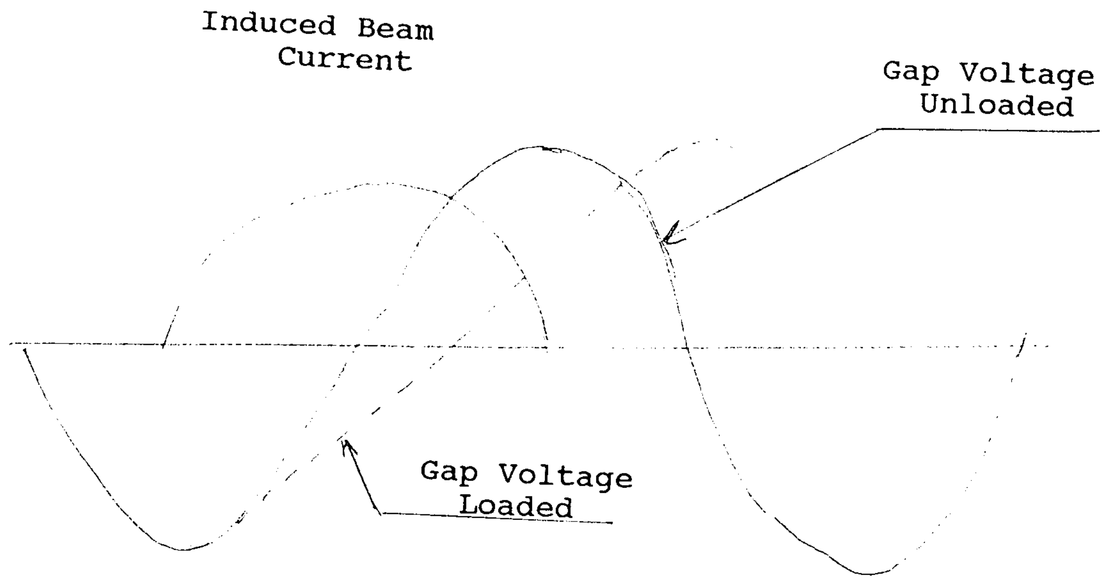


FIGURE 3A

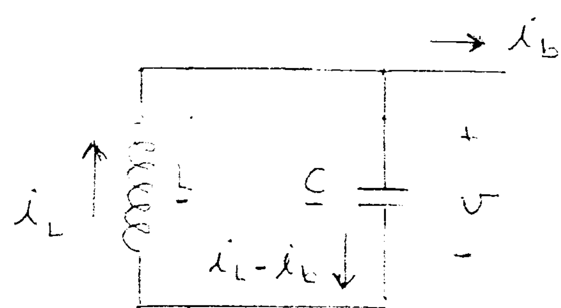


FIGURE 3B

RELATIONSHIP BETWEEN BEAM CURRENT
AND ACCELERATING VOLTAGE AT INJECTION

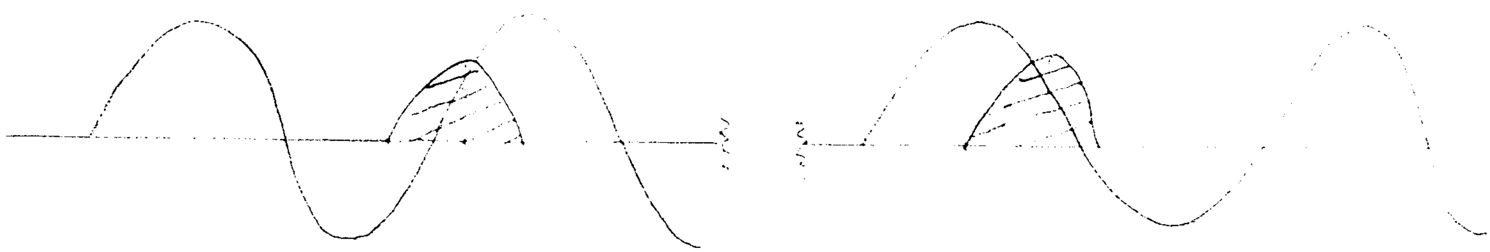


