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SOME LONGITUDINAL PARAMETERS FROM BO0STER COMMISSIONING

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Collider Accelerator Department

Brookhaven National Laboratory

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Some Longitudinal Parameters from Booster Commissioning

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1 Introduction

Longitudinal emittance is the area occupied by a bunch of particles in the longitudinal phase space spanned by $(\frac{\Delta E}{\omega_{rf}}, \phi)$. By measuring the bunch length, the synchronous phase and synchrotron oscillation frequency, one can calculate the longitudinal emittance. The constancy of the longitudinal emittance is a measure of adiabaticity of the RF parameters throughout the whole acceleration cycle.

The beam from Linac injected into Booster undergoes quadrupole motion, the whole bunch in longitudinal phase space rotates at the synchrotron oscillation frequency. The bunch length at a quarter of a synchrotron oscillation is a measure of the energy spread from the Linac.

In this report, we evaluate the energy spread of the beam in Linac and the longitudinal emittance from measurements on bunch length, synchronous phase and synchrotron oscillation frequency.

2 Measurements

All measurements were done with the wall current monitor at D6. The radial loop was on, so the synchronous particle traveled on the designed orbit. Figure 1 shows the whole cycle (injection, acceleration and deceleration); also in Figure 1, the solid curve is a measure of the magnetic field. Figure 2 shows the quadrupole motion, the synchrotron frequency is measured, from which the voltage per turn acting on protons can be deduced. Figure 3 is a measurement of the bunch length at a quarter of the synchrotron oscillation. Figures 4, 5 and 6 are measurements (not from the same pulse) of the bunch lengths at points A,B and C in Figure 1. Finally, Figure 7 is the mountain range picture taken from the injection. Ideally, we would prefer to make these measurement from the same pulse to calculate quantities we are interested in.

The RF frequency at injection is 2.5MHz and RF frequencies at two points A and B are also known, they are 2.6MHz and 4.0MHz respectively. The frequency at point C is calibrated through points A and B.

Before we analyse the data, let's define some useful symbols, their relations and some measured data.

$E_0(m_0)$	proton rest energy and mass, $E_0 = m_0c^2$.
E_s	synchronous energy, $E_s = E_0\gamma$.
ϕ_s	synchronous phase, $V_{rf}\sin\phi_s = C\rho\dot{\mathcal{B}}$.
η	slip factor $\eta = \gamma_{tr}^{-2} - \gamma_s^{-2}$.
β	ratio of speed of a synchronous particle to that of light, $\beta = \frac{Cf_{rf}}{hc}$.
$\gamma_s(\gamma)$	$\gamma_s = \frac{1}{\sqrt{1-\beta^2}}$.
γ_{tr}	γ at transition, $\gamma_{tr} = 4.88$.
h	RF harmonic number, $h = 3$.
e	electric charge.
f_{rf}	RF frequency.
f_s	revolution requency of a synchronous particle, $f_{rf} = hf_s$.
V_{rf}	voltage per turn.
C	circumference of Booster, $C = 201.78m$.
c	speed of light in vacuum.
ρ	magnetic bending radius, $\rho = 13.75099m$.
$\mathcal{B}$ $(\dot{\mathcal{B}})$	magnetic field and rate of change of magnetic field.
τ_s	a quarter of the period of a synchrotron oscillation, $\tau_s = 62.0\mu s$.
ν_s	synchrotron tune, $\nu_s = \frac{1}{4\tau_s f_s} = \frac{h}{4\tau_s f_{rf}}$.
A	$A = \frac{\eta\omega_{rf}^2}{E_s\beta^2}$.
B	$B = \frac{eV_{rf}}{2\pi h}$.
A_0	$A_0 = 16\sqrt{\frac{B}{\lvert A\rvert}}$, stationary bucket area.
A_b	$A_b = A_0\alpha_b(\phi_s, \phi_2)$, bunch area.
ϕ_{12}	$\phi_{12} = \phi_2 - \phi_1$, bunch length in radian.
τ	bunch length in time.
τ_Δ	$\frac{1}{2}$ of the bunch length at $\frac{1}{4}$ of a synchrotron oscillation, $\phi_\Delta = 2\pi f_{rf}\tau_\Delta$, $\tau_\Delta = 72.4ns$.
ϕ_Δ	$\frac{1}{2}$ of the bunch length at $\frac{1}{4}$ of a synchrotron oscillation in radian.
τ_i	bunch length at injection, $\tau_i = 185.0ns$.
τ_A	bunch length at point A, $\tau_A = 221.0ns$.
τ_B	bunch length at point B, $\tau_B = 82.5ns$.
τ_C	bunch length at point C, $\tau_C = 199.0ns$.
ϵ_i	longitudinal emittance at injection, $\epsilon_i = 2\tau_i\Delta E$.
ϵ_{fl}	longitudinal emittance after filamentation.
ϵ_A	longitudinal emittance at point A.
ϵ_B	longitudinal emittance at point B.
ϵ_C	longitudinal emittance at point C.
ΔE_i	one half of the energy spread in Linac.

