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LONGITUDINAL IMPEDANCE OF THE AGS BOOSTER AND INSTABILITY GROWTH

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1 ABSTRACT

The various sources of longitudinal impedance in the AGS booster are identified and the effects of the total impedance are estimated. In particular, the effects of the (non-accelerating) band II cavities during the proton acceleration cycle are explored. Uncertainties are large, but the author has found no way of tuning the band II cavities so that both the dipole and the quadrupole coupled bunch modes have e-folding times greater than a few milliseconds. Shorting out the band II cavities during proton operation would still yield some unstable modes, but the e-folding times would be $\gtrsim$100 ms, which seems small enough to be acceptable.

2 INTRODUCTION

The AGS booster is a fast-cycling proton and heavy ion synchrotron. The booster will accelerate protons from $E_k = 200$ MeV kinetic energy to $E_k =$ 1500 MeV in 60 ms. For heavy ions the acceleration time is about an order of magnitude larger with an output kinetic energy $E_k \sim 500$ MeV per nucleon.

This note is concerned with the collective effects that can appear during proton acceleration. For the purposes of this study, the magnetic field in the booster is assumed to ramp linearly with time and radial steering of the proton beam is not considered. The author feels that the inclusion of various RF gymnastics should not have a significant effect on the main results.

The radius of the beam is taken to be 3 cm at injection, consistent with the aperture of the machine. If the transverse phase space is well matched throughout acceleration, the radius of the beam will shrink. However, preliminary results[1] indicate that significant mismatch may exist, and lead to emittance growth. For simplicity, the radius of the beam is assumed to be 3 cm throughout the cycle. In any case, the longitudinal dynamics depend only logarithmically on the beam radius, via the space charge impedance.

In the absence of coherent forces, the length of the bunch should decrease during acceleration[2], owing to the approximate conservation of the longitudinal action. This bunch shortening is included here in a somewhat ad-hoc manner. The idea is to first calculate the bunch length as a function time, assuming conservation of longitudinal action. Once the unperturbed bunch length is found, the growth rates and frequency shifts are calculated as functions of time. The integrals of the growth rates with respect to time are then used to estimate the total growth of an initial perturbation. The reliability of this procedure depends on how adiabatic the motion is, which is beyond the scope of this note. However, if the motion is significantly non-adiabatic, problems are likely. The unperturbed beam parameters assumed are shown in Tables 1 and 2. In Table 2 the synchrotron tune is ν_s, the rms bunch length is σ_l, and the *fractional* rms momentum deviation is σ_p. For these numbers, the unperturbed distribution was taken as a gaussian: 92% of the beam lies within the effective length of $2\sqrt{\pi}\sigma_l$. The effective length at injection corresponds to 190° of RF phase. For a parabolic distribution, with σ_l and σ_p scaled appropriately, the results are similar to those for a gaussian bunch.

3 LONGITUDINAL IMPEDANCE ESTIMATE

Since the bunches in the booster are much longer than the radius of the vacuum chamber and the Lorentz factors are small, the impedance above the cutoff frequency of the pipe has a small effect on the general character of the beam dynamics.

For small Lorentz factors, the space charge impedance is important. At injection, the space charge impedance is capacitive with $|Z/n| = 670\Omega$.

Table 1: Unperturbed Constant Beam Parameters

Machine Radius	32.114 m
harmonic number	3
momentum compaction	0.0453
beam radius	3 cm
pipe radius	8 cm
protons per bunch	5.0×10^{12}
synchronous RF phase	$13°$
RF voltage	80 kV

Table 2: Unperturbed Varying Beam Parameters

time ms	ν_s	E_k MeV	σ_p	σ_l m	f_{rf} MHz
0.0	0.00804	200.	0.00395	10.00	2.526
10.0	0.00524	370.	0.00312	8.95	3.109
20.0	0.00371	569.	0.00270	7.99	3.491
30.0	0.00277	787.	0.00246	7.15	3.743
40.0	0.00214	1016.	0.00231	6.42	3.913
50.0	0.00169	1255.	0.00222	5.79	4.032
60.0	0.00136	1500.	0.00216	5.24	4.117

The space charge impedance drops as acceleration proceeds and ends up at $|Z/n| = 50\Omega$ at extraction. The other broad band impedance present is the resistive wall impedance. For these calculations, the resistive wall impedance was negligible. Other than the resistive wall impedance and the space charge impedance, the bulk of the low frequency impedance comes from the RF cavities and, to a lesser extent, modes associated with the kicker magnets and the pick-up electrodes (PUEs).

