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Effects of quadrupole gradient errors in the AGS Booster

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Collider Accelerator Department

Brookhaven National Laboratory

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EFFECTS OF QUADRUPOLE GRADIENT ERRORS IN THE AGS BOOSTER

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Abstract

We examine the effects of two types of quadrupole gradient errors in the AGS Booster on beta variations and tune shifts. Using PATRIS we find that the effective quadrupole errors, resulting from the sextupole crossing by the uncorrected distorted closed orbit, outweigh in their importance the errors in the field gradient strength for the quadrupole. With $(\Delta K/K)_{rms} = 10^{-3}$ for the latter, we find that the rms values for the stopbands are $(\delta\nu_x)_{rms} = 0.0035$ and $(\delta\nu_y)_{rms} = 0.0038$. These values, in our opinion, do not invoke any need for the half-integer stopband compensation.

Introduction

The importance of quadrupole gradient errors has been known for many years[1]. They not only cause beta variations and shift tunes away from their design values, but also entail stopbands, i.e. small regions of potential instability in the proximity of integral and half-integral values for the tunes of an unperturbed machine. Since particles during acceleration can become trapped in these regions and eventually get lost, it is important to determine sizes of these regions, i.e. the so-called stopband widths.

PATRIS currently handles two sources of gradient errors. First source are effective quadrupole gradient errors as a result of the crossing of the sextupole by the distorted closed orbit[2]. A brief discussion of these effects can be found in Appendix. Second source are true uncontrolled modulations of the quadrupole gradient field as we move from one quadrupole to another.

PATRIS handles these problems statistically, by running over 21 distributions of random errors and computing the rms values of various quantities of interest. More details are given in Ref.2. Here we would only mention that the distributions for the two kinds of quadrupole gradient errors are independent, hence the code runs in fact over 21 pairs of distributions if an evaluation of combined effects of these two kinds of errors is sought. PATRIS evaluates and prints the values of beta variations at one particular location of the lattice, and also tune shifts for each of these 21 distributions (or pairs of distributions). This is followed by evaluating and printing their rms values.

It is worthwhile to mention that there are analytic means for evaluating tune shifts and beta variations resulting from quadrupole gradient errors[1]. PATRIS, however, does not rely on these analytic approximations. It evaluates tune shifts and beta variations exactly, where by the term "exactly" we mean fully reevaluating these shifted quantities from the actual transfer map in the presence of errors. Therefore, the degree of accuracy is in fact determined and limited by the linearized equations of motion for the bend and quad, the kick approximation for the sextupole, and by the modeling of closed orbit distortions, which are obtained with kick-modeled errors and with nonlinearities not taken into account[1].

Relative Importance of the Two Sources of Quadrupole Errors in the Booster

First interesting question was to see which source of quadrupole errors is more important in the Booster. To get an answer, we ran the two cases under the most similar conditions, i.e. with the same rms values for all the errors in the lattice, $\Delta K/K$ as well as those that give rise to closed orbit distortions[2]. The rms values were taken to be 0.3×10^{-3} in the appropriate units. These values for the closed orbit errors have been found as the upper limits for the Fermilab correcting scheme to be very likely to work.[2]

Table 1 gives the results for beta variations and tune shifts solely due to pure gradient field errors in the quadrupole, $(\Delta K/K)_{rms} = 0.3 \times 10^{-3}$, with no closed orbit distortions. The beta variations are calculated in correspondance with the center of the horizontally focusing quadrupoles. Table 2 gives the same for the other case, i.e. quadrupole gradients have their ideal values but the crossings of the sextupoles by the distorted closed orbit (see Appendix) are giving rise to effective quadrupole errors. The two cases are also plotted together on Figure 1 and Figure 2. One thing is apparent. The effective quadrupole errors, stemming from the uncorrected distorted closed orbit, are more important than the true quadrupole gradient errors. This is good since this allows us to be more generous as far as the $(\Delta K/K)_{rms}$ values are concerned and, as will be shown in the sequel, obviates the need for the half-integer stopband compensation.