3 Results

The voltage per turn is found as follows

$$V_{rf} = \frac{2\pi\nu_s^2\beta^2 E_s}{eh|\eta|\cos\phi_s}, \tag{1}$$

where $\phi_s = 2.3°$. The voltage per turn is a constant throughout the whole cycle.

The magnetic field and RF frequency are related

$$\begin{aligned} \mathcal{B} &= \frac{E_0}{e\rho c}\frac{\beta}{\sqrt{1-\beta^2}} \\ \beta &= \frac{Cf_{rf}}{hc} \end{aligned} \tag{2}$$

The RF frequencies at point A and B are measured, so the magnetic fields at those two points are determined, the scale of magnetic field in Figure 1 is also determined. The RF frequency at point C is calculated by finding the magnetic field at that point.

In order to calculate $\alpha_b(\phi_s, \phi_2)$ [1], we need to find ϕ_2 from the bunch length ϕ_{12}. ϕ_2 is solved from the following two equations

$$\phi_{12} = \phi_2 - \phi_1 \tag{3}$$

$$\cos\phi_2 - \cos\phi_1 + (\phi_2 - \phi_1)\sin\phi_s = 0 \tag{4}$$

Let's write down the Hamiltonian [1] for an asynchronous particle

$$\mathcal{H}(\phi, W) = \frac{1}{2}AW^2 + B[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s] \tag{5}$$

where $W = \frac{\Delta E}{\omega_{rf}}$ the Hamiltonian is a constant of motion. Suppose the energy spread at injection is ΔE_i ($W_i = \frac{\Delta E_i}{\omega_{rf}}$). Take a point (W_i, ϕ_s) in the phase space, at injection $H = \frac{1}{2}AW_i^2$, after a quarter of synchrotron oscillation, that point goes to $(0, \phi_\Delta)$ and $H = B[\cos\phi_\Delta - \cos\phi_s + (\phi_\Delta - \phi_s)\sin\phi_s]$. So we can find the energy spread of the beam from Linac by the following relation

$$\frac{1}{2}AW_i^2 = B[\cos\phi_\Delta - \cos\phi_s + (\phi_\Delta - \phi_s)\sin\phi_s] \tag{6}$$

With the initial energy spread and the bunch length at injection, we can evaluate the initial longitudianl emittance ϵ_i. Further more, we can also calculate the emittance after filamentation. To do so, we notice the constancy of the Hamiltonian for an initial phase point (W_i, ϕ_i) and the phase point $(0, \phi_2)$, where ϕ_2 is the extreme phase point. The emittance, being the area enclosed by the trajectory of an extreme particle, can then be evaluated $\epsilon_{fl} = 16\sqrt{\frac{B}{|A|}}\alpha_b(\phi_s, \phi_2)$ [1]. Since the filamentation reduces the particle density near the extreme particle trajectory, the measurement of the bunch length through the measurement of the line charge densities is less than that of the extreme particle's. It is expected that the measured emittance would be less than the area enclosed by an extreme particle.

