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DESIGN and ERROR ANALYSIS of the QUADRUPOLE PICK-UP COILS

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U.S. Department of Energy

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BOOSTER TECHNICAL NOTE
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Design and Error Analysis of the Quadrupole Pick-up Coils

for use with the Booster Gauss Clock

Arden Warner

May 15, 1991

INTRODUCTION

Pickup coils for use with the booster quadrupole Gauss clocks[1] have been designed and a detailed error analysis done. The coils of the basic Gauss clock unit will be mounted in the two booster quadrupole reference magnets No. QH25 and QV25 located in building 930A. The coils will provide a voltage which is proportional to $\dot{B}$. The number of coil windings and form dimensions have been chosen to produce 1 volt/1Tesla/sec.

This report is divided into three parts. Part I describes the design and documents the calculation done to determine the form dimensions and the number of windings. In Part II a general formalism is developed for calculating errors and Part III looks at four types of possible errors; due to (1) Lateral motion of the coils (left-right), (2) the effect of a non-zero B_3 term (sextupole term), (3) Errors due to rotation of the coils, and (4) errors due to differences in the coil width.

We shall adopt the nomenclature used in Tech note 174 by E. Bleser[2].
The nomenclature is as follows:

$$B_y(x) = B_0 + B_1 \bullet x + B_2 \bullet x^2 + B_3 \bullet x^3 + ...$$

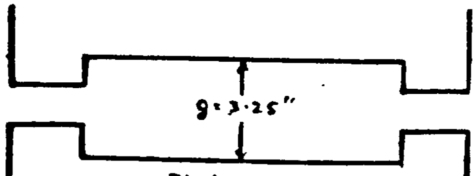
$$B_x(x) = A_0 + A_1 \bullet x + A_2 \bullet x^2 + A_3 \bullet x^3 + ...$$

In a Quadrupole the only allowed terms are B_1, B_5... etc.

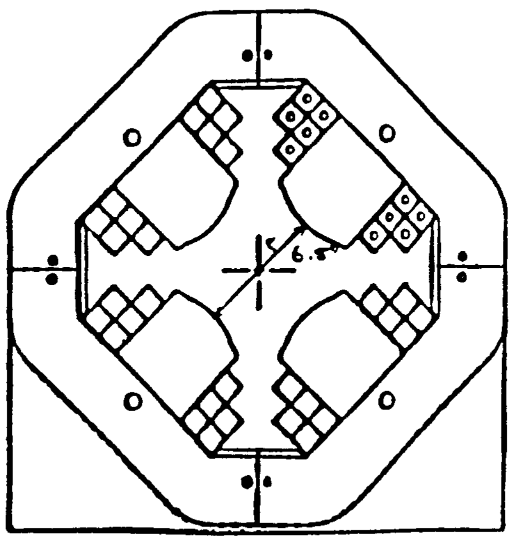
Part I

For each magnet a pair of coils are wound to lengths of 0.77 metres for the short quad, and 0.78 metres for the long quad, 0.3 metres beyond their magnetic lengths. The coils are wound on a form made of extren using #30 (12 mil diameter) wire. The dimensions of the form are chosen to maximize the signal and to reduce the effects of higher order terms. Each pair of coils are set parallel to each other a fixed distance apart which is adjustable prior to mounting in the magnets. Once mounted and pinned into position in the beam pipes at some predetermined location equidistance from the magnetic center of the quadrupole, the signals from the coils are electrically summed and are dependent on $\dot{B}$ and the coil characteristics.

Determining $\dot{B}$ (at 3.2 cm from Quadrupole Centre)



Dipole magnet



Quadrupole magnet.

For a dipole of gap g

$$B_0 = \frac{0.4\pi \times NI}{g}$$

For Booster dipole N=16 Turns

g=3.25 inches=0.08255m

I=current in amps.

For a quadrupole of radius r_q

$$4\pi \times NI = \int_0^{r_q} B_1 r \, dr = \frac{B_1 r_q^2}{2}$$

Where B_1=gradient in Tesla/m

r_q=0.08255m (for Booster)

N=5 turns

$B_1 r_q$=Tesla at pole tip

$$\frac{B_1}{B_0}\left(m^{-1}\right) = \frac{0.4\pi \times 10I}{(0.08255m)^2} \times \frac{0.08255m}{0.4\pi \times 16I} = \frac{10}{16} \times \frac{1}{0.08255m}$$

For quadrupole coils at r=0.32m from magnetic center.

$$\frac{B_1}{B_0} = \frac{10}{16} \times \frac{3.2cm}{3.25" \times 2.54cm} = \frac{dB_1}{dt} \Big/ \frac{dB_0}{dt}$$

