

# Longitudinal stability of individual bunches in the AGS - Booster

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LONGITUDINAL STABILITY OF INDIVIDUAL BUNCHES  
IN THE AGS - BOOSTER

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(1)

To estimate the longitudinal stability of individual bunches in the AGS Booster one calculates the following complex quantity

$$U' - iV' = -i \frac{2e I_p \beta^2 (Z/n)}{\pi |Z| E (\Delta E/E)_{FWHM}} \cdot \frac{Q}{A} \quad (1)$$

where

$e$  , electron charge

$Q$  , particle charge state

$A$  , particle mass number

$\beta$  , particle velocity to light velocity ratio =  $v/c$

$Z$  , the complex beam-environment coupling impedance

$n$  , the harmonic number of the instability

$E$  , the particle total energy /a.m.u.

$(\Delta E/E)_{FWHM}$  , the full-width half-maximum relative bunch energy spread

(2)

Also

$$\eta = \gamma_T^{-2} - \gamma^{-2}$$

where

$$\gamma = E/E_0$$

$E_0$  , particle rest energy /a.m.u.

and  $\gamma_T$  is  $\gamma$  evaluated at the Booster transition energy -

Finally  $I_p$  is the bunch peak current

$$I_p = \frac{Ne\beta c Q}{\sqrt{2\pi} \sigma}$$

$N$ , number of particles in a bunch

$\sigma$ , rms bunch length

We are assuming bunches with bi-gaussian distribution -

(3)

If we define

$$I_{\infty} = \frac{Nec}{2\pi R} Q, \text{ average current per bunch} \\ \text{for } \beta \rightarrow 1$$

$$B = \frac{\sqrt{2\pi} \sigma}{2\pi R}, \text{ bunching factor}$$

$$S = \pi \sigma_z \delta, \text{ rms bunch area}$$

$$\text{with } \sigma_z = \sigma / \beta c$$

$$\text{and } \delta = \Delta E / 2.355, \text{ rms energy spread}$$

$$\text{and } f_{\infty} = c / 2\pi R, \text{ revolution frequency for } \beta \rightarrow 1$$

Then we can also write (1) as

$$U' - iV' = -0.18i \frac{e I_{\infty} E_0 \beta \gamma B (Z/n)}{f_{\infty}^2 |n| S^2} \cdot \frac{Q}{A} \quad (2)$$

In our notation  $Z = X + iY$  with  $X \geq 0$  a resistance and  $Y$  is positive for a capacitive reactance and negative for an inductive reactance.

In the AGS-Booster we can conceive three modes of operation

1. Acceleration of protons from 200 MeV to 1.5 GeV with 3 bunches - Each bunch has  $5 \times 10^{12}$  protons and an rms bunch area of 0.10 eV-sec.
2. Acceleration of heavy-ions for RHIC in one single bunch - The beam parameters are those appearing in the RHIC CDR and also reported in Table I - Each heavy-ion beam is accelerated to  $\beta_{\gamma} = 0.9$  that is  $\beta = 0.669$  - The individual bunch rms area is ~~also~~ 0.05 eV-s/amu.
3. Acceleration of heavy-ion for fixed target experiments out of the AGS - In this mode there are 3 bunches each with an intensity three times smaller than in the previous mode for RHIC, and each with an rms bunch area of 0.015 eV-sec/amu. - The acceleration is also up to  $\beta = 0.669$ .

For all these modes of operation, the final energy is always below the Booster transition energy ( $\gamma_T = 4.5$ ).