$\epsilon_A(eVs)$	$\epsilon_B(eVs)$	$\epsilon_C(eVs)$	$\Delta E_i(MeV)$	$\epsilon_i(eVs)$	$\epsilon_{fl}(eVs)$
0.55	0.54	0.52	1.2	0.43	0.65

Table 1: Calculated longitudinal emittance and energy spread in Linac

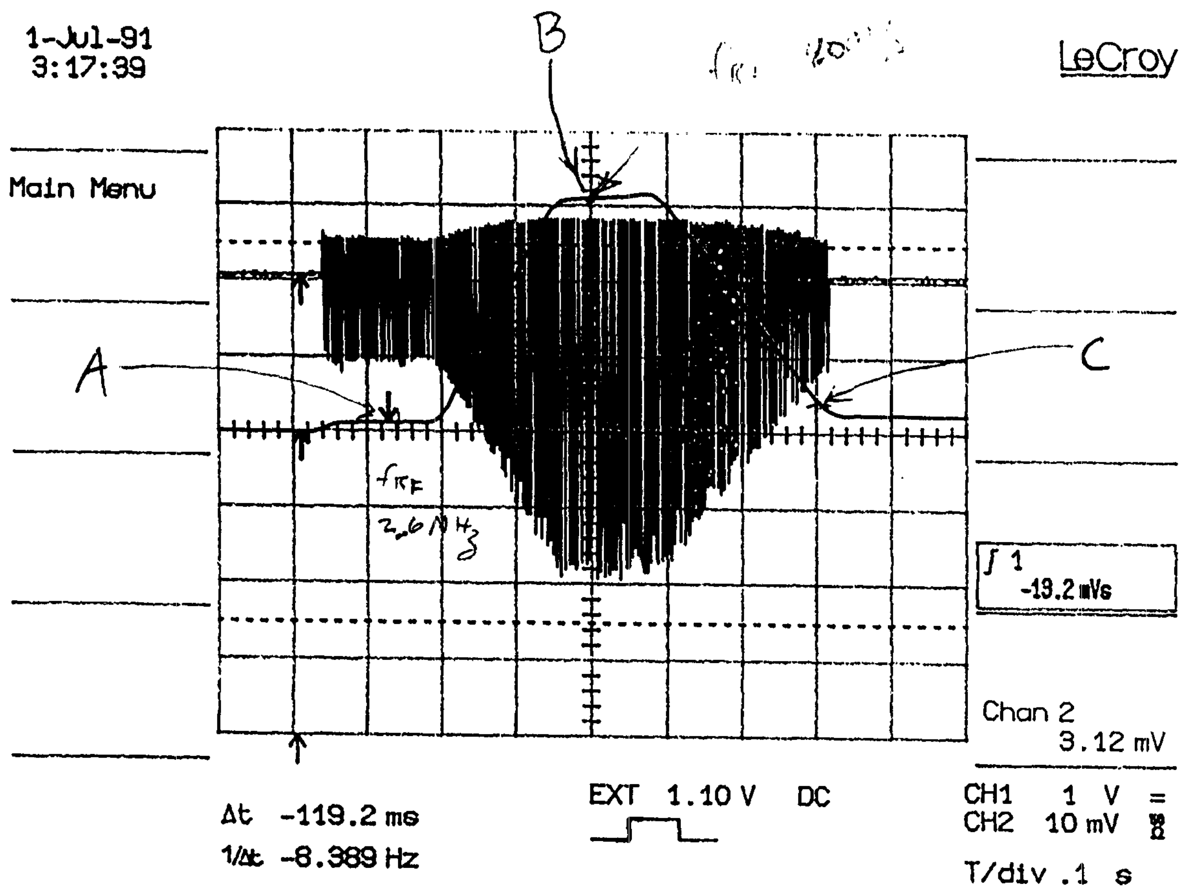
The final results are listed in Table 1.

The fact that the emittance maintains almost as a constant at three distinct points in a cycle tells us that the adiabaticity is good. The capture process enlarges the emittance by a factor approximately 30%.

References

[1] D-P Deng. Longitudinal motion—data presentation. AGS/AD/Tech. 351, BNL, 5 1991.

