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BUMPS IN THE AGS BOOSTER

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BOOSTER TECHNICAL NOTE
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BUMPS IN THE AGS BOOSTER

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The equilibrium orbit in a ring machine may have a set of permanent or time dependent bumps or deformations. Betatron oscillations take place around this bumpy or deformed orbit. Bumps are obtained with appropriate steering magnets or kickers; a steering device can be an independent magnet or can be physically realized by means of additional coils on the main bending magnets.

Let us consider a bump of order n (obtained with n magnets). The pseudoharmonic oscillation of the equilibrium orbit around a non perturbed orbit can be written as[1]

$$
\begin{aligned}
x_c(s) &= \frac{\sqrt{\beta(s)}}{2 \sin \pi\nu} \sum_{i=1}^{n} \sqrt{\beta_i}\, \theta_i \cos\left(\phi(s) - \phi_i \pm \pi\nu\right) \\
x_c'(s) &= -\frac{\alpha(s)}{\beta(s)} x_c(s) - \frac{1}{2\sqrt{\beta(s)} \sin \pi\nu} \sum_{i=1}^{n} \sqrt{\beta_i}\, \theta_i \sin\left(\phi(s) - \phi_i \pm \pi\nu\right)
\end{aligned}
\tag{1}
$$

where x_c is the horizontal or vertical displacement, α, β, ϕ are Twiss functions calculated at s and at the location of the i -th kick, and ν is the tune

$$\alpha = -\frac{1}{2}\beta' \quad , \quad \phi = 2\pi\,\mu$$

The $\pm$ sign is "+" if $\phi(s) > \phi_i$ and "-" if $\phi(s) \le \phi_i$.

After an n -bump, we want the equilibrium orbit back on the unperturbed orbit

$$
\begin{aligned}
x_c(s_n) &\equiv x_n = 0 \\
x_c'(s_n) &\equiv x_n' = 0
\end{aligned}
\tag{2}
$$

[1] M.Sands, *The Physics of Electron Storage Rings*. Proc. Varenna School, Academic Press, N Y 1971, p.305

This takes care of two of equations (1). We can satisfy n -2 more conditions. Generally, for a 3-bump we ask a given displacement $x(\underline{s})$ at position $\underline{s}$ (within the bump), and for a 4-bump a given displacement and angle $x'(\underline{s})$.

It is interesting to verify that the conditions (2) with equations (1) imply (we drop, from now on, the suffix "c")

$$x_1 = 0$$

$$x_1' = \theta_1$$

To find the kicks θ_i for an n -bump, we will solve the system

$$\begin{pmatrix} 0 \\ 0 \\ x(\underline{s}) \\ x'(\underline{s}) \\ \vdots \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots\cdots & A_{1n} \\ A_{21} & A_{22} & \cdots\cdots & \\ A_{31} & \cdots\cdots\cdots & & \\ \cdots\cdots\cdots & & & \end{bmatrix} \begin{pmatrix} \sqrt{\beta_1}\theta_1 \\ \sqrt{\beta_2}\theta_2 \\ \vdots \\ \sqrt{\beta_n}\theta_n \end{pmatrix} \tag{3}$$

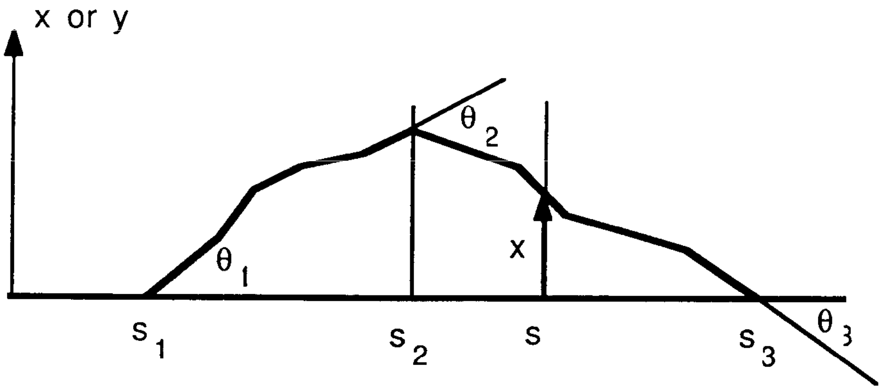
where the first two rows correspond to conditions (2).

A solution of (3) has the general form

$$\theta_i = c_i\, x(\underline{s}) + d_i\, x'(\underline{s}) + \;.....$$

with c_i , d_i , ... coefficients built with the elements of the matrix inverse of $[A]$ of equations (3). We will write explicit expressions for a 3- and a 4-bump.