Table I. Heavy-Ion Beams for RHIC

	Carbon	Sulfur	Copper	Iodine	Gold
A	12	32	63	127	197
Q	6	14	21	29	33
$N \times 10^9$	22	6.7	4.7	3.2	2.2
$\beta_{\text{injection}}$	0.1262	0.1002	0.0782	0.0595	0.0463
$\beta_{\text{extraction}}$	← 0.669			→	
S eV-s/amu	← 0.05			→	
$Z/n$ in Kohn from space charge					
@ injection	1.47	1.86	2.40	3.16	4.06
@ extraction			0.70	→	



(6)

The largest contribution to the coupling impedance for the Booster is the "space charge"

$$\frac{Z}{n} = i \frac{Z_0 g}{2\beta\gamma^2} \quad Z_0 = 377 \text{ ohm}$$

where  $g = 1 + 2 \log b/a$  is 1 at injection and 4.5 at extraction for all mode of operation. The value of this impedance for the heavy-ion cases is given in Table I. For the proton beam case

$$\begin{aligned} Z/n &= i 226 \text{ ohm at injection} \\ &= i 136 \text{ ohm at extraction} \end{aligned}$$

This impedance is so large that it is hard to imagine an inductive wall impedance of the same magnitude.

Provisional Conclusion: If there is no resistance, the reactance being positive (capacitive) and the accelerating cycle always below the Booster transition energy the individual bunches are always stable.

(7)

Only the presence of a resistance in the coupling impedance can cause the bunches to be unstable - We can calculate the tolerances on  $X/n$  -

Observe that the energy dependence of  $V'$  is given by the quantity

$$\frac{\beta \gamma B(X/n)}{|z|} \sim \gamma B$$

for the space charge impedance and since one is so well below the transition energy for all cases - Since  $B$  decreases with increasing energy, it is seen that  $V'$  has indeed only a very weak dependence with energy. We will take  $B = 0.3$  at injection and  $B = 0.03$  at top energy for all cases - The results are given in Table II - We show the values of  $V'$  with space charge at injection and extraction for each case - Based on the stability diagram shown in Fig. 1 we can then infer the maximum allowed values for  $V'$  - Since we are below the transition energy  $\text{sign}(K_0) > 0$  -

The choice for  $V'$  depends critically on the shape of the energy distribution.

The range of  $U'$  for the proton beam during the acceleration cycle is shown in Fig. 1 - With the exception of a truncated cosine distribution (8) and a ~~first~~ first-order parabola distribution (9), the beam bunch is always stable provided  $V' < 0.4$ , the limit being set by a second-order parabola distribution (7) at top energy - This corresponds to the resistive impedance limit ~~if~~  $X/n < 60 \text{ ohms}$ .

Heavy-ion bunches are even more stable than proton bunches. The range of  $U'$  for all heavy-ion cases is also shown in Fig. 1 - At very most  $U' = 0.11$  for Carbon at injection in fixed-target mode. All the distribution considered in Fig. 1 are stable provided  $V' < 0.5$ . The tolerance on the resistive impedance is very high: tens of Kohn? It is possible to double the number of heavy ions per bunch and reduce considerably the initial bunch area.

Table II - Bunch Stability Requirement in the Booster

	$U'$		$V'$	$X/\eta$
	Injec.	Extr.		
Proton	0.67	0.90	0.4	60 ohm
<u>Fixed Target:</u>				
Carbon	0.11	0.066	0.5	5.3 Kohm
Sulfur	0.066	0.041	0.5	8.5
Copper	0.052	0.033	0.5	10.6
Iodine	0.034	0.021	0.5	16.7
Gold	0.019	0.012	0.5	29.2
<u>RHIC:</u>				
Carbon	0.029	0.018	0.5	19 Kohm
Sulfur	0.018	0.011	0.5	32
Copper	0.014	0.009	0.5	39
Iodine	0.009	0.006	0.5	58
Gold	0.005	0.003	0.5	117

