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TOLERANCES FOR THE QUADRUPOLES IN LINAC AND HEBT

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The quadrupoles in the linac and HEBT form a nearly periodic harmonic focusing lattice. Its elementary cell is sketched in Fig. 1. The departure from pure periodicity stems from a gradual increase of the drift length ℓ and focal lengths $1/q$ from the entrance of the linac towards the inflector.

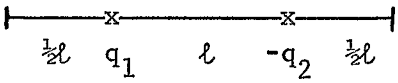

Fig. 1

For the betatron phase advance per cell one finds

$$\cos (\Delta\Psi_{cell}) = \left(1 + \frac{\ell q_1}{2} \right) \left(1 - \frac{\ell q_2}{2} \right) + \frac{\ell q_1}{2} - \frac{\ell q_2}{2} - \frac{\ell q_1}{2} \, \frac{\ell q_2}{2} \quad .$$

It may be noted that if ℓq_1 and ℓq_2 are kept constant, the phase advance per cell is independent of cell length and particle energy.

Nominally one has

$$q_1 = q_2 = q$$

hence

$$\cos (\Delta\Psi_{cell}) = 1 - 2 \left(\frac{\ell q}{2} \right)^2$$

$$\sin \left(\frac{\Delta\Psi_{cell}}{2} \right) = \frac{\ell q}{2} \quad .$$

If the quadrupoles depart from their nominal strengths by dq one finds for the change in phase advance

$$d \, \Delta\Psi_{cell} = \frac{1 - \sin \left(\frac{\Delta\Psi}{2} \right)}{\cos \frac{\Delta\Psi}{2}} \, \frac{dq_1}{q_1} - \frac{1 + \sin \frac{\Delta\Psi}{2}}{\cos \frac{\Delta\Psi}{2}} \, \frac{dq_2}{q_2} \quad .$$

Inside the linac, quadrupole pairs are electrically in series, so that $dq_1/q_1 = dq_2/q_2 = dq/q$ in response to power supply variations. Using this we have

$$d\ \Delta\Psi_{cell} = -\ 2\ \tan\left(\frac{\Delta\Psi_{cell}}{2}\right)\frac{dq}{q} \quad .$$

Since there are 286/2 = 143 mutually independent quadrupole pairs one expects for the error in phase advance in the whole accelerator due to random errors in the power supplies of rms magnitude dq/q

$$d\ \Delta\Psi_{1\ \ell inac} = 2\ \tan\left(\frac{\Delta\Psi_{cell}}{2}\right)\sqrt{143}\ \frac{dq}{q}$$

$$\approx 24\ \tan\left(\frac{\Delta\Psi_{cell}}{2}\right)\frac{dq}{q} \quad .$$

If the quads are individually adjustable, the phase advance error due to random adjustment errors dq/q is much larger and becomes

$$d\ \Delta\Psi_{2\ \ell inac} = \sqrt{\frac{1 + \sin^2\left(\frac{\Delta\Psi_{cell}}{2}\right)}{\cos^2\left(\frac{\Delta\Psi_{cell}}{2}\right)}}\ \sqrt{286}\ \frac{dq}{q}$$

$$\approx 17\sqrt{\frac{1 + \sin^2\left(\frac{\Delta\Psi_{cell}}{2}\right)}{\cos^2\left(\frac{\Delta\Psi_{cell}}{2}\right)}} \cdot \frac{dq}{q} \quad .$$

Some numerical results are given in Table I.

Table I

$\Delta\Psi_{cell}$ (rad)	$\frac{d\ \Delta\Psi_{1\ \ell inac}}{dq/q}$	$\frac{d\ \Delta\Psi_{2\ \ell inac}}{dq/q}$
$\pi/2$	24	29
$\pi/4$	10	20
small	$12\ \Delta\Psi_{cell}$	17

Apparently a small $\Delta\Psi_{cell}$ helps to reduce the error in phase advance due to power supply variation.

The HEBT system contains 9 cells up to Q_{18}, the first quadrupole of the AGS matching section. The quads are mutually independent and the phase advance per cell is $\pi/2$ rad. This yields for the expected error in phase advance

$$d\ \Delta\Psi_{HEBT} = 7.2\ dq/q \quad .$$

Assuming that the linac runs at $\pi/4$ rad/cell, the net expected error due to power supply variation is

$$d\ \Delta\Psi_{system} = \sqrt{10^2 + 7.2^2}\ \frac{dq}{q} = 12.3\ \frac{dq}{q} \quad .$$

If we set a limit $\Delta\Psi_\ell$ and require 95% certainty that the actual $|\Delta\Psi_{system}| \leq \Delta\Psi_\ell$ we have to impose

$$\frac{dq}{q} \leq \frac{1}{25}\ \Delta\Psi_\ell \quad ,$$

If this criterion for the rms error in the power supplies is satisfied the actual error in phase advance will stay within the specified bounds for 95% of the time. With

$$\Delta\Psi_\ell = 0.1 \text{ rad}$$

$$\frac{dq}{q} = \frac{0.1}{25} = 0.004 \quad .$$

Because of the tight coupling between the horizontal and vertical components of the focusing forces satisfying our criterion for one direction implies that the motion in the other is also within tolerance.

JC:ph

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