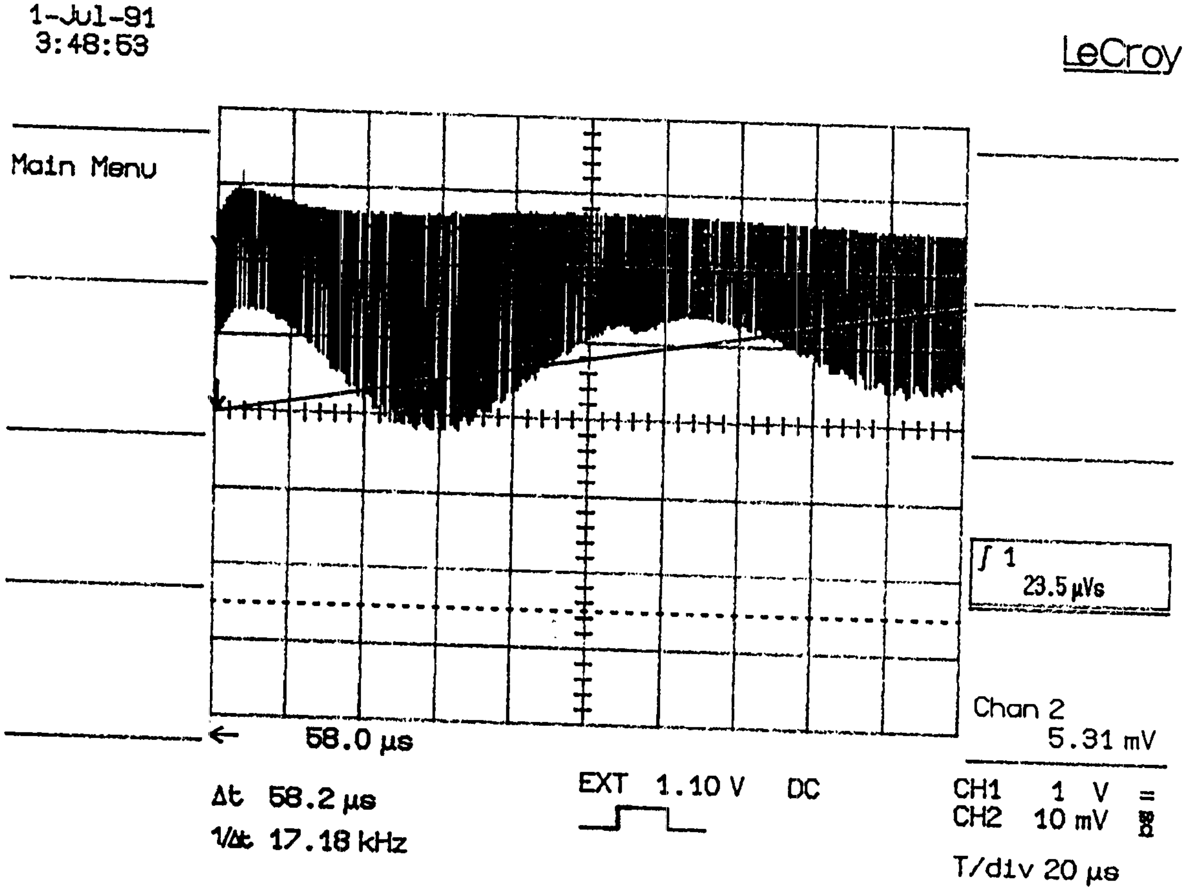
Fig. 1



D6 -20dB
for HP3562
Disk files
WBEAM01-0

Trigger at Tϕ

Fig. 2