The measured value of $\frac{dB_0}{dt}$ (dipole) is 9.5 T/sec

$$\frac{dB_1}{dt} = 9.5T/\sec \times \frac{10}{16} \times \frac{0.32m}{0.08255m} = 2.3T/\sec$$

The emf generated in the pickup coils is

$$V_q = \frac{dB_1}{dt} \bullet W_q N_q \bullet L_q \times 2coils$$

where:

W_q=coil width (cm)
N_q=#of turns
L_q=Length of coil (cm)

$$\frac{dB_1}{dt} = \dot{B}_1 (T/sec)$$

$$A_{eff} = \text{Effective Area} = W_q \times N_q \times L_q$$

The form was designed to have an effective area of 7157.41 cm^2 for the short Quadrupole and 7355.04 cm^2 for the long quadrupole. The emf generated with these characteristics is 3.29 volts and 3.39 volts respectively, and after scaling to 1 volt/1Tesla/sec the number of windings where 49 and 48. Fig. 2 shows the form dimensions.

Part II

General Formalism (for calculating errors)

In the above calculation I have assumed that the B1 term is dominant and that the other terms are small i.e.

$$B_5\, x^5 << B_1\, x$$

let me explicitly show that this is the case and at the same time develop a general formalism for calculating errors.

The B that is used in the calculation is the average B over the area of the coil. So lets calculate B_{avg}. See Fig. 3.

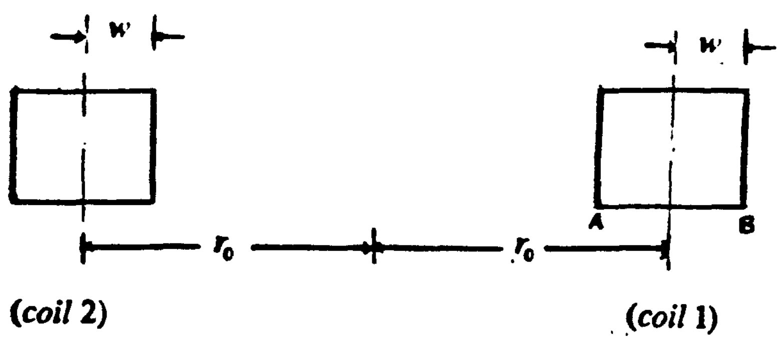


Fig. 3.

$$B = \sum B_n r^n = B_0 + B_1 r + B_2 r^2 + B_3 r^3 + ...$$

$$B_{avg} = \frac{1}{2w} \int_{r_0 - w(\equiv A)}^{r_0 + w(\equiv B)} \left(\sum B_n r^n\right) dr$$

$$= \frac{1}{2w} \sum B_n \frac{r^{n+1}}{n+1} \Big|_{r_0 - w}^{r_0 + w}$$

$$= \sum \frac{B_n\left[(r_0 + w)^{n+1} - (r_0 - w)^{n+1}\right]}{2(n+1)w}$$

$$= B_0 \frac{(r_0 + w) - (r_0 - w)}{2w} + B_1 \frac{(r_0 + w)^2 - (r_0 - w)^2}{4w} + B_2 \frac{(r_0 + w)^3 - (r_0 - w)^3}{6w} + B_3 \frac{(r_0 + w)^4 - (r_0 - w)^4}{8w} + ...$$

Similarly for coil 2

$$B_{avg} = \frac{1}{2w} \int_{-r_0 - w}^{-r_0 + w} \left(\sum B_n r^n\right) dr \text{ etc.....(only the limits are different)}$$

$$B_{avg}\ (coil\ 1) = B_0 + B_1 r_0 + B_2 \left(r_0^2 + \frac{w^2}{3}\right) + B_3\left(r_0^3 + r_0 w^2\right) + ...$$

finite width correction

$$B_{avg}\ (coil\ 2) = B_0 - B_1 r_0 + B_2 \left(r_0^2 + \frac{w^2}{3}\right) - B_3\left(r_0^3 + r_0 w^2\right) + ...$$

Average $\qquad 0 + B_1 r_0 + 0 + B_3 \left(r_0^3 + r_0 w^2\right) +$

therefore the average B is:

$$B_1 r_0 + B_3 \left(r_0^3 + r_0 w^2\right) + ...$$

it is known from measurement that the higher order terms are small compared to B_1. See tech note 175 by E. Bleser[3].

