

PHASE SPACE ELLIPSES FOR A MODEL AGS

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August 1969

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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Upton, L.I., N.Y.

AGS DIVISION TECHNICAL NOTE

No. 65

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PHASE SPACE ELLIPSES FOR A MODEL AGS

The accompanying graphs are plots of the AGS phase space ellipses for the horizontal (H) and vertical (V) directions, obtained on the CDC-6600 and the Calcomp 565 plotter.

The calculation assumes a model horizontal $\Psi(s)$ function¹ of the following form:

$$\Psi(s) = \Psi_L \left[\frac{s}{L} + \frac{\tau}{2\pi} \left(\cos \frac{2\pi}{L} s - 1 \right) + \frac{\sigma}{2\pi} \left(\cos \frac{6\pi}{L} s - 1 \right) \right] \text{ where:}$$

$$L = \frac{2\pi R}{60}; \quad R = \text{radius of machine} = 5057.266 \text{ inches}$$

$$\Psi_L = \frac{2\pi \eta}{60}; \quad \text{and for a form factor } \frac{\beta_{\max}}{\beta_{\text{av}}} = 1.5$$

$$\tau = 3/8$$

$$\sigma = 1/72$$

This phase function has a periodicity of four magnets (1/60 of the circumference), and agrees well with the results obtained from the BEAM program at intermediate field. The corresponding β function is of the form:

$$\beta(s) = \frac{R/\eta}{1 - \tau \sin \frac{2\pi}{L} s - 3\sigma \sin \frac{6\pi}{L} s}$$

Each set of eight graphs gives the phase space ellipses in either the horizontal or vertical direction for one particle at eight points within the super-superperiod (1/60 of the circumference). The following table lists the graphs shown:

<u>Graph Numbers</u>	<u>ν</u>	<u>Y_{\max} (inches)</u>
1-8	8.0 (H)	2.00
9-16	8.0 (V)	1.00
17-24	8.5 (H)	2.00
25-32	8.5 (V)	1.00
33-40	9.0 (H)	2.00
41-48	9.0 (V)	1.00

In general, the program is designed to plot graphs for any ν value and any value of Y_{\max} , where Y_{\max} is the maximum value of the transverse excursion of the particle taken at the point where β itself is maximum. Thus, at no point in the machine is the greatest transverse excursion of the particle any larger than Y_{\max} .

The phase invariant, W , is set at Y_{\max}^2/β_{\max} , and the equation of each ellipse is:

$$\gamma y^2 + 2\alpha y y' + \beta y'^2 = W$$

A diagram of a super-superperiod with the horizontal β function superimposed is given in Fig. 1.

The phase space ellipses are computed and plotted at the indicated points, A through H.

The graphs contain all parameters pertinent to the calculation, as well as the area (emittance) of the ellipses; that is:

$$A = \pi W \text{ rad-in.}$$

References

1. J.C. Herrera, BNL Accel. Dept. Int. Rept. AGS DIV 69-3 (1969).

Distr:

Department Administration
AGS Division Physicists

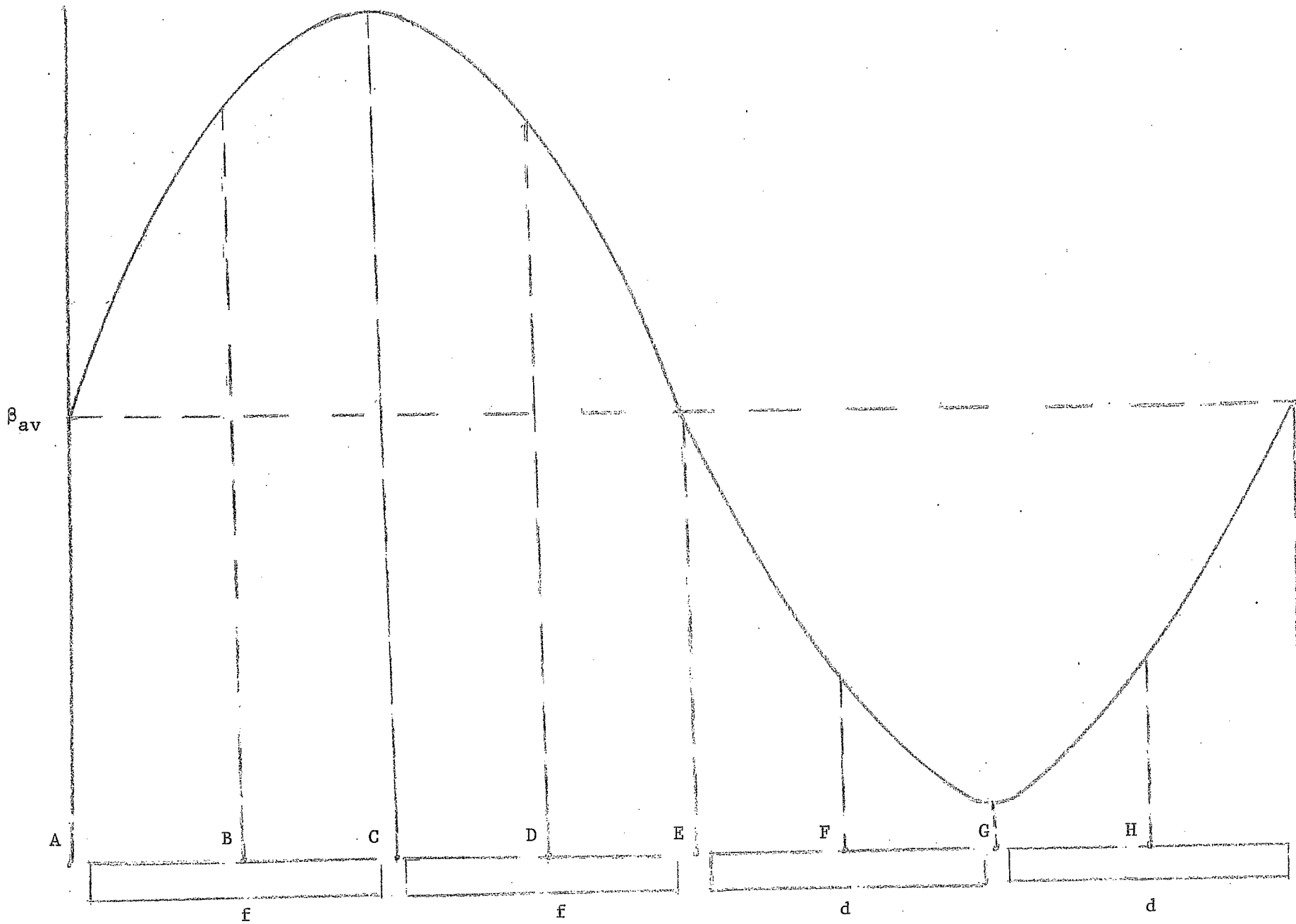
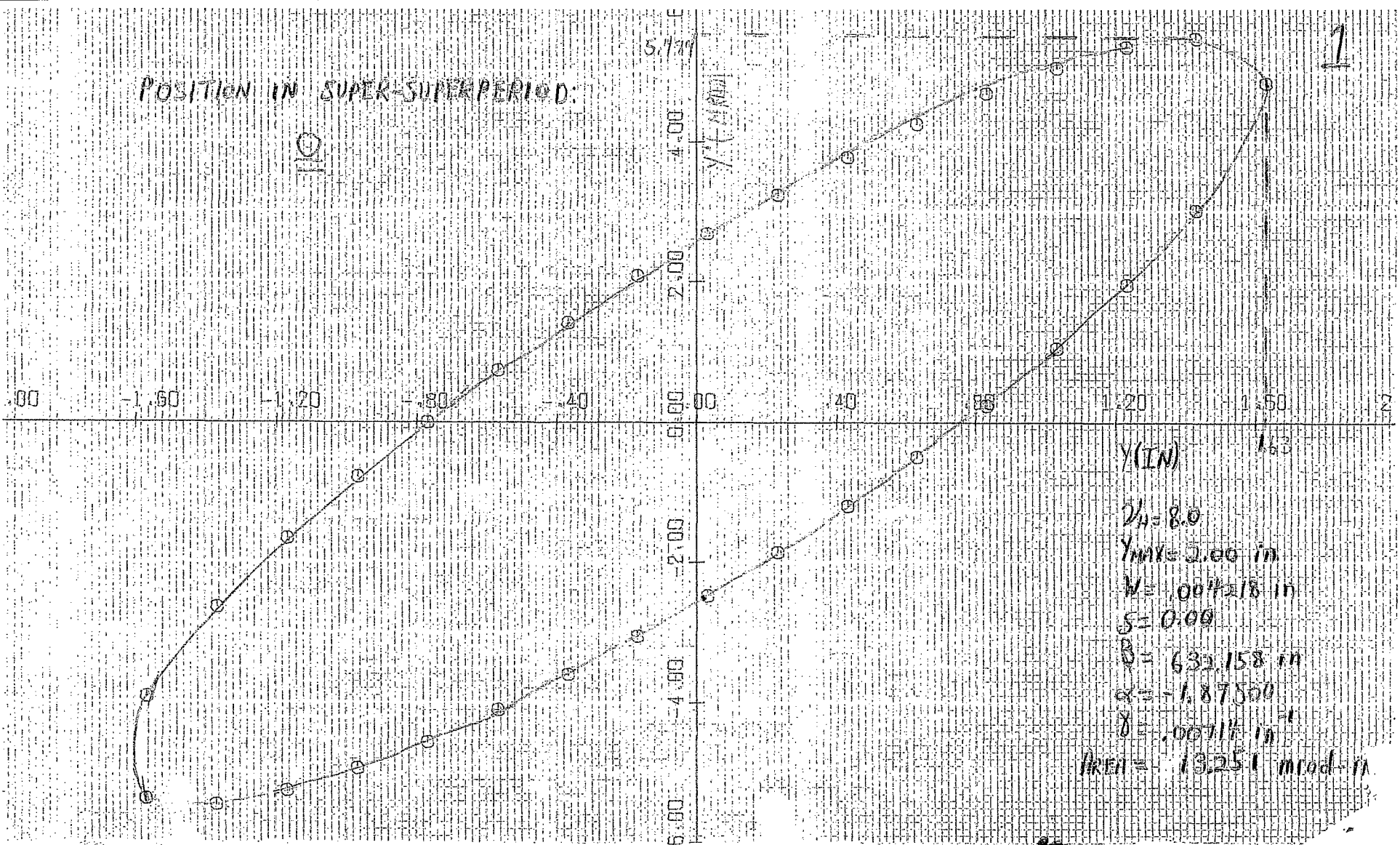