2745 µs Rampenlänge

Fig. 3

Trigger at
2745 from
peaker
3.562
no effect

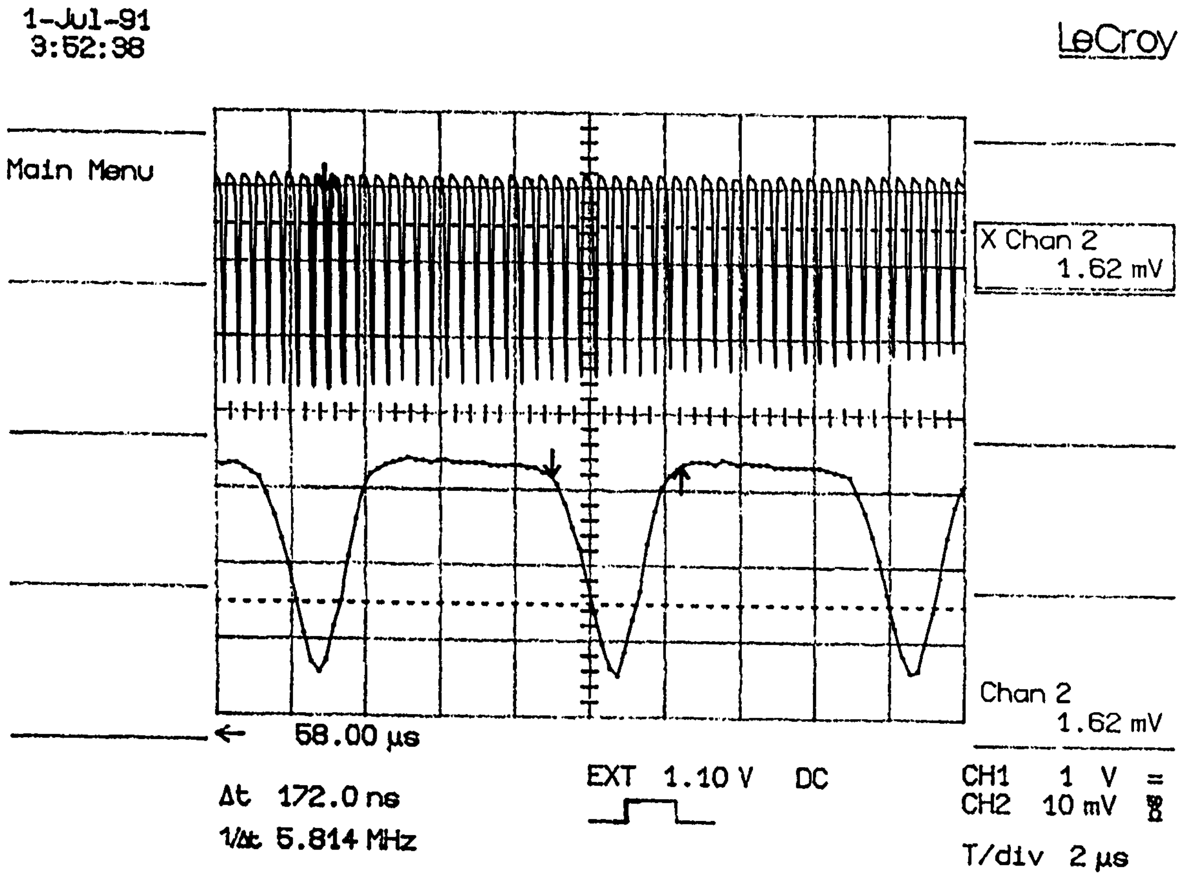