Effects of 10^{-3} RMS Values for $\Delta K/K$

Since we have seen that the effects of the true gradient errors are less dramatic than those coming from the effective errors, we have adopted much more generous values for the former ones. We examine the case $(\Delta K/K)_{rms} = 10^{-3}$, a value for the relative rms errors that should not be difficult to maintain in practice.

The results of a run with $(\Delta K/K)_{rms} = 10^{-3}$ and with no closed orbit distortions are given in Table 3. If compared with Table 2, this table reveals that even now the effects of pure gradient errors just start competing with the effective errors arising from 0.3×10^{-3} rms values for the lattice errors that cause closed orbit distortions. In Table 4, we present a mixed case, i.e. beta variations and tune shifts due to the combination of the two effects. The picture is similar, except that now the rms values approximately double.

From now on we focus our attention on the pure $(\Delta K/K)_{rms} = 10^{-3}$ case. This we do because of the fact that the closed orbit will inevitably have to be corrected, with a subsequent large reduction of the effective quadrupole errors, resulting now from the crossings of the sextupoles by the corrected closed orbit. Under such operating circumstances, the effects of the true quadrupole errors are expected to prevail. Statistically the rms values for the stopband width around the nearest integer or half-integer tune values are just twice the rms values of the tune shifts. From the rms values of the tune shifts, on the bottom of Table 3, we get thus the rms values for the stopband width. They are

$$(\delta\nu_x)_{rms} = 0.0035 \qquad \text{and} \qquad (\delta\nu_y)_{rms} = 0.0038 \quad .$$

These values are small, especially when compared with the large tune spread (~ 0.3) due to space charge at injection. It is true nevertheless that during the early stage of acceleration it may not be possible to avoid that part of the beam sweeps through a half-integer stopband. One can estimate the expected growth of the emittance for these particles. This is given by

$$\frac{\Delta\epsilon}{\epsilon} \sim \left[e^{2\pi(\delta\nu)n} - 1 \right] \quad ,$$

where n is the number of turns the particle spends inside the stopband of width $\delta\nu$. If ν' is the sweeping rate, then $n = (\delta\nu)/\nu'$ and

$$\frac{\Delta\epsilon}{\epsilon} \sim \left[e^{2\pi(\delta\nu)^2/\nu'} - 1 \right] \quad .$$

For a small emittance increase, say 10%, we have a minimum requirement on the crossing speed ν'. If we take $\delta\nu = 0.004$, we obtain

$$\nu' \gtrsim 20\pi(\delta\nu)^2 \sim 0.001 \quad .$$

Since, in our opinion, this condition is easily satisfied, we do not foresee any need to compensate for the half-integer stopbands.

Appendix

Here we give a sketch of derivation of the effective quadrupole error, resulting from the crossing of the sextupole by the distorted closed orbit. For the sake of brevity, we consider just one degree of freedom.

The differential equation of motion, in the presence of a sextupole, is given by

$$X'' + K(s)X = \frac{1}{2}\,\alpha\,X^2 \quad . \tag{A.1}$$

If both transverse degrees of freedom are considered, then the right hand side of (A.1) contains also a $-\alpha Y^2/2$ term. Furthermore, for several thin lens modeled sextupoles in the lattice, the right hand side of (A.1) would look like

$$\frac{1}{2}\sum_i \alpha_i \delta\,(s - s_i)\left[X^2(s) - Y^2(s)\right] \quad . \tag{A.2}$$

However, (A.1) is sufficient to illustrate the point. It is satisfied by any particle whose transverse coordinates are measured from the design orbit. Therefore, it is satisfied by the free betatron motion in the absence of errors

$$X''_\beta + K(s)X_\beta = \frac{1}{2}\,\alpha\,X^2_\beta \quad , \tag{A.3}$$

as well as by the closed orbit in the presence of errors elsewhere in the lattice

$$X''_{CO} + K(s)X_{CO} = \frac{1}{2}\,\alpha\,X^2_{CO} \quad . \tag{A.4}$$

Now consider betatron oscillations around the distorted closed orbit, i.e. we set

$$X = X_{CO} + X_\beta \tag{A.5}$$

and insert this into (A.1). We obtain

$$(X_{CO} + X_\beta)'' + K(s)\,(X_{CO} + X_\beta) = \frac{1}{2}\,\alpha\,(X_{CO} + X_\beta)^2 \quad . \tag{A.6}$$