Fig. 1

POSITION IN SUPER-SUPERPERIOD:

1

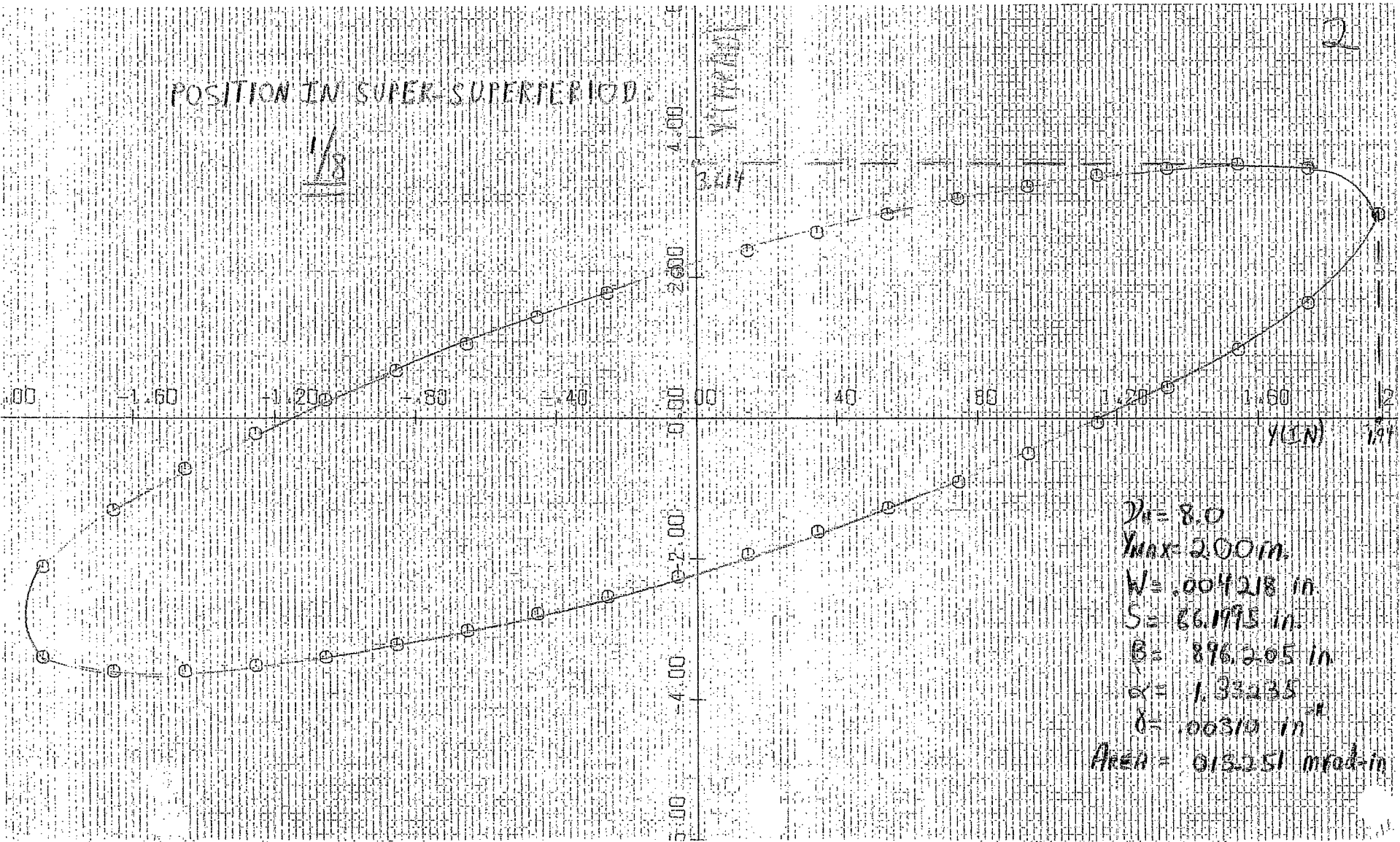


$Y (IN) \quad 1.63$
 $2A = 8.0$
 $Y_{max} = 2.00 \text{ in}$
 $W = .004218 \text{ in}$
 $S = 0.00$
 $B = 632.158 \text{ in}$
 $\alpha = -1.87500$
 $\delta = .00717 \text{ in}^2$
 $A_{REN} = 13.25 \text{ mod-in}$