Fig. 4

140 ms
bam Ty

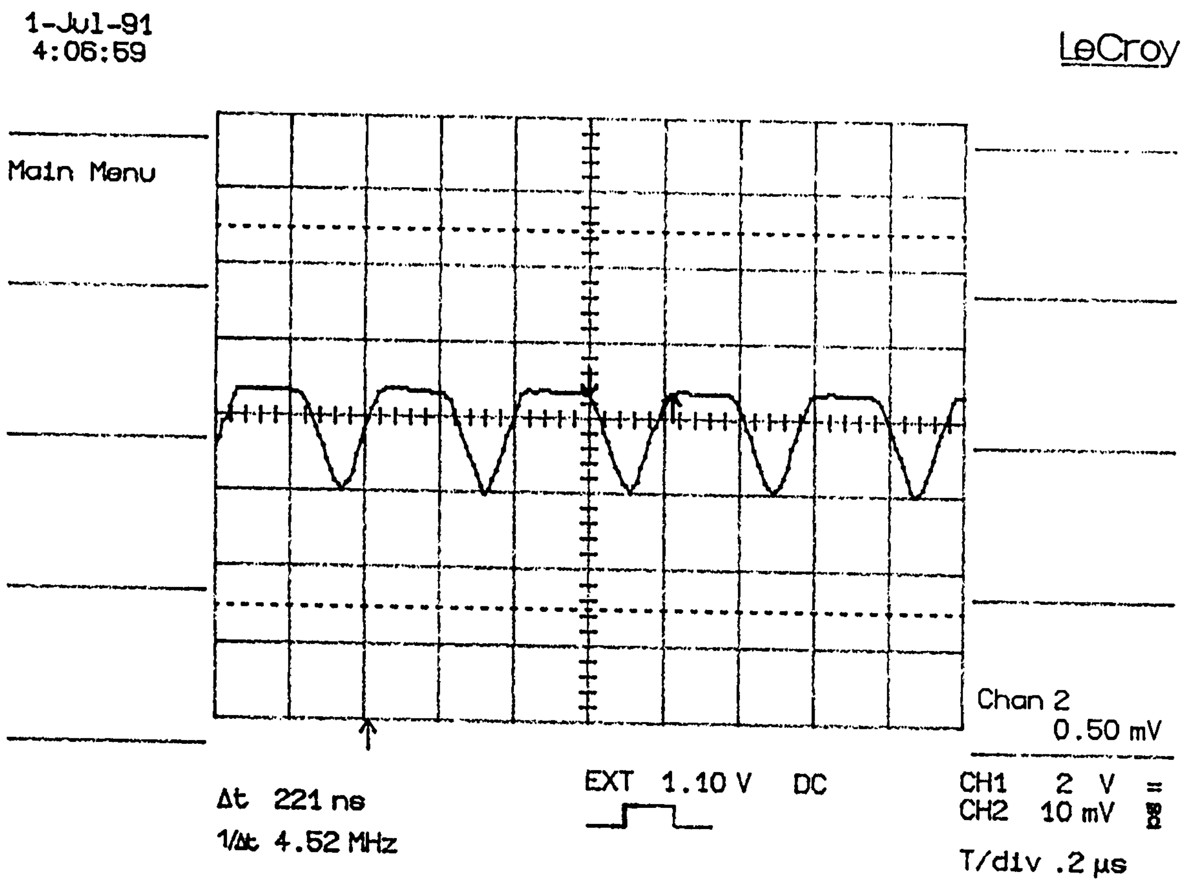
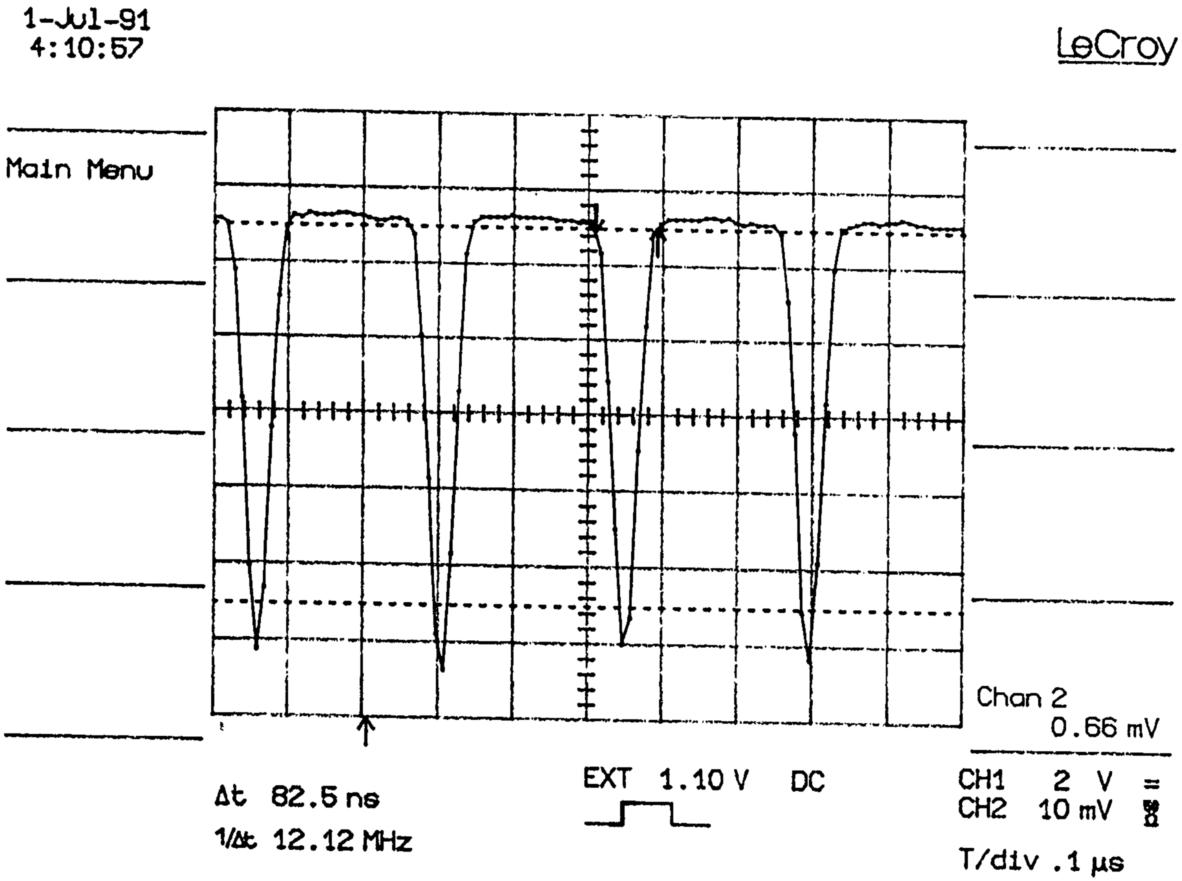