3-bump



In this case, the first two equations of the system (3) for the conditions (2) are

$$0 = \sqrt{\beta_1}\theta_1 \cos(\phi_3 - \phi_1 - \pi\nu) + \sqrt{\beta_2}\theta_2 \cos(\phi_3 - \phi_2 - \pi\nu) + \sqrt{\beta_3}\theta_3 \cos\pi\nu$$

$$0 = \sqrt{\beta_1}\theta_1 \sin(\phi_3 - \phi_1 - \pi\nu) + \sqrt{\beta_2}\theta_2 \sin(\phi_3 - \phi_2 - \pi\nu) - \sqrt{\beta_3}\theta_3 \sin\pi\nu$$

They can be solved for the kick ratios

$$\frac{\theta_2}{\theta_1} = -\frac{\sqrt{\beta_1}}{\sqrt{\beta_2}}\frac{\sin(\phi_3 - \phi_1)}{\sin(\phi_3 - \phi_2)} \quad , \quad \frac{\theta_3}{\theta_1} = \frac{\sqrt{\beta_1}}{\sqrt{\beta_3}}\frac{\sin(\phi_2 - \phi_1)}{\sin(\phi_3 - \phi_2)} \qquad (4)$$

The third equation of (3) is

$$x = \sqrt{\beta_1}\theta_1 \cos(\phi - \phi_1 - \pi\nu) + \sqrt{\beta_2}\theta_2 \cos(\phi - \phi_2 \pm\pi\nu) + \sqrt{\beta_3}\theta_3 \cos(\phi - \phi_3 + \pi\nu)$$

where

$$x \equiv x(\underline{s}) \ , \quad \phi \equiv \phi(\underline{s})$$

and we use "+" or "-" whether $\phi > \phi_2$ or $\phi \le \phi_2$ ("-" in the figure).

With equations (4), obtain

$$x = \sqrt{\beta\beta_1}\,\theta_1 \frac{\sin(\phi_2 - \phi_1)\sin(\phi_3 - \phi)}{\sin(\phi_3 - \phi_2)} \qquad \textit{if } \phi > \phi_2$$

$$x = \sqrt{\beta\beta_1}\,\theta_1 \sin(\phi - \phi_1) \qquad \textit{if } \phi < \phi_2 \tag{5}$$

In a 3-bump, the equilibrium orbit angle at s can be found from the second of eqs. (1)

$$x' = -\sqrt{\frac{\beta_3}{\beta}}\,\theta_3 \left(\cos(\phi_3 - \phi) + \alpha \sin(\phi_3 - \phi)\right) \qquad \text{if } \phi > \phi_2$$

$$x' = \sqrt{\frac{\beta_1}{\beta}}\,\theta_1 \left(\cos(\phi - \phi_1) - \alpha \sin(\phi - \phi_1)\right) \qquad \text{if } \phi < \phi_2 \tag{6}$$

We can verify from eqs. (6) that

$$x_1'^{(-)} = 0\,, \quad x_1'^{(+)} = \theta_1 \quad , \quad x_2'^{(+)} - x_2'^{(-)} = \theta_2 \quad , \quad x_3'^{(-)} = -\,\theta_3 \quad , \quad x_3'^{(+)} = 0$$

The expressions (4), (5) and (6) can be used to find kicks and x', once x is given. However, it is simpler to refer back to equations (3) and invert $[A]$ to write

$$\theta_i = c_i x \quad , \quad c_i = \frac{A^{-1}_{i3}}{\sqrt{\beta_i}} \quad , \quad i = 1,2,3$$

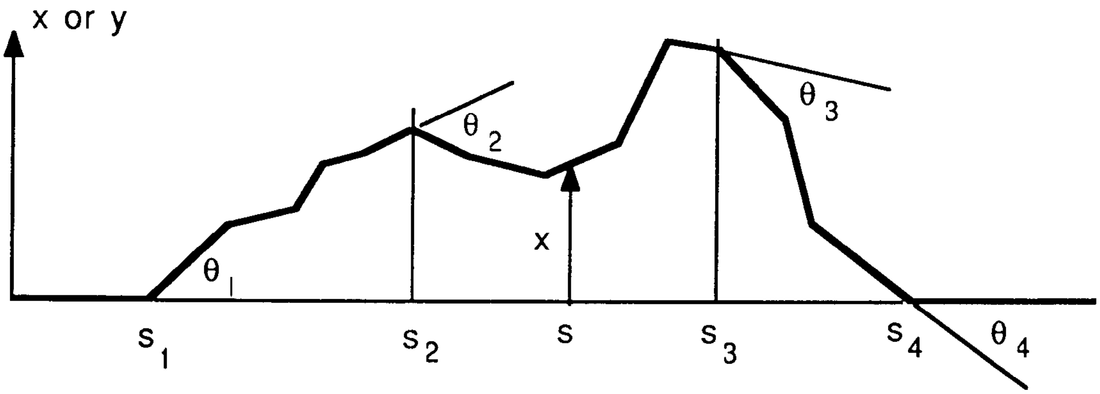
$$x' = c_4 x \quad , \quad c_4 = -\frac{\alpha}{\beta} - \frac{1}{2\sqrt{\beta}\sin \pi\nu} \sum_{i=1}^{3} \sqrt{\beta_i}\, c_i \tag{7}$$

