BNL-105145-2014-TECH

Booster Technical Note No. 98;BNL-105145-2014-IR

Longitudinal stability of individual bunches in the AGS - Booster

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November 1987

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U.S. Department of Energy

USDOE Office of Science (SC)

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LONGITUDINAL STABILITY OF INDIVIDUAL BUNCHES IN THE AGS - BOOSTER

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To estimate the longitudinal stability of individual bunches in the AGS Booster one calculates the following complex quantity

$$U' - i V' = -i \frac{2 e I_p \beta^2 (Z/n)}{\pi |\eta| E \, (\Delta E/E)_{FWHM}} \cdot \frac{Q}{A} \qquad (1)$$

where

e , electron charge
Q , particle charge state
A , particle mass number
β , particle velocity to light velocity ratio $= v/c$
Z , the complex beam-environment coupling impedance
n , the harmonic number of the instability
E , the particle total energy /a.m.u.
$(\Delta E/E)_{FWHM}$, the full-width half-maximum relative bunch energy spread

Also

$$\eta = \gamma_T^{-2} - \gamma^{-2}$$

where

$\gamma = E/E_0$

E_0 , particle rest energy /a.m.u.

and γ_T is γ evaluated at the Booster transi= tion energy.

Finally I_p is the bunch peak current

$$I_p = \frac{N e \beta c Q}{\sqrt{2\pi}\,\sigma}$$

N, number of particles in a bunch

σ, rms bunch length

We are assuming bunches with bi-gaussian distribution.

If we define

$$I_\infty = \frac{Nec}{2\pi R} Q$$, average current per bunch for $\beta \to 1$

$$B = \frac{\sqrt{2\pi}\,\sigma}{2\pi R}$$, bunching factor

$$S = \pi \sigma_\tau \delta$$, rms bunch area

with $\sigma_\tau = \sigma / \beta c$

and $\delta = \Delta E / 2.355$, rms energy spread

and $f_\infty = c / 2\pi R$, revolution frequency for $\beta \to 1$

then we can also write (1) as

$$U' - iV' = -0.18\, i \, \frac{e I_\infty E_0 \beta \gamma B (Z/n)}{f_\infty^2 |\eta| S^2} \cdot \frac{Q}{A} \qquad (2)$$

In our notation $Z = X + iY$ with $X > 0$ a resistance and Y is positive for a capacitive reactance and negative for an inductive reactance.

In the AGS-Booster we can conceive three modes of operation

1. Acceleration of protons from 200 MeV to 1.5 GeV with 3 bunches. Each bunch has 5×10^{12} protons and an rms bunch area of 0.10 eV-sec.

2. Acceleration of heavy-ions for RHIC in one single bunch. The beam parameters are those appearing in the RHIC CDR and also reported in Table I. Each heavy-ion beam is accelerated to $\beta\gamma = 0.9$ that is $\beta = 0.669$. The individual bunch rms area is 0.05 eV-s/amu

3. Acceleration of heavy-ion for fixed target experiments out of the AGS. In this mode there are 3 bunches each with an intensity three times smaller than in the previous mode for RHIC, and each with an rms bunch area of 0.015 eV-sec/a.m.u.. The acceleration is also up to $\beta = 0.669$.

For all these modes of operation, the final energy is always below the Booster transition energy ($\gamma_T = 4.5$).

Table I. Heavy-Ion Beams for RHIC

	Carbon	Sulfur	Copper	Iodine	Gold
A	12	32	63	127	197
Q	6	14	21	29	33
$N \times 10^9$	22	6.7	4.7	3.2	2.2
$\beta_{injection}$	0.1262	0.1002	0.0782	0.0595	0.0463
$\beta_{extraction}$	←		0.669		→
S eV-s/amu	←		0.05		→
Z/n in Kohm from space charge					
@ injection	1.47	1.86	2.40	3.16	4.06
@ extraction			0.70		→

The largest contribution to the coupling impedance for the Booster is the "space charge"

$$\frac{Z}{n} = i \frac{Z_0 g}{2\beta\gamma^2} \qquad Z_0 = 377 \text{ ohm}$$

where $g = 1 + 2\log b/a$ is 1 at injection and 4.5 at extraction for all mode of operation. The value of this impedance for the heavy-ion cases is given in Table I. For the proton beam case

$$Z/n = i\ 226 \text{ ohm at injection}$$
$$= i\ 136 \text{ ohm at extraction}$$

This impedance is so large that it is hard to imagine an inductive wall impedance of the same magnitude.

<u>Provisional Conclusion</u>: If there is no resistance, the reactance being positive (capacitive) and the accelerating cycle always below the Booster transition energy the individual bunches are always stable.

Only the presence of a resistance in the coupling impedance can cause the bunches to be unstable. We can calculate the tollerances on X/n.

Observe that the energy dependence of U' is given by the quantity

$$\frac{\beta\gamma B(Y/n)}{|\eta|} \sim \gamma B$$

for the space charge impedance and since one is so well below the transition energy for all cases. Since B decreases with increasing energy, it is seen that U' has indeed only a very weak dependence with energy. We will take $B = 0.3$ at injection and $B = 0.03$ at top energy for all cases. The results are given in Table II. We show the values of U' with space charge at injection and extraction for each case. Based on the stability diagram shown in Fig. 1 we can then infere the maximum allowed values for V'. Since we are below the transition energy $\text{sign}(K_0) > 0$.

The choice for V' depends critically on the shape of the energy distribution.