Part III

Error due to Lateral motion of the coil

Suppose the coil is not located at r_0 but at $r_0+\varepsilon$ which could be the result of a temperature change or mechanical error, how does this affect our measurement?

$$B_{avg}\ (coil\ 1) = B_0 + B_1(r_0+\varepsilon) + B_2\left((r_0+\varepsilon)^2 + \frac{w^2}{3}\right) + B_3\left[(r_0+\varepsilon)^3 + (r_0+\varepsilon)w^2\right] + ...$$

$$B_{avg}\ (coil\ 2) = B_0 - B_1(r_0-\varepsilon) + B_2\left((r_0-\varepsilon)^2 + \frac{w^2}{3}\right) - B_3\left[(r_0-\varepsilon)^3 + (r_0-\varepsilon)w^2\right] + ...$$

Average $= \quad 0 + B_1 r_0 + 2B_2\varepsilon r_0 + B_3\left[r_0^3 + 3\varepsilon^2 r_0 + r_0 w^2\right] + ...$

Relative change $= \dfrac{2B_2 r_0\varepsilon + 3B_3 r_0\varepsilon^2}{B_1 r_0 + B_3(r_0^3 + r_0 w^2)}$ $\quad \begin{matrix}(what\ we\ get\ at\ (r_o+\varepsilon)) \\ (what\ we\ get\ at\ (r_o))\end{matrix}$

Dividing by $B_1 r_0$ we get

$$\text{Relative change} = \frac{2\left(\frac{B_2}{B_1}\right)\varepsilon + 3\left(\frac{B_3}{B_1}\right)\varepsilon^2}{1+\frac{B_3}{B_1}(r_0^2+w^2)}$$

$$= \frac{2\left(\frac{B_2}{B_1}\right)\varepsilon + 3\left(\frac{B_3}{B_1}\right)\varepsilon^2}{1+\left(\frac{B_3}{B_1}\right)r_0^2\left(1+{w^2}/{r_0^2}\right)}$$

The values of B_3/B_1 and B_2/B_1 are reported in tech note 175 by E. Bleser.

$\dfrac{B_3}{B_1} = 1.20E-07\ cm^{-2}$ and $\dfrac{B_2}{B1} = 3.37E - 06\ cm^{-1}$

relative change ~ $2\left(\dfrac{B_2}{B_1}\right)\varepsilon$

terms with ε^2 are very small and are neglected i.e. For ε ~ 1mil=2.54×10^{-3} cm, ε^2 ~6-2×10^{-6}
relative change ~ 2x-3.37 x 10^{-6} x 2.54 x 10^{-3}
= 1.6 x 10^{-8} ~ 10^{-6}%

Error due to non zero B_3 term

The field as it stands is given by:

$$B_1 r_0 \; + \; B_3\left(r_0^3 + r_0 w^2\right)$$

where we have set $B_3=0$ leaving us with $B_1 r_0$. What is the effect are this B_3 term? The change due to it $= B_3\left(r_0^3 + r_0 w^2\right)$

relative change $= \dfrac{B_3\left(r_0^3 + r_0 w^2\right)}{B_1 r_0} \quad = \left(\dfrac{B_3}{B_1}\right)\left(r_0^2 + w^2\right)$

remember: $r_0 = 3.2$ cm from magnetic center.

For $w = 1.2$ cm

relative change $= -1.20 \times 10^{-7} \left((3.2)^2 + (1.2)^2\right)$

$= -1.3 \times 10^{-6} \sim 1$ ppm

Error due to rotation

Error due to rotation of one coil about an axis through its center. It is possible that one coil or even both coils are rotated in some way. This would introduce a cosine dependent component into one coil. i.e.

$$B_n = B_n \cos\theta + A_n \sin\theta$$

Assuming

$$\left(\frac{2\pi\theta}{360}\right)^3 << 1$$

$$B_n = B_n\left[1 - \left(\frac{2\pi\theta}{360}\right)^2\right] + A_n\frac{2\pi\theta}{360}$$

The original Flux was $B_1 r_0$

The actual flux is $\left(B_1 + A_1\dfrac{2\pi\theta}{360}\right) r_0$

relative change $= \left(\dfrac{A_1}{B_1}\right)\dfrac{2\pi\theta}{360}$

$= -7.4 \times 10^{-5}\dfrac{6.2}{360}\theta$

$= -1.25 \times 10^{-6}\,\theta \sim 1$ppm/degree.

Error due to a difference in the width of the coils

As mentioned before, the emf generated in the coils are given by:

B_{avg} (coil 1) = $B_o w + B_1 r_o w + ...$
B_{avg} (coil 2) = $B_o w - B_1 r_o w + ...$

$$\text{Error} = \frac{B_1 r_0 (w+\varepsilon) - B_1 r_0 w}{B_1 r_o w} = \frac{\varepsilon}{w} = 10^{-3} \text{ for 1mil.}$$

Part IV

Conclusion

The degree of confidence given by these calculations are based on the accuracy of the measured data reported. Since the quadrupoles are so very precise and are beating the allowed tolerances by a factor of five, any errors in the above calculations should be attributed to the author and not the data. Thanks to E. Bleser and G. Danby for pointing me in the right direction and reviewing my calculations, J. Geller, D. Mangra for the design work and help in understanding the Gauss clock.