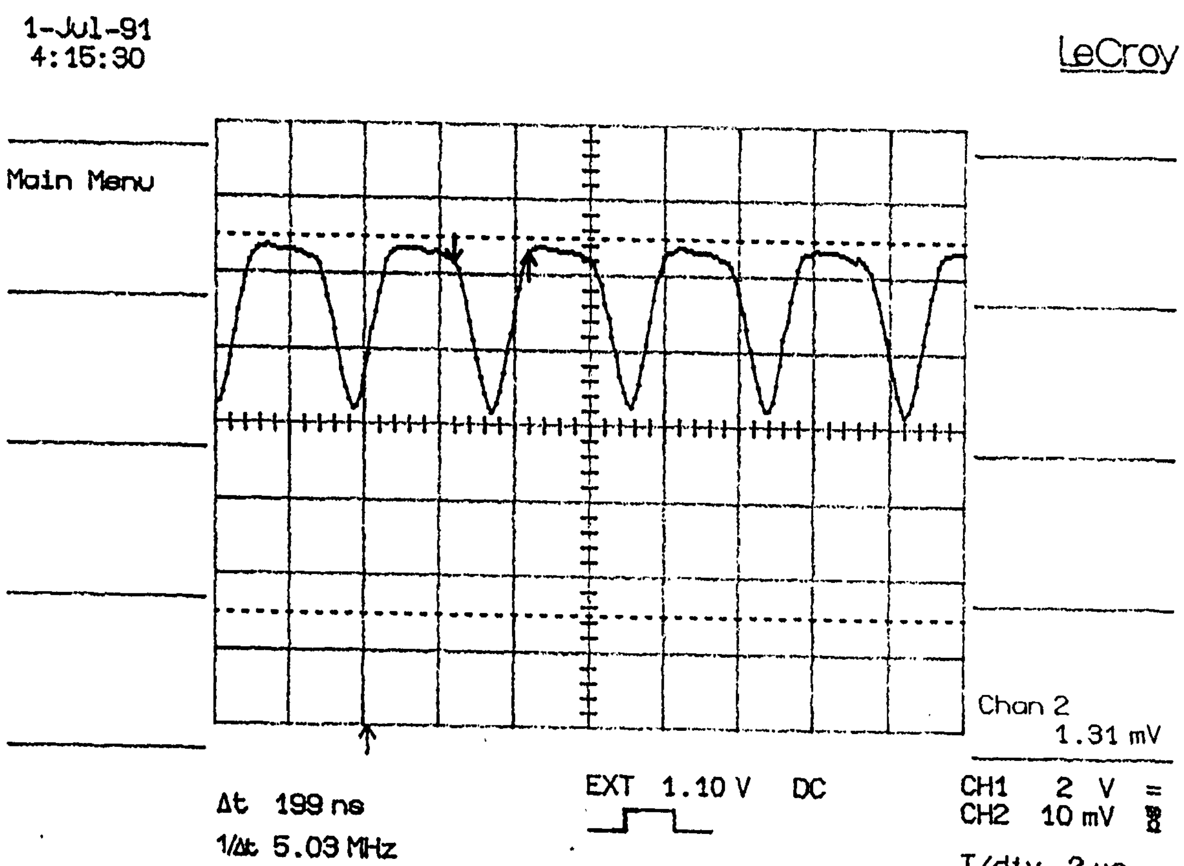
Fig. 5



$E_m = .6$ ev.s
400 ms from T0

