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S. White

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Collider-Accelerator Department Brookhaven National Laboratory Upton, NY 11973

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S. White, W. Fischer, Y. Luo BNL, Upton, NY, USA

Abstract

Electron lenses for head-on compensation are currently under commissioning and foreseen to be operational for the 2015 polarized proton run. These devices will provide a partial compensation of head-on beam-beam effects and allow to double the RHIC proton luminosity [1]. This note reviews the optics constraints related to beam-beam compensation and summarizes the the current lattice options for proton operation at 100 GeV.

INTRODUCTION

The RHIC collider is currently operating between the 2/3 and 7/10 resonances with a beam-beam parameter of approximately 0.015 leaving little space for significant increase in luminosity. The RHIC luminosity upgrade program [1] aims at an increase of the luminosity by a factor of 2. In order to accommodate the significant increase in beam-beam tune spread it was decided to install electron lenses to compensate for the beam-beam non-linearities and effectively reduce the tune spread at constant bunch intensity. This technology was first developed at the Tevatron where it was tested for head-on compensation [2] and then successfully used for long-range compensation, abort gap cleaning [3] and collimation studies [4].



Figure 1: Layout of the RHIC collider. The colliding IPs are denoted by the red stars, the head-on compensation by the green star.

The RHIC collider consists of two rings where the beams are colliding in IP6 and IP8 as shown in Fig. 1. The two electron lenses, one for each ring, are located close to IP10. Studies regarding dynamic aperture were performed and showed improvements for high beam-beam parameter [5]. In additon to the requirements on the electron charge density distribution to achieve compensation of the beam-beam force, specific optics constraints apply and dedicated lattice developments are necessary. This note reviews recent developments and the current status of optics studies for 100 GeV proton operation.

OPTICS CONSTRAINTS

Due to its non-linearity the beam-beam force can drive non-linear resonances [6]. In the presence of multiple beam-beam interactions, the strength of these resonances depends on the phase advance between IPs. It can be shown [5] that in order to compensate provide compensation of the non-linear resonances the electrons lenses have to be place at a phase advance of $k\pi$ (where k is an integer) from the beam-beam interaction to be compensated for. It is preferable to have as few as possible non-linear elements between the electron lens and the beam-beam interaction, hence, in the case of RHIC the phase advance is constrained between IP8 and IP10. Phase errors may enhance the resonances driving terms, it is therefore essential to enforce this condition.

When a proton bunch interacts with the electron beam it will drive Larmor oscillations of the electrons along the interaction region resulting in an *s*-dependent kick onto the proton bunch. This can be interpreted as an electron lens impedance comparable to or larger than the machine impedance. Its strength depends on the electron lens parameters and under certain conditions can lead to transverse mode coupling instabilities (TMCI). This effects was studied in details in [7] where it is shown that the *s*-dependent momentum change of the protons can be modeled with the following wake function

$$\Delta p_{x,y} = W[\Delta_{x,y}\sin(ks) \pm \Delta_{y,x}(1 - \cos(ks))] \quad (1)$$

where Δ_x and Δ_y are the offset of the source in the horizontal and vertical planes respectively and W is a constant depending on both the beam-beam parameters of the electron and proton beams and the solenoid field B. A similar equation is also valid for the vertical plane y. The variable k is defined as

$$k = \frac{\omega_L}{(1 + \beta_e)c} \tag{2}$$

where β_e is the relativistic β of the electron beam, c is the speed of light and ω_L is the Larmor angular frequency defined as

$$\omega_L = \frac{eB}{\gamma_e m}.\tag{3}$$

Using this wake function and considering uniform and equal transverse distributions for the proton and electron beams, it is possible to analytically derive the TMCI threshold and hence the required solenoid field to insure stability. This threshold can be expressed expressed as [7]

$$B_{th} = 1.3 \frac{e N_p \xi_e}{r^2 \sqrt{\Delta Q Q_s}}.$$
(4)