This now splits into two equations. One is just (A.4), the equation for the closed orbit, while the other becomes

$$X''_\beta + K(s)X_\beta = \frac{1}{2}\,\alpha\,(2X_{CO}X_\beta + X^2_\beta) \quad , \tag{A.7}$$

which is equivalent to

$$X''_\beta + [K(s) - \alpha X_{CO}]\, X_\beta = \frac{1}{2}\ \alpha X^2_\beta \quad . \qquad (A.8)$$

The last equation is an equation for betatron motion in the presence of a modified quadrupole

$$K(s) \rightarrow K(s) - \alpha X_{CO} \quad . \qquad (A.9)$$

Therefore, the crossing of a sextupole of strength α with the distorted closed orbit, whose measure is X_{CO}, indeed produces an effective gradient error, as claimed. This effect is calculated with PATRIS.

References

1. E. Courant & H. Snyder, Annals of Physics 3, 1 (1958).

2. J. Milutinovic & A.G. Ruggiero, Closed Orbit Analysis for the AGS Booster, Booster Technical Note No. 107.

Tables

1. This table represents the tune shifts and beta variations solely due to pure quadrupole gradient errors. The rms value for $\Delta K/K$ was taken 0.3×10^{-3}. The 21 rows in the table correspond to the 21 distributions of random errors, whereas the bottom line represents the rms values over these 21 distributions. The random number sequences were cut at 2.5σ.

2. This table represents the tune shifts and beta variations solely due to the crossing of the sextupoles by the distorted closed orbit. The rms values for all the lattice errors that give rise to closed orbit distortions were taken to be 0.3×10^{-3} in the appropriate units.

3. This table represents exactly the same as Table 1, except that the rms value for $\Delta K/K$ is now 1.0×10^{-3}.

4. This table represents the tune shifts and beta variations due to combined effects of the sextupole crossings by the distorted closed orbit and of pure quadrupole gradient errors. The rms values for the closed orbit errors are 0.3×10^{-3}, while those for the pure gradient errors are 1.0×10^{-3}. Therefore, this table represents combined effects of conditions that were separately presented in Table 2 and Table 3.

Figure Captions

1. This figure represents the tune shift plots based on the tune shift columns (column 3 and 5) from Table 1 and Table 2. Therefore, the figure portrays the relative importances of the two kinds of quadrupole errors, since the runs were performed under the most similar conditions. The prevalence of the effects of the sextupole crossings by the distorted closed orbit is obvious.