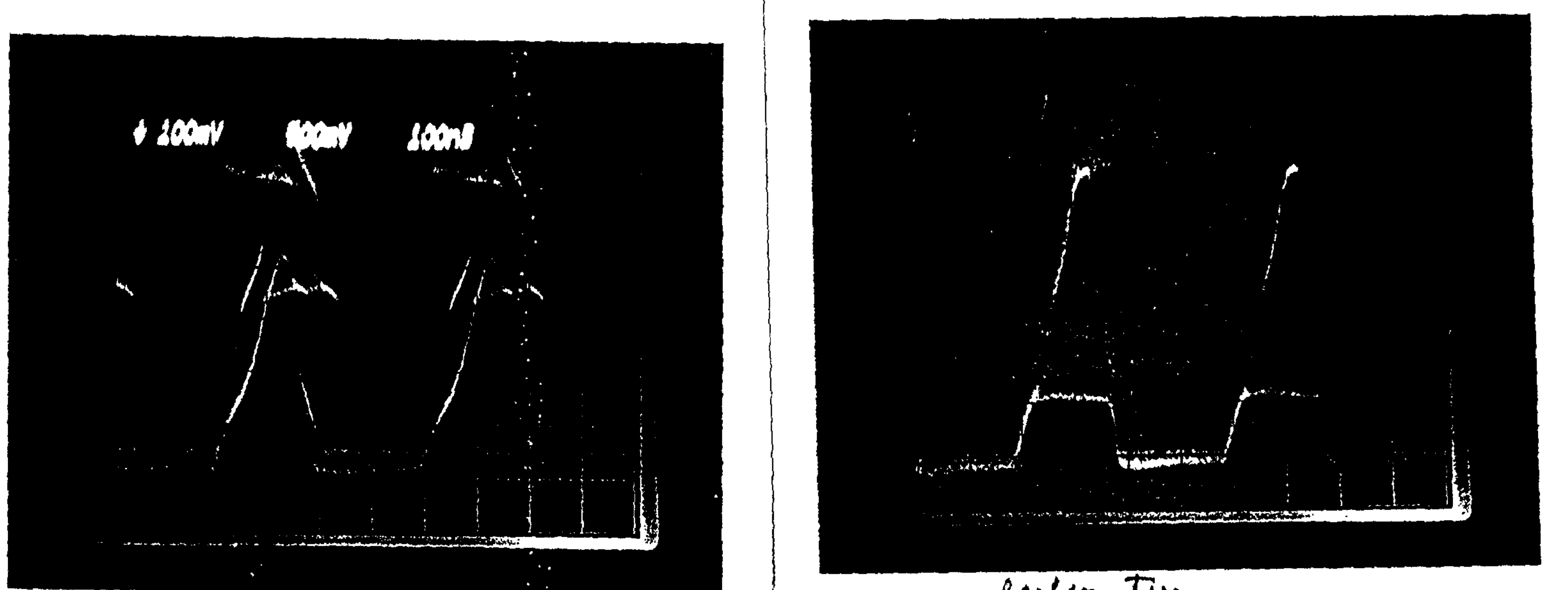
710
Tϕ

Fig. 6



710 ms
from Tϕ

Fig. 7



early turn