where N_p is the proton bunch intensity, ξ_e is the electron lens beam-beam parameter, r is the radius of the beam $(r \approx 2\sigma \text{ for Gaussian distribution}), \Delta Q$ is the separation between horizontal and vertical tune and Q_s is the synchrotron tune. Using typical RHIC parameters (N_p = 3.0×10^{11} protons per bunch, $\xi_e = 0.011$, $\Delta Q = 0.01$, $Q_s = 5.0 \times 10^{-4}$ and $r \approx 0.8 \,\mathrm{mm}$) a threshold field of 14 T is found which is approximately a factor 2 above the design field of 6 T. Increasing the beam size at the electron lens would mitigate this instability. Although not as critical for 100 GeV operation thanks to the lower energy (large beam size) and bunch intensity, having a β -function as large as possible at the center of the electron lens is therefore desirable. It should be noted that numerical simulations [8] showed that a transverse bunch-by-bunch damper could cure these instabilities. This device is currently under construction in RHIC.

CHROMATICITY

Past studies have shown that, as long as the beam-beam tune spread remains far enough from low order betatron resonances, the colliding proton beam lifetime is limited by the off-momentum aperture. In the process of developing a new lattice, dedicated effort was made to minimize the non-linear chromaticity.

For non-zero chromaticity, off-momentum particles will experience tune shifts and optics distortion with respect to the on-momentum, reference particle. The resulting momentum dependent tunes and β -functions can be expressed as:

$$Q(\delta_p) = Q_0 + Q'\delta_p + \frac{1}{2}Q''\delta_p^2 + \frac{1}{6}Q'''\delta_p^3 + \dots,$$
 (5)

$$\beta(\delta_p) = \beta_0 + \beta' \delta_p + \frac{1}{2} \beta'' \delta_p^2 + \frac{1}{6} \beta''' \delta_p^3 + \dots, \quad (6)$$

where Q_0 and β_0 are the unperturbed tune and β -function. The first and second order chromaticities can be expressed as:

$$Q'_{x,y} = -\frac{1}{4\pi} \int_0^C K_{x,y}(s)\beta_{x,y}(s)ds,$$
(7)

$$Q_{x,y}'' = -\frac{1}{4\pi} \int_0^C K_{x,y}(s) \beta_{x,y}'(s) ds,$$
(8)

with

$$K_x = K_1(s) - K_2(s)D_x(s),$$
 (9)

and

$$K_y = -K_1(s) + K_2(s)D_x(s).$$
(10)

 K_1 corresponds to the quadrupole strength, K_2 the sextupole and D the dispersion. Here, we have neglected the contribution from bending magnets. The off-momentum β -function β' is given by:

$$\beta'_{x,y}(s_1) = \frac{\beta_{x,y}(s_1)}{2\sin(2\pi Q_{x,y})} \int_0^C K_{x,y}(s)\beta_{x,y}(s) \quad (11)$$
$$\cos(2|\phi_{x,y}(s) - \phi_{x,y}(s_1)| - 2\pi Q_{x,y})ds.$$

From these equation one can see that a proper arrangement of chromatic sextupole would allow to correct both the linear and non-linear chromaticities. This topic has been largely described in the literature and will not be further discussed. However, for squeezed optics, the β functions at the final focusing quadrupoles are significantly increased giving rise to chromatic aberrations which may become challenging to correct. In order to improve the offmomentum aperture it is therefore interesting to investigate optics design allowing for passive compensation of nonlinear chromaticity. Within this scope, two options were considered for the design of the the electron lens lattice which basic principles will be discussed in the following section.