FIGURE 4

STABLE PHASE ANGLE AT TRANSITION

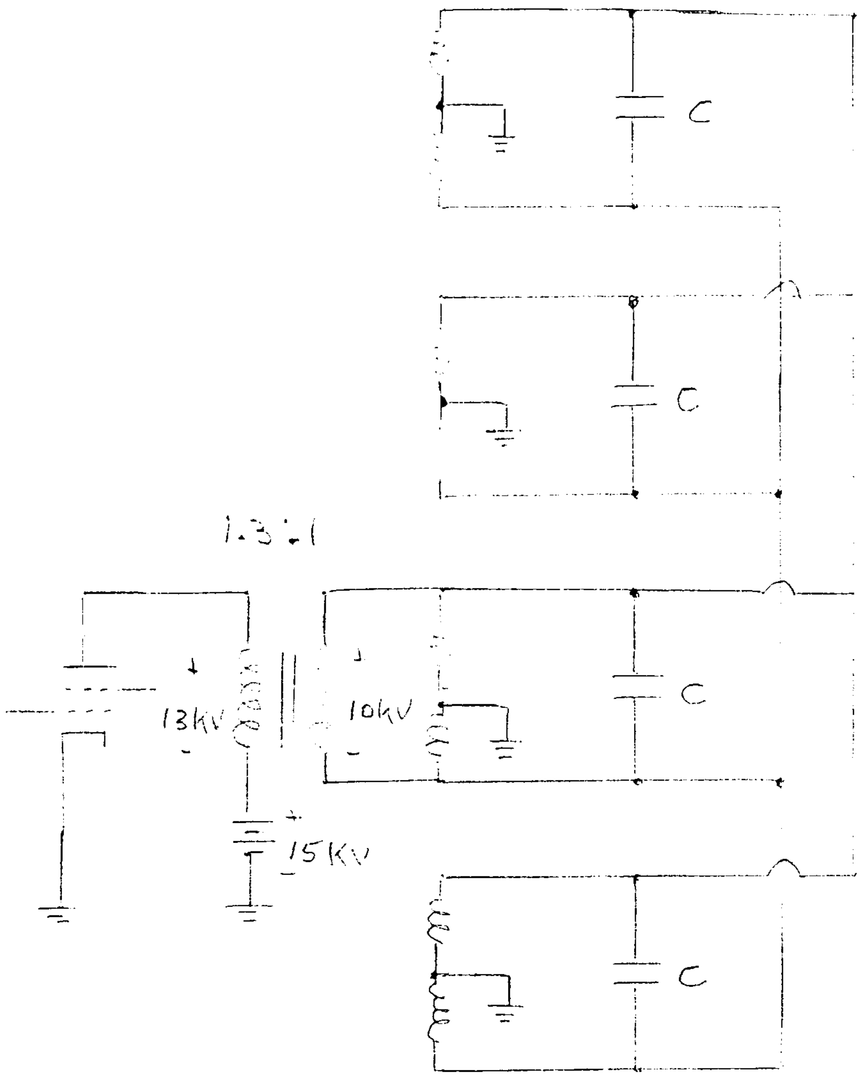


FIGURE 5

POWER AMPLIFIER DRIVING EXISTING AGS CAVITY

FIGURE 6A

LOAD LINE AT MAXIMUM ACCELERATION

PL = 154 KW.

