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PHASE SPACE ELLIPSES FOR A MODEL AGS

G. Fiske

August 1969

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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Accelerator Department
BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, L.I., N.Y.

AGS DIVISION TECHNICAL NOTE

No. 65

G. Fiske
August 26, 1969

PHASE SPACE ELLIPSES FOR A MODEL AGS

The accompanying graphs are plots of the AGS phase space ellipses for the horizontal (H) and vertical (V) directions, obtained on the CDC-6600 and the Calcomp 565 plotter.

The calculation assumes a model horizontal $\Psi(s)$ function[1] of the following form:

$$\Psi(s) = \Psi_L \left[\frac{S}{L} + \frac{\tau}{2\pi}\left(\cos\frac{2\pi}{L}S - 1\right) + \frac{\sigma}{2\pi}\left(\cos\frac{6\pi}{L}S - 1\right)\right] \text{ where:}$$

$L = \frac{2\pi R}{60}$; R = radius of machine = 5057.266 inches

$\Psi_L = \frac{2\pi\nu}{60}$; and for a form factor $\frac{\beta_{max}}{\beta_{av}} = 1.5$

$\tau = 3/8$

$\sigma = 1/72$

This phase function has a periodicity of four magnets (1/60 of the circumference), and agrees well with the results obtained from the BEAM program at intermediate field. The corresponding β function is of the form:

$$\beta(s) = \frac{R/\nu}{1 - \tau \sin\frac{2\pi}{L}S - 3\sigma \sin\frac{6\pi}{L}S}$$

Each set of eight graphs gives the phase space ellipses in either the horizontal or vertical direction for one particle at eight points within the super-superperiod (1/60 of the circumference). The following table lists the graphs shown:

Graph Numbers	ν	Y_{max} (inches)
1-8	8.0 (H)	2.00
9-16	8.0 (V)	1.00
17-24	8.5 (H)	2.00
25-32	8.5 (V)	1.00
33-40	9.0 (H)	2.00
41-48	9.0 (V)	1.00

In general, the program is designed to plot graphs for any ν value and any value of Y_{max}, where Y_{max} is the maximum value of the transverse excursion of the particle taken at the point where β itself is maximum. Thus, at no point in the machine is the greatest transverse excursion of the particle any larger than Y_{max}.

The phase invariant, W, is set at Y_{max}^2/β_{max}, and the equation of each ellipse is:

$$\gamma y^2 + 2\alpha y y' + \beta y'^2 = W$$

A diagram of a super-superperiod with the horizontal β function superimposed is given in Fig. 1.

The phase space ellipses are computed and plotted at the indicated points, A through H.

The graphs contain all parameters pertinent to the calculation, as well as the area (emittance) of the ellipses; that is:

$$A = \pi W \text{ rad-in.}$$

References

1. J.C. Herrera, BNL Accel. Dept. Int. Rept. AGS DIV 69-3 (1969).

Distr:

Department Administration
AGS Division Physicists

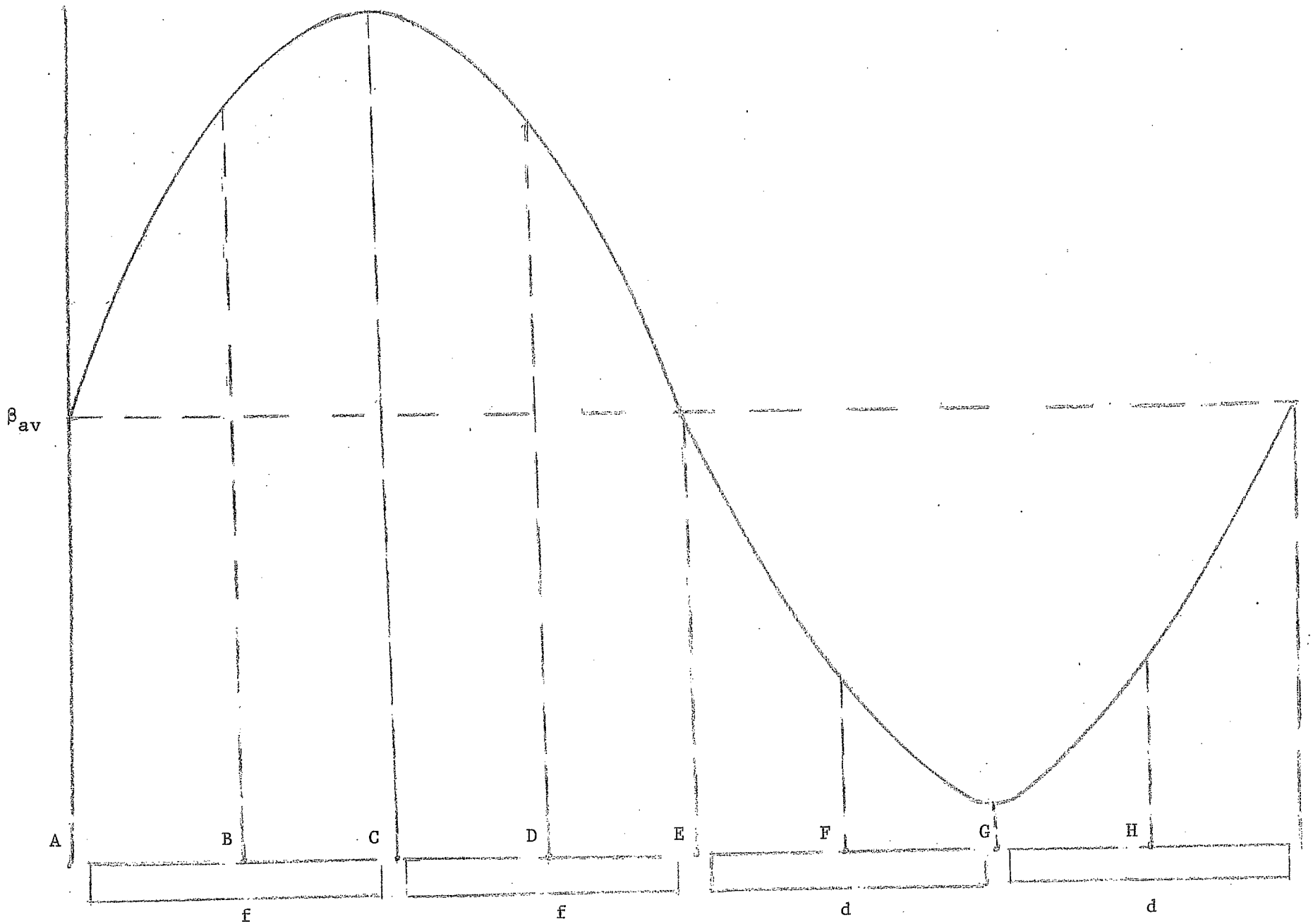
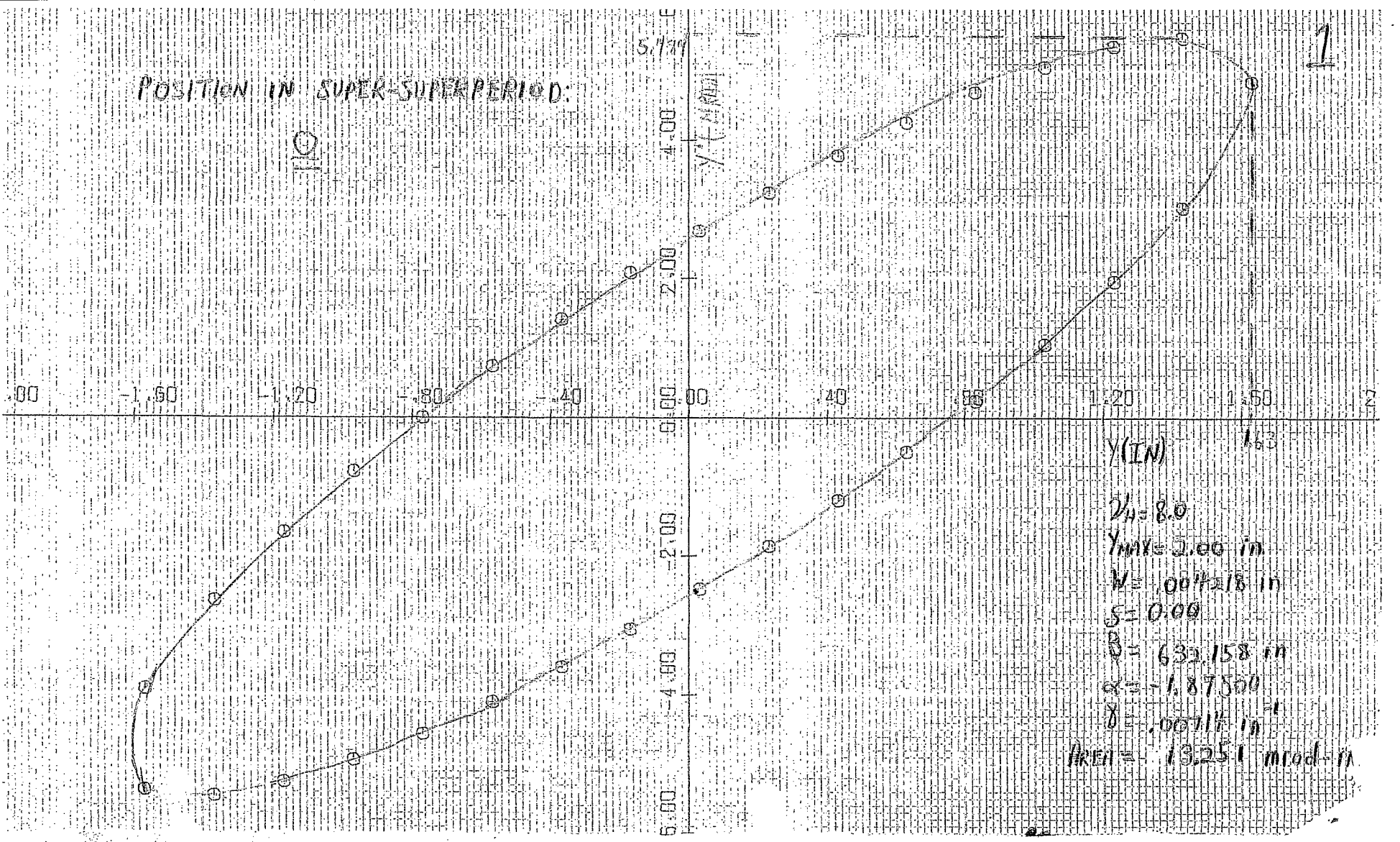


Fig. 1



POSITION IN SUPER-SUPERPERIOD:
1
5.479
4.00
2.00
0.00
-2.00
-4.00
-6.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2
Y(IN)
1.63
ν_H = 8.0
Y_{MAX} = 2.00 in
W = .004218 in
S = 0.00
β = 632.158 in
α = -1.87500
γ = .00714 in^{-1}
AREA = 13.251 mrad-in

POSITION IN SUPER-SUPERPERIOD:

1/8

$Y_0 = 8.0$

$Y_{MAX} = 2.00$ in.

$W = .004218$ in.

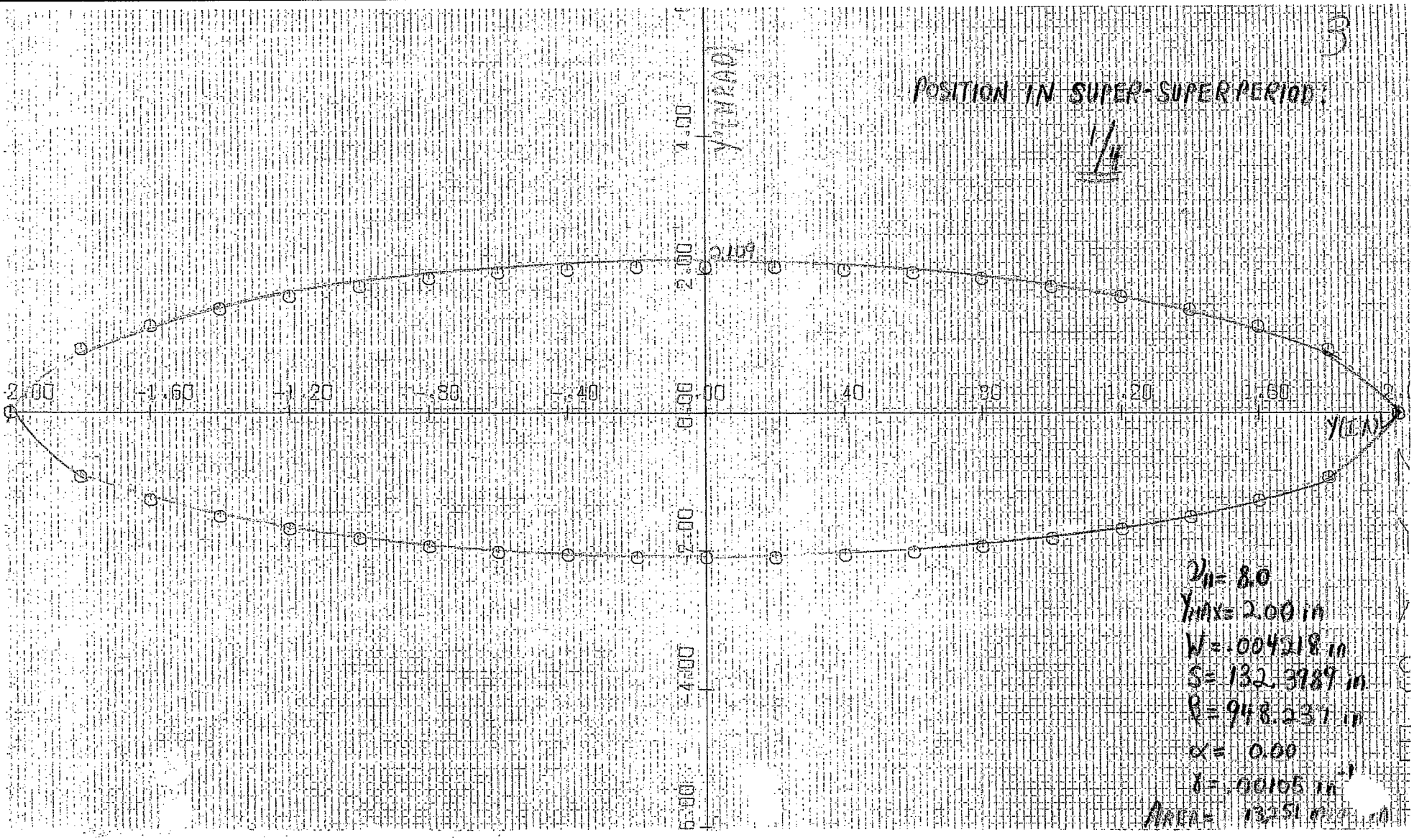
$S = 66.1995$ in.

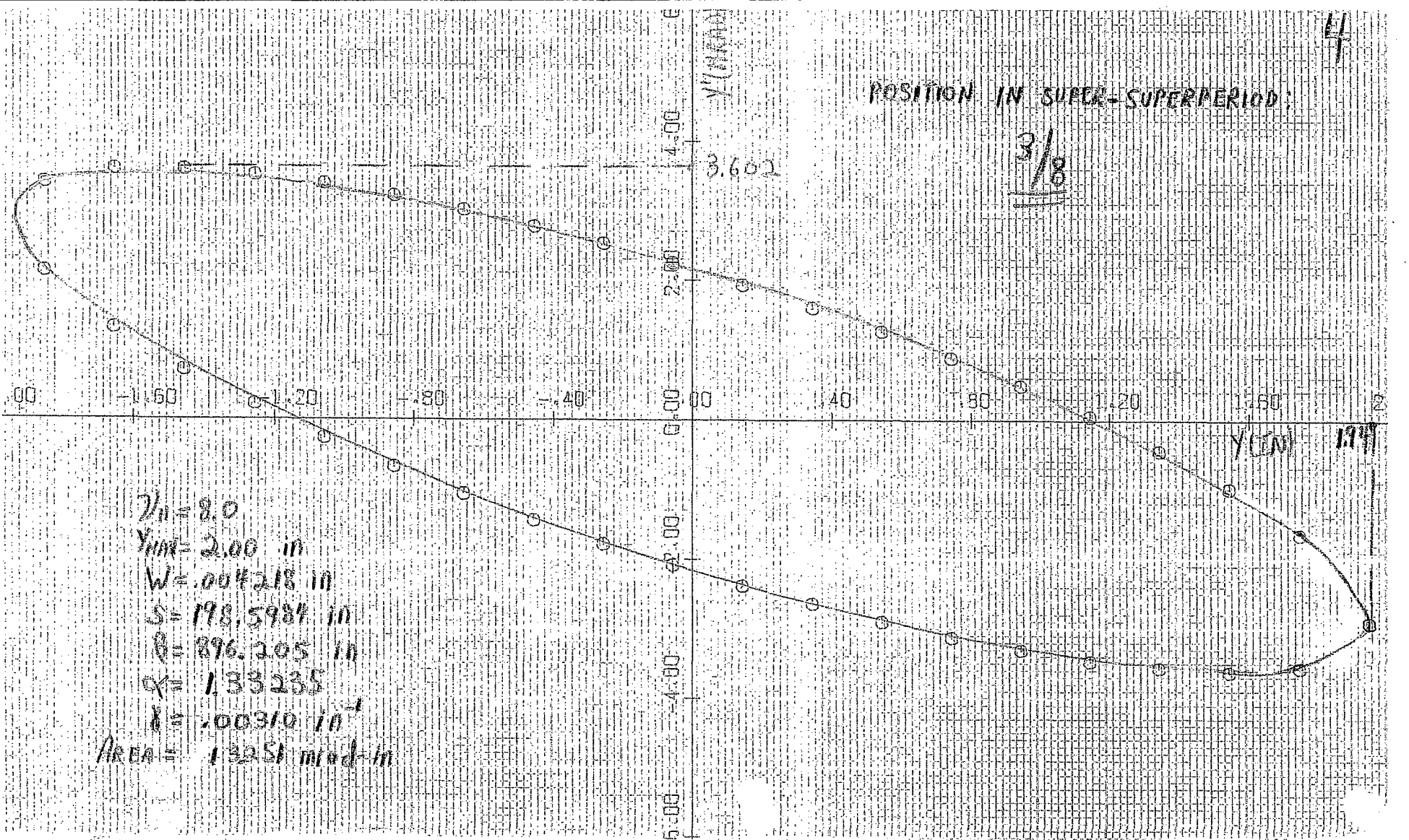
$\beta = 896.205$ in.