POSITION IN SUPER-SUPERPERIOD

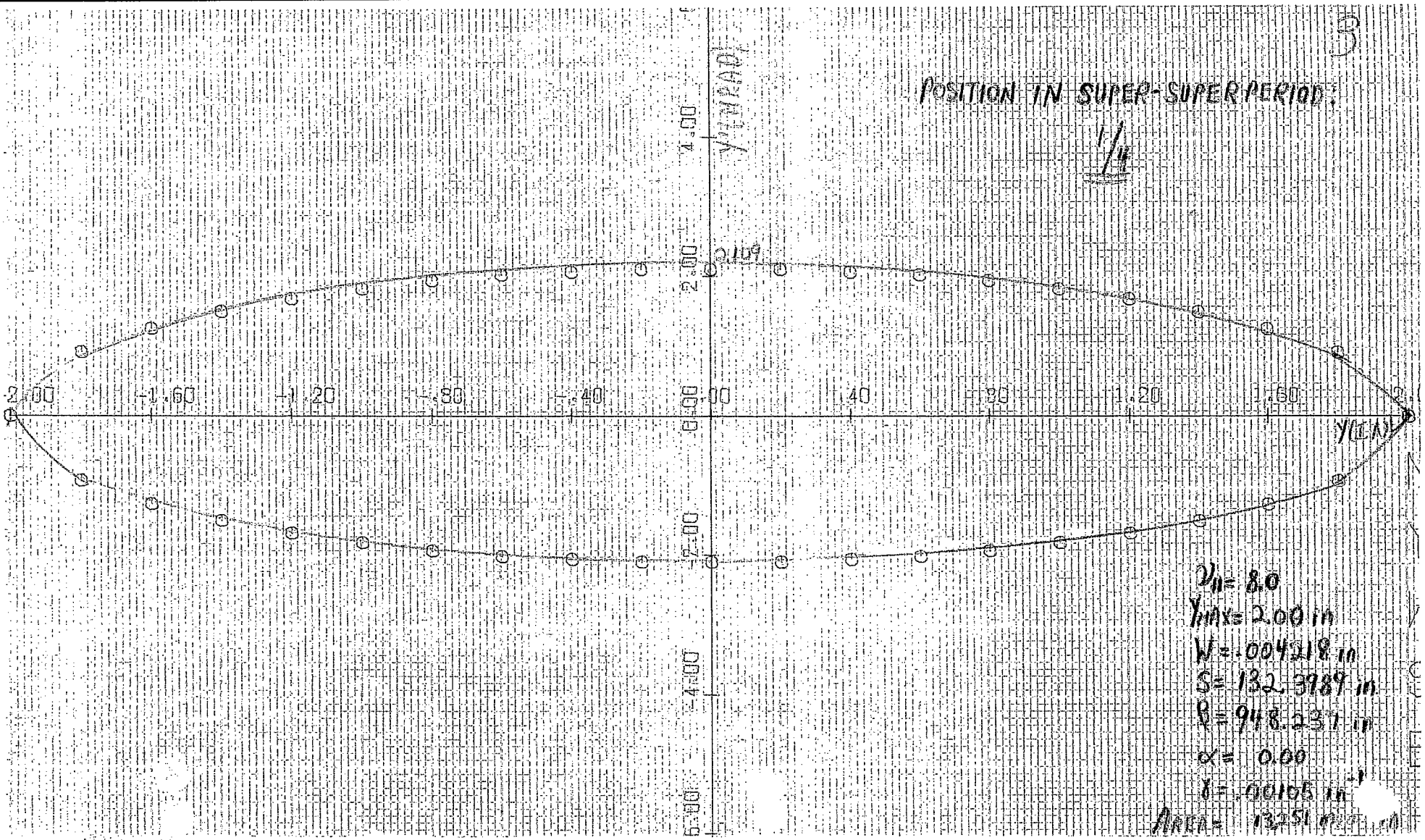
1/8

2



POSITION IN SUPER-SUPER PERIOD

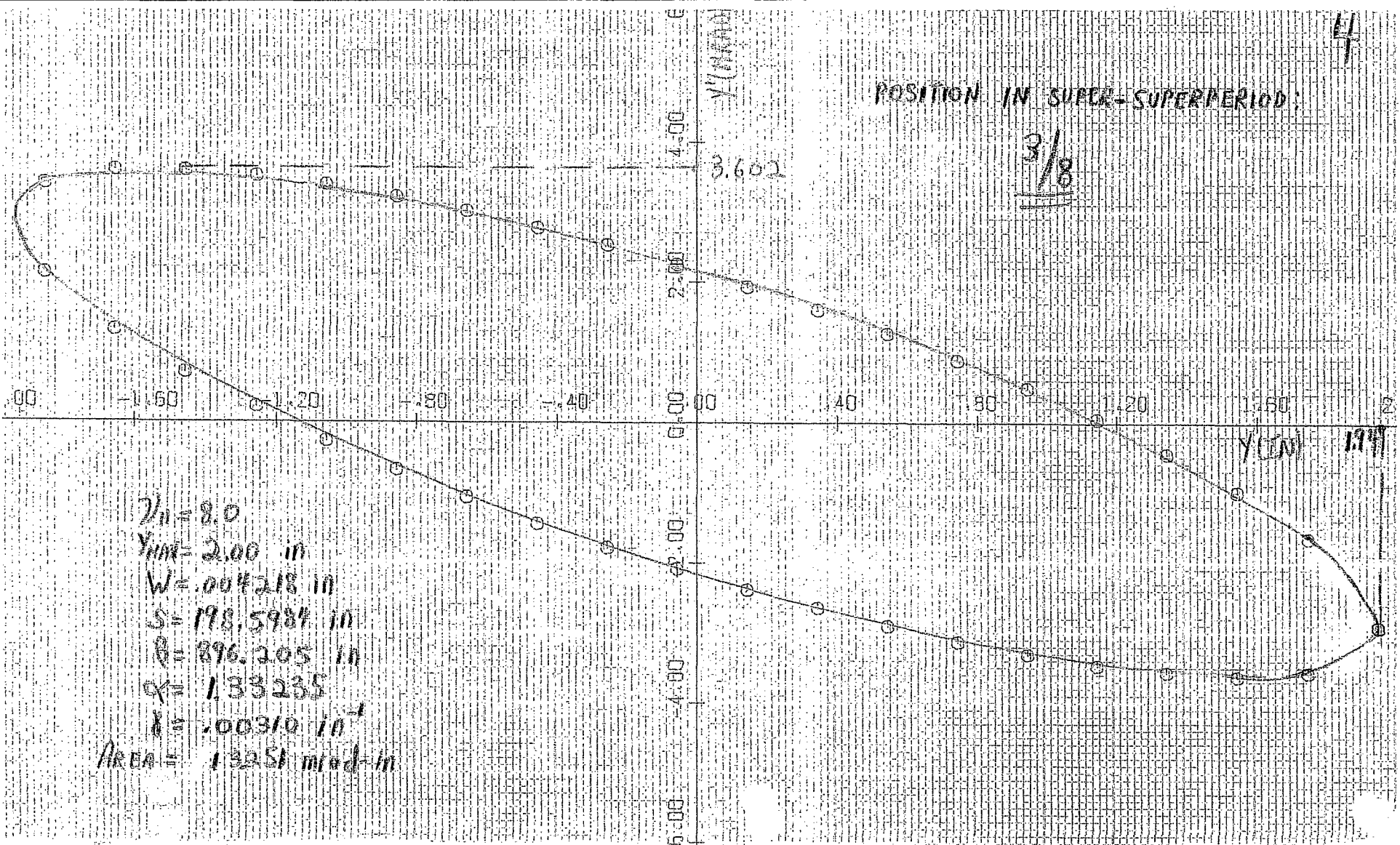
$\frac{1}{4}$



$\psi_0 = 8.0$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = .004218 \text{ in}$