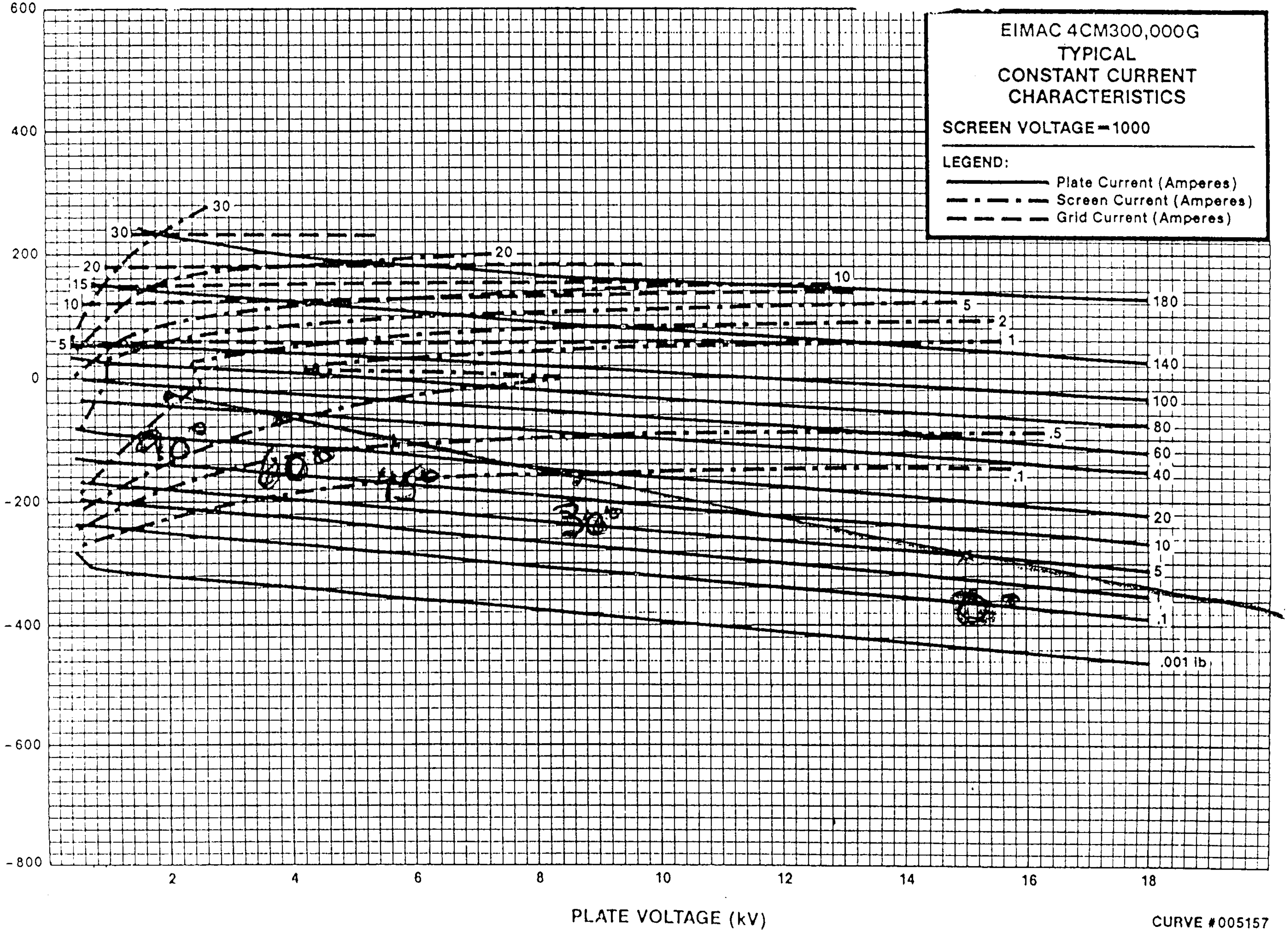