It is desirable to find the displacement at other azimuths s_j ($j = 5 \ldots 4 + n_o$) within the bump, i.e. where n_o beam position monitors (PUE's) are located. From (1) and (7) obtain

$$x_j \equiv x^0(s_j) = c_j x \quad , \quad c_j = \frac{\sqrt{\beta(s_j)}}{2\sin \pi\nu} \sum_{i=1}^{3} \sqrt{\beta_i}\, c_i \cos\left(\phi(s_j) - \phi_i \pm \pi\nu\right) \tag{8}$$

In conclusion, for a 3-bump we will need $4 + n_o$ coefficients: 3 for the kicks, 1 for x' and n_o for the PUE's.

4-bump



In this case, we have an extra degree of freedom and we are allowed to ask e.g. for a displacement x and an angle x' at a given location. Let us re-write eq. (3) in a more convenient form

$$\begin{pmatrix} 0 \\ 0 \\ x \\ x' + \frac{\alpha}{\beta} x \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \sqrt{\beta_1}\,\theta_1 \\ \sqrt{\beta_2}\,\theta_2 \\ \sqrt{\beta_3}\,\theta_3 \\ \sqrt{\beta_4}\,\theta_4 \end{pmatrix}$$

For the kicks we will define eight coefficients, as follows

$$\theta_i = c_i x + d_i x'$$

$$c_i = \frac{A^{-1}_{i3} + \frac{\alpha}{\beta} A^{-1}_{i4}}{\sqrt{\beta_i}} \;, \quad d_i = \frac{A^{-1}_{i4}}{\sqrt{\beta_i}} \quad , \quad i = 1,2,3,4 \tag{9}$$

The displacement at the PUE's are calculated from eqs. (8) as for a 3-bump, with the sum extended from $i = 1..4$

$$x_j \equiv x^0(s_j) = c_j x + d_j x' \ , \quad j = 5..4+no$$

$$c_j = \frac{\sqrt{\beta(s_j)}}{2 \sin \pi\nu} \sum_{i=1}^{4} \sqrt{\beta_i}\, c_i \cos\left(\phi(s_j) - \phi_i \pm \pi\nu\right)$$

$$d_j = \frac{\sqrt{\beta(s_j)}}{2 \sin \pi\nu} \sum_{i=1}^{4} \sqrt{\beta_i}\, d_i \cos\left(\phi(s_j) - \phi_i \pm \pi\nu\right)$$

In conclusion, for a 4 bump we have $8 + 2 \times n_o$ coefficients: eight for the kicks and the rest for the PUE's.

Implementation in the AGS Booster

In the AGS Booster a certain number of 3- and 4-bumps can be conveniently pre-set corresponding to specific operations.

Injection is accomplished by the concurrent action of a "slow" horizontal 3-bump, obtained with extra pole windings on some bending magnets, and a "fast" 3-bump, obtained with fast horizontal kickers. Here, the bump location e.g. for protons is the stripping foil, and observation points are two PUE's within the range of the bumps. Fine tuning of the injection, both horizontal and vertical, can be obtained with 4-bumps, using steering correction magnets in the lattice, that allow one to specify both displacement and angle of the stored beam at the foil.

Aperture scan at some quadrupole location can be accomplished with 3-bumps in the vicinity of the quadrupole, and observing the beam at some PUE's located within the bump.

We have analyzed some of these cases. For each, a string of physical device names is read from a file in the Apollo network and the appropriate coefficients calculated and appended to the file. We can always add strings to the file to calculate new bumps.

An input string is composed by:

bump identifier;
bump type;
bump order, 3 or 4;
if horizontal or vertical;
name of bump location, e.g the foil;
names of the bump magnets,
number of PUE's, n_o , names of the PUE's.

A coefficient string has

the appropriate identifier;
a sequence of $4+n_o$ coefficients, if a 3-bump;
a sequence of $8+2*n_o$ coeffs, if a 4-bump.

The coefficients for a certain number of default strings have been calculated and their values tested by the model MAD. Strings and coefficients are given in Table I. Results are shown in the following figures, referred to the booster lattice, and with the location of the bump and of the magnets marked on the plots.