 $S = 132.3989 \text{ in}$
 $R = 948.237 \text{ in}$
 $\alpha = 0.00$
 $\delta = .00105 \text{ in}^{-1}$
 $AREA = 13251.027 \text{ in}^2$

POSITION IN SUPER-SUPERPERIOD:

3/8

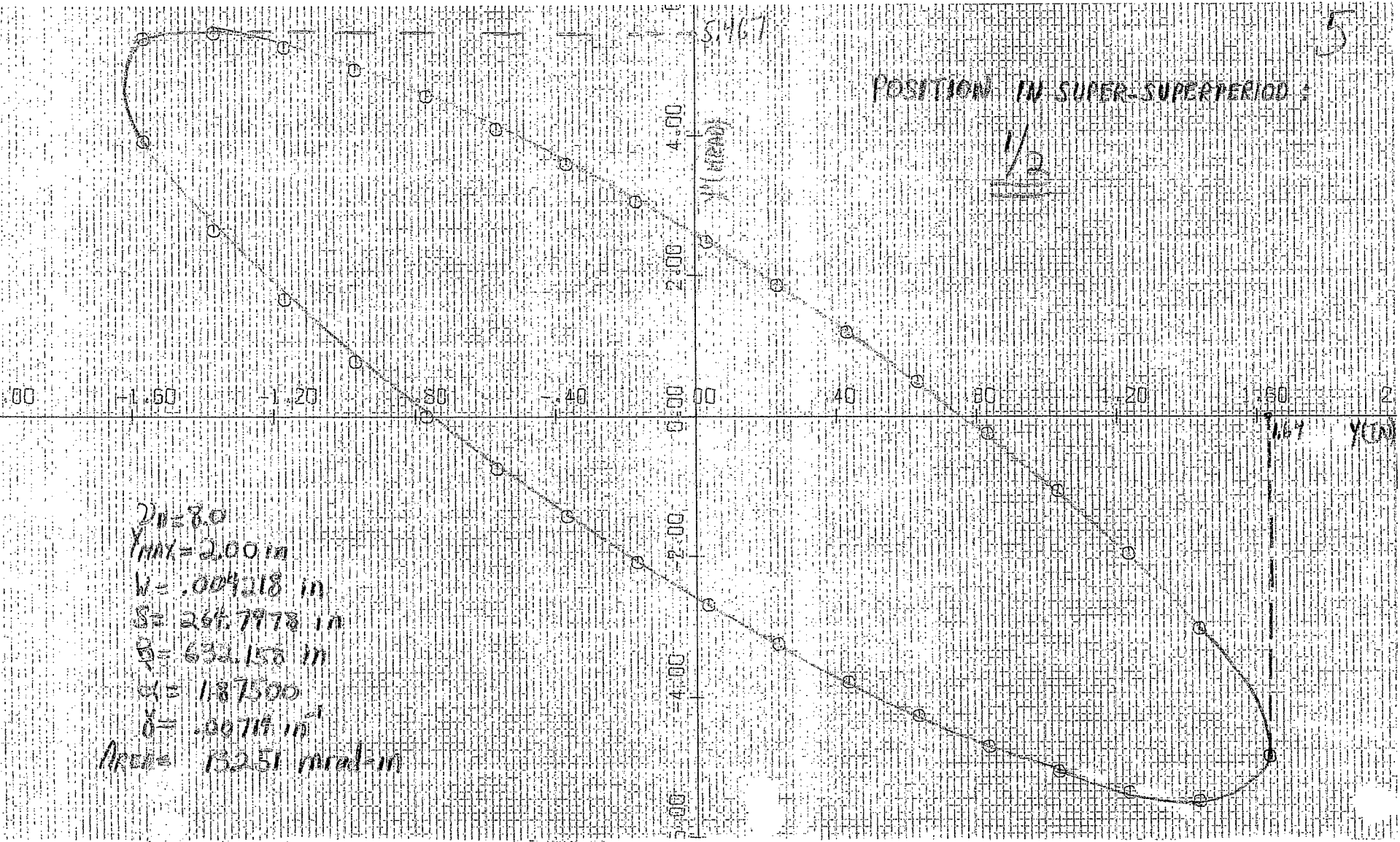


$\lambda_1 = 8.0$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = .004218 \text{ in}$
 $S = 198.5984 \text{ in}$
 $\theta = 896.205 \text{ in}$
 $Q = 1.33235$
 $k = .00310 \text{ in}^{-1}$
 AREA = 13251 mod-in