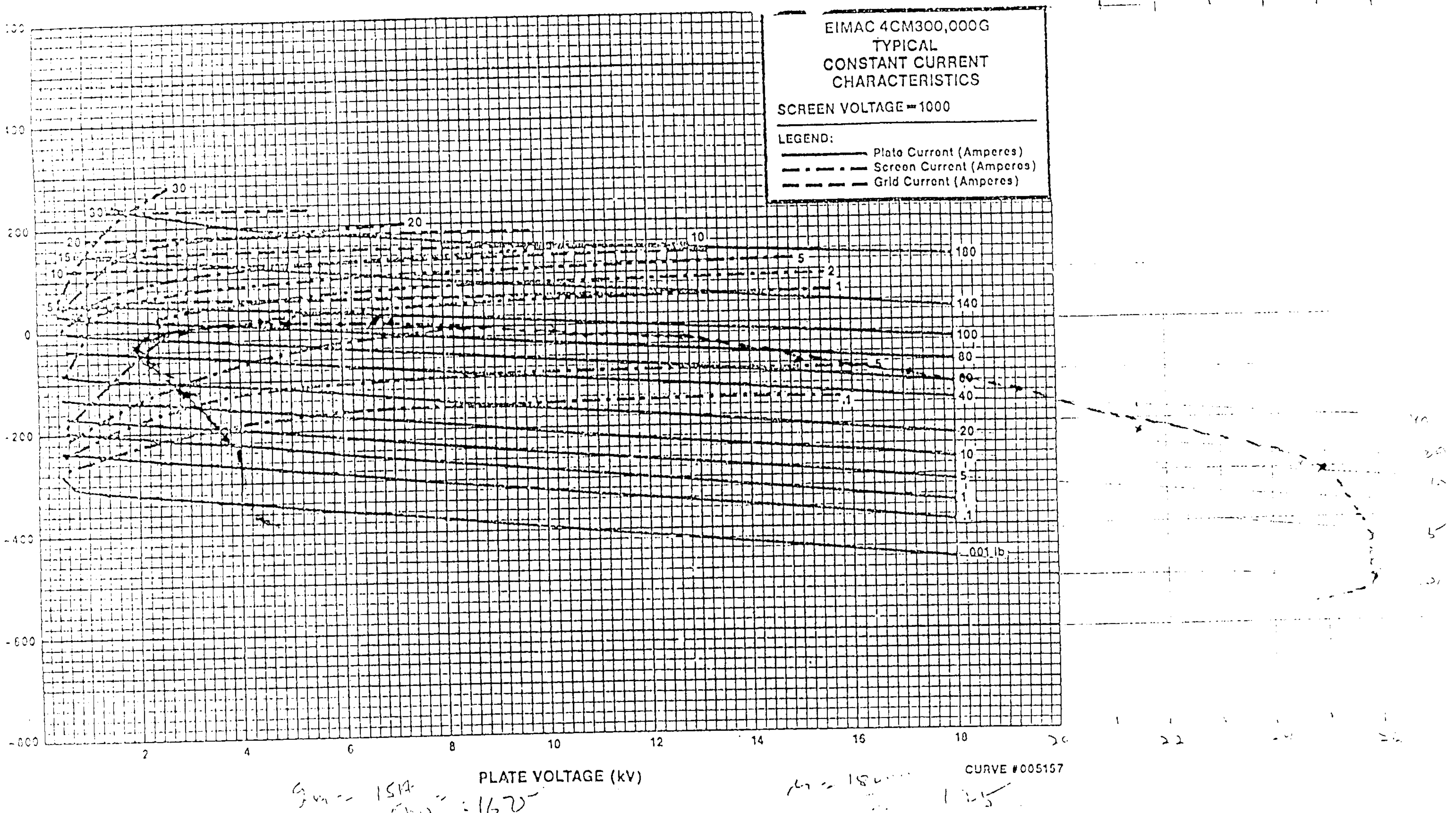