The range of U' for the proton beam during the acceleration cycle is shown in Fig. 1. With the exception of a truncated cosine distribution (8) and a first-order parabola distribution (9), the beam bunch is always stable provided V' < 0.4, the limit being set by a second-order parabola distribution (7) at top energy. This corresponds to the resistive impedance limit X/n < 60 ohm.

Heavy-ion bunches are even mor stable than proton bunches. The range of U' for all heavy-ion cases is also shown in Fig. 1. At very most U' = 0.11 for Carbon at injection in fixed-target mode. All the distribution considered in Fig. 1 are stable provided V' < 0.5. The tollerance on the resistive impedance is very high: tens of Kohm? It is possible to double the number of heavy ions per bunch and reduce considerably the initial bunch area.

Table II – Bunch Stability Requirement in the Booster

	U′ Injec.	U′ Extr.	V′	X/n
Proton	0.67	0.90	0.4	60 ohm
Fixed Target:				
Carbon	0.11	0.066	0.5	5.3 Kohm
Sulfur	0.066	0.041	0.5	8.5
Copper	0.052	0.033	0.5	10.6
Iodine	0.034	0.021	0.5	16.7
Gold	0.019	0.012	0.5	29.2
RHIC:				
Carbon	0.029	0.018	0.5	19 Kohm
Sulfur	0.018	0.011	0.5	32
Copper	0.014	0.009	0.5	39
Iodine	0.009	0.006	0.5	58
Gold	0.005	0.003	0.5	117

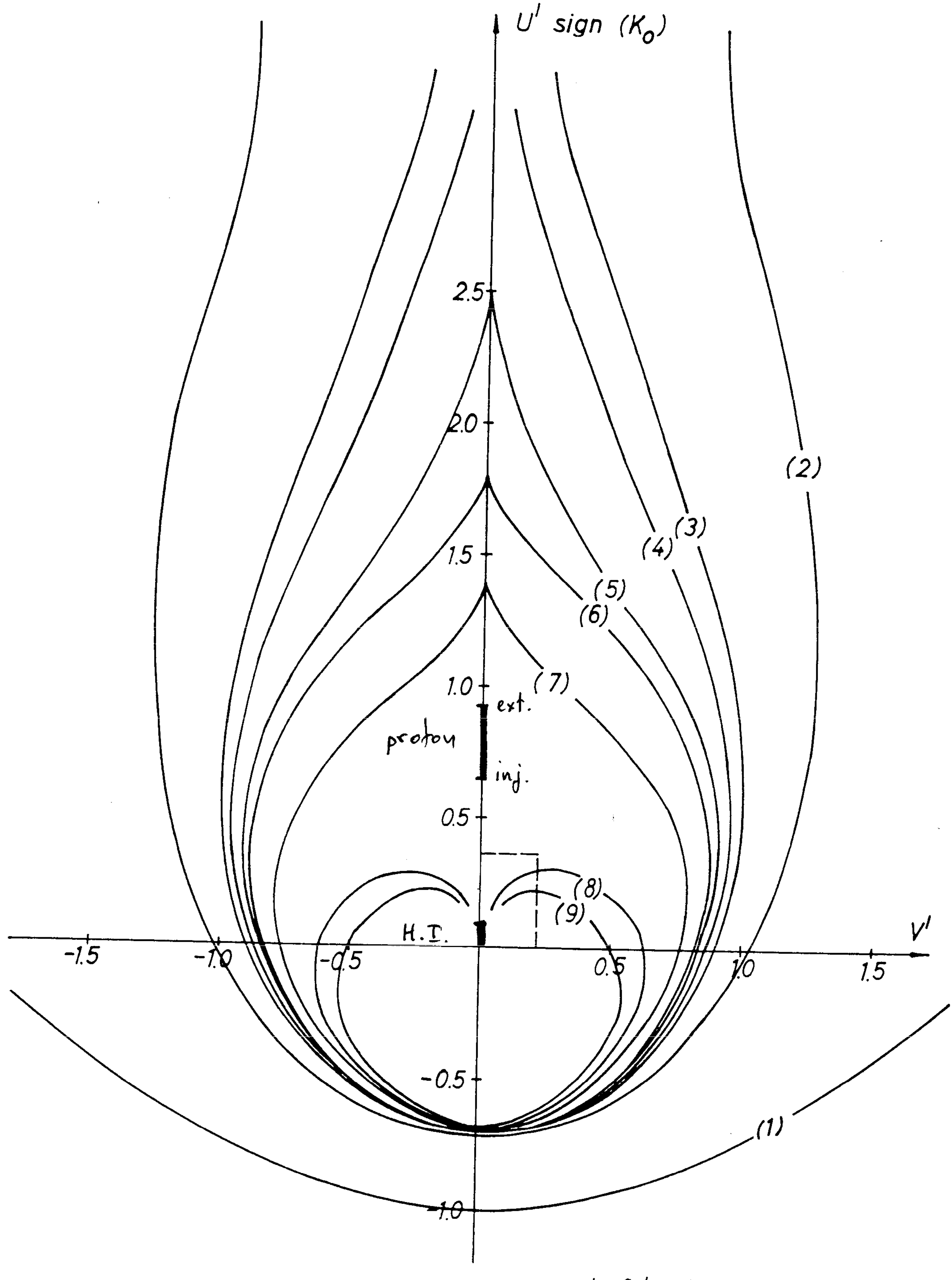

Fig. 1. Stability Diagram

Distributions in Fig. 1

1	Lorentzian
2	Gaussian
3	5th-order Parabola
4	4th-order Parabola
5	3rd-order Parabola
6	Squared Cosine
7	2nd-order Parabola
8	Truncated Cosine
9	1st-order Parabola