2. This figure represents the same as Figure 1, except that now the beta variations are plotted instead of tune shifts.

Table 1

CPU-TIME SO FAR = 908
CPU-TIME SO FAR = 908
CPU-TIME SO FAR = 909

CLOSED ORBIT ANALYSIS WITH SEXTUPOLES AND QUAD GRADIENT RANDOM ERRORS

TUNE SHIFTS AND BETA VARIATIONS

	HORIZONTAL		VERTICAL	
	DBETA/BETA	D-TUNE	DBETA/BETA	D-TUNE
1	-0.35476e-02	0.55241e-03	0.29170e-02	-0.13557e-02
2	0.43207e-02	-0.10812e-02	-0.31254e-02	0.14542e-02
3	0.11038e-02	0.54187e-04	0.12501e-02	0.95397e-04
4	-0.22860e-02	-0.17222e-03	-0.71782e-03	0.24907e-03
5	0.67692e-03	0.39814e-03	-0.14491e-02	-0.52129e-03
6	-0.29296e-03	-0.97914e-04	-0.50180e-03	0.27791e-03
7	0.34119e-02	0.50694e-03	-0.57992e-03	-0.24967e-03
8	0.49322e-03	-0.49643e-03	-0.10156e-02	-0.13271e-03
9	-0.18210e-02	0.47249e-03	-0.17633e-02	-0.30961e-03
10	0.83095e-03	0.72723e-03	0.12636e-02	0.54162e-03
11	-0.40623e-03	-0.34155e-03	-0.34543e-03	0.36843e-03
12	0.13381e-02	-0.20267e-03	-0.33651e-02	0.30274e-03
13	-0.15704e-02	-0.10737e-02	0.47897e-03	0.27603e-03
14	0.73957e-03	-0.43012e-03	0.18004e-02	0.35648e-04
15	-0.12470e-03	0.19212e-03	-0.20205e-02	-0.65603e-03
16	-0.92125e-03	0.67178e-04	-0.14671e-02	0.96054e-04
17	0.12594e-02	-0.62996e-03	-0.10971e-02	0.78587e-03
18	0.70191e-03	-0.48674e-03	0.14419e-02	0.56559e-03
19	0.47445e-02	0.11499e-03	-0.35991e-02	-0.15212e-03
20	0.14163e-02	0.56702e-03	-0.14924e-02	-0.14760e-03
21	-0.59241e-03	0.12439e-03	0.54414e-02	-0.20653e-03
rms values	0.20846e-02	0.52208e-03	0.22136e-02	0.57495e-03

CPU-TIME SO FAR = 1440

END OF THE RUN PATRIS VRSN 3.2

Table 2

CLOSED ORBIT ANALYSIS WITH SEXTUPOLES

TUNE SHIFTS AND BETA VARIATIONS

	HORIZONTAL		VERTICAL	
	DBETA/BETA	D-TUNE	DBETA/BETA	D-TUNE
1	0.35986e-02	0.67646e-04	0.20248e-02	0.55865e-04
2	0.87661e-02	0.28109e-02	-0.36719e-02	0.31661e-03
3	-0.34446e-02	0.13982e-02	0.12921e-01	-0.12674e-02
4	-0.51444e-02	-0.30722e-02	0.10382e-01	0.32857e-02
5	0.34473e-02	0.23414e-03	-0.10523e-01	0.82913e-03
6	0.83141e-02	0.15707e-02	-0.84879e-02	0.79469e-03
7	-0.54302e-02	0.28144e-04	0.63170e-02	0.19825e-02
8	0.17500e-01	0.69144e-02	-0.68454e-02	-0.15292e-02
9	-0.20588e-02	0.56023e-03	0.70723e-02	-0.14308e-02
10	-0.56593e-02	-0.33207e-02	-0.76399e-02	0.34624e-02
11	-0.25092e-02	-0.12864e-02	-0.46559e-02	0.58659e-03
12	0.96144e-02	0.13351e-02	-0.28951e-02	0.17469e-02
13	0.11041e-01	0.52351e-02	-0.24901e-02	-0.23369e-02
14	-0.16015e-03	-0.17859e-02	-0.65663e-02	0.36037e-02
15	-0.57906e-02	-0.40243e-04	0.59524e-02	-0.57185e-03
16	-0.42458e-03	-0.18543e-03	0.42070e-02	0.16501e-02
17	-0.87577e-02	-0.47354e-02	-0.84466e-02	0.15660e-02
18	0.14788e-02	-0.15164e-02	-0.71469e-03	0.29325e-02
19	0.17838e-01	0.78069e-02	-0.12690e-01	-0.30189e-02
20	-0.19558e-02	-0.40741e-03	-0.10504e-03	-0.10229e-03
21	0.60399e-02	0.25056e-02	-0.31512e-02	-0.18314e-02
rms values	0.79777e-02	0.32183e-02	0.72304e-02	0.20317e-02