NON-LINEAR CHROMATICITY COMPENSATION

Two options featuring passive non-linear chromaticity compensation were studied:

- Adjusting the phase advance between low- β insertions
- The Achromatic Telescopic Squeeze (ATS) optics [10]

As shown in Fig. 2, the RHIC low- β optics feature asymmetric properties and, as is generally the case for low- β insertions there is a phase advance of approximately π between the peak β -functions left and right of the IP. Using the these properties we can derive the following relations regarding phase advance, gradients and β -functions:

$$\beta_{L6}K_{1L6} \approx \beta_{R8}K_{1R8}$$
(12)
$$\Delta \phi_{L6 \to R6} = \Delta \phi_{L8 \to R8} \approx \pi$$

Tuning the lattice in such a way that the phase advance between the two low- β insertions equal $\pi/2$ we get additional phase relations:

$$\Delta \phi_{L6 \to R8} = k \frac{\pi}{2}, \qquad (13)$$

$$\Delta \phi_{R6 \to L8} = k \frac{\pi}{2},$$

where k is an odd integer. Plugging these relation into Eq. (8) we can see that the contribution from the triplets to the terms $K_1\beta$ will cancel out, providing a passive compensation of the chromatic aberrations. It should be noted that similarly to the electron lens, non-linear beam-beam resonances from the two pp interactions will also be compensated with this option.



Figure 2: β -functions and quadrupole strengths across the RHIC low- β insertions.

An alternative to tis method is the use the ATS optics. The ATS optics were developed at CERN to allow for chromatic corrections with very low- β insertions as required for the LHC upgrade. It uses a β -beat wave propagating through the arcs and low- β insertions to further reduce the β -function at the IP without changing the chromatic properties of the lattice. As long as the pre-squeeze optics (before the wave is applied) is properly corrected, or naturally features low non-linear chromaticity, further squeezing using the β wave will not degrade the situation. The target β^* for the electron lens lattice at 100 GeV being 0.85 m, this should allow to design a lattice with a naturally very low non-linear chromaticity. In order the achieve the ideal ATS squeeze several constraints have to be fulfilled:

- The two RHIC low-β insertions are consecutive, the β-beat will therefore propagate through both insertions. In order to squeeze them both at the same time the phase advance between the two IPs has to be π
- In order to insure proper chromaticity correction a 90°
 FODO lattice is required and a phase advance of π/2

is required between the IP and one of the sextupole families in both planes

These constraints have to be fulfilled before the β -beat wave is applied, i.e. for the pre-squeeze optics.



Figure 3: ATS lattice for 100 GeV protons in RHIC Blue ring.

Figure 3 shows the ATS lattice for the RHIC Blue RHIC. The β -beat wave is launched in IR4 and closed in IR10. The phase advance constraints could only be fulfilled on either side of IR6 and IR8 due to power supply limits. The performance of this optics will be discussed in more details in the following sections.

$\pi/2$ BETWEEN LOW- β INSERTIONS



Figure 4: Blue and Yellow lattices for 100 GeV protons with $\pi/2$ between low- β insertions.

Figure 4 shows the β -functions and dispersion for the 100 GeV proton lattice with a phase advance of $\pi/2$ between the two low- β insertions. β^* was set to 0.85 m as used in past 100 GeV proton runs. The phase advance was matched to π between IP8 and the electron lenses on either side of IP10. The β -function at the electron lens is approximately 10 m. In order to achieve a phase advance of $\pi/2$ between IP6 and IP8 the integer part of the tune was change to (27,29) and (27,28) in the Blue and Yellow rings respectively. This has no consequences for injection as the transition γ is reduced in this case but the FODO cell phase advance is reduced to 75° which may have detrimental effects on the dynamic aperture. It should be noted that phase shifters had to used in the Yellow ring to adjust the phase advance between IP8 and the electron lens to π , this is seen on the bottom plot of Fig. 4 where the arc between IP8 and IP10 has a lower dispersion.



Figure 5: Blue and Yellow chromatic amplitude functions for 100 GeV protons with $\pi/2$ between low- β insertions.