Table I. Physical device strrings and coefficients for 6 preset bumps in the AGS Booster. (March 1991 Booster data. Tunes 4.649 and 4.568)

identifier	type	order	bump location	magnets	PUE's	c	d
SIB	TDH	3 h	IJFOIL	TDHC4	PUEHC6	0.127984	
				TDHC8	PUEHC8	0.011850	
slow				TDHD1		0.139391	
injection						0.207353	
bump						1.12470	
						1.09020	
FIB	KDH	3 h	IJFOIL	IJKDHC3	PUEHC4	0.144593	
				IJKDHC7	PUEHC6	0.016422	
fast				IJKDHD1	PUEHC8	0.128142	
injection						0.138972	
bump						0.474574	
						1.08357	
						0.418075	
HIB4	DVC	4 h	IJFOIL	DHCC2	PUEHC2	0.169406	0.936067
				DHCC4	PUEHC4	-0.029924	-0.70500
horizontal				DHCC6	PUEHC6	0.104271	0.96937
fine tune				DHCC8		0.090956	-0.026716
injection						0.052118	0.287985
						1.94531	10.5830
						1.03208	-0.303145
ASQVA3	DVC	3 v	QVA3	DVCA1	PUEVA3	0.088224	
aperture				DVCA3		-0.064613	
scan of				DVCA5		0.088224	
odd quad						-0.174374	
						1.00488	
ASQHA4	DHC	3 h	QHA4	DHCA2	PUEHA4	0.088532	
aperture				DHCA4		-0.063780	
scan of				DHCA6		0.089753	
even quad						-0.167088	
						1.00185	

VIB4 vertical fine tune injection	DHC	4 v	IJFOIL	DVCC3 DVCC5 DVCC7 DVCD1	PUEVC5 PUEVC7	0.074245 0.125079 -0.057184 0.179454 0.900870 2.04402	0.301902 -0.714295 0.663102 -0.493189 3.28699 -5.61751

Use of the Table:

For a 3-Bump, input x [meters]:

$$\left.\begin{aligned} \theta_1 &= c_1 x \\ \theta_2 &= c_2 x \\ \theta_3 &= c_3 x \end{aligned}\right\} \quad \text{kicks}$$

$$x' = c_4 x \qquad \text{orbit angle at s}$$

$$\left.\begin{aligned} x_1^0 &= c_5 x \\ x_2^0 &= c_6 x \\ &\cdots\cdots \\ &\cdots\cdots \end{aligned}\right\} \quad \text{displacement at PUE's}$$

For a 4-Bump, input x [m] and x' [rad]:

$$\left.\begin{aligned} \theta_1 &= c_1 x + d_1 x' \\ \theta_2 &= c_2 x + d_2 x' \\ \theta_3 &= c_3 x + d_3 x' \\ \theta_4 &= c_4 x + d_4 x' \end{aligned}\right\} \quad \text{kicks}$$

$$\left.\begin{aligned} x_1^0 &= c_5 x + d_5 x' \\ x_2^0 &= c_6 x + d_6 x' \\ &\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots \end{aligned}\right\} \quad \text{displacement at PUE's}$$

Fig.1. SIB = Slow Injection Bump. Horizontal, order-3.
x = 10 mm, resulting x' = 2.074 mrad.