POSITION IN SUPER-SUPERPERIOD :

$\frac{1}{2}$

S:467



$2n = 8.0$
 $Y_{max} = 2.00 \text{ in}$
 $W = .004218 \text{ in}$
 $S = 266.7978 \text{ in}$
 $B = 632.158 \text{ in}$
 $\alpha = 1.87500$
 $\delta = .00717 \text{ in}^2$
 $Area = 132.51 \text{ inrad}^2$

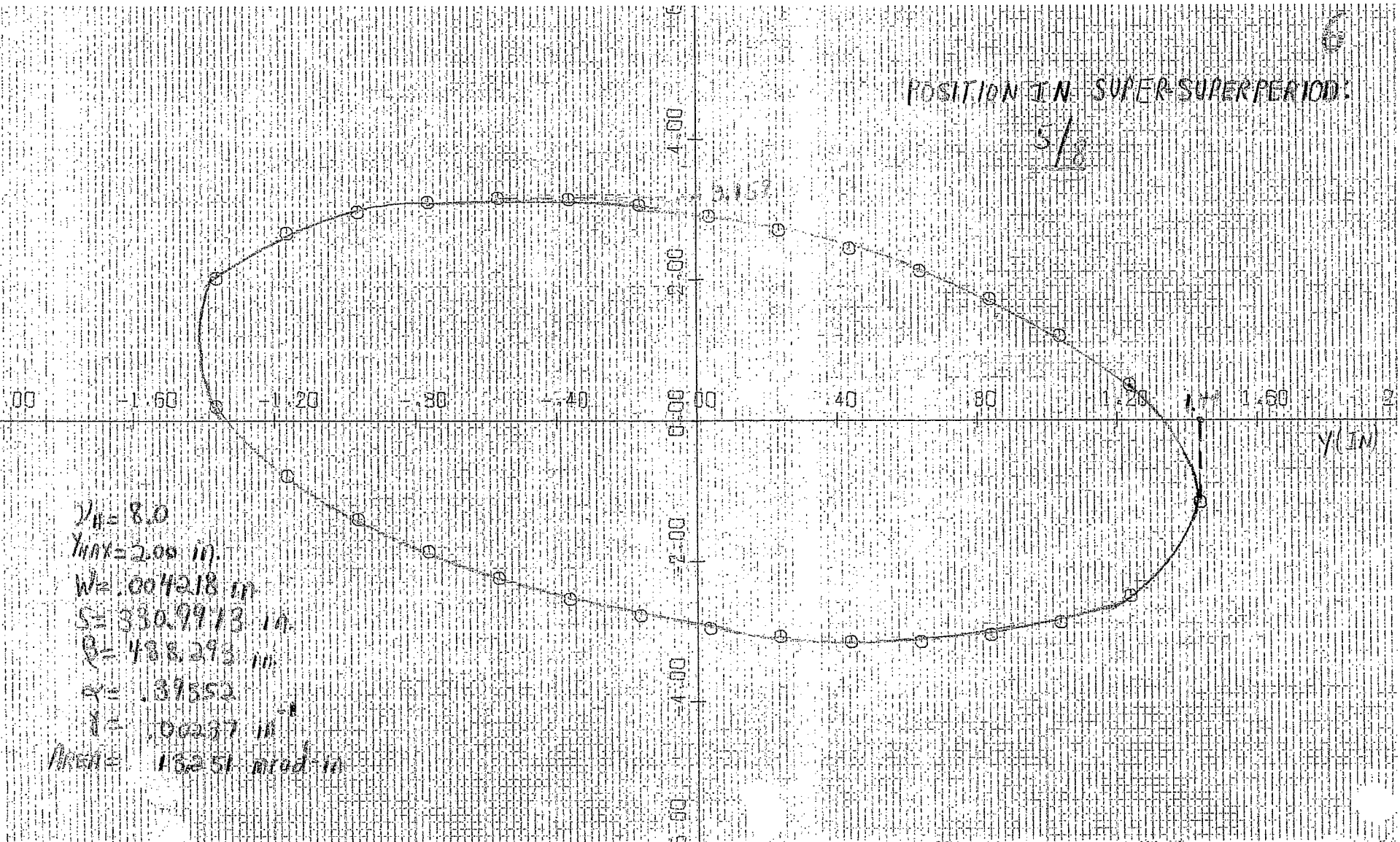
S:467

Y (in)