Figure 5 show the chromatic amplitude functions for the Blue and Yellow rings as defined in the MADX code [11].

$$W_x = \sqrt{\left(\frac{1}{\beta_x}\frac{\partial\beta_x}{\partial(\delta_p\beta)}\right)^2 + \left(\frac{\partial\alpha_x}{\partial(\delta_p\beta)} - \frac{\alpha_x}{\beta_x}\frac{\partial\beta_x}{\partial(\delta_p\beta)}\right)^2}.$$
(14)

These function can be interpreted as a representation of the off-momentum optics distortion. It is clearly observed the the W-functions are large between IP6 and IP8 and small everywhere else showing that, as expected, setting the phase advance $\pi/2$ between IP6 and IP8 allows for a local compensation of chromatic effects.

Figure 6 shows the tune as function of momentum offset for the 2012 lattice which was used until now and the $\pi/2$ lattice. The chromaticity was set to Q' = 2 in this case. A clear improvement is observed in all cases without any optimization using the 24 sextupole families, further improvement are possible if necessary.

Although this lattice offers advantage regarding chromatic effects, some limitations were reached mainly due to power supply limits. The β -function at the electron lens could not be increased further than 10 m which may not be sufficient in case instabilities are observed. The Yellow ring required the use of phase shifters which are breaking the ring symmetry which in return appears to be driving the 3rd order resonance as shown in Fig. 7. On this plot, the horizontal tune was placed close to the 2/3 resonance and the phase shifters current was gradually in-



Figure 6: Blue and Yellow non-linear chromaticity for 100 GeV protons with $\pi/2$ between low- β insertions.

creased. The initial amplitude for all case was 8σ . While at small phase shifter currents the single particle motion is behaving nicely, it clearly degrades as the current increases indicating a possible poor dynamic aperture or lifetime.



Figure 7: Impact of the phase shifters on the 8σ phase space close to the 3^{rd} order resonance.

In order to overcome these difficulties it was decided to take a new approach and adopt the ATS optics which should allow to relax to current of the power supplies in IR6 IR8 by involving more insertions in the squeezing process.

90° FODO AND INJECTION OPTICS

The current RHIC lattice features a 85° FODO, the first and essential step for the ATS optics is therefore to design a 90° FODO lattice.

In order to achieve a 90° FODO lattice the current in the arc quadrupoles was increased in both planes. This results in an increase of the integer part of the tunes by one unit from (28,29) to (29,30). As a result, the average dispersion is decreased, as seen in Fig. 8 and the transition γ (γ_t) is



Figure 8: Comparison between the 85° and 90° FODO lattices. In this case β^* was arbitrarily matched to 10 m in all IPs.

increased from 23.3 to 25.0. The RHIC injection γ for proton is 25.4 and using this optics for injection would lead to run very close to transition which is not a desirable mode of operation but the integer part of the tune has to remain constant over the whole cycle. In order to overcome this issue, an new injection optics was designed by reducing the current in the arc quadrupoles and compensating for the tune change using the matching insertions. The best configuration was found with $\beta^*=7.5$ m in all IPs and leaving α^* as a free parameter to allow for more flexibility. Using this method γ_t was reduced to 23.5 which is very close to the current running conditions, the 90° FODO lattice would then gradually be matched during the energy ramp once the beam is operated far away from transition.

Matching the FODO cell phase advance to 90° in both planes enforces the current in the main focusing and defocusing arc quadrupoles to be approximately equal. RHIC is currently equipped with a unipolar power supply to allow for different currents in the focusing and defocusing tune quadrupoles. If the currents are initially equal in these two quadrupole families this would allow for tune corrections in only one direction. This lattice is therefore not compatible with the current design a would required the installation of a bipolar power supply. A promising option would be to use an H-bridge polarity inverter which is currently under study [12].