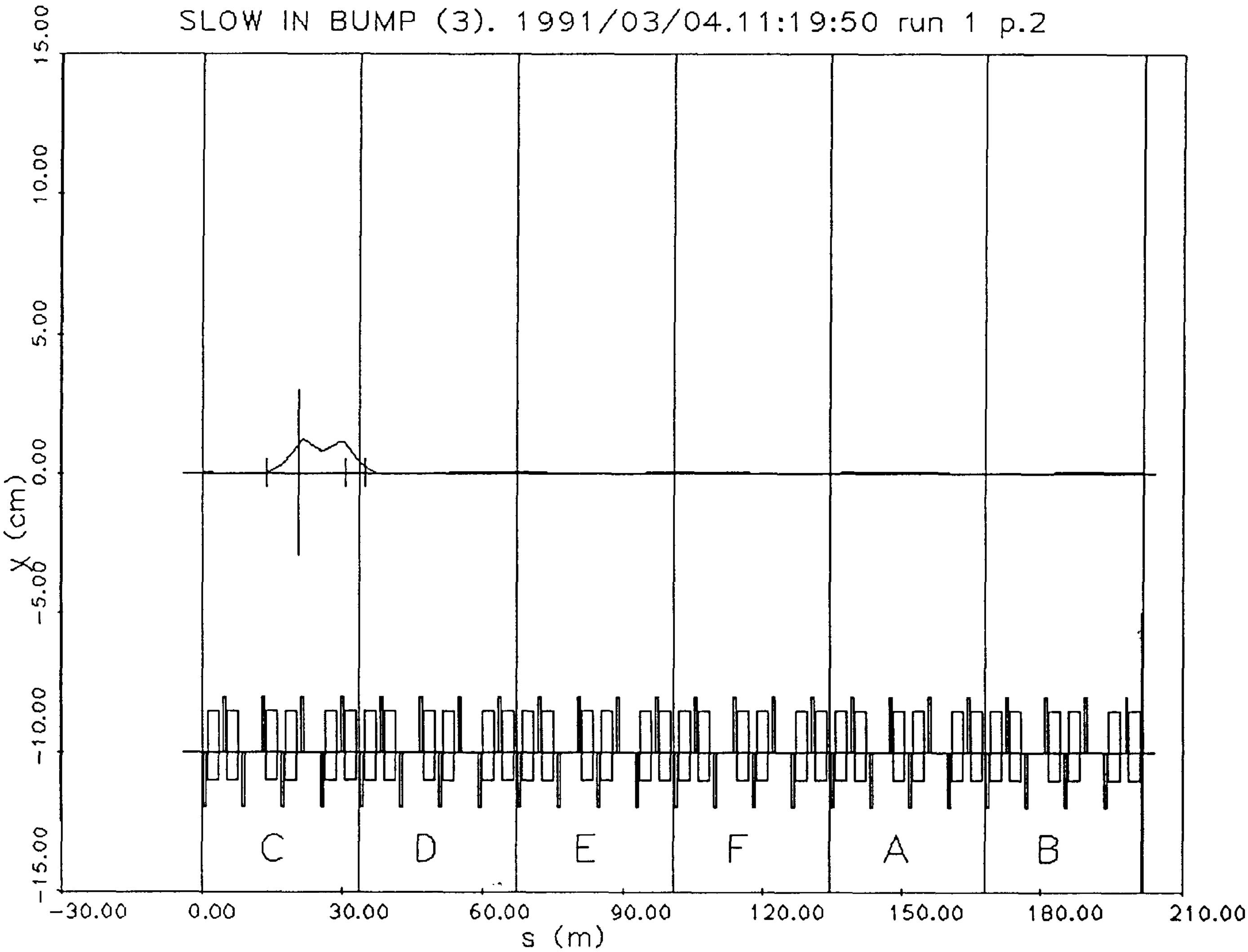


Fig.2. FIB = Fast Injection Bump. Horizontal, order-3.
x = -25 mm, resulting x' = 3.474 mrad.