CPU-TIME SO FAR = 26535

END OF THE RUN PATRIS VRSN 3.2

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Table 3

F

CPU-TIME SO FAR = 91(
CPU-TIME SO FAR = 91(
CPU-TIME SO FAR = 911

CLOSED ORBIT ANALYSIS WITH SEXTUPOLES AND QUAD GRADIENT RANDOM ERRORS

TUNE SHIFTS AND BETA VARIATIONS

	HORIZONTAL		VERTICAL	
	DBETA/BETA	D-TUNE	DBETA/BETA	D-TUNE
1	-0.11780e-01	0.18326e-02	0.97063e-02	-0.45210e-02
2	0.14390e-01	-0.35978e-02	-0.10456e-01	0.48463e-02
3	0.37226e-02	0.17673e-03	0.42411e-02	0.31546e-03
4	-0.75451e-02	-0.57493e-03	-0.23590e-02	0.82729e-03
5	0.23638e-02	0.13295e-02	-0.46165e-02	-0.17379e-02
6	-0.96185e-03	-0.32890e-03	-0.15199e-02	0.92146e-03
7	0.11538e-01	0.16975e-02	-0.18785e-02	-0.82676e-03
8	0.16498e-02	-0.16602e-02	-0.33528e-02	-0.44314e-03
9	-0.60671e-02	0.15829e-02	-0.59238e-02	-0.10455e-02
10	0.28225e-02	0.24282e-02	0.41833e-02	0.18057e-02
11	-0.13622e-02	-0.11397e-02	-0.11151e-02	0.12218e-02
12	0.44152e-02	-0.66520e-03	-0.11282e-01	0.10195e-02
13	-0.51977e-02	-0.35820e-02	0.16418e-02	0.91958e-03
14	0.25303e-02	-0.14409e-02	0.60092e-02	0.11009e-03
15	-0.38929e-03	0.63953e-03	-0.67337e-02	-0.21914e-02
16	-0.30462e-02	0.22140e-03	-0.49596e-02	0.32122e-03
17	0.41684e-02	-0.20949e-02	-0.37716e-02	0.26212e-02
18	0.23229e-02	-0.16219e-02	0.48380e-02	0.18837e-02
19	0.15953e-01	0.38870e-03	-0.11939e-01	-0.50631e-03
20	0.48294e-02	0.18945e-02	-0.46442e-02	-0.49437e-03
21	-0.19786e-02	0.40957e-03	0.18229e-01	-0.67873e-03
rms values	0.69730e-02	0.17409e-02	0.73803e-02	0.19169e-02