PRE-SQUEEZE AND COLLISION OPTICS

The pre-squeeze optics is the at which all phase advances constraints besides the one from IP8 to the electron lens have to be set. In order to ease the matching β^* at IP6 and IP8 was chosen such that the phase advance between IP6 and IP8 is not too far from π , this value was found to be approximately 2 m in Blue and was set to the same value in Yellow. This value is not optimal for Yellow and a dispersion beating wave was allowed between IP6 and IP8 and will be carried over to the collision optics as seen in Fig. 9.

The main difficulty was found to be matching the phase advance to the sextupoles due to strong dependency between the power supplies left and right of the IP. For this reason it was chosen to match the phase advance to the sextupoles on one side of each IP only. Even with these relaxed constraints it was not possible to exactly match the phase advance to the sextupoles, one can therefore expect partial compensation only. The non-linear chromaticity being very small with 2 m β^* no additional effort was made to further improve the situation, improvement are therefore possible and should be investigated using the 24 sextupole families available in RHIC.



Figure 9: Blue and Yellow ATS lattices for 100 GeV protons.

Figure 9 shows the β -functions and dispersion for the 0.85 m ATS lattice. The β functions in the arc are increased by approximately a factor 2.5 from IR4, where the β wave is started, to IR10 where the β wave is closed. A mentioned above, dispersion is allowed between IP6 and IP8 in Yellow to adjust the phase advance to π between the two IPs. The β -function at the electron lens in this example is 20 m. For practical reasons, the phase advance to electron lens in Yellow is set to $\pi/2$ with respect the IP6 instead of IP8, no detrimental effects were observed compared to the case where the phase advance is adjusted with respect to IP8.

Figure 10 shows the chromatic amplitude functions for the Blue and Yellow rings. The benefits of the ATS scheme, i.e. increased β -function in one sextupole family and decreased in the other for the arcs adjacent to the low- β insertions is observed as the W-function is decreased in steps each time a strong sextupole is reached. Further improvements regarding non-linear chromaticity are possible by varying the sextupole currents in these same arcs an keeping the linear chromaticity constant using the other arcs sextupole.

However, as observed in Fig. 11 (Q'=2.0) the non-linear chromaticity is well below the one from the 2012 lattice



Figure 10: Blue and Yellow ATS chromatic amplitude functions.

and about the same as the lattice with $\pi/2$ between the low- β insertions. As this was believe to be satisfying to achieve a good dynamic aperture no further effort was made to improve the corrections. Better correction scheme could be studied while doing the final optimizations on this lattice.



Figure 11: Blue and Yellow non-linear chromaticity for ATS optics.

PUSHING THE LIMITS

In past runs it was found that although decreasing β^* would improve the peak luminosity, the lifetime would be strongly degraded resulting in lower integrated luminosity. It was then decided to remain with $\beta^*=0.85$ m in order to optimize the overall machine performance. Looking at Figs. 6 or 11 for the 2012 lattice at 0.85 m one could think that the rather high non-linear chromaticity was a major contributor to this degradation as it significantly increases when β^* is decreased. Having now lattice solutions with intrinsically small non-linear chromaticity it could be interesting to revive this study and probe the minimum β^* for these new optics solutions. Within this scope, the ATS squeeze was pushed to 0.5 m while fulfilling all electron lens lattice constraints and power supply limits and still leaving some margins for eventual optics corrections.



Figure 12: Blue chromatic amplitude functions and nonlinear chromaticity with $\beta^*=0.5$ m.

Figure 12 shows the chromatic amplitude functions and non-linear chromaticity with $\beta^*=0.5$ m in the Blue ring. Although the performance are slightly degraded with respect to the 0.85 m the non-linear chromaticity remains a factor 2 below the 0.85 m 2012 lattice. The degradation is mostly observed in the vertical plane due to the imperfect matching of the phase advance to the sextupoles, however this could be easily fixed by varying the current in the sextupoles located in the arcs adjacent to IP6 and IP8. These good results are rather encouraging and it would be worth investigating this solution for future RHIC runs as a possible way to improve luminosity performance.