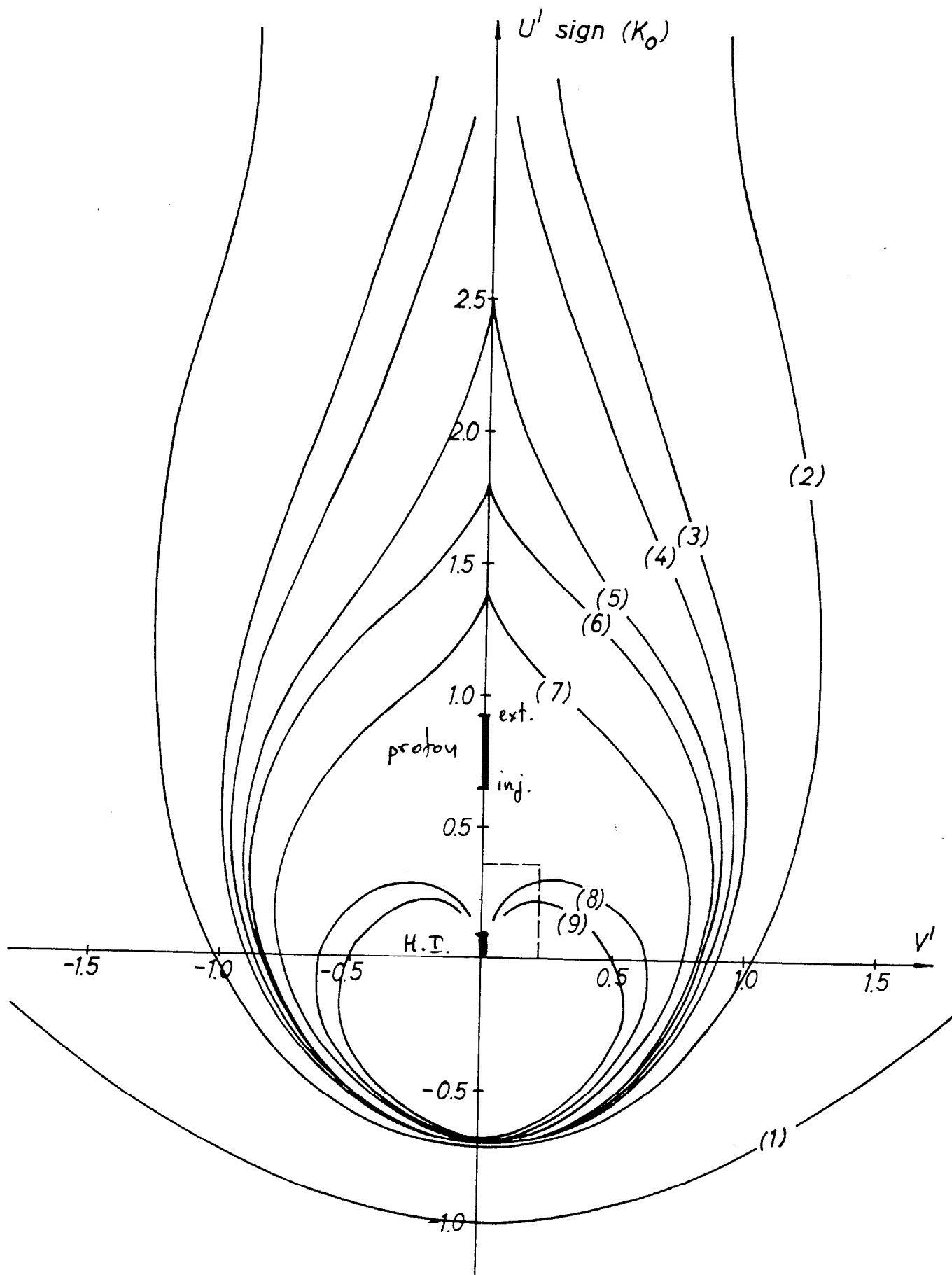


Fig. 1. Stability Diagram

## Distributions in Fig. 1

- 1 Lorentzian
- 2 Gaussian
- 3 5th-order Parabola
- 4 4th-order Parabola
- 5 3rd-order Parabola
- 6 Squared Cosine
- 7 2nd-order Parabola
- 8 Truncated Cosine
- 9 1st-order Parabola