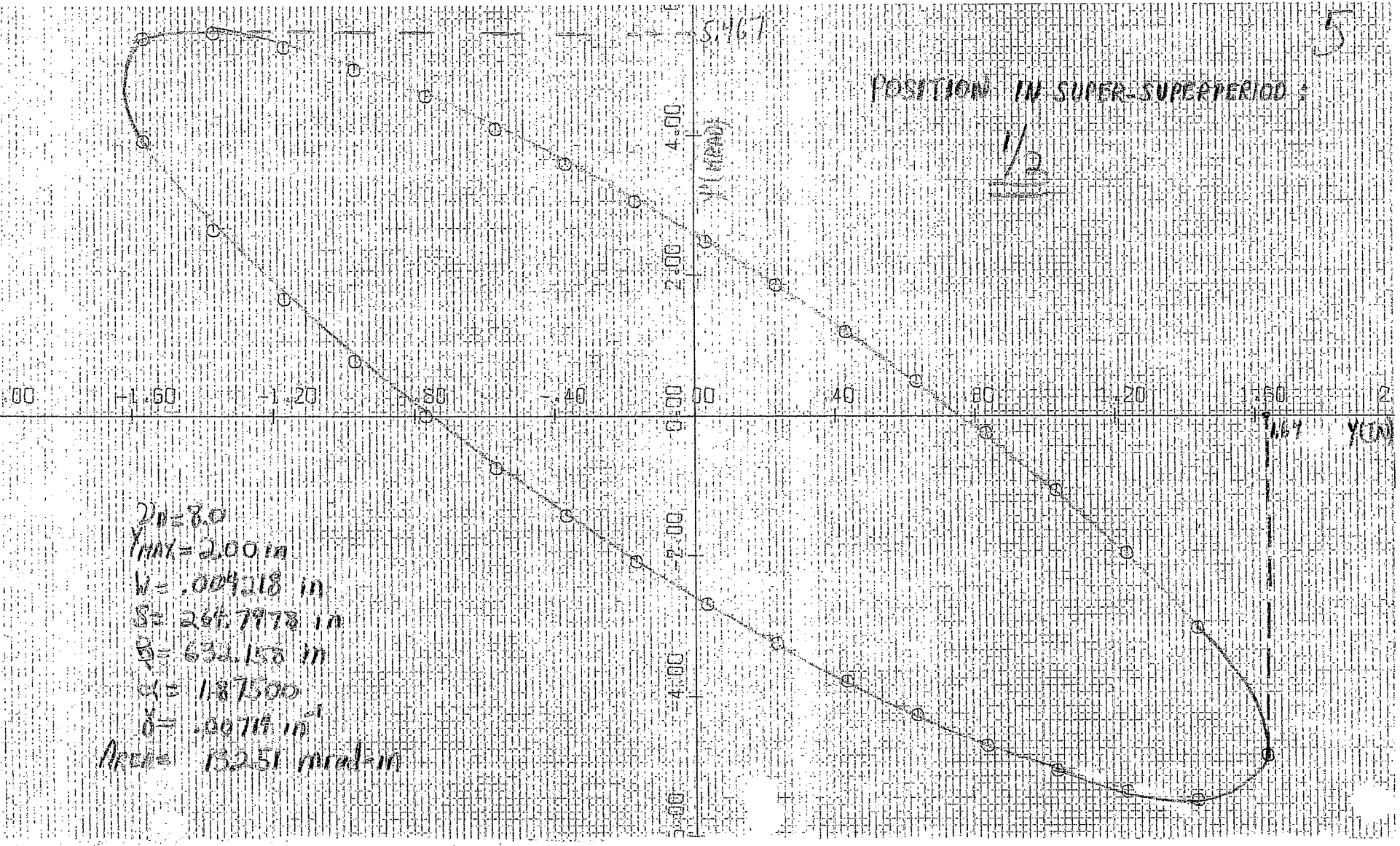
$\alpha = 1.33235$

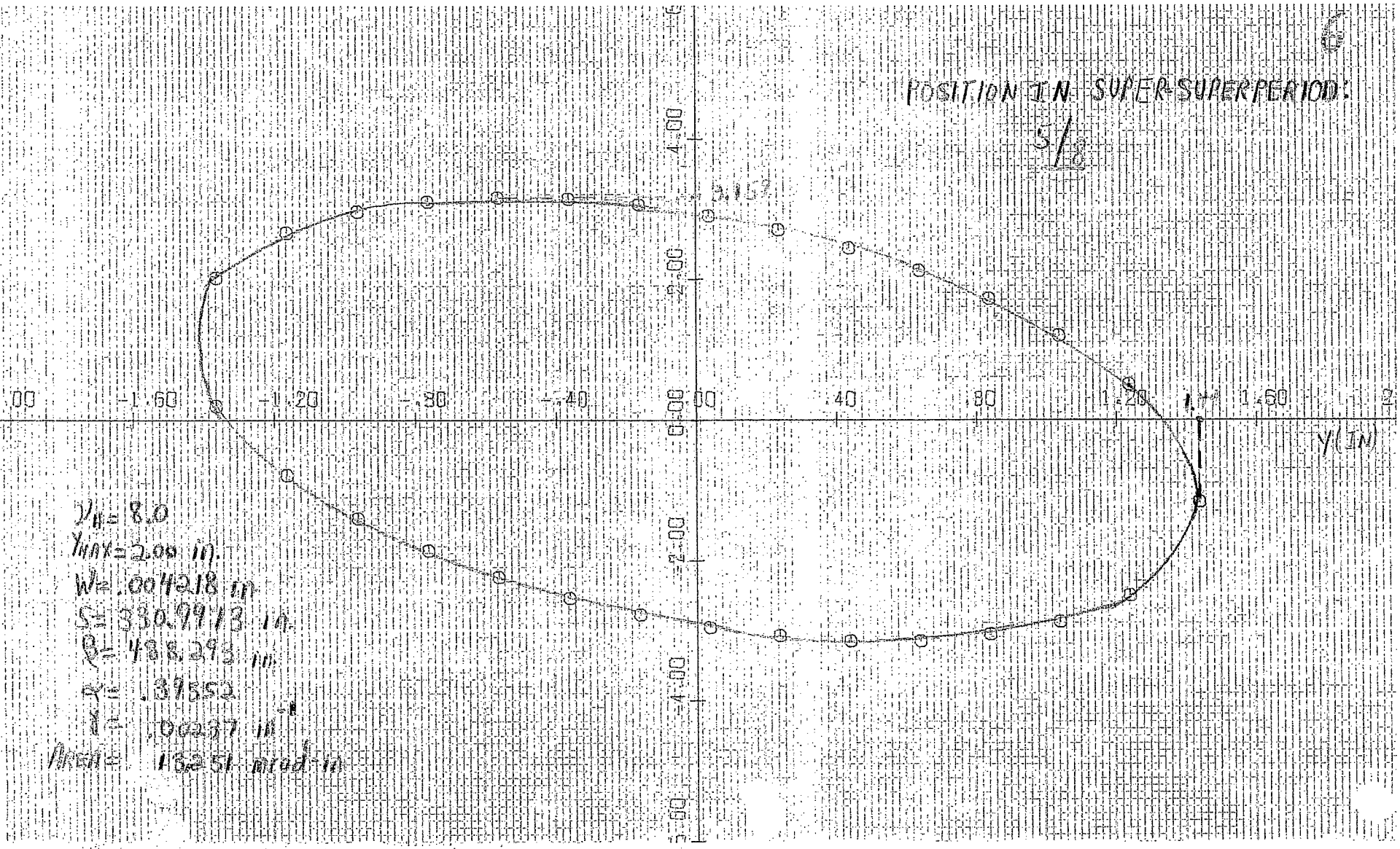
$\delta = .00810$ in^{-1}

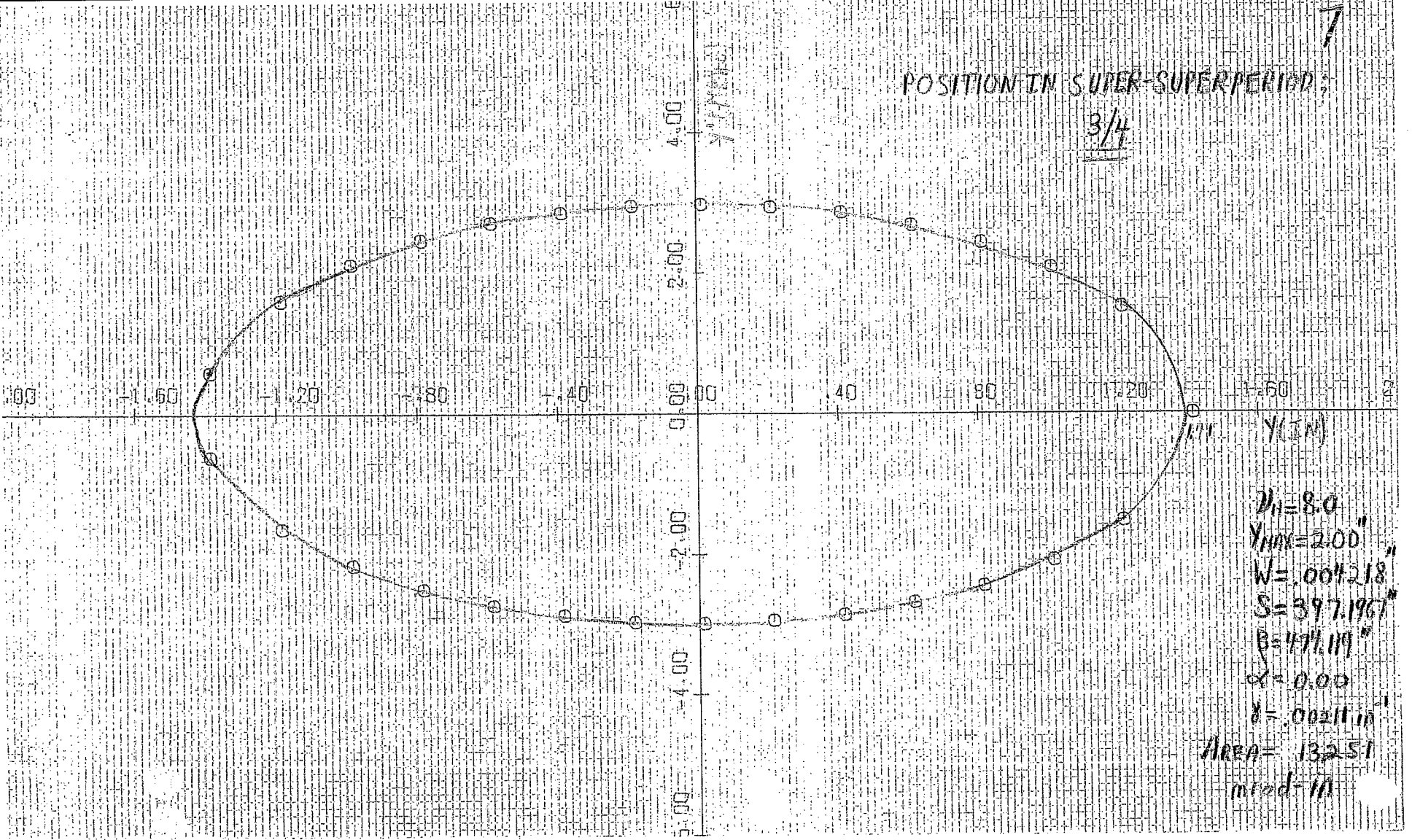
Area = 018.251 mrad-in

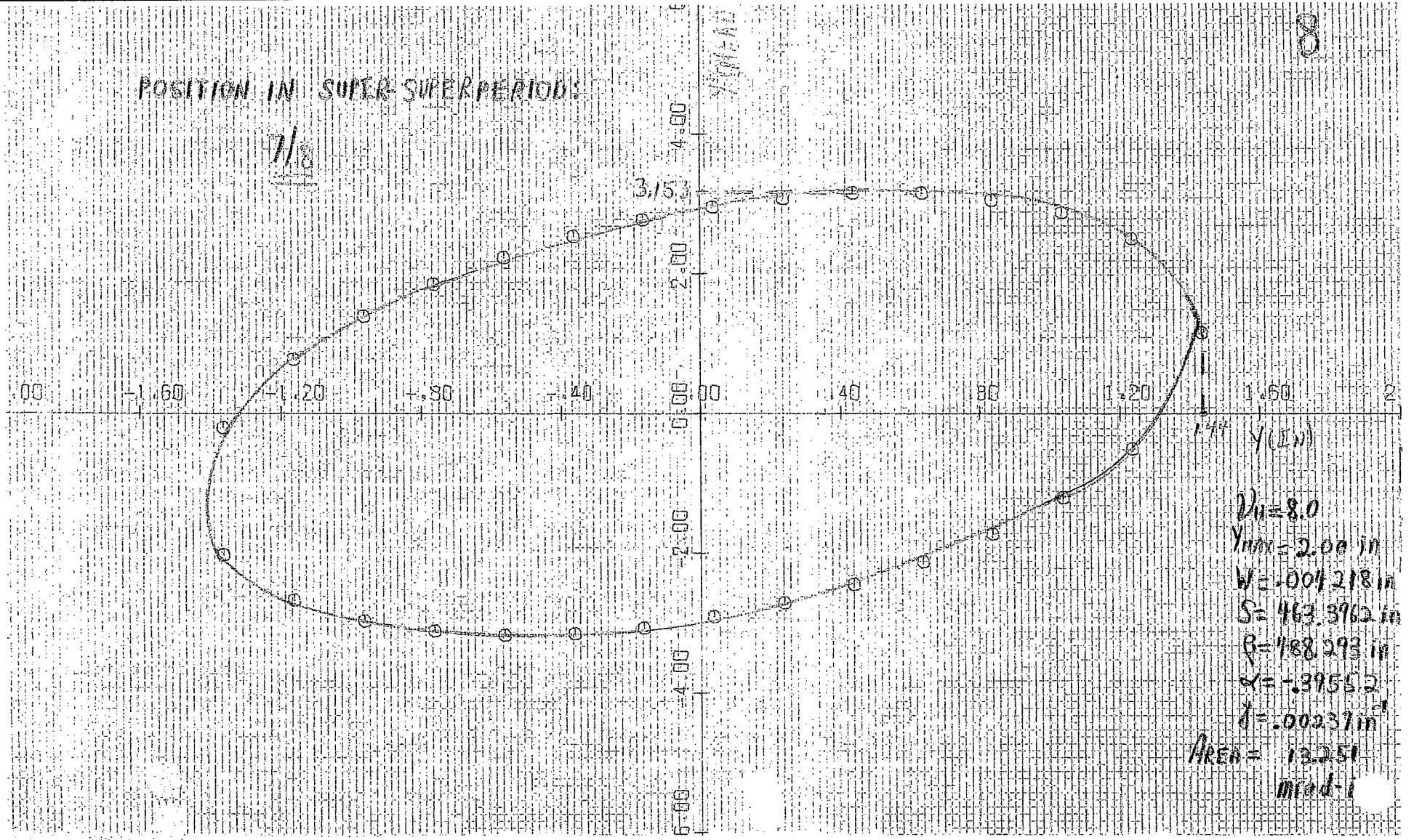

3
POSITION IN SUPER-SUPER PERIOD:
1/4
Y(IN)
-2.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
4.00
2.00
-2.00
-4.00
ν_{11}= 8.0
Y_{MAX}= 2.00 in
W= .004218 in
S= 132.3989 in
ρ= 948.237 in
α= 0.00
γ= .00105 in^{-1}
AREA= 13251


4
POSITION IN SUPER-SUPERPERIOD:
3/8
Y'(MRAD)
4.00
3.602
2.00
0.00
-2.00
-4.00
-6.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
Y(IN)
1.94
$\nu_n = 8.0$
$Y_{MAX} = 2.00$ in
$W = .004218$ in
$S = 198.5984$ in
$\beta = 896.205$ in
$\alpha = 1.33235$
$\gamma = .00310$ in^{-1}
AREA = 13251 mrad-in


5
POSITION IN SUPER-SUPERPERIOD :
1/2
5.467
4.00
2.00
0.00
-2.00
-4.00
-5.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2
1.64
Y(IN)
ν_H = 8.0
Y_{MAX} = 2.00 in
W = .004218 in
S = 264.7978 in
β = 632.158 in
α = 1.87500
γ = .00719 in^{-1}
AREA = 13.251 mrad-in

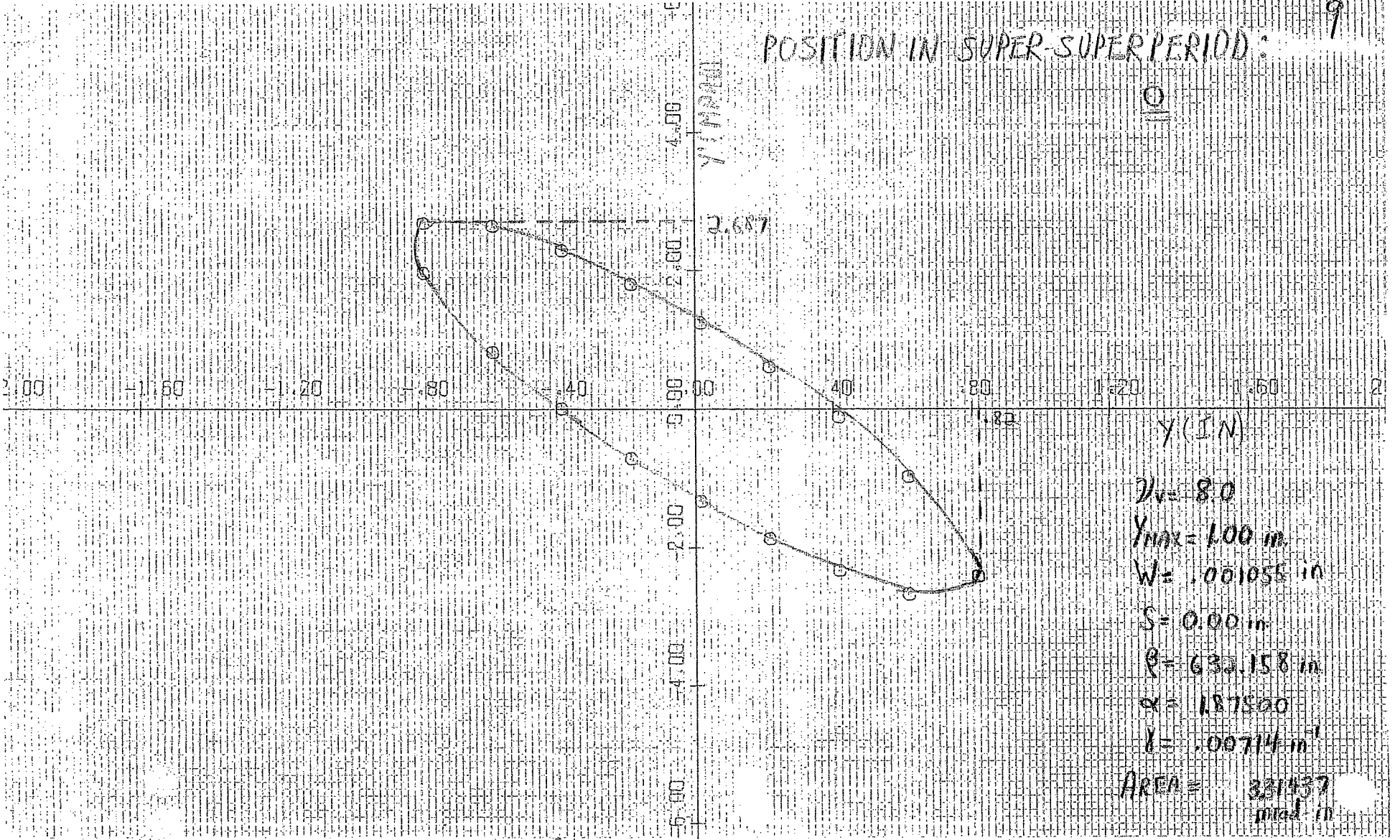

6
POSITION IN SUPER-SUPERPERIOD:
5/8
4.00
2.00
0.00
-2.00
-4.00
-5.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
Y(IN)
ν_H = 8.0
Y_{MAX} = 2.00 in.
W = .004218 in
S = 330.9973 in.
β = 488.293 in
α = .39552
γ = .00237 in^{-1}
AREA = 13.251 mrad-in


7
POSITION IN SUPER-SUPERPERIOD;
3/4
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
Y(IN)
ν_H=8.0
Y_{MAX}=2.00"
W=.004218"
S=397.1967"
β=474.119"
α=0.00
γ=.00211 in⁻¹
AREA= 13.251
mrad-in



8
POSITION IN SUPER-SUPERPERIOD:
7/8
3.153
4.00
2.00
0.00
-2.00
-4.00
-6.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
1.44
Y(IN)
$\nu_H = 8.0$
$Y_{MAX} = 2.00$ in
W = .004218 in
S = 463.3962 in
β = 488.293 in
α = -.3955 2
γ = .00237 in⁻¹
AREA = 13.251
mrad-i

POSITION IN SUPER-SUPER PERIOD: 0



Y(IN)

$\mathcal{D}_V = 8.0$

$Y_{MAX} = 1.00$ in.

$W = .001055$ in

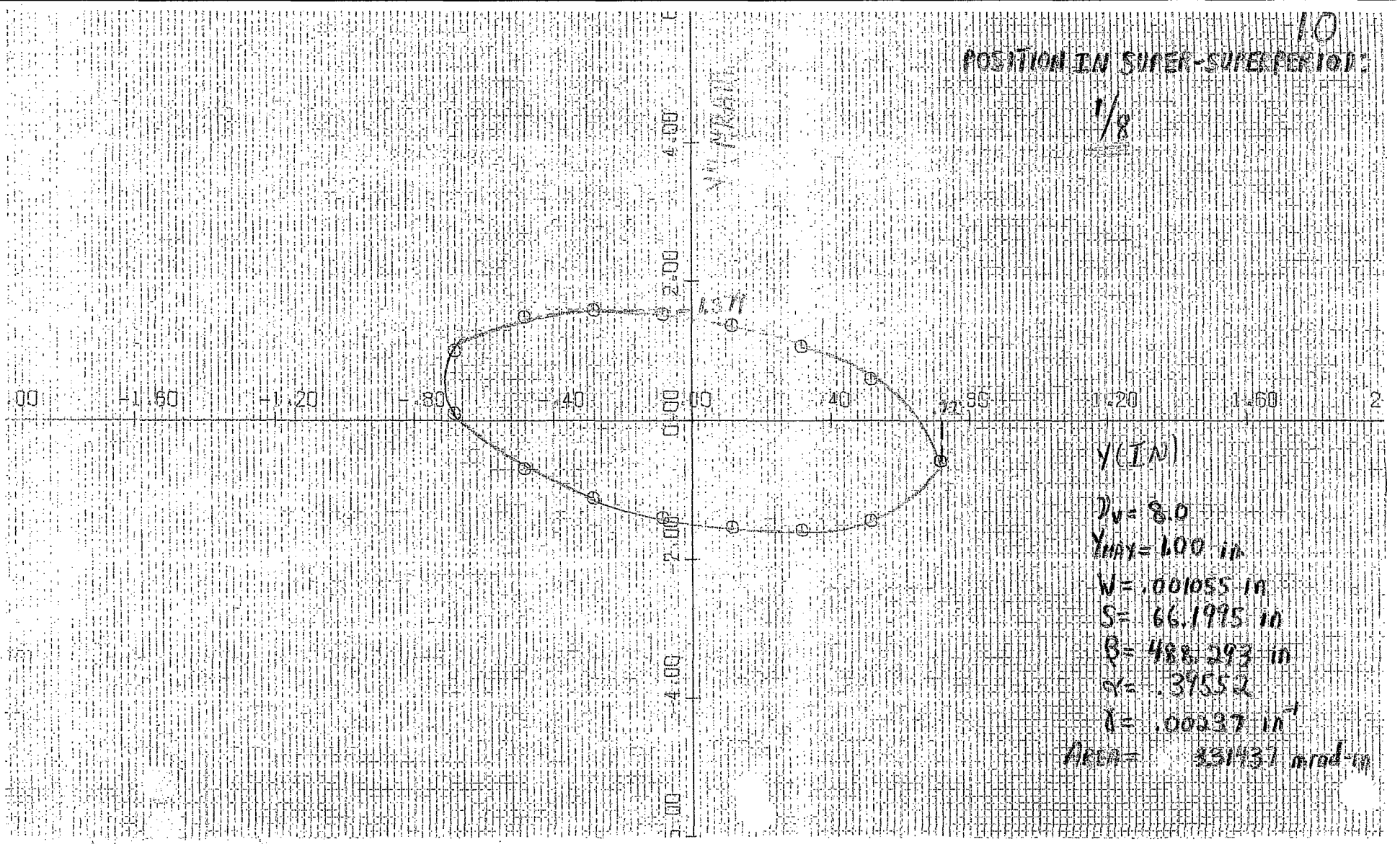
$S = 0.00$ in

$\beta = 633.158$ in

$\alpha = 1.87500$

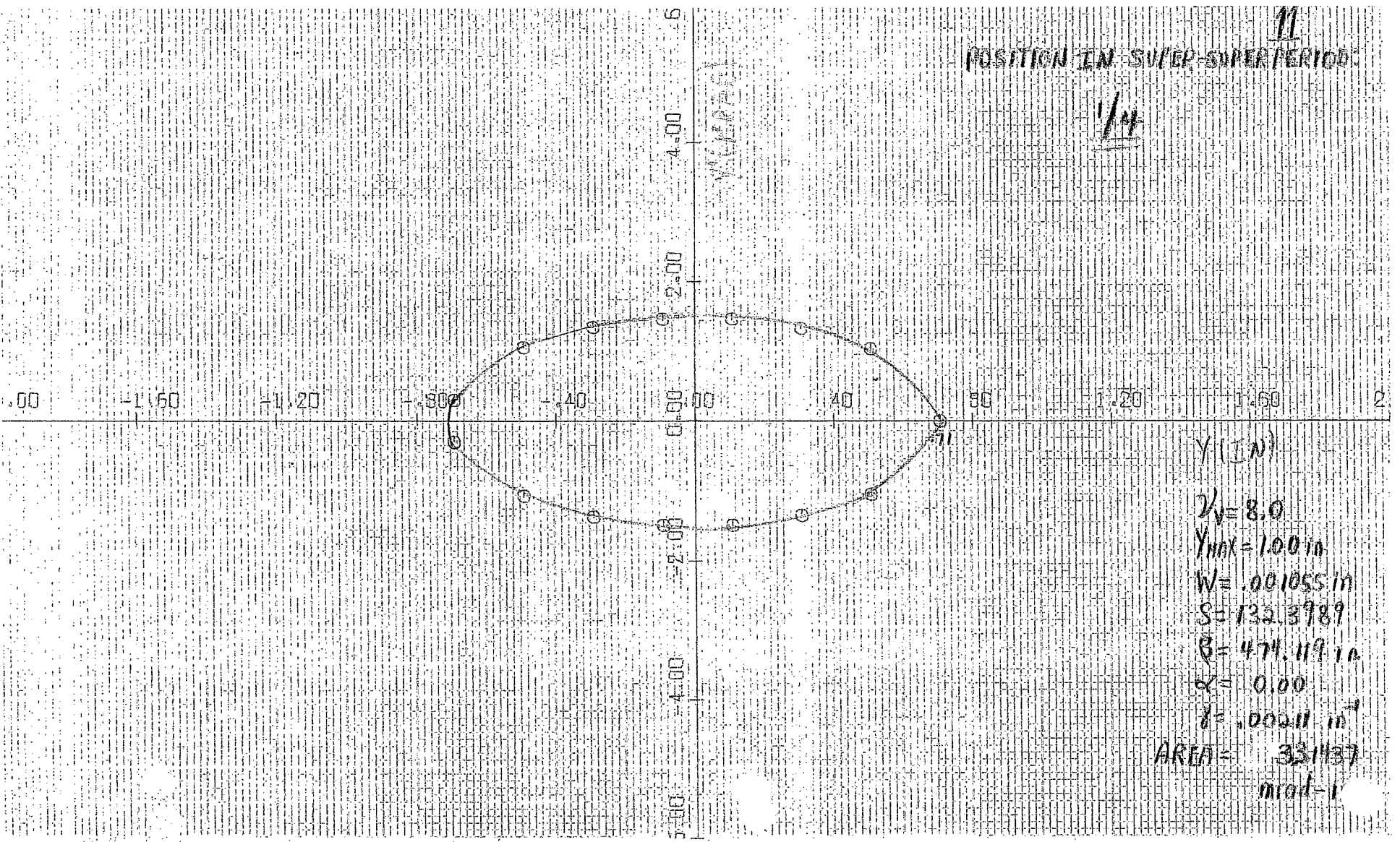
$\delta = .00714$ in^{-1}

AREA = 331437 mod in


10
POSITION IN SUPER-SUPERPERIOD:
1/8
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2
Y(IN)
ν_v = 8.0
Y_{MAX} = 1.00 in
W = .001055 in
S = 66.1995 in
β = 488.293 in
α = .39552
γ = .00237 in^{-1}
AREA = 3.31437 mrad-in