CPU-TIME SO FAR = 14382

END OF THE RUN PATRIS VRSN 3.2

Table 4

CLOSED ORBIT ANALYSIS WITH SEXTUPOLES AND QUAD GRADIENT RANDOM ERRORS

TUNE SHIFTS AND BETA VARIATIONS

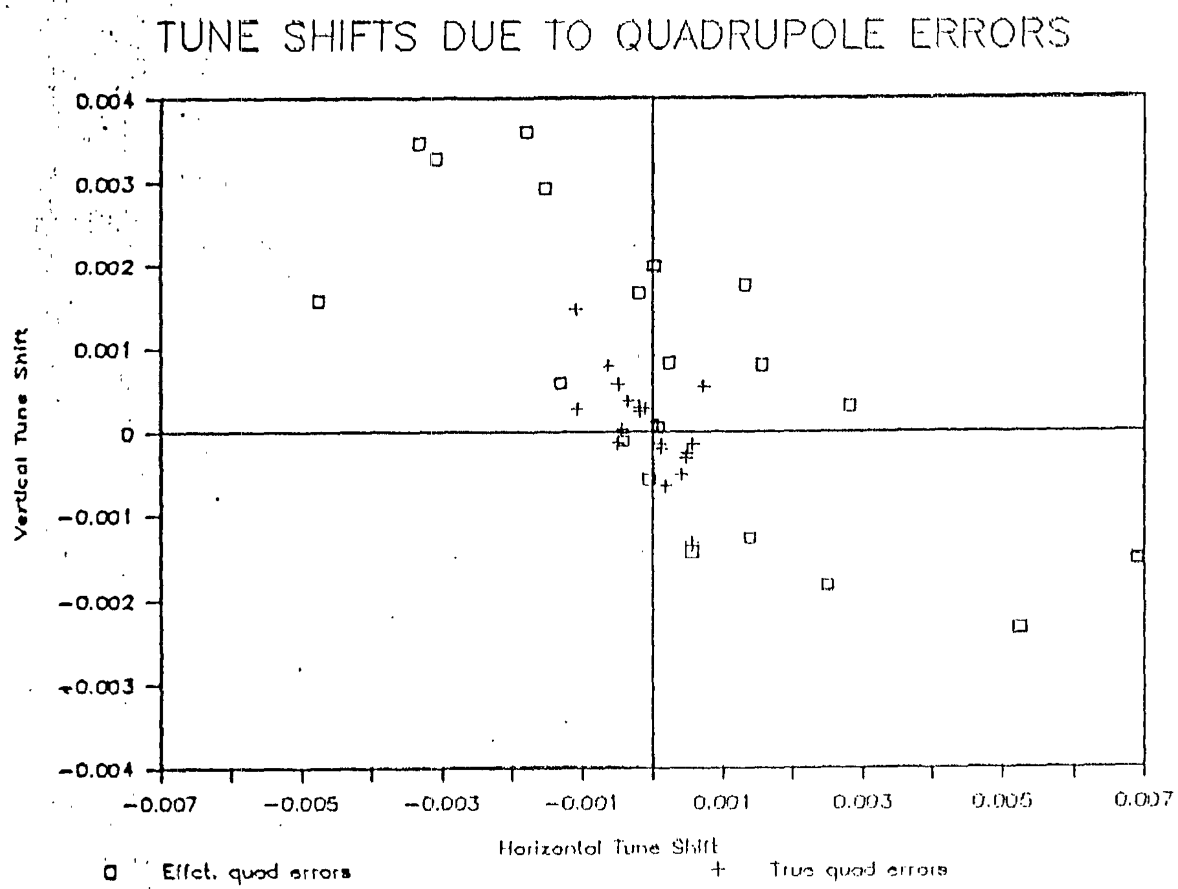
	HORIZONTAL		VERTICAL	
	DBETA/BETA	D-TUNE	DBETA/BETA	D-TUNE
1	-0.78571e-02	0.17984e-02	0.12209e-01	-0.44787e-02
2	0.24075e-01	-0.76306e-03	-0.13726e-01	0.51578e-02
3	0.12923e-03	0.15637e-02	0.17362e-01	-0.93518e-03
4	-0.12597e-01	-0.36729e-02	0.83346e-02	0.40855e-02
5	0.54437e-02	0.15199e-02	-0.15472e-01	-0.91374e-03
6	0.69191e-02	0.12534e-02	-0.95715e-02	0.16539e-02
7	0.54869e-02	0.16917e-02	0.41424e-02	0.11139e-02
8	0.18827e-01	0.52332e-02	-0.10323e-01	-0.19677e-02
9	-0.80759e-02	0.21285e-02	0.11167e-02	-0.24706e-02
10	-0.31384e-02	-0.86737e-03	-0.35389e-02	0.52613e-02
11	-0.39809e-02	-0.24596e-02	-0.60179e-02	0.18073e-02
12	0.13838e-01	0.62885e-03	-0.13969e-01	0.27972e-02
13	0.63508e-02	0.16920e-02	-0.56653e-03	-0.13595e-02
14	0.22869e-02	-0.32459e-02	-0.29780e-03	0.36884e-02
15	-0.60012e-02	0.68637e-03	-0.78242e-03	-0.27566e-02
16	-0.30503e-02	0.18927e-04	-0.61107e-03	0.19842e-02
17	-0.39124e-02	-0.68046e-02	-0.12197e-01	0.42120e-02
18	0.37130e-02	-0.31853e-02	0.42450e-02	0.48103e-02
19	0.33510e-01	0.81236e-02	-0.24286e-01	-0.35164e-02
20	0.29759e-02	0.15043e-02	-0.44457e-02	-0.58802e-03
21	0.42154e-02	0.29096e-02	0.15062e-01	-0.25598e-02
rms values	0.11862e-01	0.32607e-02	0.10984e-01	0.32009e-02

CPU-TIME SO FAR = 2645:

1

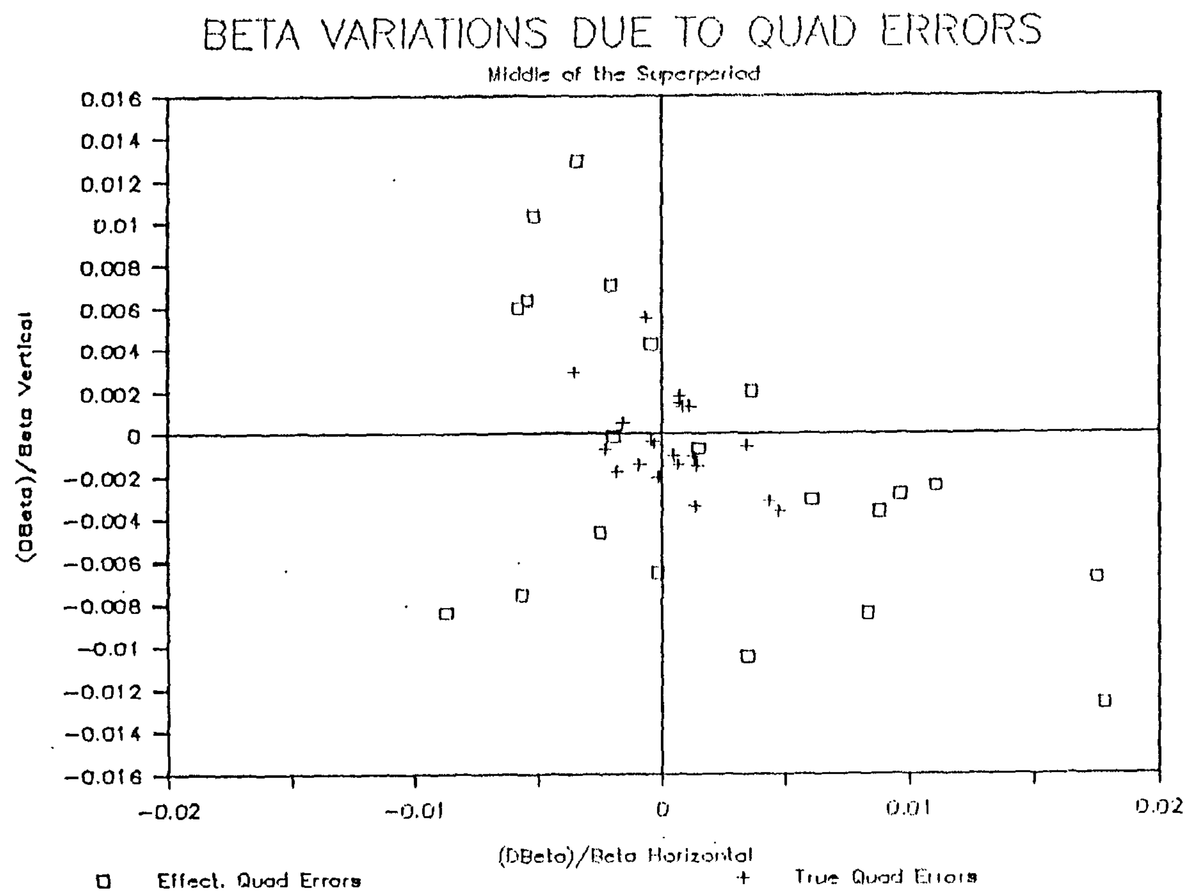
END OF THE RUN PATRIS VRSN 3.2

Figure 1



TUNE SHIFTS DUE TO QUADRUPOLE ERRORS
Vertical Tune Shift
0.004
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
-0.007
-0.005
-0.003
-0.001
0.001
0.003
0.005
0.007
Horizontal Tune Shift
Effct. quad errors
+ True quad errors

Figure 2



BETA VARIATIONS DUE TO QUAD ERRORS
Middle of the Superperiod
(DBeta)/Beta Vertical
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
−0.002
−0.004
−0.006
−0.008
−0.01
−0.012
−0.014
−0.016
−0.02
−0.01
0
0.01
0.02
(DBeta)/Beta Horizontal
Effect. Quad Errors
True Quad Errors