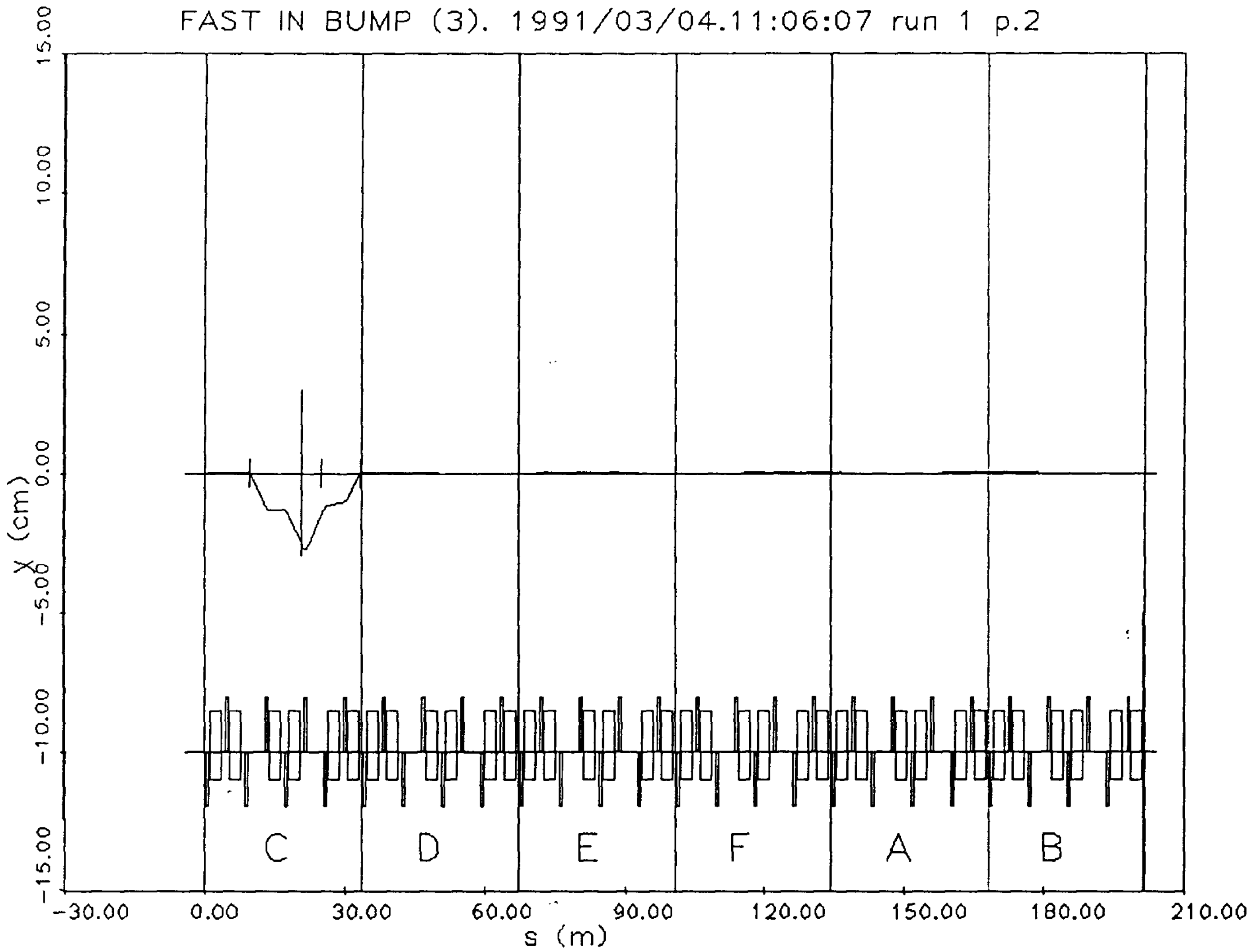


Fig.3. HIB4 = Fine Injection Bump. Horizontal, order-4. x = 1 mm, x' = 0.