POSITION IN SUPER-SUPERPERIOD

1/4



$\nu_V = 8.0$

$Y_{MAX} = 1.001$ in

$W = .001055$ in

$S = 132.3787$

$\beta = 471.1916$

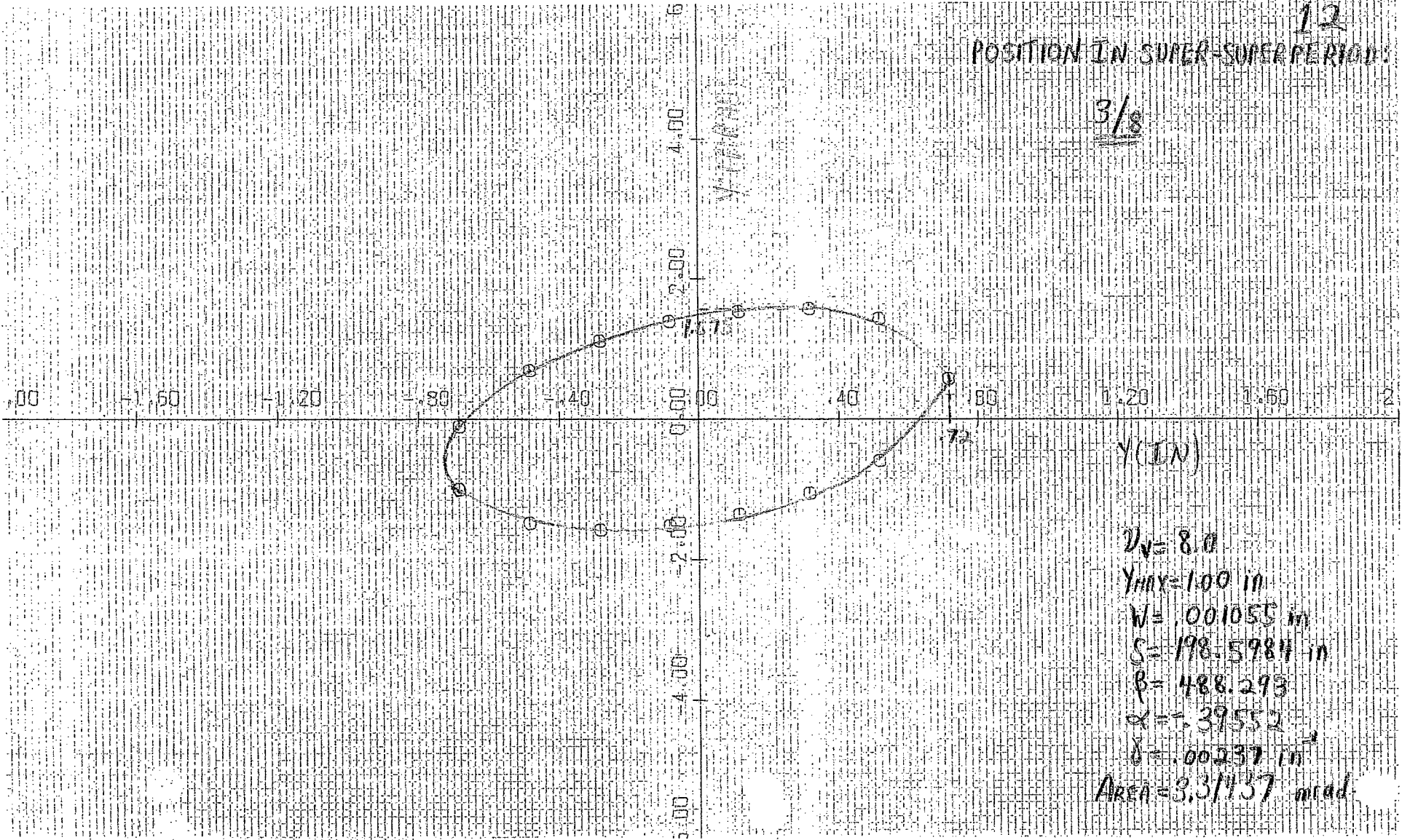
$\alpha = 0.00$

$\delta = .0022$ in^{-1}

AREA = 33.1137 mrad-r

POSITION IN SUPER-SUPERPERIOD

3/8



$D_V = 8.0$

$Y_{MAX} = 1.00$ in

$W = .001055$ in

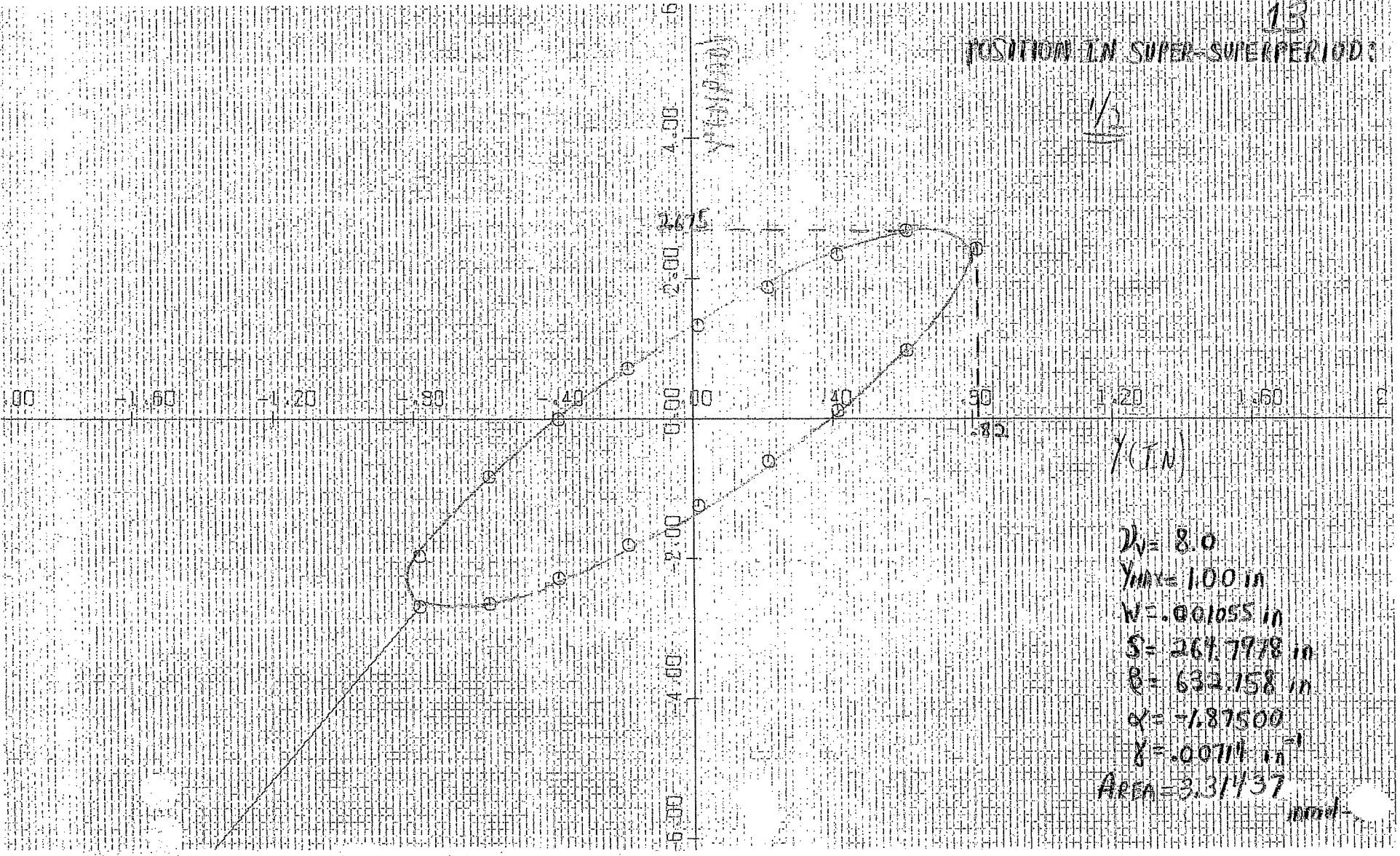
$\zeta = 198.5984$ in

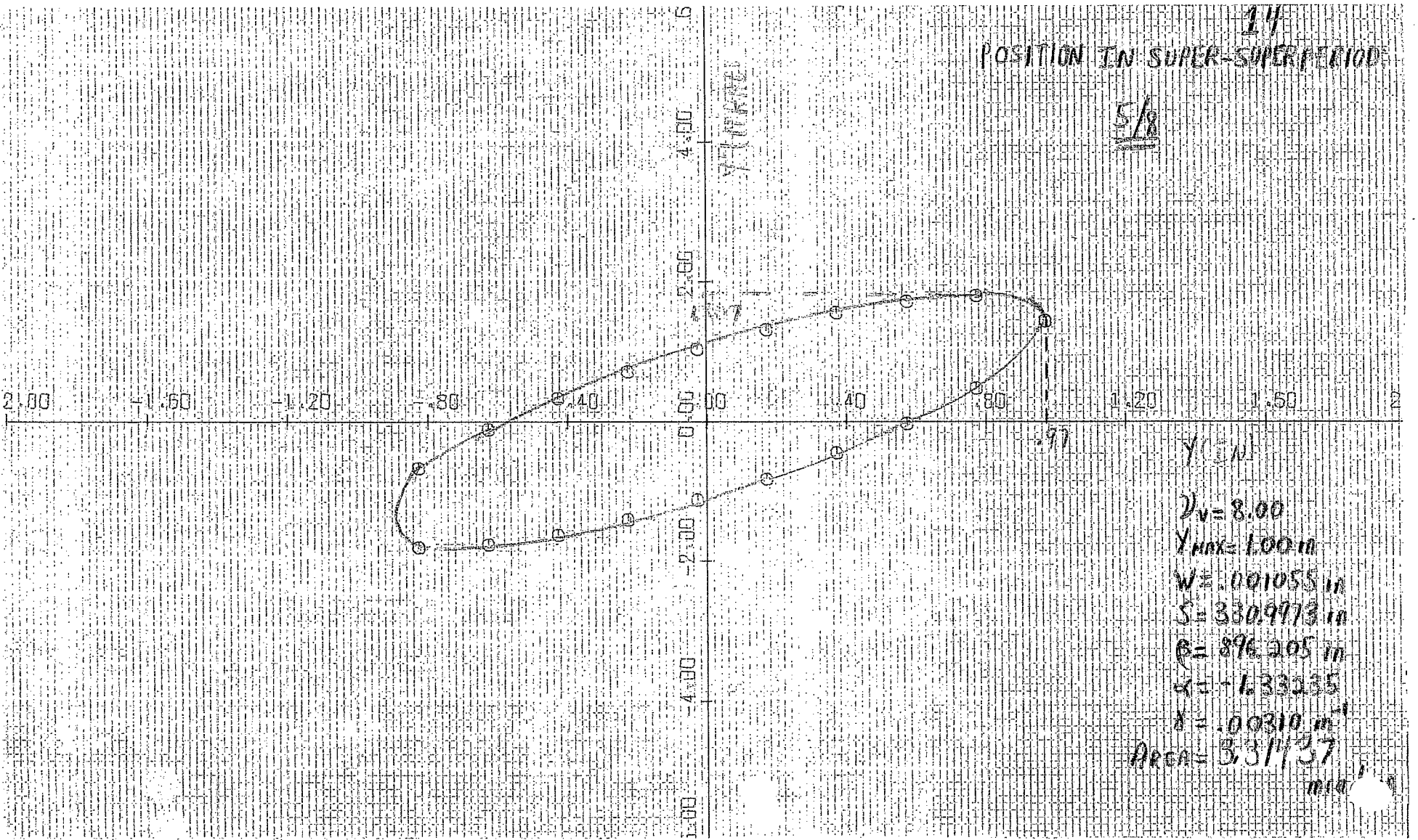
$\beta = 488.293$

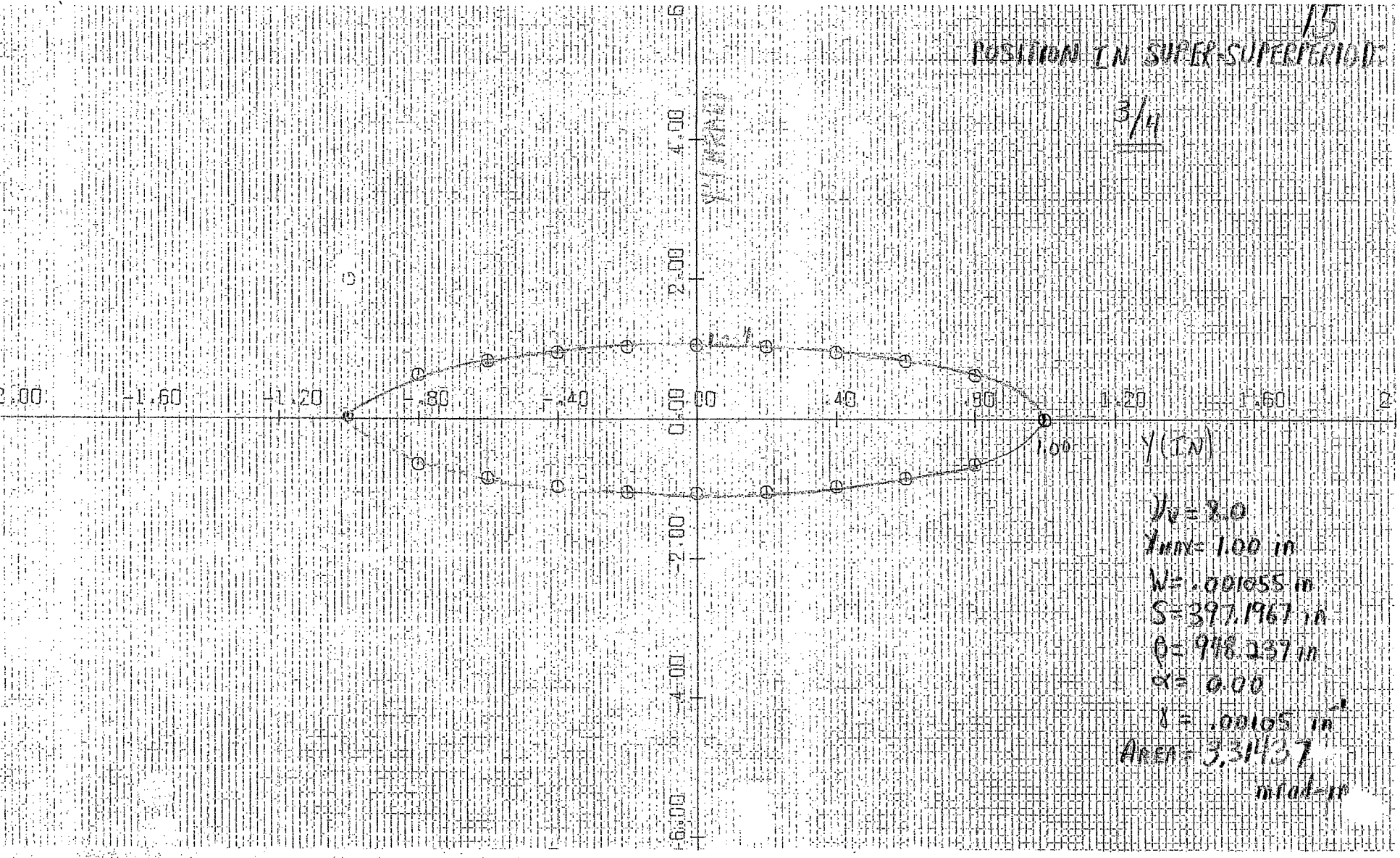
$\alpha = -.39552$

$\delta = .00237$ in

AREA = 3.31437 mrad


13
POSITION IN SUPER-SUPERPERIOD:
1/3
Y(IN)
2.675
.82
ν_V = 8.0
Y_{MAX} = 1.00 in
W = .001055 in
S = 264.7978 in
β = 632.158 in
α = -1.87500
γ = .00714 in^{-1}
AREA = 3.31437
mrad


14
POSITION IN SUPER-SUPERPERIOD
5/8
Y(IN)
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
.97
ν_v = 8.00
Y_{MAX} = 1.00 in
W = .001055 in
S = 330.9973 in
β = 896.205 in
α = -1.33235
γ = .00310 m^{-1}
AREA = 3.31437
mm


15
POSITION IN SUPER-SUPERPERIOD:
3/4
Y (IN)
1.00
ν_v = 8.0
Y_{MAX} = 1.00 in
W = .001055 in
S = 397.1967 in
β = 948.237 in
α = 0.00
γ = .00105 in⁻¹
AREA = 3.31437
mrad-in

POSITION IN SUPER-SUPERPERIOD:

7/8

Y (LN)