FIGURE 6B

LOAD AT PHASE TRANSITION

$P_L =$ 154KW



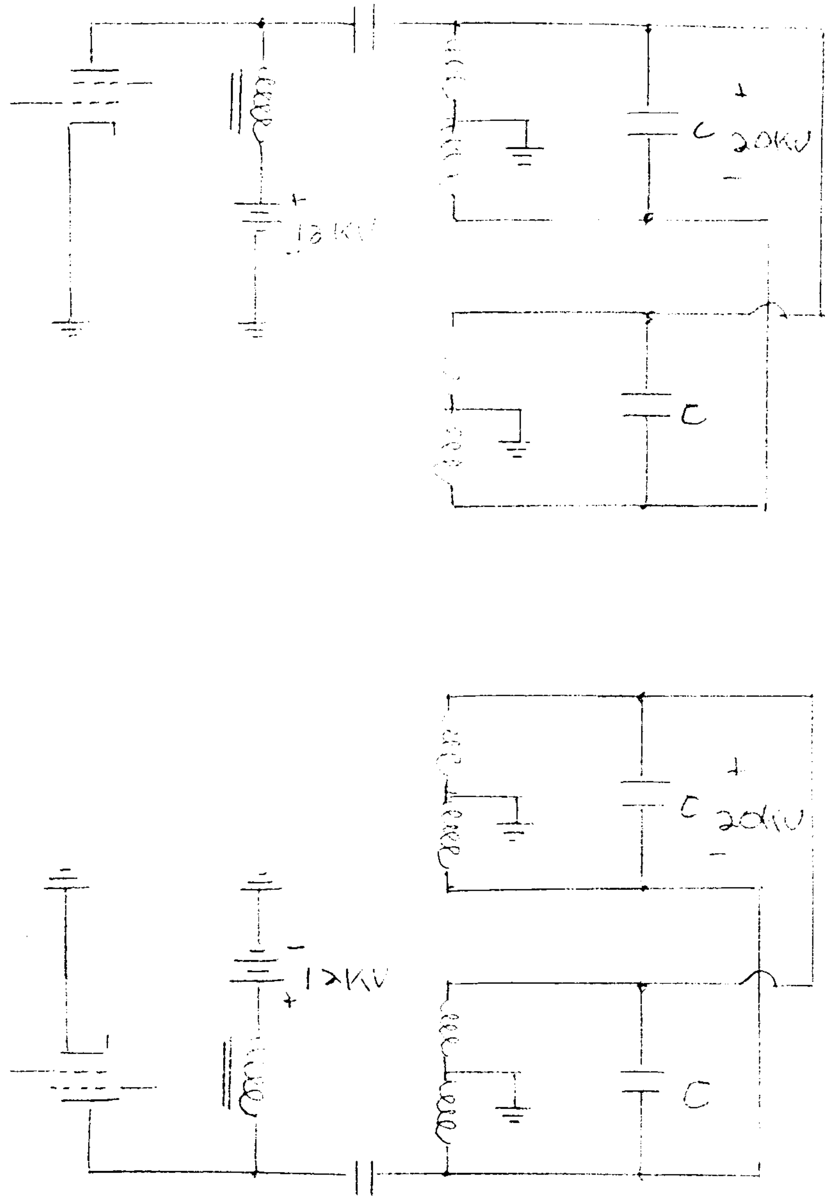


FIGURE 7

POWER AMPLIFIER DRIVING MODIFIED AGS CAVITIES

TWO STATIONS PER CAVITY

THE GRIDS ARE EXCITED PUSH-PULL

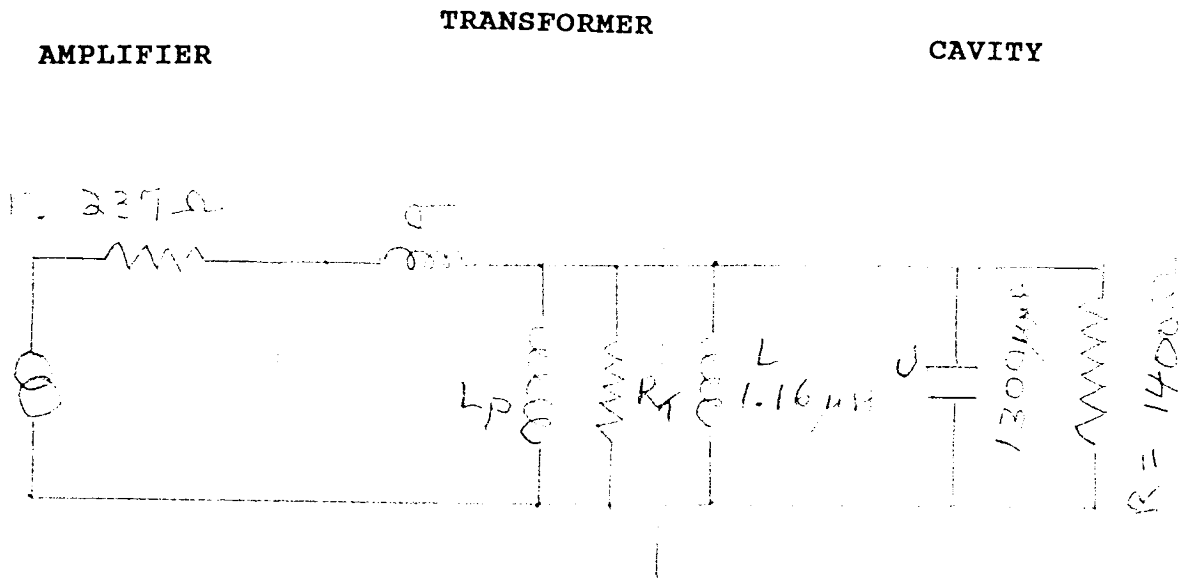


FIGURE 8

EQUIVALENT CIRCUIT OF LOADED TRANSFORMER

NO HEAT SINK COMPOUND USED

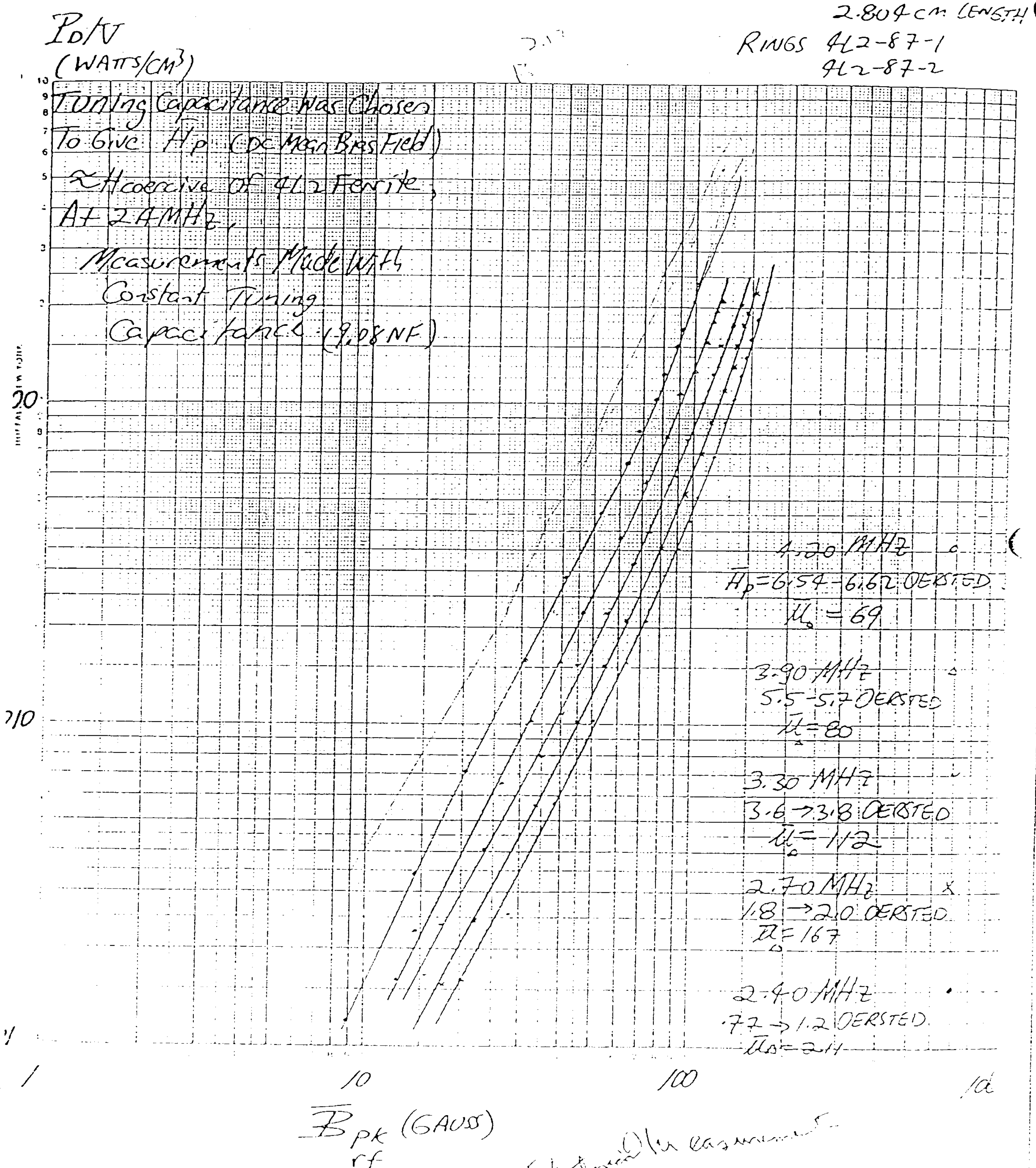
$T_{SET} = 81°F$

9/14, 9/16, 9/17/87

PHILIPS 4L2 FERRITE

2 RINGS: 50 CM O.D.
20 CM I.D.
2.804 CM LENGTH

RINGS 4L2-87-1
4L2-87-2



4L2 LOSS MEASUREMENTS

FIGURE 9

REFERENCES

M. Meth, A. Ratti; Specifications and Design of RF Power Amplifiers for Proton Cavity, AD Booster Tech. Note 92, September, 21, 1987.

M. Plotkin; Booster Proton Cavity with Voltage Reduction During the Cycle, AD Booster Tech. Note 85, July 29, 1987.