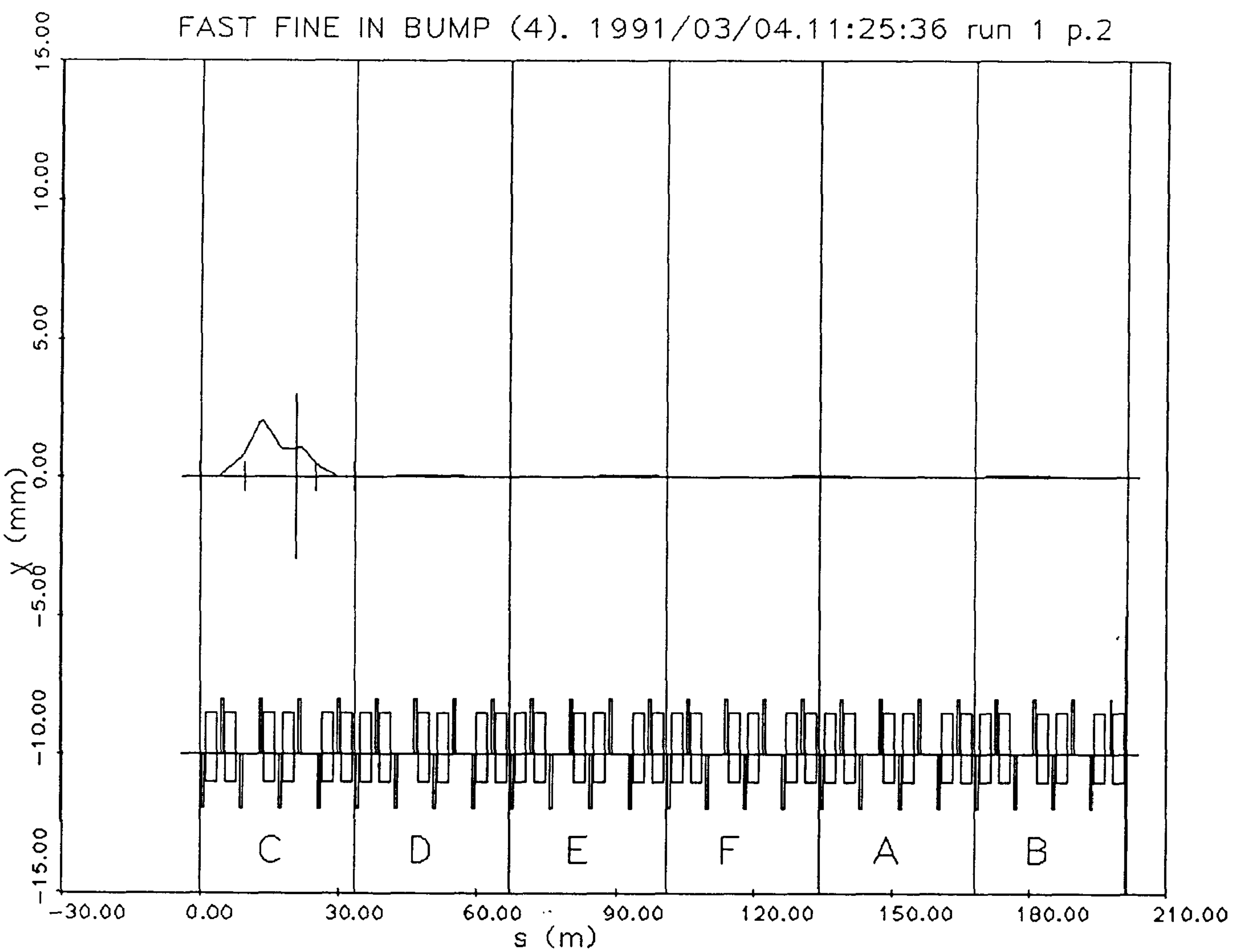


Fig.4. ASQVA3 = Aperture Scan on Odd Quadrupole. Vertical, order-3.
y = 10 mm, resulting y' = -1.744 mrad.

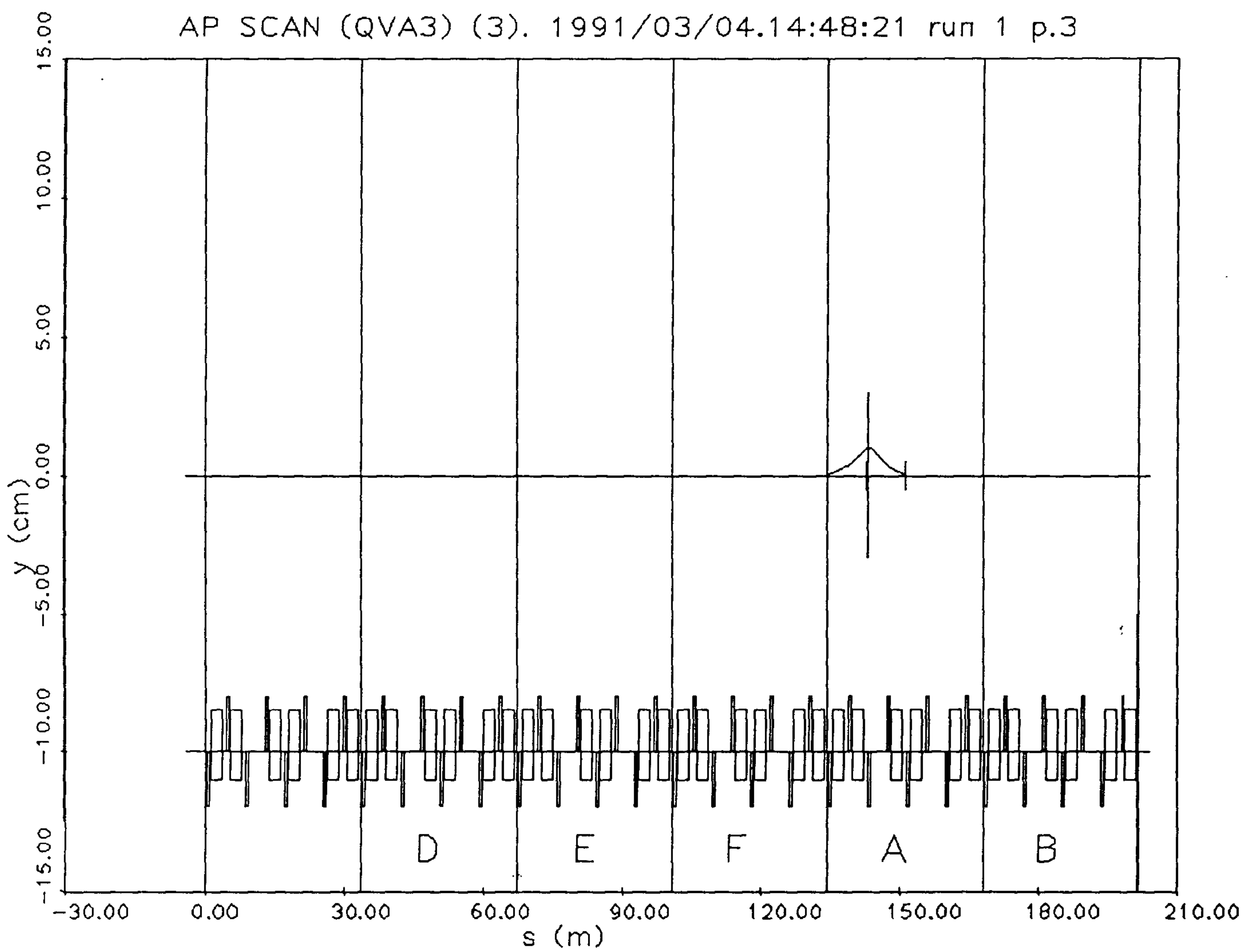


Fig.5. ASQHA4 = Aperture Scan on Even Quadrupole. Horizontal, order-3.
y = 10 mm, resulting y' = -1.671 mrad.