$\nu_s = 8.0$

$Y_{MAX} = 1.00$ in

$W = .001055$ in

$S_1 = 463.3962$ in

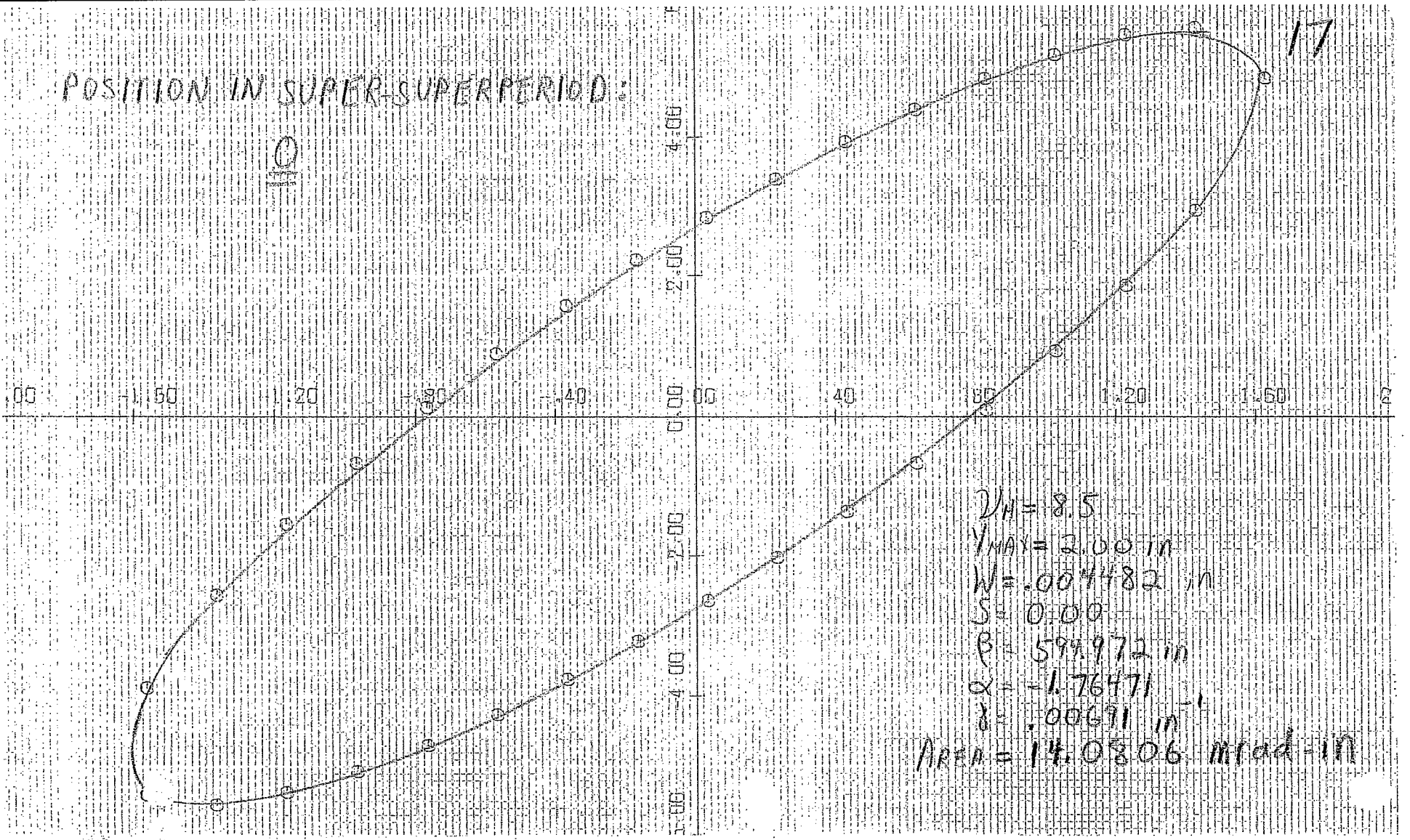
$\beta = 876.205$ in

$\alpha = 1.33235$

$\gamma = .00310$ in^{-1}

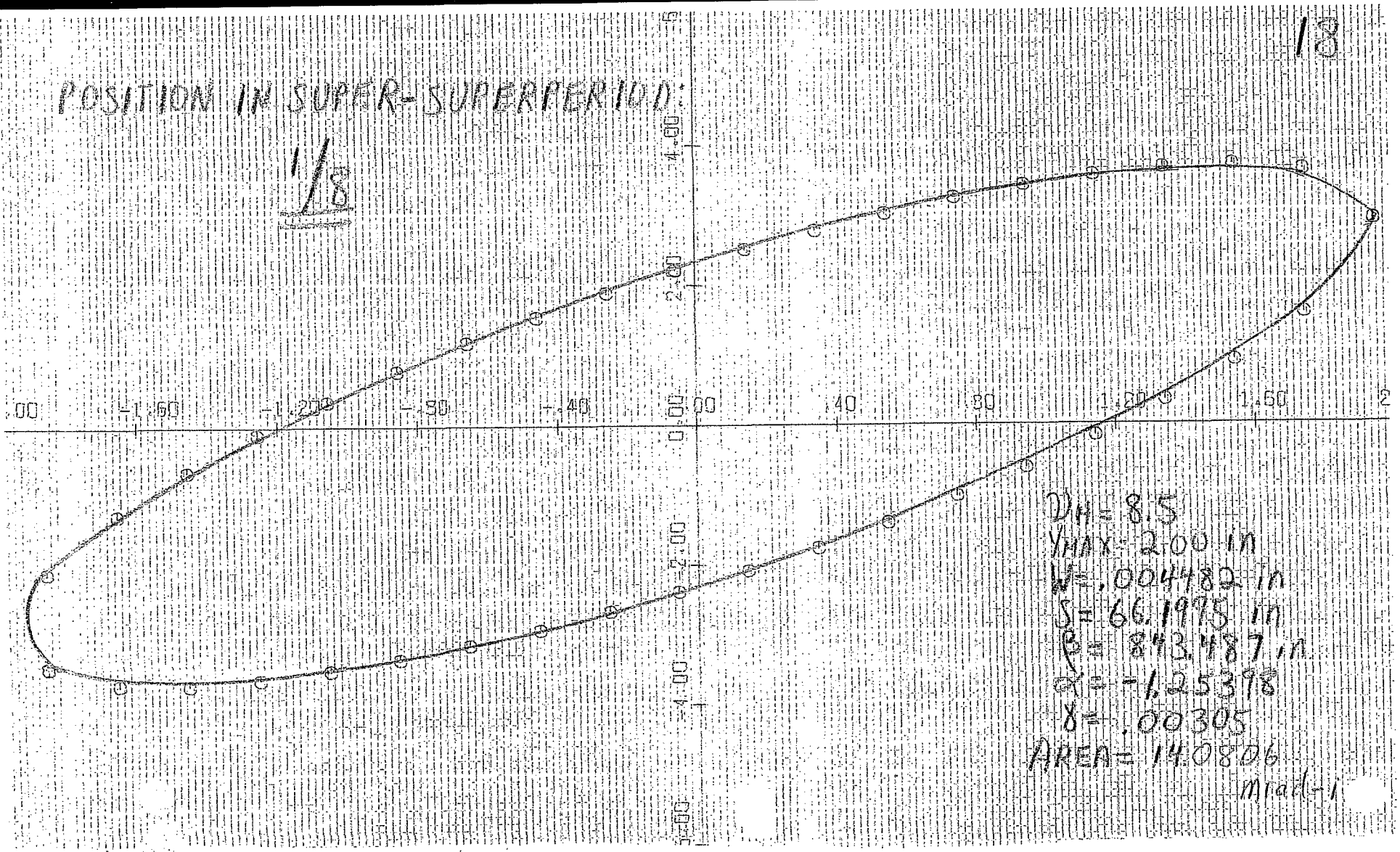
AREA = 3.31737 in^2

POSITION IN SUPER-SUPERPERIOD: $\frac{0}{III}$



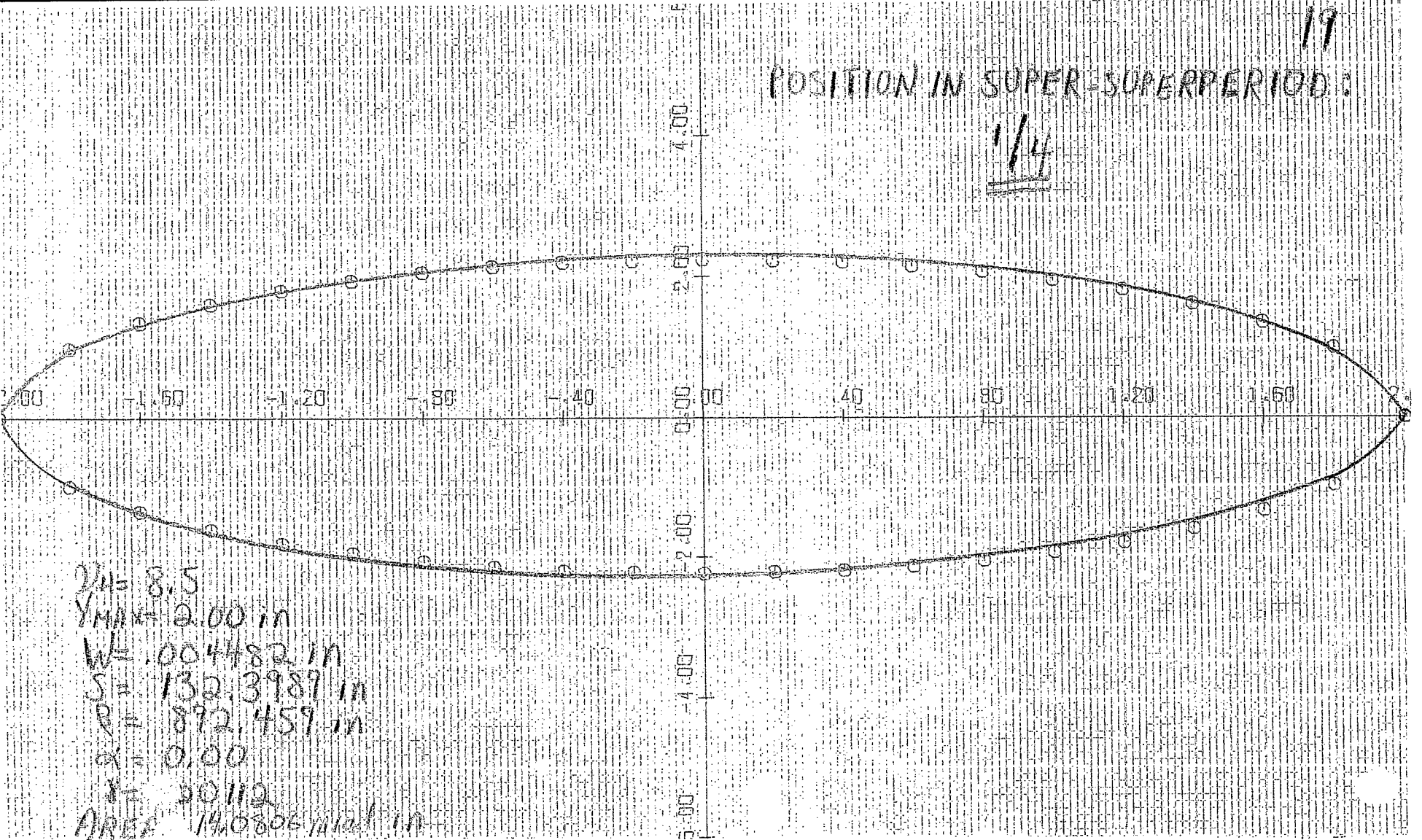
POSITION IN SUPER-SUPERPERIOD

1/18

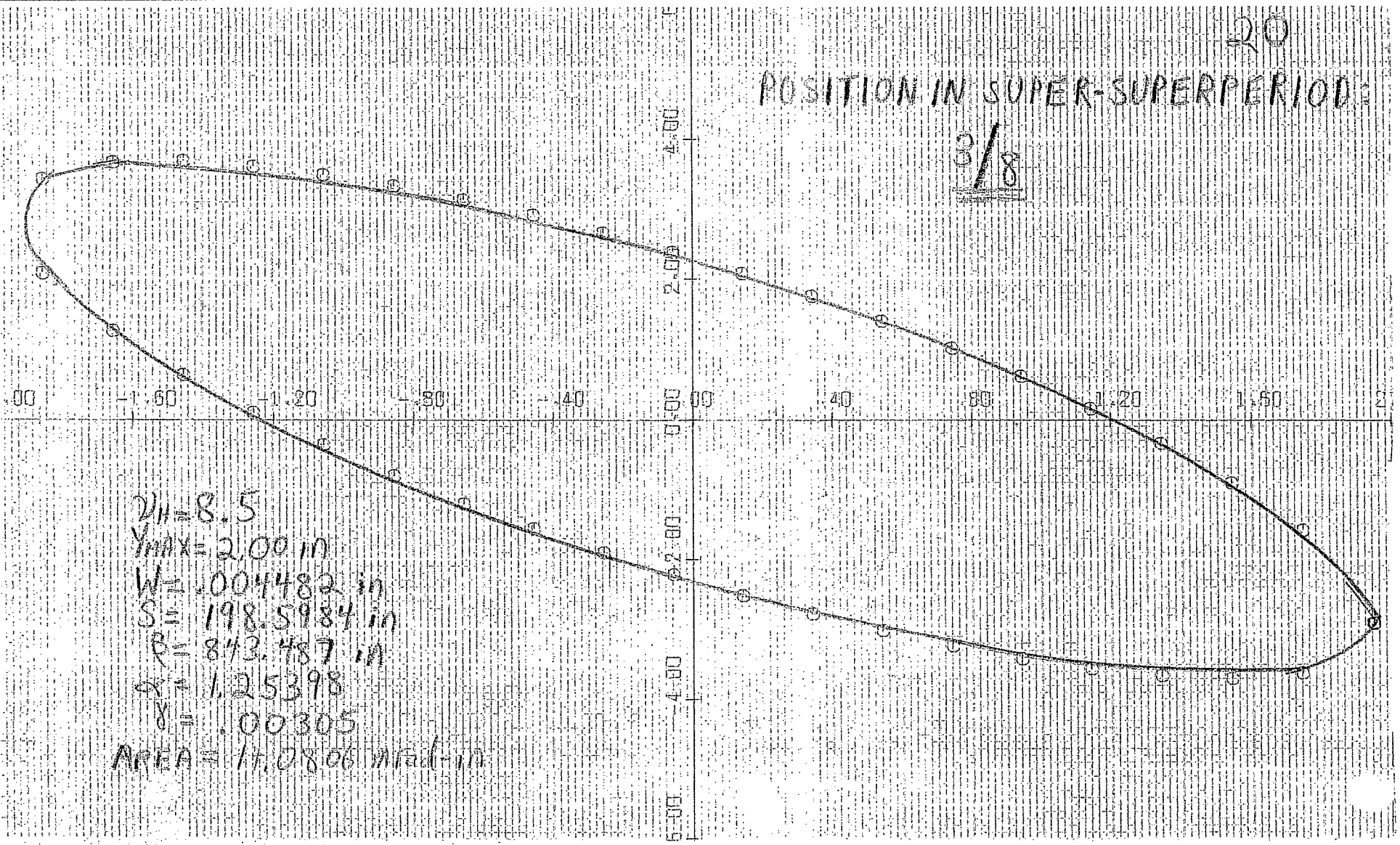


POSITION IN SUPER-SUPERPERIOD:

1/4

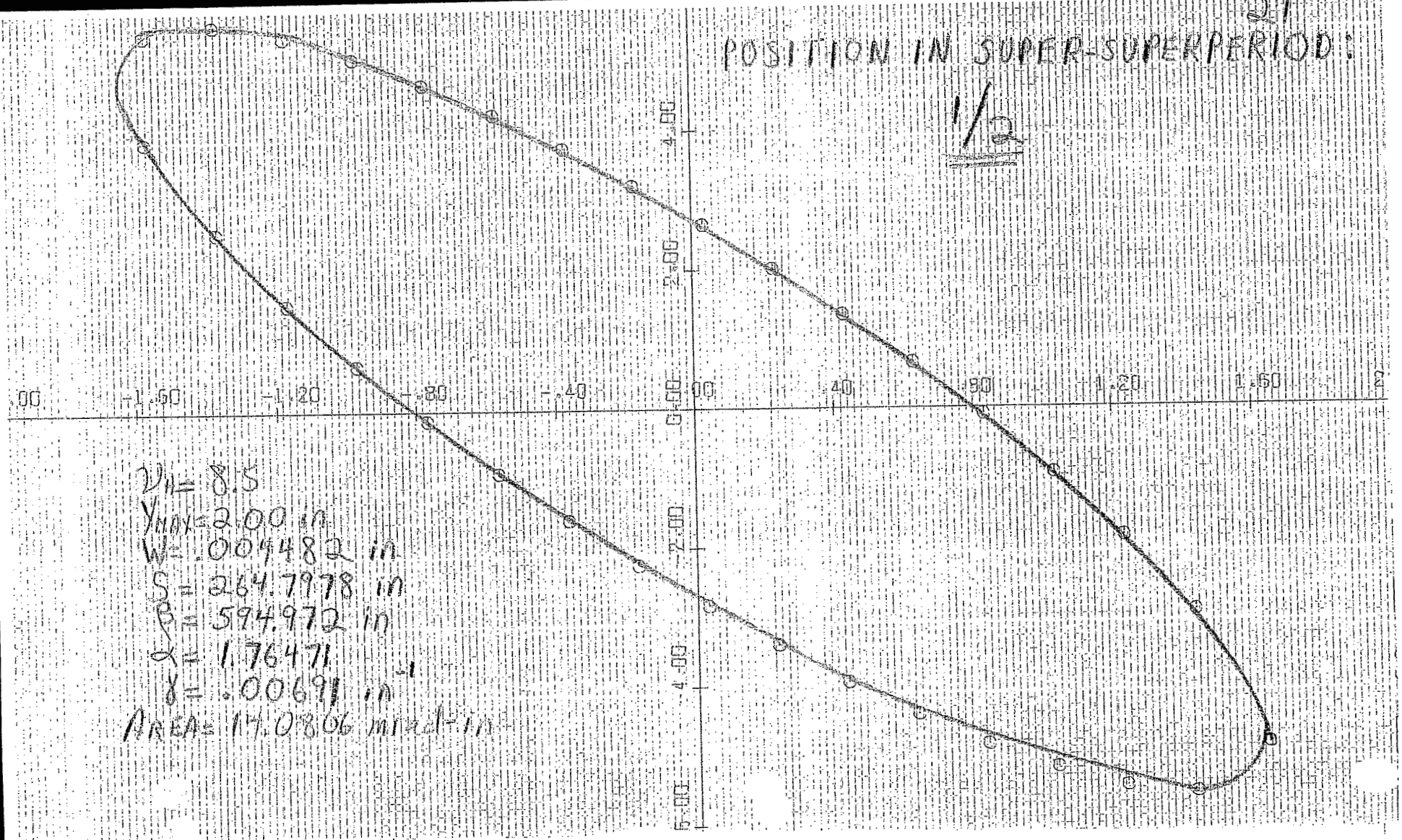


ν_H = 8.5
Y_{MAX} = 2.00 in
W = .004482 in
S = 132.3989 in
ρ = 892.459 in
α = 0.00
γ = 20112
AREA 14.0806 mm² in


20
POSITION IN SUPER-SUPERPERIOD:
3/8
4.00
2.00
0.00
-2.00
-4.00
-6.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
$\nu_H = 8.5$
$Y_{MAX} = 2.00$ in
$W = .004482$ in
$S = 198.5984$ in
$\beta = 843.487$ in
$\alpha = 1.25398$
$\gamma = .00305$
AREA = 14.0806 mrad-in

POSITION IN SUPER-SUPERPERIOD:

1/6



$\nu_H = 8.5$

$Y_{MAX} = 2.00$ in

$W = .004482$ in

$S = 264.7978$ in

$D = 594.970$ in

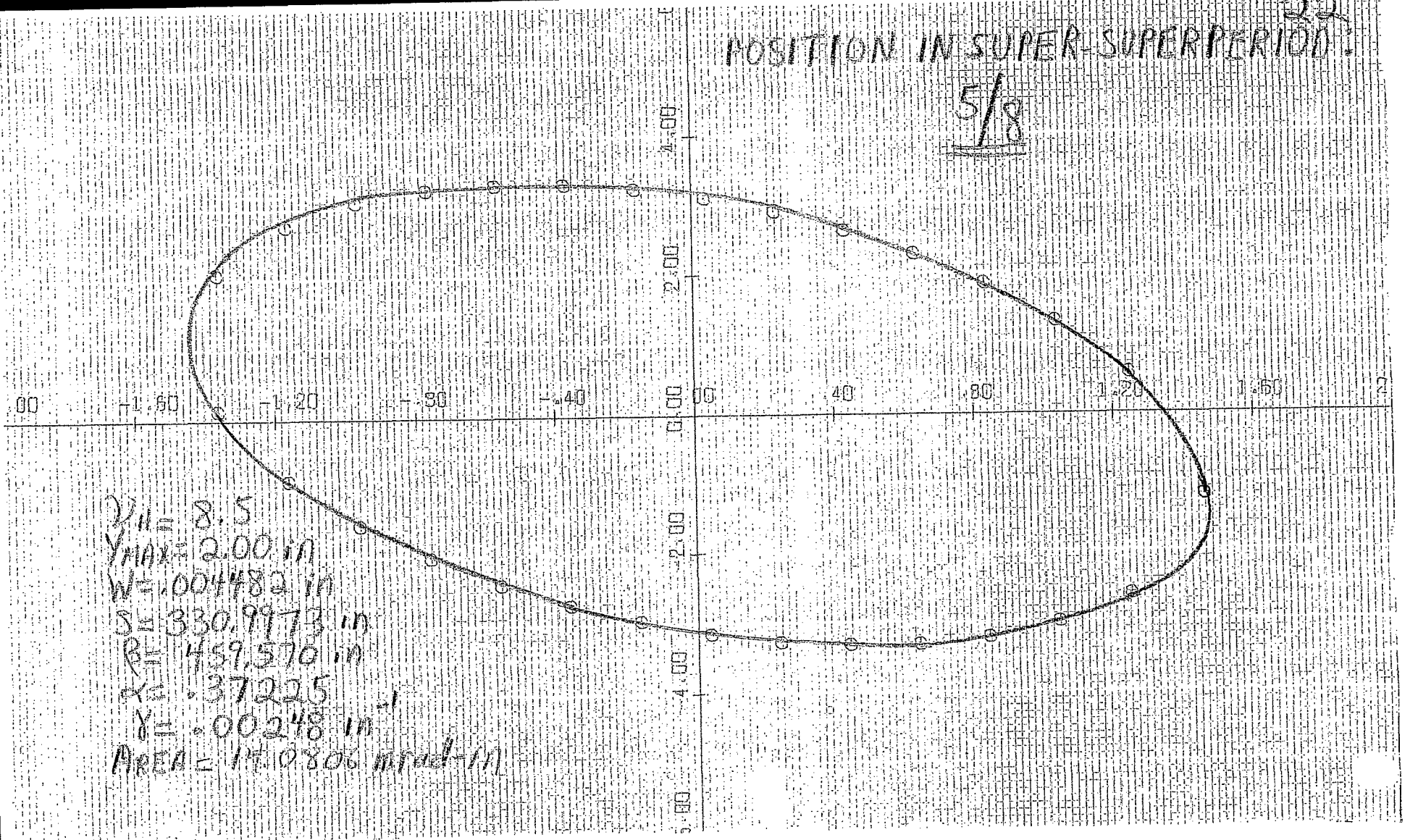
$\alpha = 1.76471$ in^{-1}

$\gamma = .00691$ in

AREA $= 117.0806$ rad-in

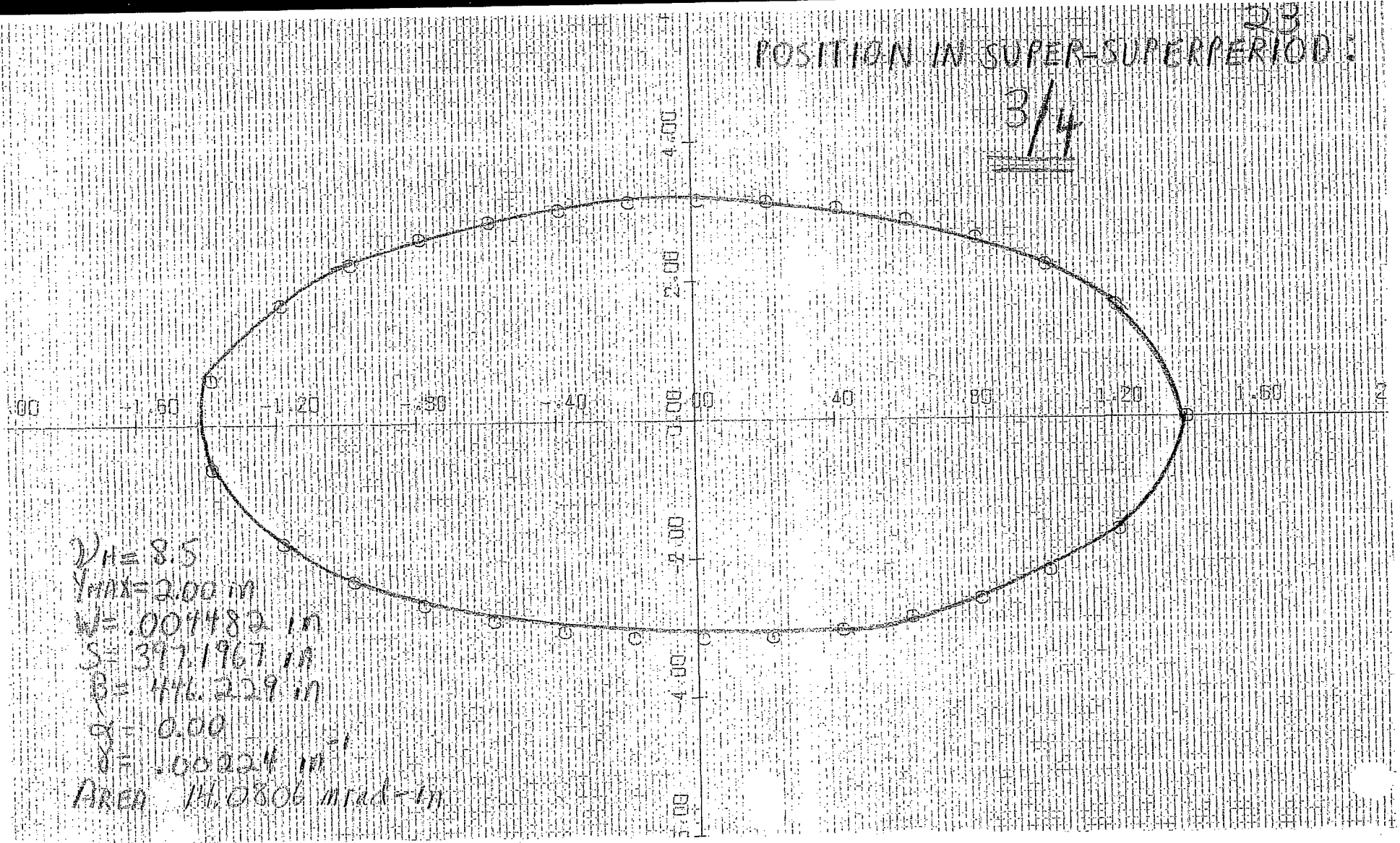
POSITION IN SUPER-SUPERPERIOD:

5/8



$\nu_m = 8.5$
$Y_{max} = 2.00$ in
$W = .004482$ in
$S = 330.9773$ in
$R = 459.570$ in
$\chi_2 = .37225$
$\gamma = .00248$ in^{-1}
AREA $= 11.0806$ mrad-in

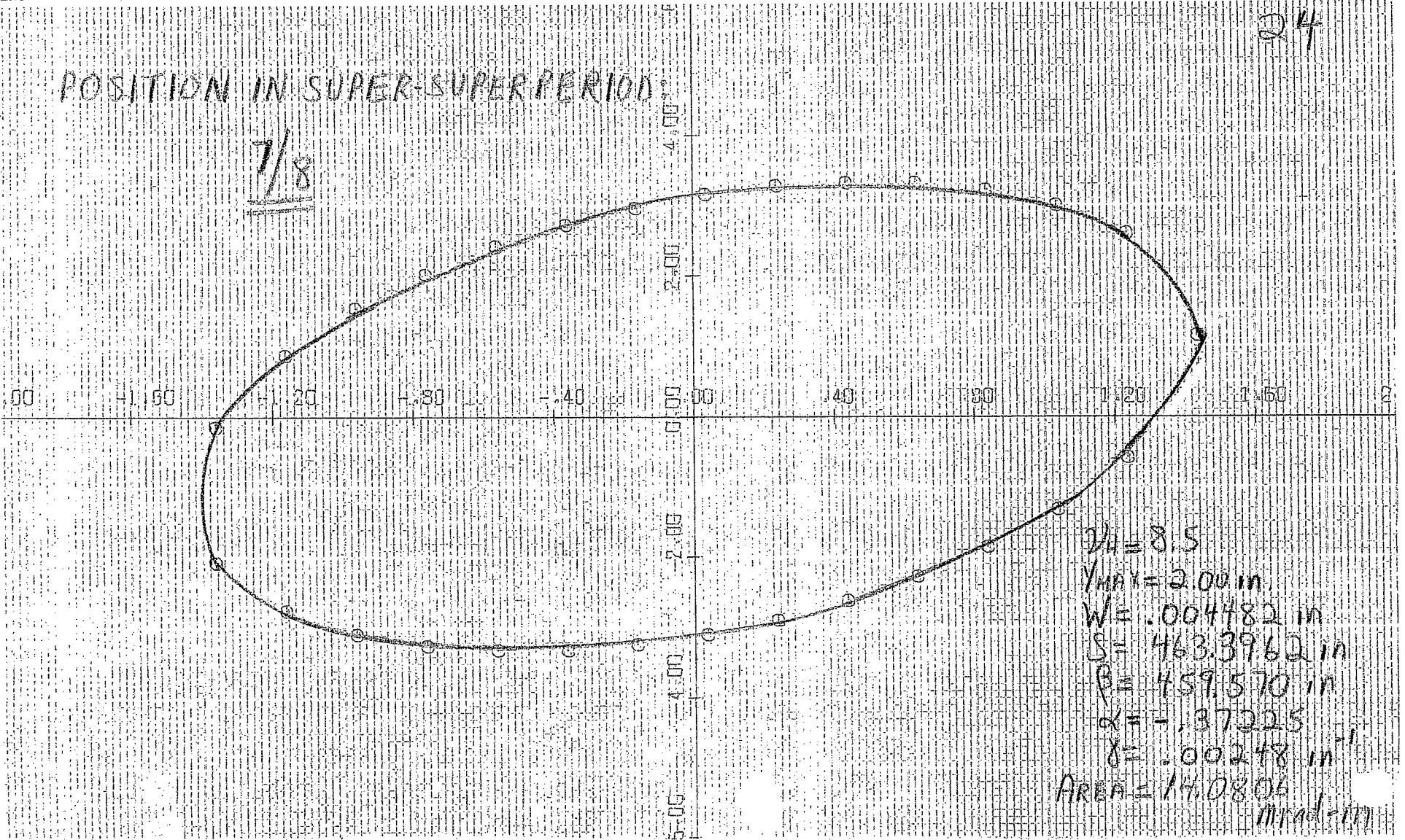
POSITION IN SUPER-SUPERPERIOD: 3/4



$\nu_H = 8.5$
$y_{max} = 2.00$ m
$W = .0074822$ m
$S = 397.1967$ m
$B = 476.3229$ m
$\alpha = 0.00$
$\delta = .00224$ m^{-1}
AREA 111.0806 mrad-m

POSITION IN SUPER-SUPER PERIOD:

7/8



POSITION IN SUPER-SUPERPERIOD:

$2\nu = 8.5$

$Y_{MAX} = 1.00$ in

$WE = .001121$ in

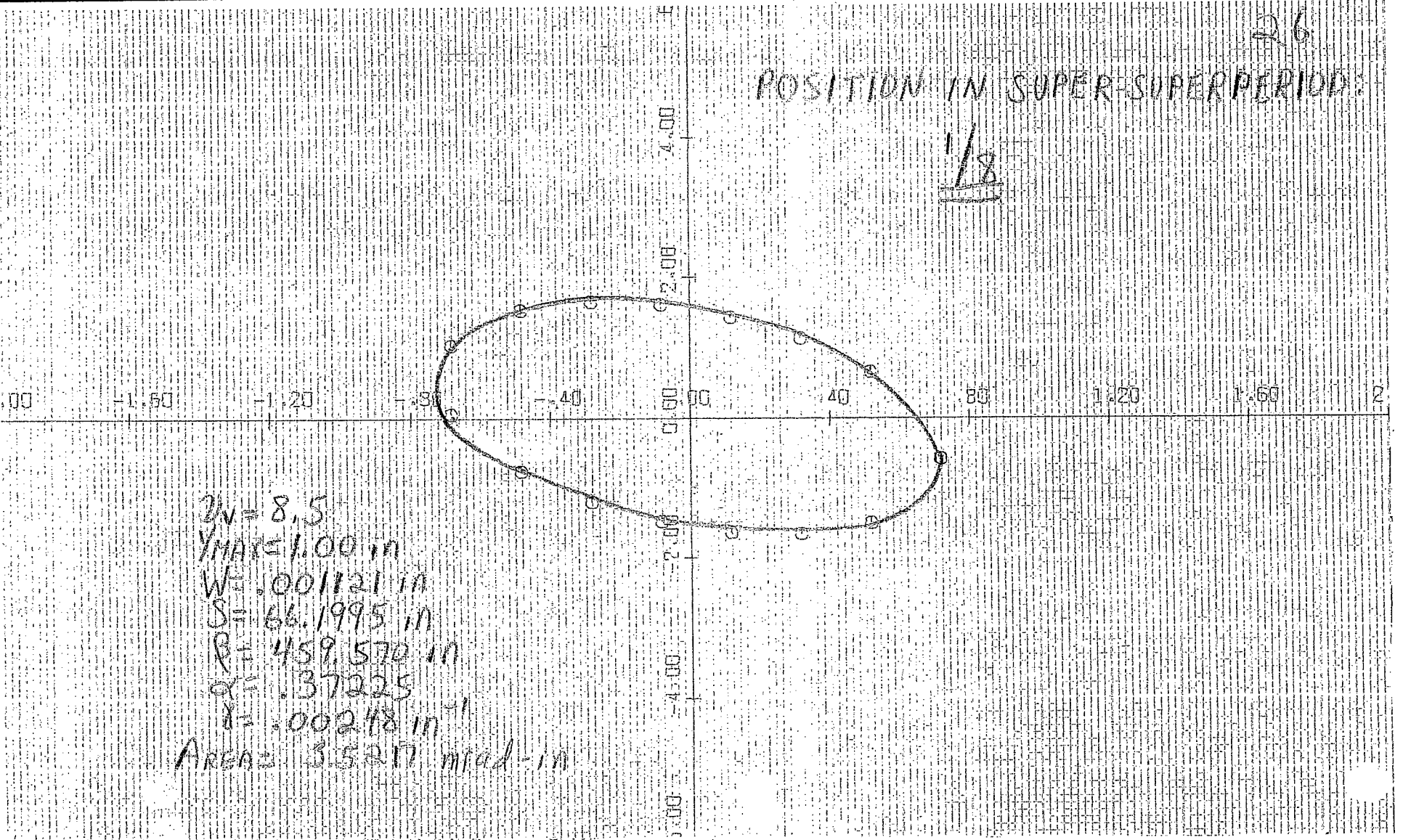
$S = 0.00$

$R = 594.973$ in

$\alpha = 1.76471$

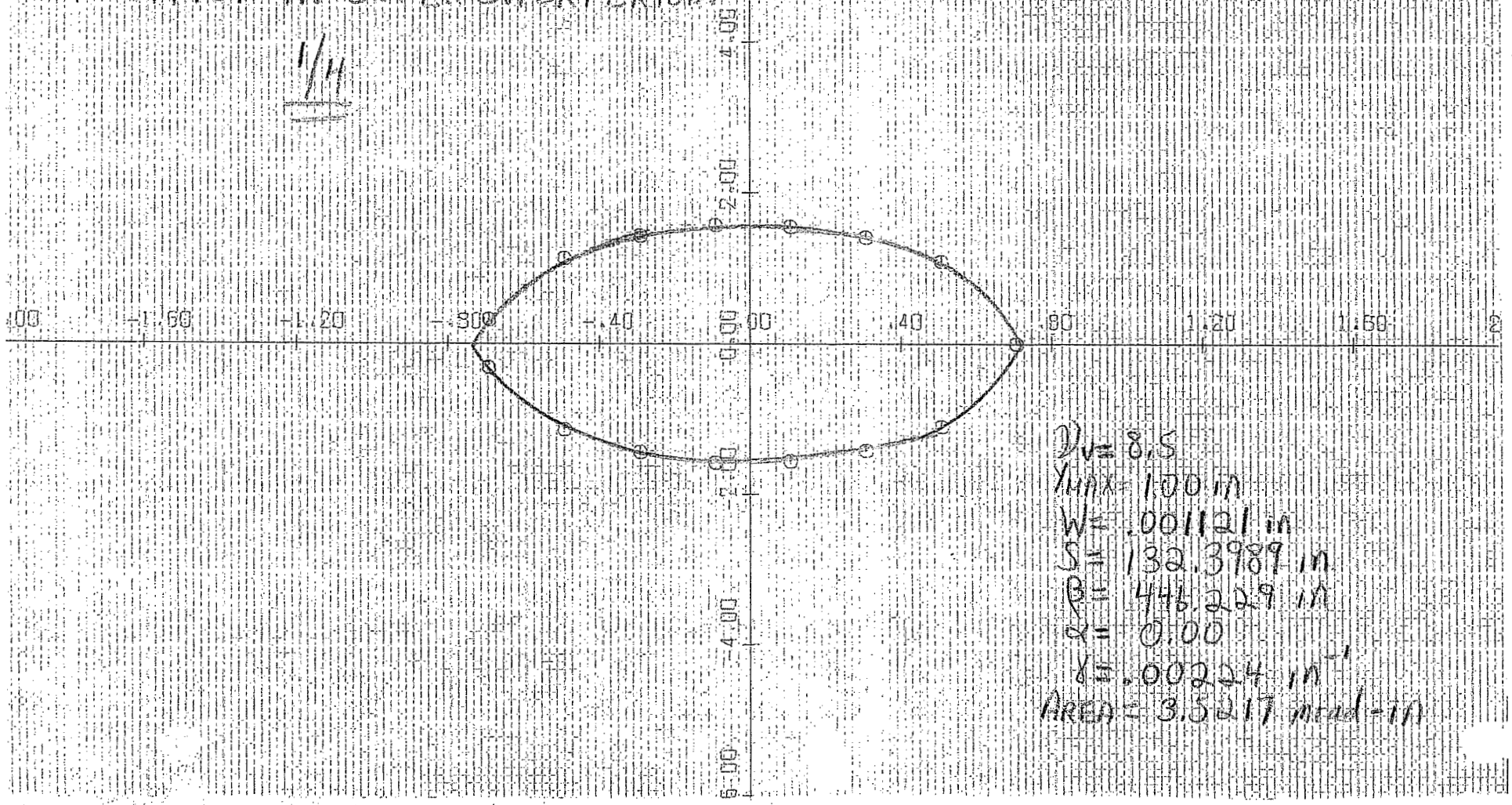
$\beta = .00691$ in^{-1}

AREA = 3.5217 mrad-in


POSITION IN SUPER-SUPERPERIOD:
1/8
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
2
$\nu_v = 8.5$
$Y_{MAX} = 1.00$ in
$W = .001121$ in
$S = 66.1995$ in
$\beta = 459.570$ in
$\alpha = .37225$
$\gamma = .00248$ in^{-1}
AREA = 3.5217 mrad-in

POSITION IN SUPER-SUPERPERIOD:

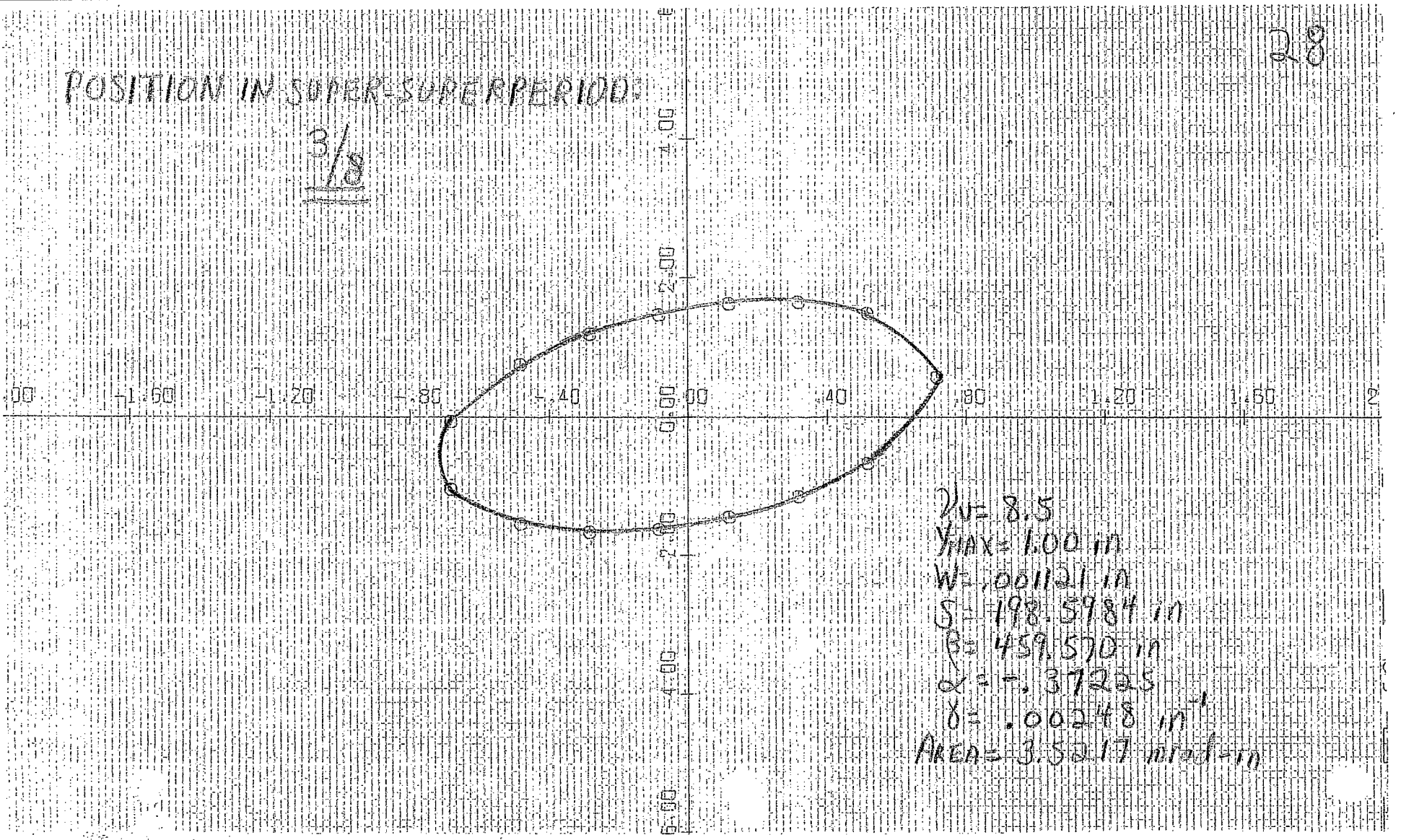
1/4



$\nu_v = 8.5$
$Y_{MAX} = 1.00$ in
$W = .001121$ in
$S = 132.3989$ in
$\beta = 446.229$ in
$\alpha = 0.00$
$\gamma = .00224$ in^{-1}
AREA = 3.5217 mrad-in

POSITION IN SUPER-SUPERPERIOD:

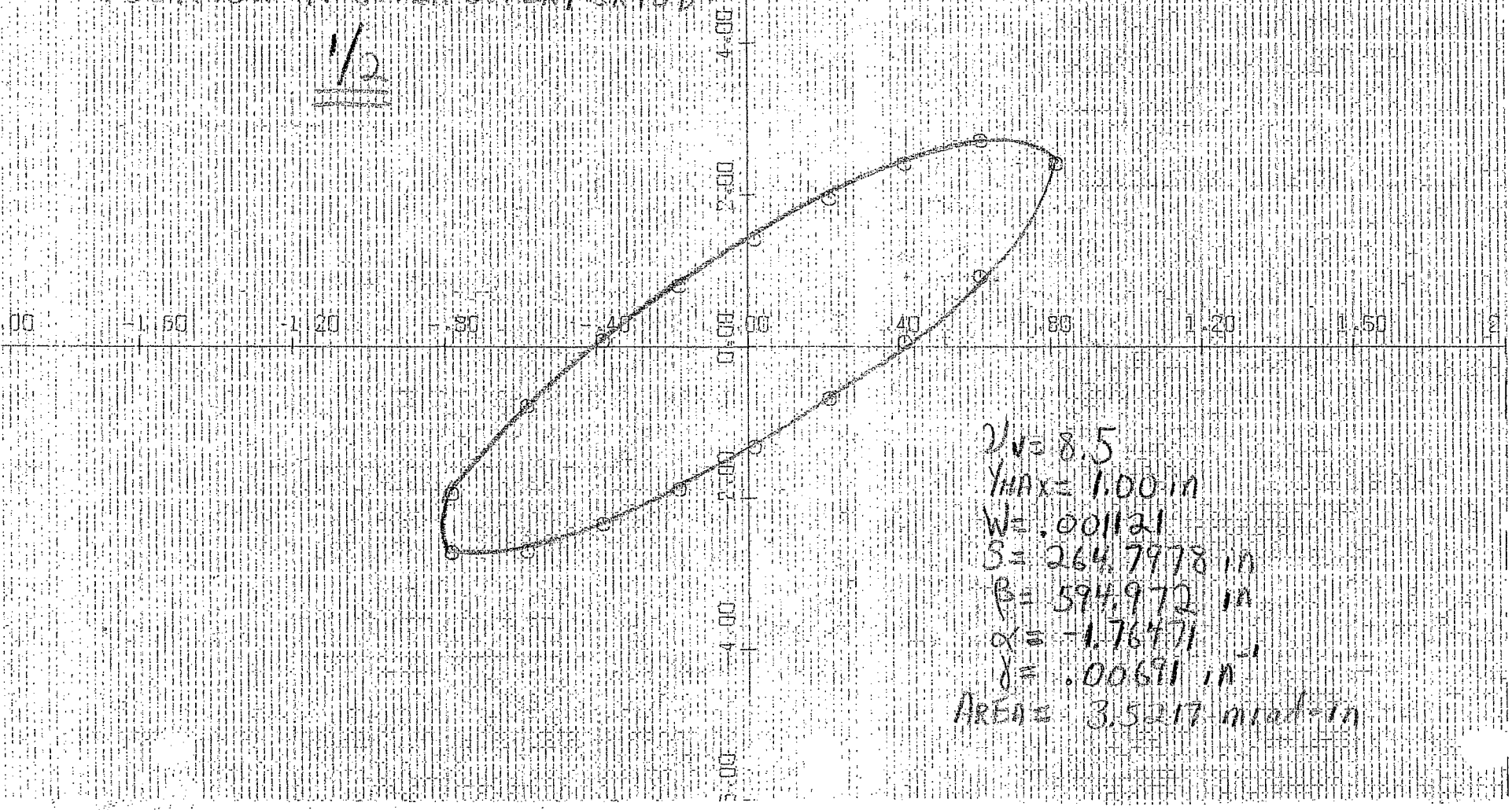
3/8



$2N = 8.5$
$Y_{MAX} = 1.00$ in
$W = .00121$ in
$S = 198.5984$ in
$\beta = 459.520$ in
$\alpha = -1.37295$
$\delta = .00248$ in^{-1}
AREA $= 3.5017$ in^2-in

POSITION IN SUPER-SUPERPERIOD:

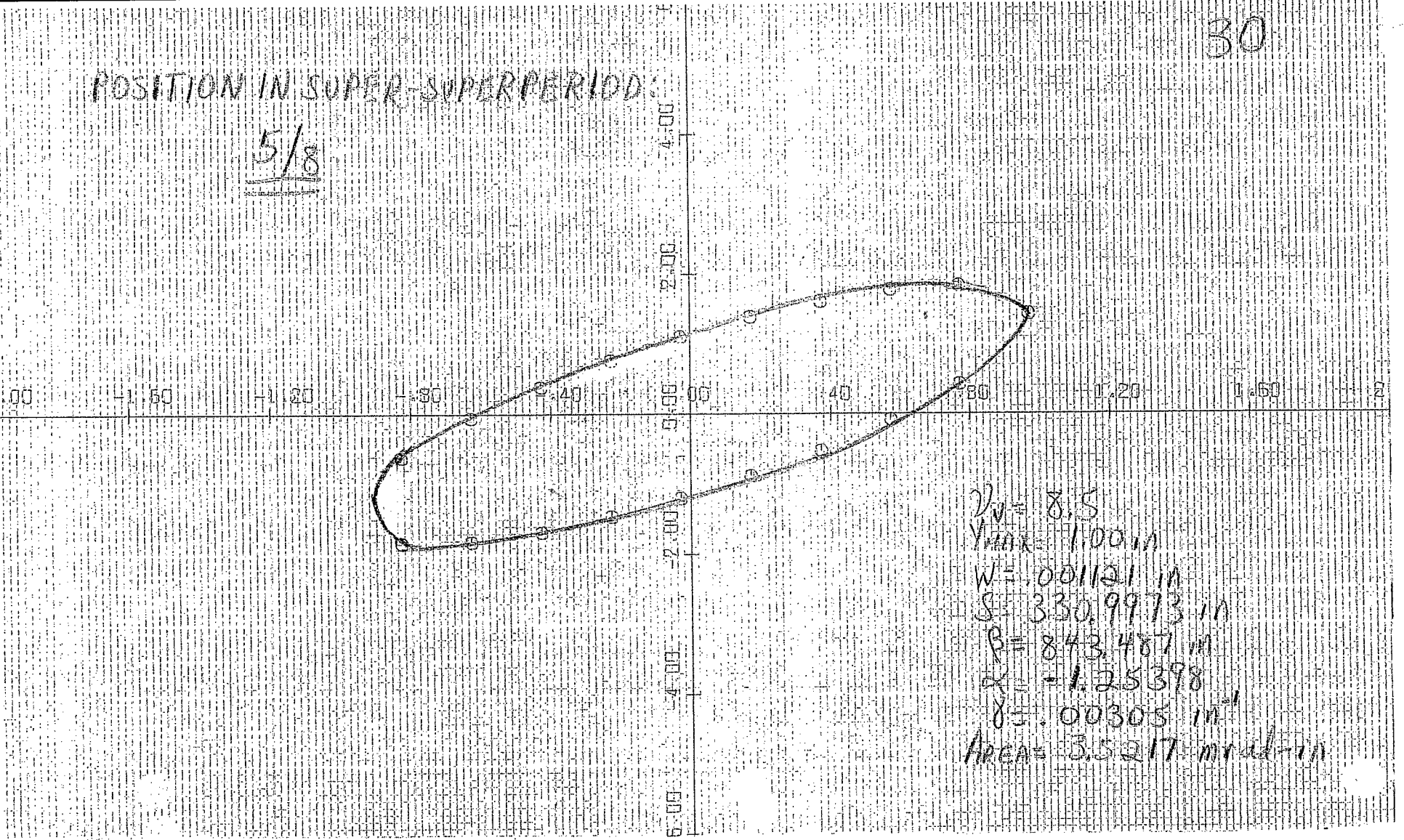
1/2



$\nu_v = 8.5$
$Y_{MAX} = 1.00$ in
$W = .001121$
$S = 264.7978$ in
$\beta = 594.972$ in
$\alpha = -1.76471$
$\gamma = .00691$ in^{-1}
AREA = 3.5217 mrad-in

POSITION IN SUPER-SUPERPERIOD:

5/8



$\nu_v = 8.5$
$Y_{MAX} = 1.00$ in
$W = .001121$ in
$S = 330.9973$ in
$\beta = 843.487$ in
$\alpha = -1.25398$
$\gamma = .00305$ in^{-1}
AREA = 3.5217 mrad-in

B1

POSITION IN SUPER-SUPERPERIOD:

3/4

$2\nu_x = 8.5$

$Y_{MAX} = 1.00$ in

$W = .00112.1$ in

$S = 397.1967$ in

$\beta = 892.459$ in

$\alpha = 0.00$

$\gamma = .0011_2$

AREA = 3.5 3.17 mrad-in

8.2

POSITION IN SUPER-SUPERPERIOD:

7/8

$D_V = 8.5$

$Y_{MAX} = 1.00$ in

$W = .001121$ in

$S = 463.3962$ in

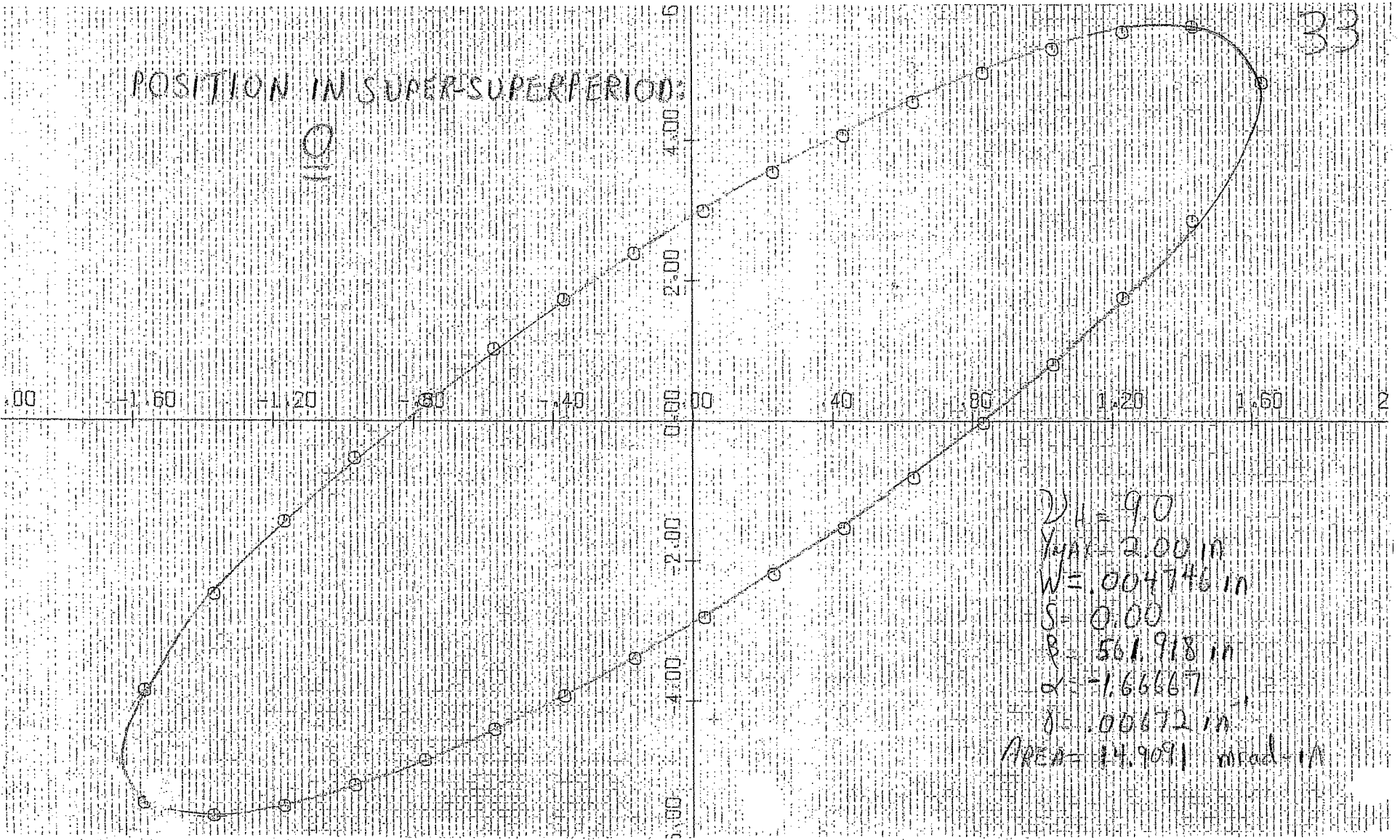
$R = 843.4871$ in

$Q = 1.25398$

$\gamma = .009305$ in^{-1}

AREA = 3.0217 mrad-in

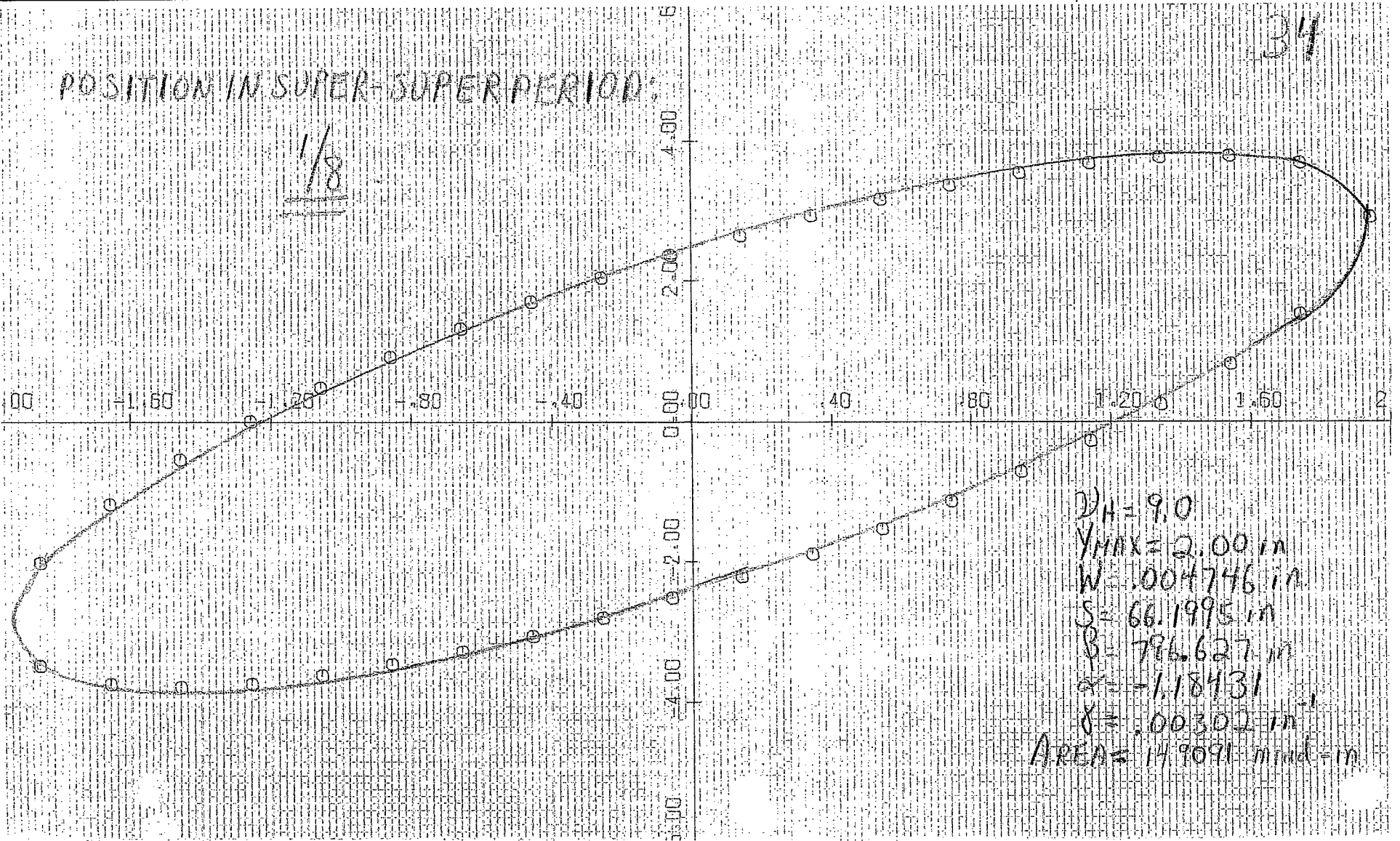
POSITION IN SUPER-SUPERPERIOD: 0



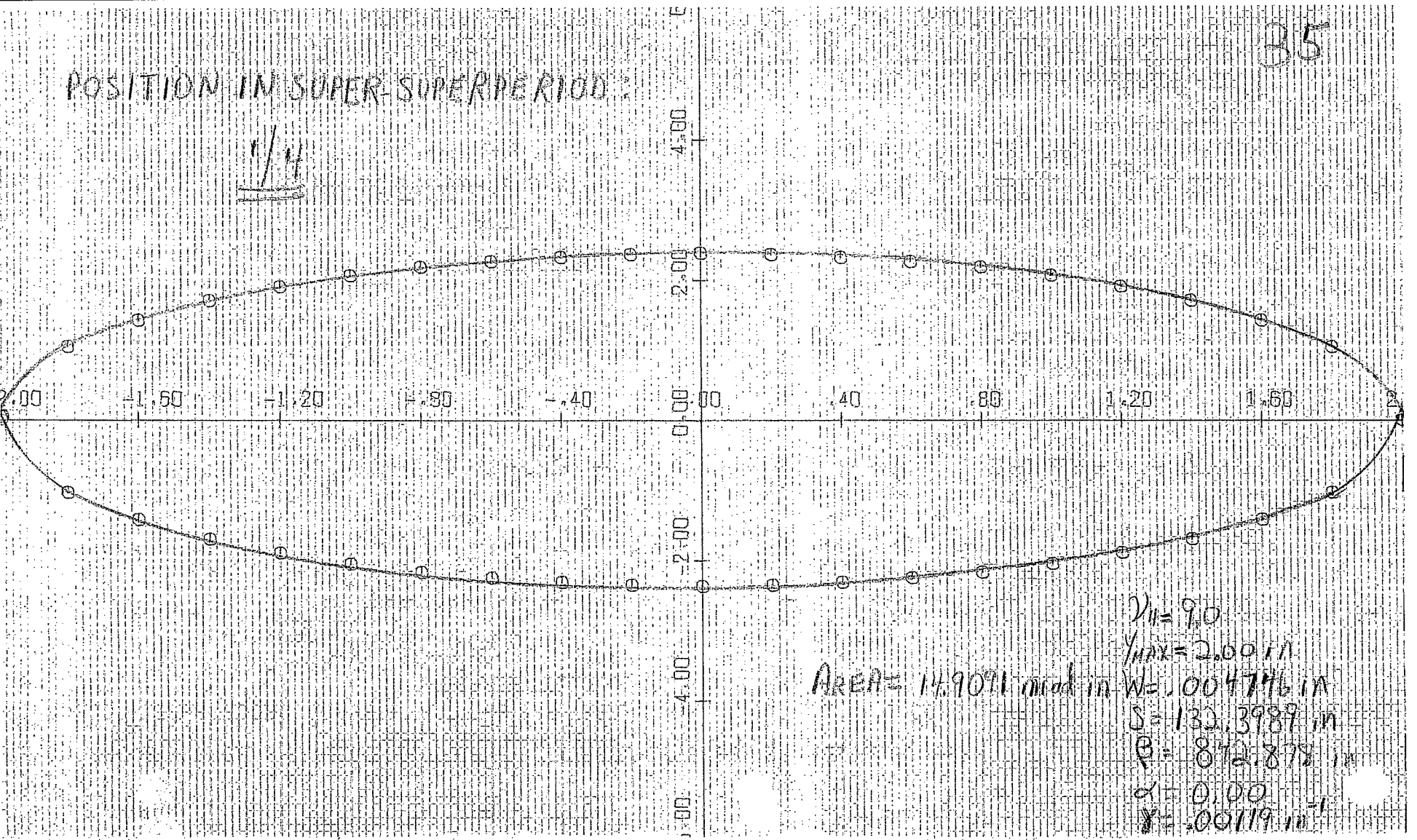
$\nu_H = 9.0$
$h_{MAX} = 2.00$ in
$W = .004746$ in
$S = 0.00$
$\beta = 561.918$ in
$\alpha = -1.66667$
$\delta = .00672$ in^{-1}
$\beta_{MEAN} = 14.9091$ mrad^{-1}

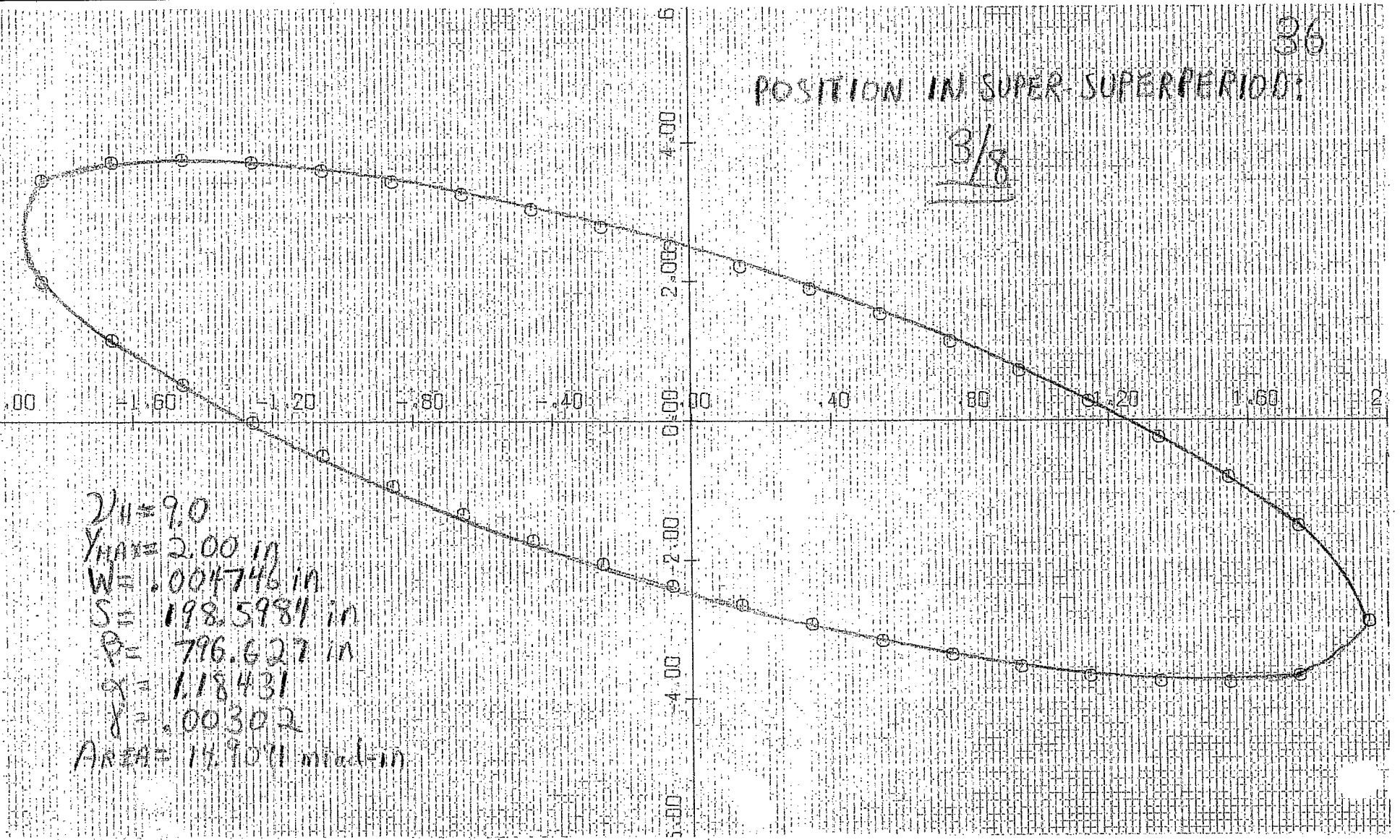
POSITION IN SUPER-SUPERPERIOD:

1/8



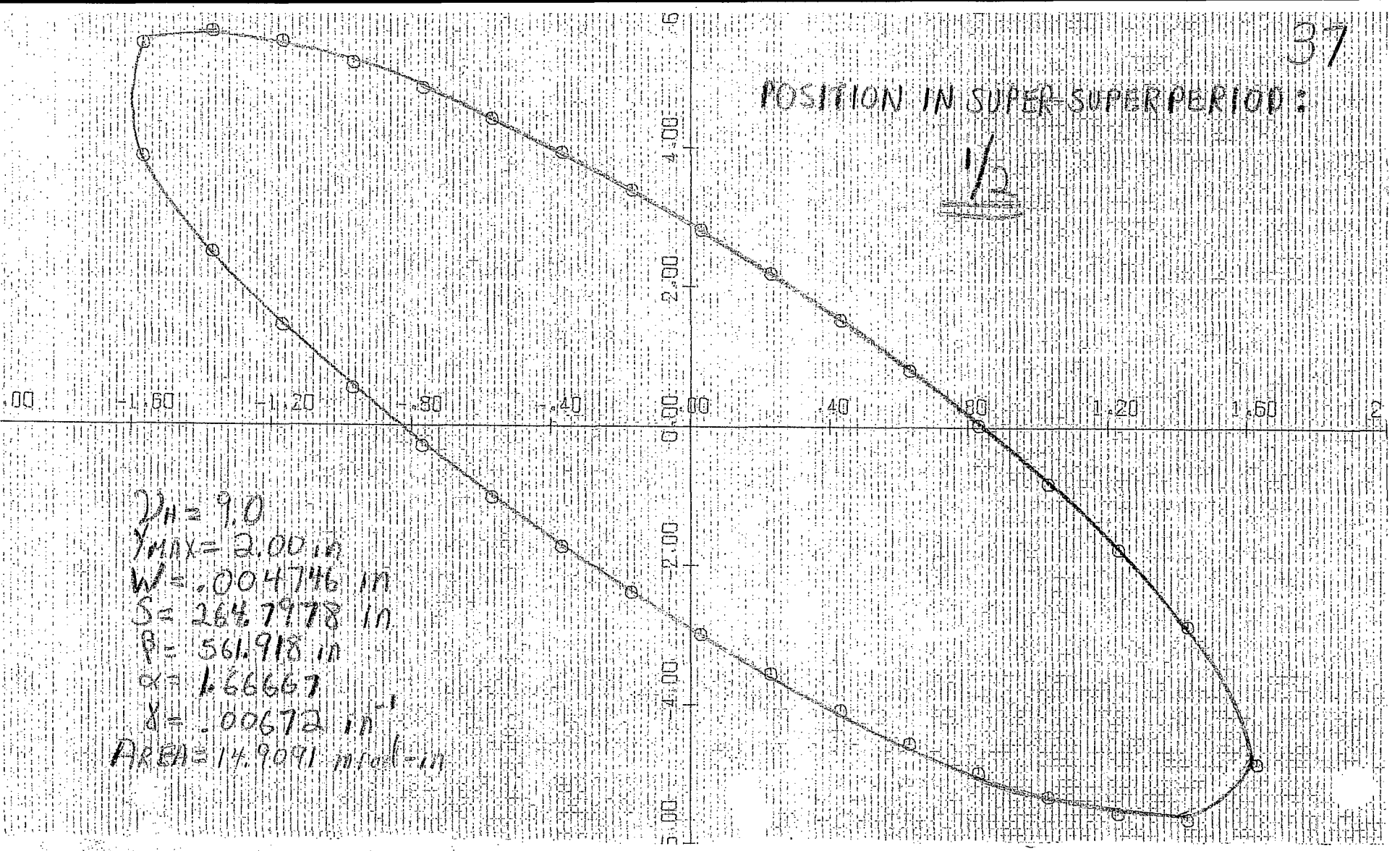
$D_H = 9.0$
$y_{MAX} = 2.00$ in
$W = .0044746$ in
$S = 66.1995$ in
$\beta = 796.627$ in
$\alpha = -1.18431$
$\gamma = .003002$ in^{-1}
AREA = 14.9091 mod-in