References

1. J. Geller, A Digital Voltage to Frequency Converter for the Booster Gauss Clock, Booster Technical Note #175, July 25, 1990.
2. E. Bleser, Short Quadrupole Production Measurements, Booster Technical Note #174, September 12, 1990.
3. E. Bleser, Booster Long Quadrupole Production Measurements, Booster Technical Note #176, September 13, 1990.

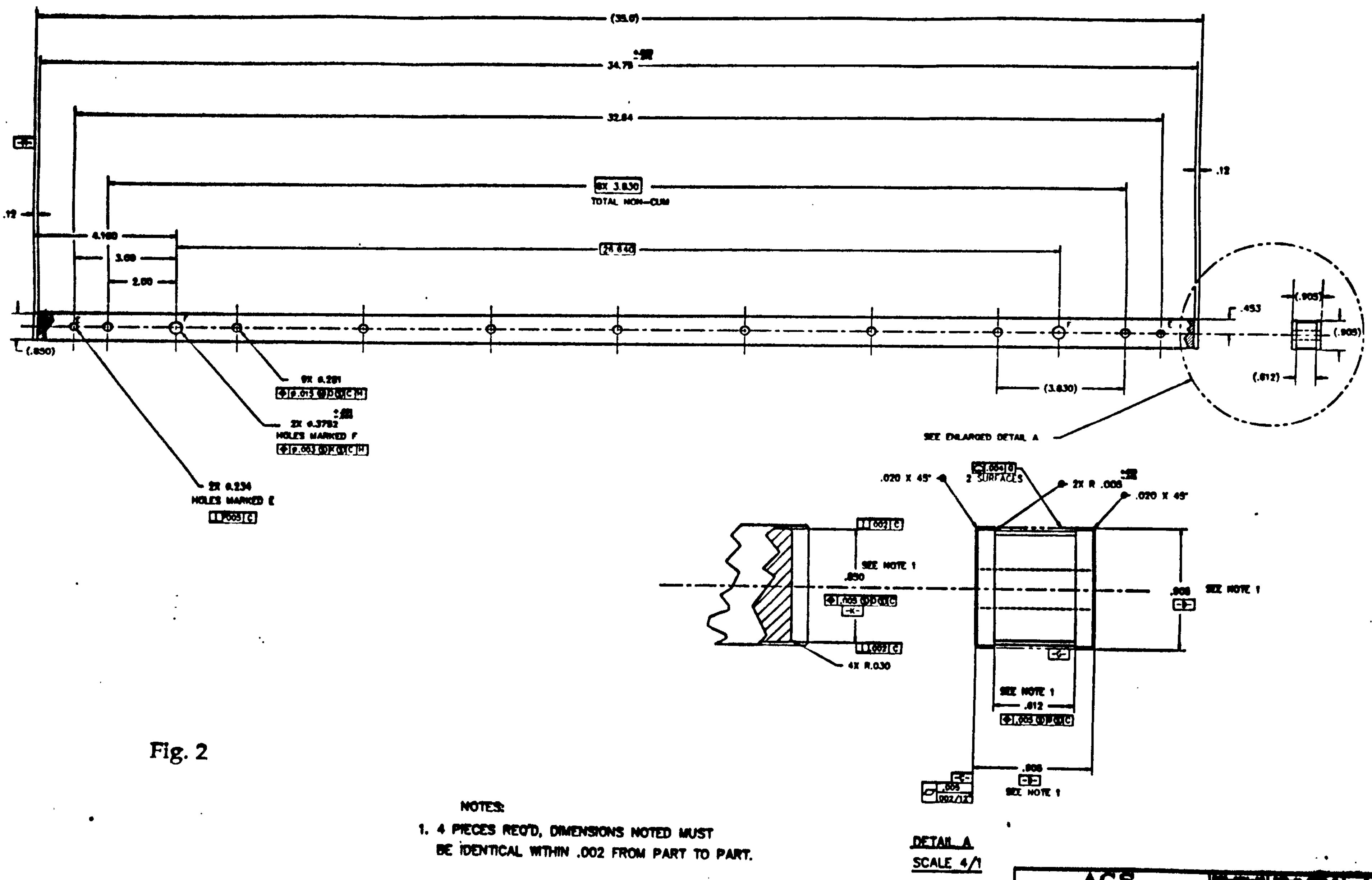


Fig. 2

NOTES:

1. 4 PIECES REQ'D, DIMENSIONS NOTED MUST BE IDENTICAL WITHIN .002 FROM PART TO PART.