It should noted that the ideal head-on compensation can only be achieved for weak hourglass effect as the electron lens is to the first order a thin beam-beam lens. Reducing β^* to 0.5 m could therefore have detrimental effects on lifetime due to imperfect beam-beam compensation. Dedicated studies are therefore required to find the right balance between beam-beam compensation and β^* for optimum luminosity performance.

DYNAMIC APERTURE STUDIES

Dynamic aperture simulations were performed using the code SimTrack developed at BNL [5]. In all case the particles are tracked over 10^6 with a momentum offset of 12.4×10^{-4} and include all multipole magnet field errors.

Figure 13 show a tune scan for the various lattices presented in this paper. The separation between the horizontal and vertical tunes is kept constant at 0.01. The benefits of low non-linear chromaticity are clearly observed both for the ATS and the $\pi/2$ lattices. This improvement is more pronounced for the Yellow beam which featured particularly high non-linear chromaticity in both planes in 2012.



Figure 13: Dynamic aperture as a function of tune without beam-beam interactions.



Figure 14: Dynamic aperture as a function of bunch intensity with beam-beam interactions.

Figure 14 shows the dynamic aperture as a function of bunch intensity with beam-beam interactions for the various lattices presented in this paper. In all case the dynamic aperture remains constant up to a bunch intensity of approximately 2.0×10^{11} p/bunch and then decreases as the beam-beam interactions become too strong. Again a clear improvement is seen in Yellow with respect to the 2012 lattice, all other options behave similarly. Yellow is slightly worse than Blue.

Figure 15 shows the dynamic aperture as a function of bunch intensity with beam-beam interactions and head-on compensation for the various lattices presented in this paper. The 2012 lattice is much worse than the other solutions as the phase advance between the beam-beam interaction and the electron lens is not matched to π . The Blue electron lens lattice feature a dynamic aperture which is constant as a function of the bunch intensity while Yellow does not perform as well which is most likely due to the dispersion or the stronger 2/3 resonance due to the phase shifters in the case of the $\pi/2$ lattice.



Figure 15: Dynamic aperture as a function of bunch intensity with beam-beam interactions and head-on compensation.

An overall degradation of 0.5 to 1.0σ of the dynamic aperture is observed in the presence of electron lens. For these simulations the low amplitude particles tunes is set to 0.67, in the presence of beam-beam compensation all particles will therefore cluster around this working point making the dynamic aperture more sensitive to the 2/3 resonance. Working point optimization would therefore be required to obtain the maximum dynamic aperture for this area of the tune diagram.

SUMMARY AND OUTLOOK

Two new lattices were developed in view of the coming 100 GeV proton run with electron lenses, one using a phase advance of $\pi/2$ between colliding IPs to compensate for chromatic aberrations from the triplets and the other one using the ATS optics. Both option show a significant improvement with respect to the 2012 lattice showing the importance of careful control of chromatic effects in RHIC.

A dynamic aperture independent of bunch intensity was achieved in the presence of head-on compensation.

Although both options perform similarly in terms of nonlinear chromaticity and dynamic aperture the ATS option offers the advantage of not requiring the phase shifters which have detrimental effects on dynamic aperture and allowing for a β -function of 20 m at the electron lens and is therefore the preferable option. The ATS option would however the installation of a new bipolar power supply for the tune quadrupoles. The ATS optics also force a phase advance of a multiple of π from the electron lens to both colliding IPs, possibly allowing for more aggressive compensation than the initially foreseen half compensation. More studies would be required to confirm this hypothesis.

The ATS optics looks like a very promising alternative to further improve the luminosity performance of RHIC even without beam-beam compensation, for AuAu runs for instance, (higher tune are also favorable for IBS) as it allows to reach lower β^* than the classical squeeze while keeping a low non-linear chromaticity and respecting all power supply limits. We therefore recommend this option to be considered for any future RHIC run.

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