POSITION IN SUPER-SUPERPERIOD:
1/4
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
$\nu_H = 9.0$
$Y_{MAX} = 2.00$ in
AREA = 14.9091 mrad in
W = .004746 in
S = 132.3989 in
β = 842.878 in
α = 0.00
γ = .00119 in⁻¹


POSITION IN SUPER-SUPERPERIOD:
3/8
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2
$\nu_H = 9.0$
$Y_{MAX} = 2.00$ in
$W = .004746$ in
$S = 198.5984$ in
$\beta = 796.627$ in
$\alpha = 1.18431$
$\gamma = .00302$
AREA = 14.9091 mrad-in

POSITION IN SUPER-SUPER PERIOD:

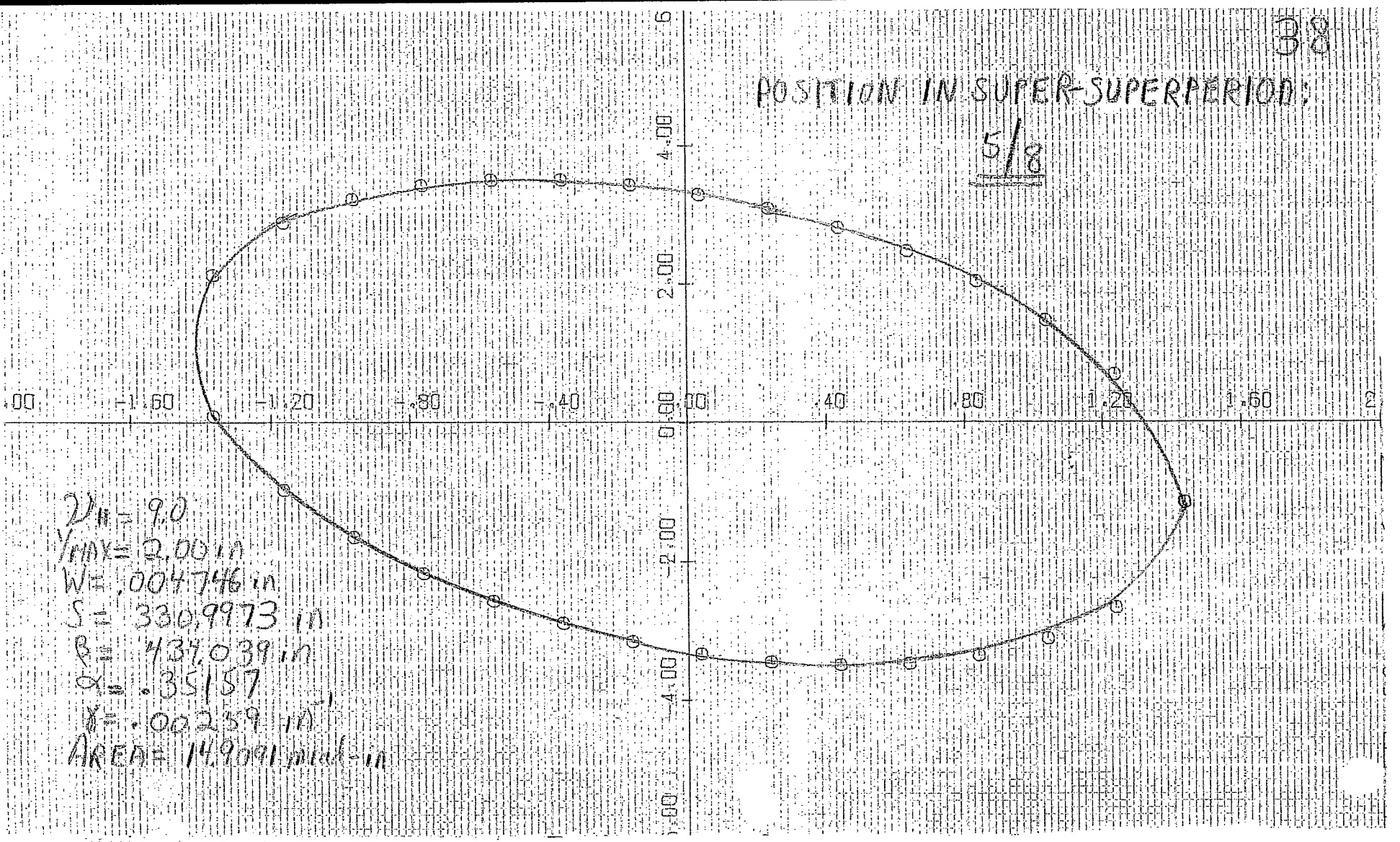
1/2



$2n = 9.0$
$Y_{max} = 2.00$ in
$W = .004746$ in
$S = 264.7978$ in
$P = 561.918$ in
$\alpha = 1.66667$
$\lambda = .00672$ in^{-1}
AREA $= 14.9091$ mod-in

POSITION IN SUPER-SUPERPERIOD:

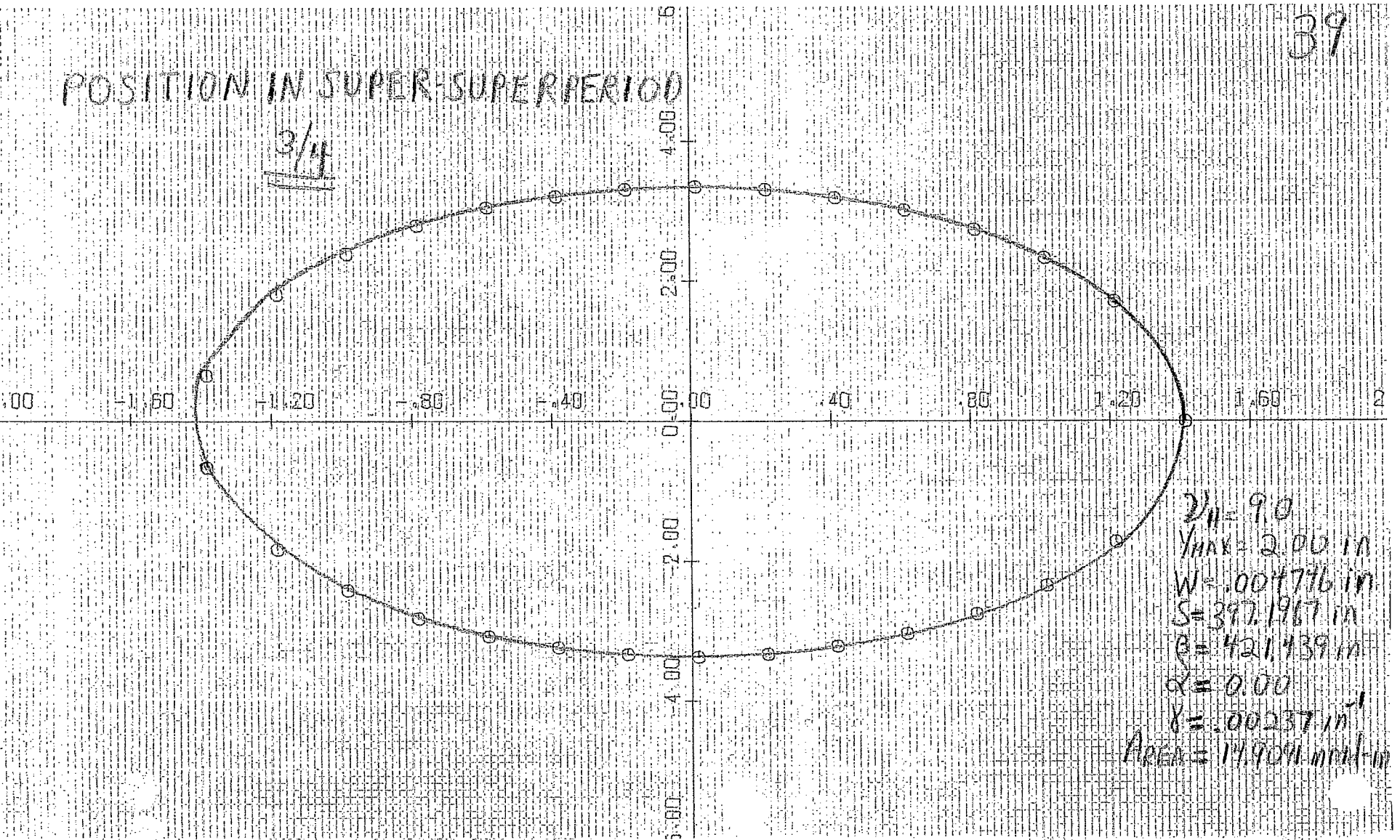
5/8



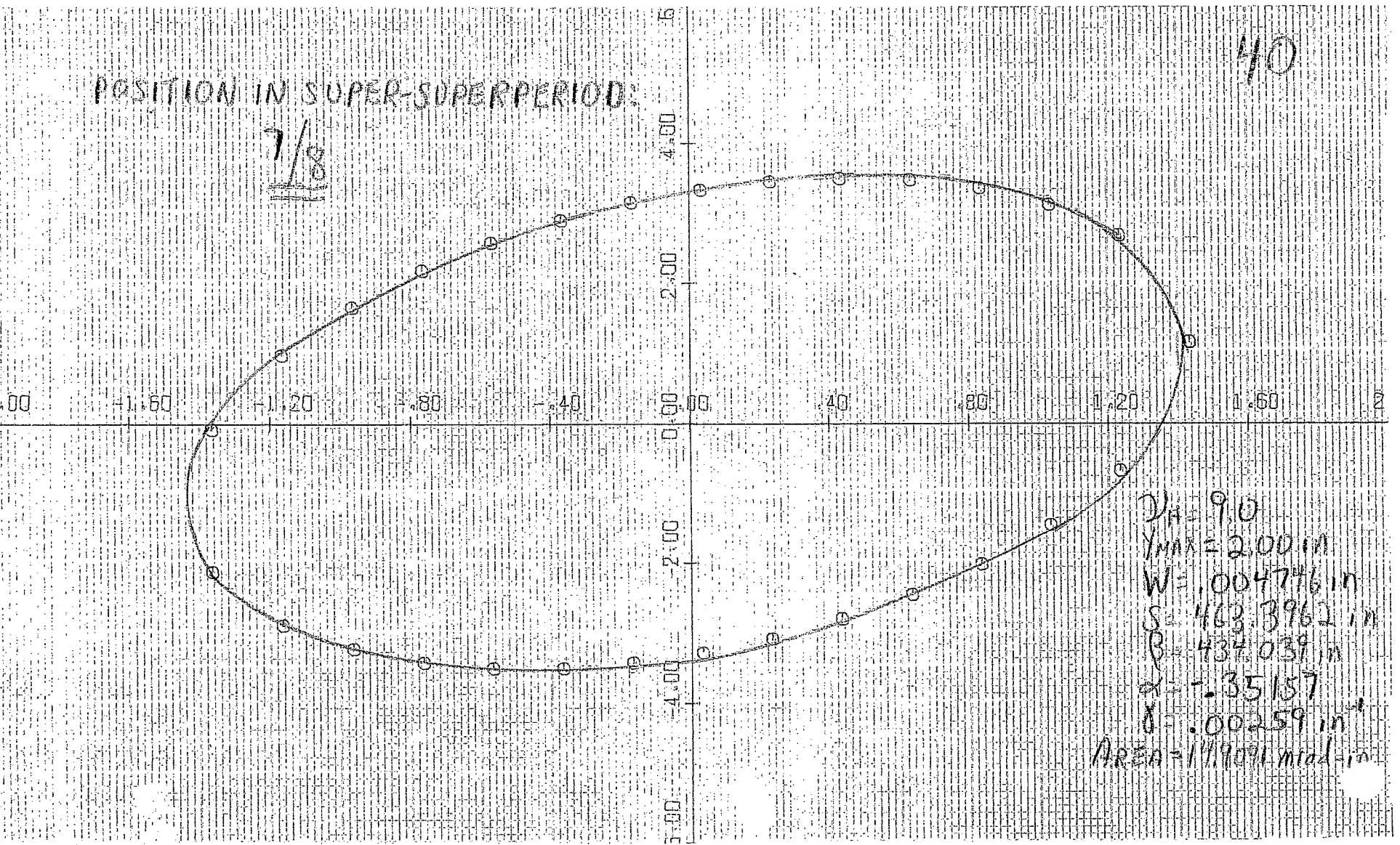
$\nu_H = 9.0$
$Y_{MAX} = 2.00$ in
$W = .004746$ in
$S = 330.9973$ in
$\beta = 437.039$ in
$\alpha = .35157$
$\gamma = .00259$ in^{-1}
AREA = 14.9091 mrad-in

POSITION IN SUPER-SUPERPERIOD

3/4

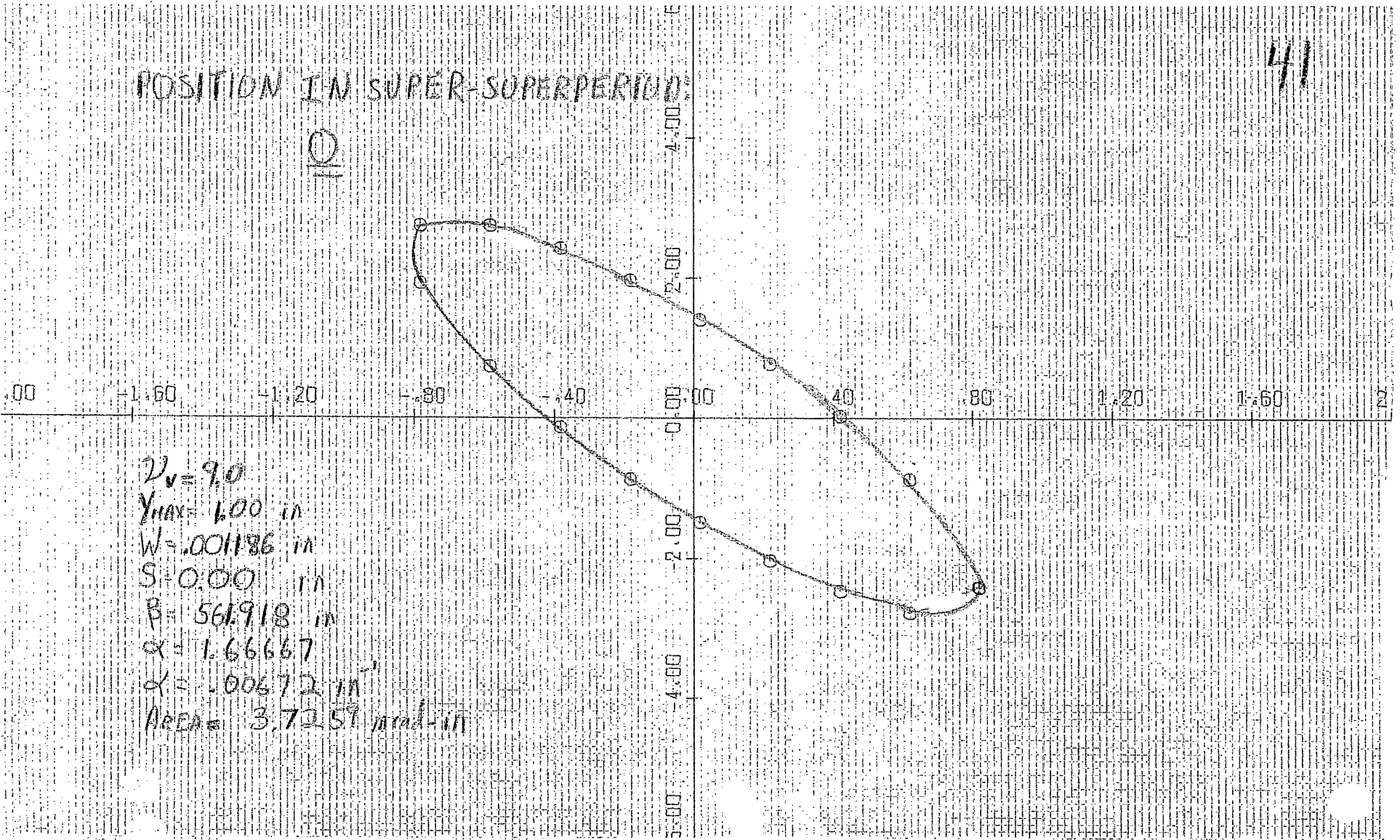


$2/\pi = 9.0$
$Y_{MAX} = 2.00$ IN
$W = .001776$ IN
$S = 397.1967$ IN
$Q = 421.1139$ IN
$\alpha = 0.00$
$\gamma = .00237$ IN^{-1}
AREA = 17.9091 MIN-IN


40
POSITION IN SUPER-SUPERPERIOD:
7/8
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2
ν_H = 9.0
Y_{MAX} = 2.00 in
W = .004746 in
S = 463.3962 in
β = 434.039 in
α = -.35157
γ = .00259 in⁻¹
AREA = 14.9091 mrad-in

POSITION IN SUPER-SUPERPERIOD:

Q1



$\nu_v = 9.0$

$y_{max} = 4.00$ in

$W = .001186$ in

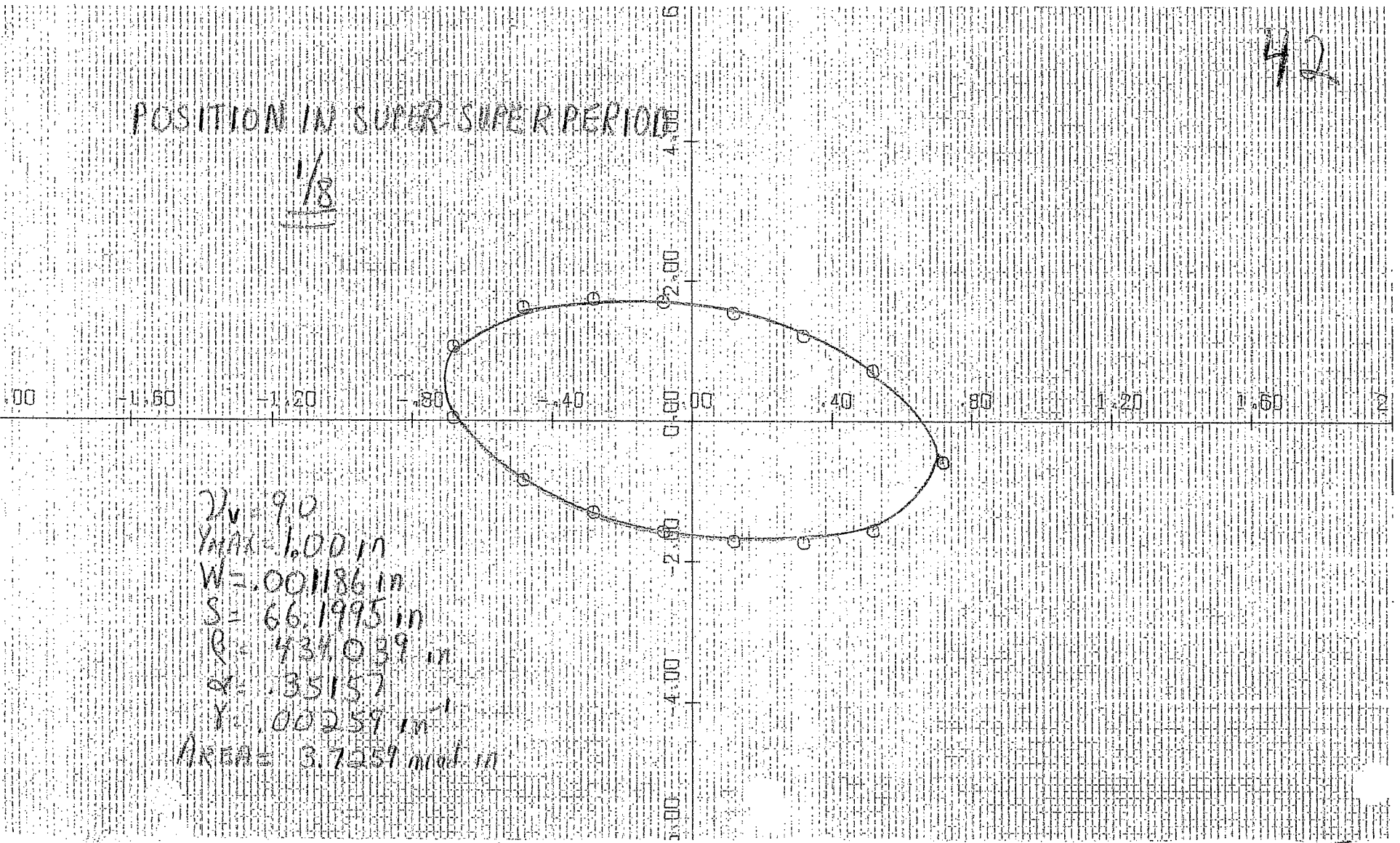
$S = 0.00$ in

$\beta = 561.918$ in

$\alpha = 1.66667$

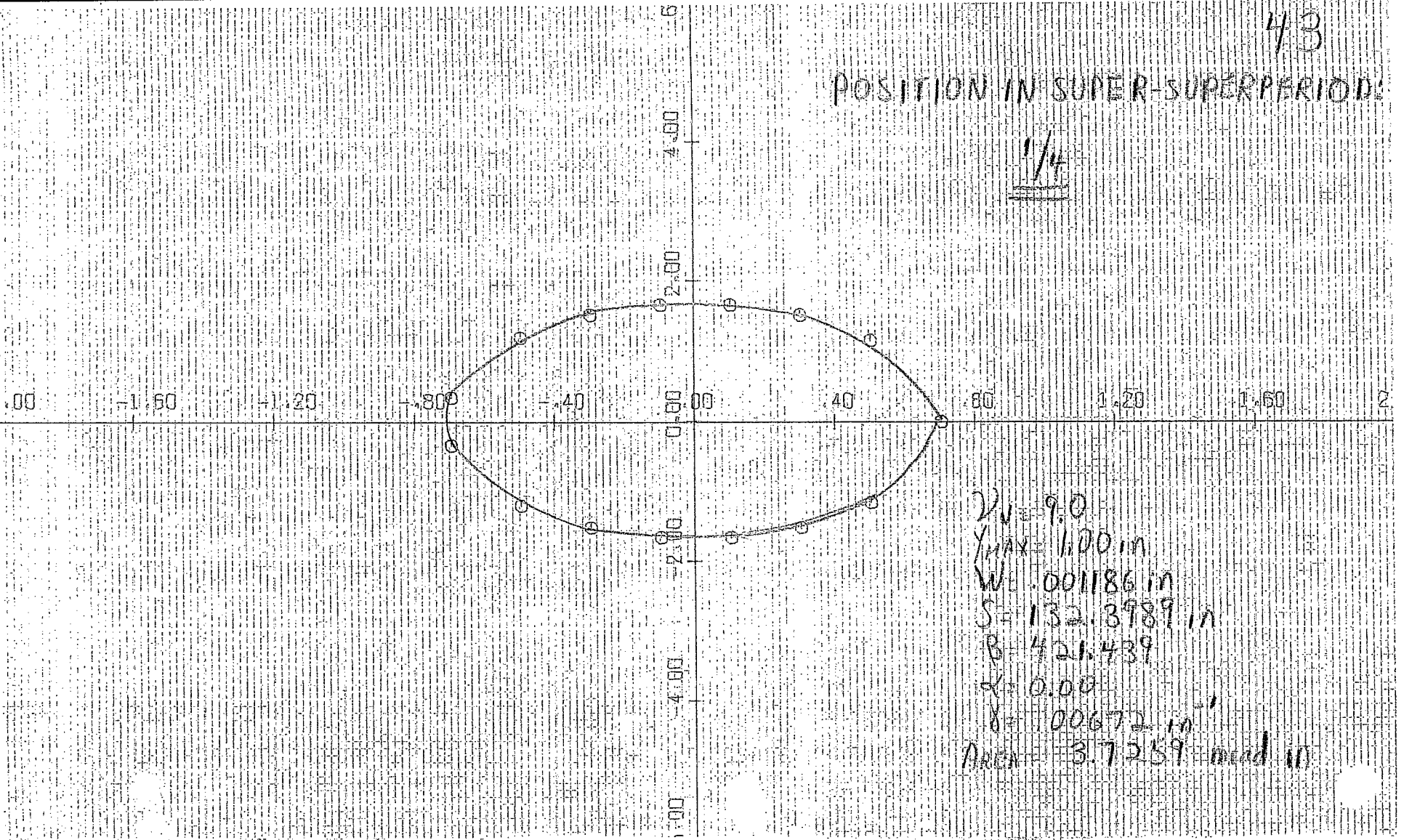
$\gamma = .006702$ in^{-1}

AREA = 3.7257 mrad-in


POSITION IN SUPER-SUPER PERIOD
1/8
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
2
ν_v = 9.0
Y_{MAX} = 1.00 in
W = .001186 in
S = 66.1995 in
β = 434.039 in
α = .35157
γ = .00259 in^{-1}
AREA = 3.7259 mrad in