The beam sees six RF cavity gaps. For protons, four band III gaps will provide the acceleration. The resonant frequency for band III varies between 2.5 MHz $\leq f_r \leq$ 4.11 MHz. Measurements on a single gap of these cavities[3] yielded a shunt impedance $\sim$ 10 $k\Omega$ with $Q \sim 55$, for no bias field. I assume that the shunt impedance and the Q are independent of bias. Since the Q is fairly small, it is likely that the spread in cavity resonant frequencies will be small compared to the width of the resonance lines. This gives a total of 40 $k\Omega$ for the accelerating mode.

There are two other RF gaps associated with the band II cavities with 0.6 MHz $\leq f_r \leq$ 2.5 MHz. No impedance measurements for band II have been made. Estimated values are[3], $Q \sim 2.5$, and $C \sim 10^{-10}$ F for each gap, with 50% uncertainty. For the purposes of calculation I take the fiducial band II shunt impedance to be 15 $k\Omega$ with $Q = 3$. Calculations were also carried out with $Q = 1.5$. The resonant frequency for the band II cavities was treated as a free parameter within the limits quoted above, reflecting the possibility of tuning the cavity during proton operation.

Using data taken on one of the kicker magnets[4], and multiplying by a factor to account for the total length of kicker in the ring, the shunt impedance due to the kickers is $\sim$ 9 $k\Omega$ with a resonant frequency of 60 MHz and $Q = 3$. Higher order modes in the band II cavities have not been measured under conditions appropriate to booster operation. However, measurements made without the power amplifier[4] suggest a shunt impedance $\sim$ 5 $k\Omega$ at a resonant frequency $\sim$ 50 MHz with $Q \sim 10$. Measurements on the pickup electrodes give $f_r = 230$ MHz, $R = 1$ $k\Omega$, $Q = 5$. The impedance budget due to resonances is summarized in Table 3.

Table 3: Low Frequency Resonant Impedance Budget

Source	$R(k\Omega)$	f_r(MHz)	Q
Band III RF	40	2.5-4.11	55
Band II RF	15	0.4-2.5	3
kickers	9	60	3
Band III parasitic	5	50	10
PUEs	1	230	5

4 COUPLED BUNCH INSTABILITIES

In the limit of first order pertubation theory, with small tune spread, the tune shifts and instability growth rates depend on the values of the impedance evaluated at the frequencies[5],

$$\omega_{s,p,a} = \omega_0(s + ph + a\nu_s). \tag{1}$$

In equation (1), ω_0 is the (angular) go around frequency; h is the harmonic number; $\nu_s = \omega_s/\omega_0$ is the synchrotron tune; and s, p, and a take on integer values. For a given mode, s and a are constant. In synchronous coordinates, the oscillation has a frequency $\omega_c = a\omega_s + \Omega$, where Ω is the coherent frequency shift which depends on s and a. In general, the instability growth rates are complicated functionals of the unperturbed distribution and the impedance. For the special case where the bunches are treated as rigid objects (dipole mode only, $a = 1$) with no synchrotron frequency spread, and in the limit of small Ω, an expression for the growth rate $\alpha = -iIm(\Omega)$ may be obtained[6],

$$\alpha = \frac{\eta q^2 N h}{2\omega_s \beta^2 T^2 E_s} \sum_{p=-\infty}^{+\infty} \omega_{s,p,a} R(\omega_{s,p,a}) \left| \frac{I(\omega_{s,p,a})}{I(0)} \right|^2 . \tag{2}$$

In equation (2), η is the frequency slip factor, q is the charge of a particle, N is the number of particles in a bunch, $\beta = v/c$ is the particle velocity measured in units of the speed of light, $T = 2\pi/\omega_0$, E_s is the total energy of a synchronous particle, $R(\omega)$ and $I(\omega)$ are the resistance and the (single) bunch current as a function of frequency *e.g.* $|I(\omega)/I(0)|^2 = \exp(-\omega^2\tau^2)$ for a gaussian bunch of rms duration τ. The center of the bunches oscillate about the synchronous particle according to

$$\phi_l^s = \phi_0 e^{-2\pi i l s/h - i\omega_c t} \tag{3}$$

where $l = 0, 1, \ldots h-1$ is the bunch index.

Equation (2) is crude and does not estimate the behavior of modes with $a > 1$. Additionally, equation 2 does not give any estimate for the *threshold* at which instability starts. The calculation of higher order synchrotron modes, and of thresholds are generally carried out using linearized pertubation theory on the Vlasov equation. In fact, the usual implementation of the theory does not account for Landau damping in a direct manner. Instead, the growth rates and frequency shifts are calculated assuming that there is *no* spread in the synchrotron tune, and the magnitude of the coherent frequency is then compared with the natural synchrotron frequency spread.