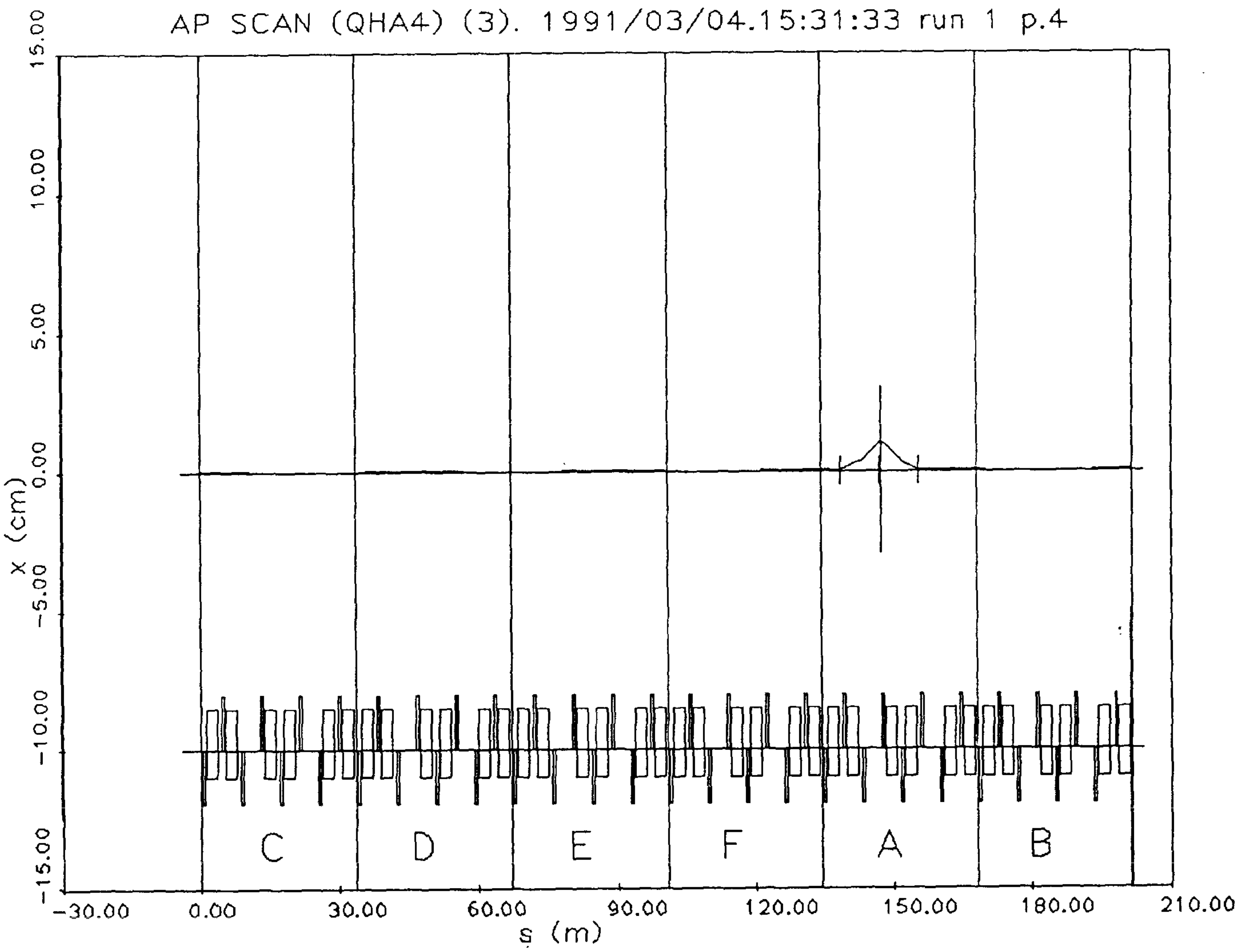


Fig.6. VIB4 = Fine Injection Bump. Vertical, order-4. y = 1 mm, y' = 0.