POSITION IN SUPER-SUPERPERIOD:

1/4



$2\nu = 9.0$

$Y_{MAX} = 1.20$ in

$W = .001186$ in

$S = 132.3989$ in

$\beta = 421.439$

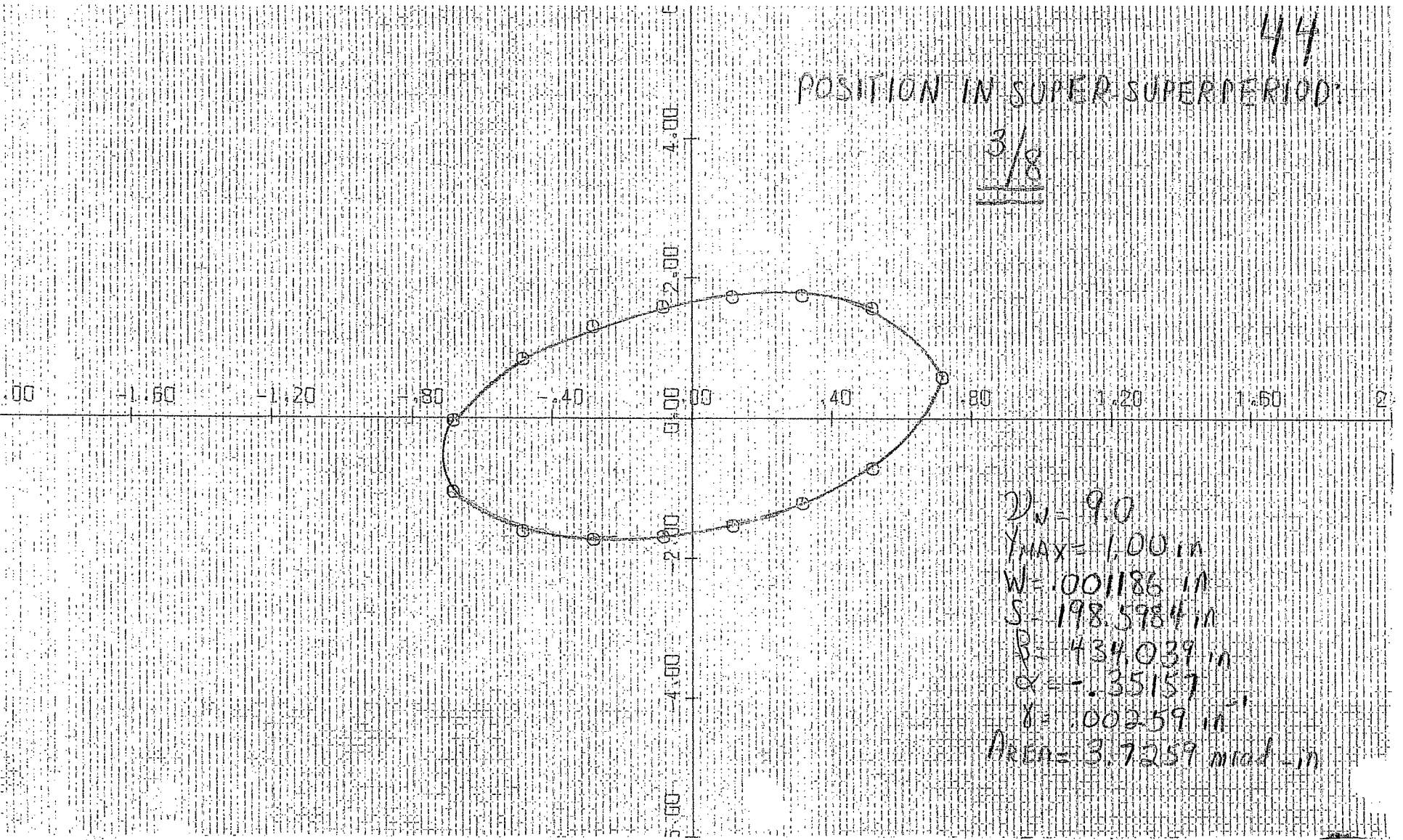
$\alpha = 0.00$

$\chi = .00672$ in^{-1}

$\eta_{MEN} = 3.7259$ mrad

POSITION IN SUPER SUPERPERIOD:

3/8



$2\nu = 9.0$

$Y_{MAX} = 1.00$ in

$W = .0011186$ in

$S = 198.5984$ in

$D = 434.039$ in

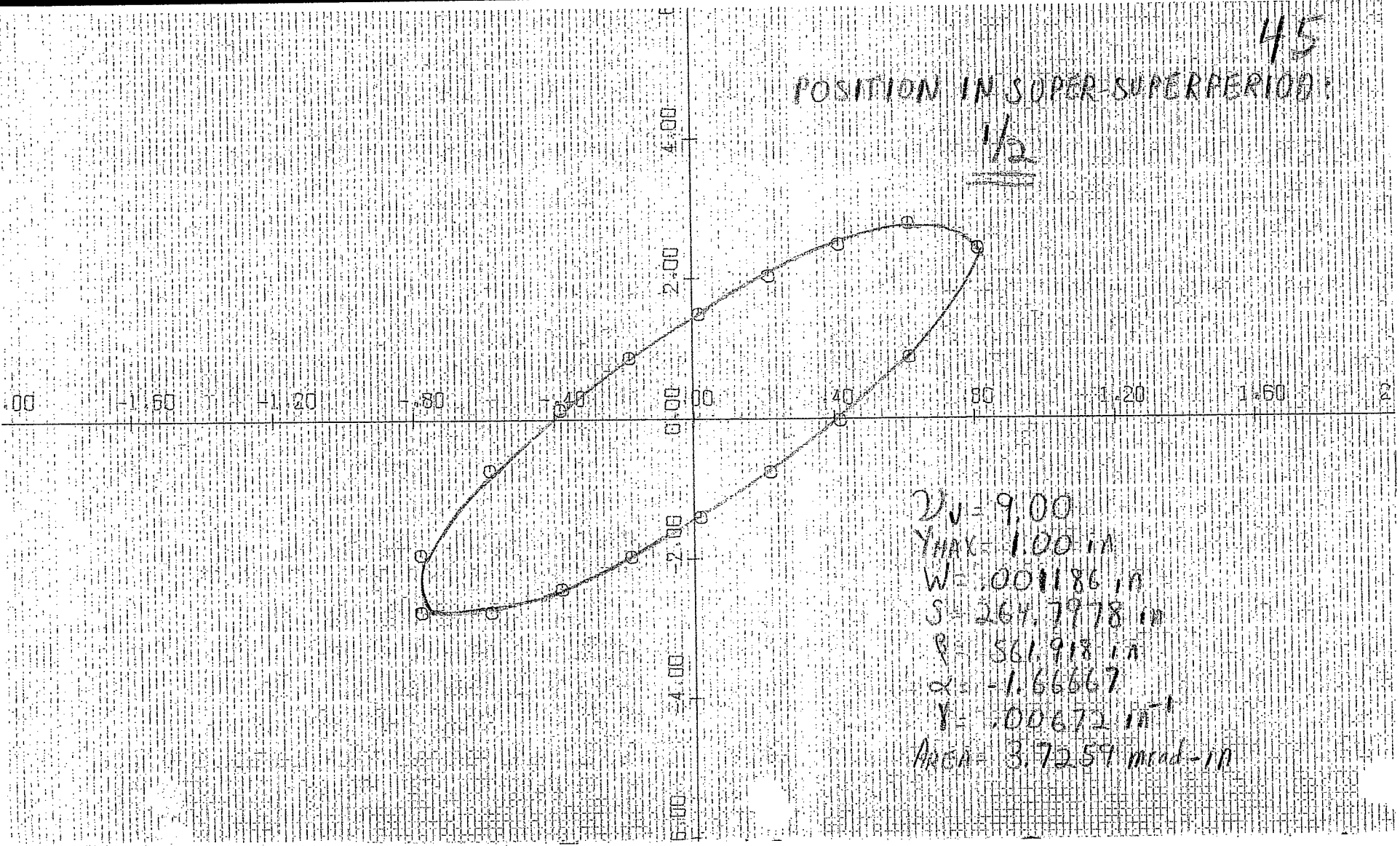
$\alpha = -.35157$

$\gamma = .002.59$ in^{-1}

$Area = 3.7259$ mrad-in

POSITION IN SUPER-SUPERPERIOD:

1/2



$2\nu = 9.00$

$Y_{MAX} = 1.00$ in

$W = .001186$ in

$S = 264.7778$ in

$R = 561.918$ in

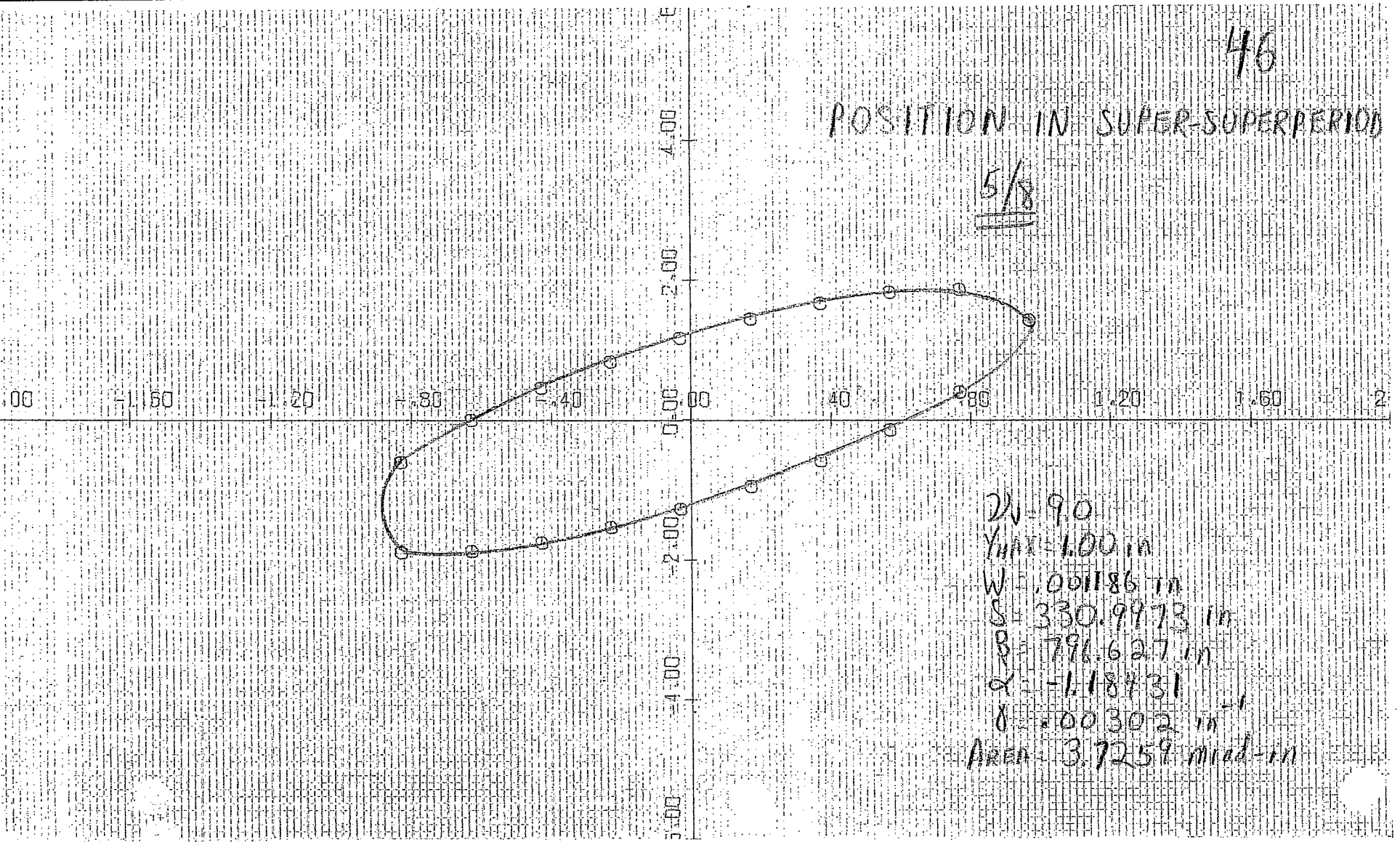
$\alpha = -1.66667$

$\gamma = .00672$ in^{-1}

$A_{RCH} = 3.7259$ mrad-in

POSITION IN SUPER-SUPERPERIOD

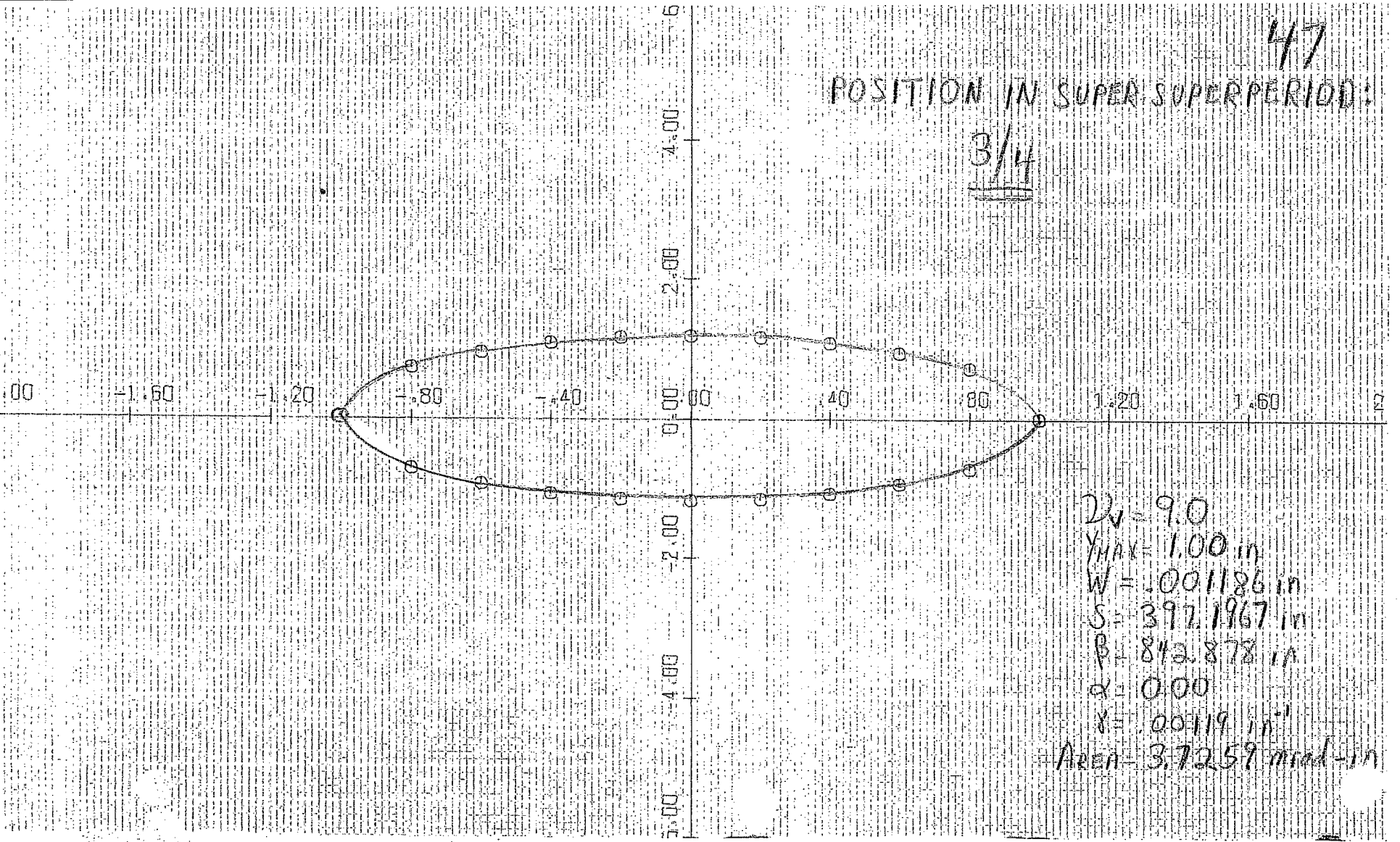
5/8



D_V = 9.0
Y_{MAX} = 1.00 in
W = .00186 in
S = 330.9973 in
R = 796.627 in
α = -1.18731
δ = .00302 in^{-1}
AREA = 3.7259 mid-in

POSITION IN SUPER SUPERPERIOD:

3/4



$2\nu = 9.0$

$Y_{MAX} = 1.00$ in

$W = .001186$ in

$S = 372.1967$ in

$\beta = 842.878$ in

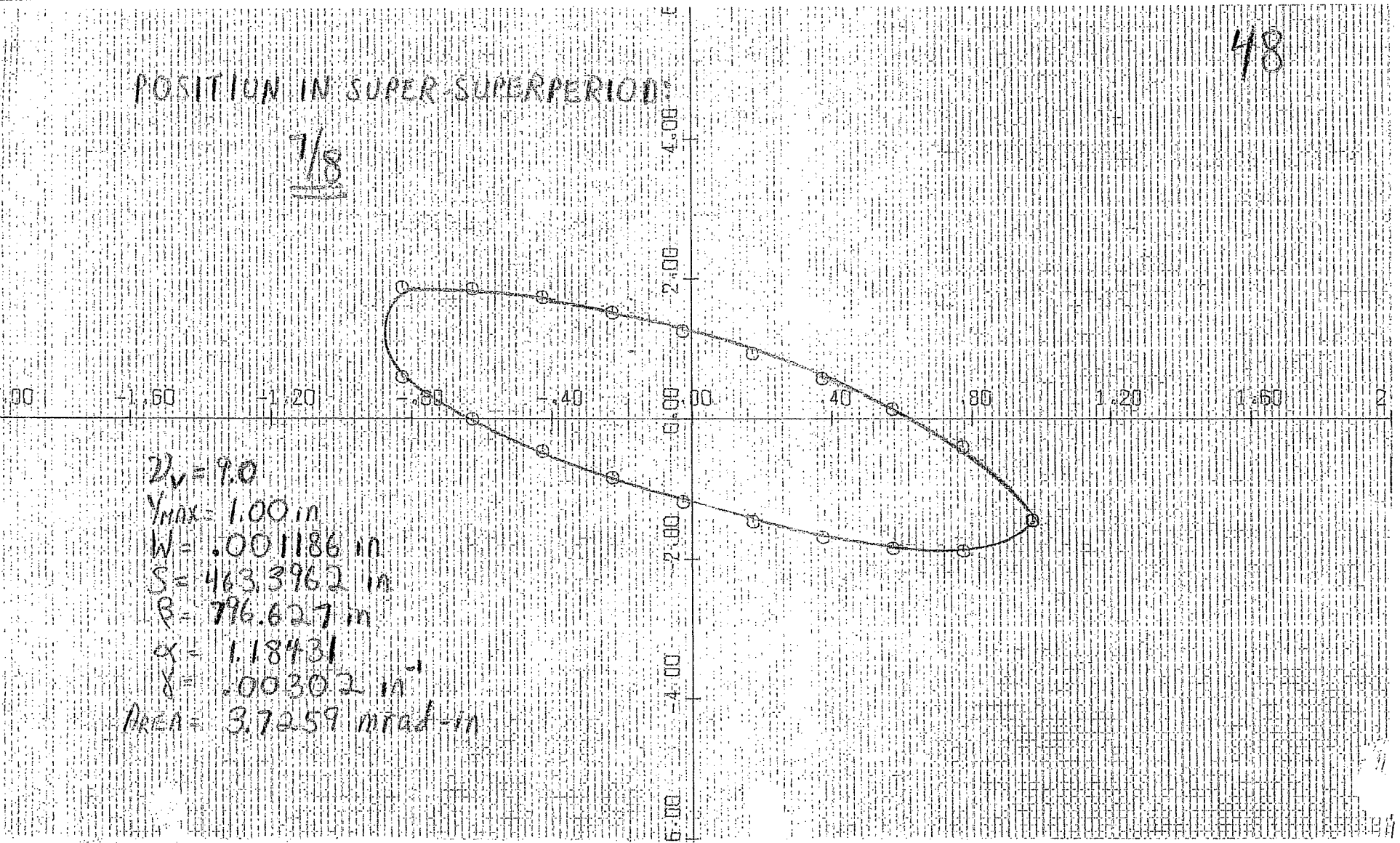
$\alpha = 0.00$

$\delta = .00119$ in^{-1}

$A_{REA} = 3.7257$ mrad-in

POSITION IN SUPER-SUPERPERIOD:

7/8



$2\nu = 9.0$
$Y_{MAX} = 1.00$ in
$W = .0011186$ in
$S = 463.3962$ in
$\beta = 796.627$ in
$\alpha = 1.18431$
$\gamma = .00302$ in^{-1}
AREA $= 3.7259$ mrad-in