5 RESULTS

I used the computer code ZAP[7] to calculate the coherent frequencies for no synchrotron tune spread. Initial calculations were carried out using a band II resonant frequency of 0.6 MHz and $Q = 3$, which corresponds to a "dead" cavity. The tune shifts and the growth rates for the unstable dipole and quadrupole modes at various phases in the acceleration cycle are given in Table 4. As well as growth rates $\alpha_{a,s}$, and tune shifts $\Delta\nu_{a,s}$, the last column of Table 4 gives the full synchrotron frequency spread of the bunch, calculated using the octupole approximation. Zotter[8] suggests that the beam is stable when the magnitude of the coherent tune shift is less than about half the natural tune spread. For the dipole mode the tune shift starts at 80% of the tune spread, and is greater than the tune spread after 10 ms. Even starting at 20 ms dipole pertubations will grow by a factor ~ 100 by extraction.

It is important to point out that the value of $Q = 3$ assumed for the band II cavities has a significant effect on the tune shifts and growth rates. Taking $Q = 1.5$ increased tune shifts by a factor ~ 1.3 and growth rates by

Table 4: Coherent Frequency Shifts

time	$\alpha_{1,2}$	$\Delta\nu_{1,2}$	$\alpha_{2,2}$	$\Delta\nu_{2,2}$	$\Delta\nu_{nat}$
(ms)	s^{-1}	$\times 10^3$	s^{-1}	$\times 10^3$	$\times 10^3$
0.0	889.	1.14	65.	1.01	1.37
10.0	338.	0.78	24.	0.74	0.72
20.0	199.	0.62	16.	0.59	0.41
30.0	139.	0.53	11.	0.49	0.24
40.0	104.	0.47	8.	0.42	0.15
50.0	82.	0.42	6.	0.37	0.10
60.0	65.	0.39	5.	0.32	0.06

a factor $\sim$ 2.3. Also, it is important to remember that the value of 15 kΩ for the band II shunt impedance is only good to 50%. All things considered it seems risky to have no way of modifying the band II impedance during proton acceleration.

For conditions just after injection it was found that a bias in the band II cavity corresponding to a resonant frequency of 1.14 MHz could cause a favorable cancellation of terms in equation 2. However, for e-folding times $\gtrsim$10 ms, the accuracy in the resonant frequency needed is $\sim$ 10 kHz, which is much smaller than the linewidth of the cavity. Therefore, the best frequency would need to be arrived at by some interative technique, since the true Q of the cavity is unknown. This result was confirmed with ZAP calculations.

Unfortunately, the resonant frequency which caused the dipole growth rates to vanish corresponded to a large growth rate in one of the quadrupole modes, with $\alpha_{2,2} = 620\mathrm{s}^{-1}$. The corresponding tune shift was 1.5×10^{-3}, which is larger than the full tune spread. Such a robust instability would guarantee beam loss. It is possible that some way of varying the resonant frequency of the band II cavities would lead to acceptable motion, but the author has not been able to find it. One possible solution is to short out the band II cavities during proton operation.

When the band II cavities are ignored (*eg.* if they are shorted out), the strongest unstable modes appear to be dominated by the fundamental cavity mode of the band III cavities, with an e-folding time $\sim$ 140 ms, which appears to be something we can live with. The kickers, PUEs and higher order cavity modes have little effect on the growth rates, since the frequency scale over which these resonances change is of order several MHz. This causes much favorable cancellation in equation 2 even though the shunt impedance is quite large. This also was confirmed by ZAP calculations.

6 FUTURE DIRECTIONS

It is important to point out that this entire study has been limited to the low intensity limit[9] wherein coupling between different synchrotron modes is ignored. From space charge alone, one expects a fractional tune shift $\Delta\nu/\nu \sim 0.2$ for high intensity operation in the booster. This is getting near the limit of the theory[9].

Transverse instabilities in the booster have not been mentioned at all. Using *coasting* beam criteria, with the resistive wall impedance and space charge, Ruggiero has found that transverse motion in the booster is unstable[10]. Higher order modes in the rf cavities and, possibly, resonant modes in the kickers need to be included in a realistic estimate. This work is in progress.

1. S. Machida, *private communication.*

2. see *eg.* W. T. Weng & S. R. Mane, 1991, BNL 52298

3. A. Ratti, *private communication.*

4. A. Ratti & T. Shea, 1991 San Francisco Conference.

5. A. Chao, 1982, SLAC-PUB-2946.

6. See A. Chao, 1982, SLAC-PUB-2946, section 2.1, then include $v < c$, multiple bunches, and finite bunch length.

7. M. Zisman, S. Chattopadhyay & J. Bisognano, 1986, LBL-21270, ESG-15.

8. B. Zotter, 1985, CERN 85-19, pp 427.

9. Y. Chin, K. Satoh & K. Yokoya, 1983, particle accelerators, V 13, p 45.

10. A. Ruggerio, 1989, Booster Design Manual, pp 2-31.