X (inrad)

POSITION IN SUPER-SUPERPERIOD:

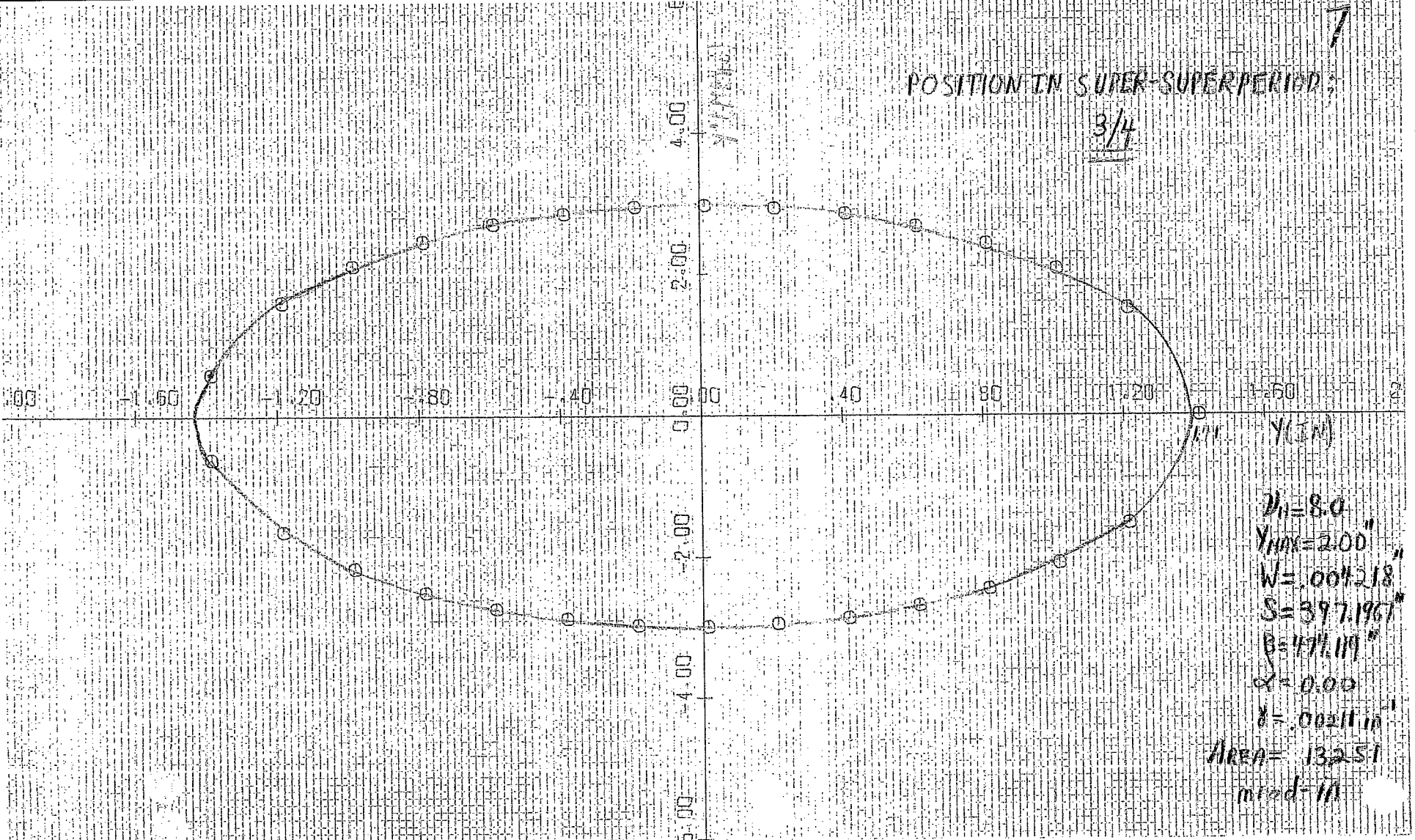
5/8



$v_H = 8.0$
 $Y_{MAX} = 2.00 \text{ in.}$
 $W = .004218 \text{ in.}$
 $S = 330.99713 \text{ in.}$
 $Q = 488.293 \text{ in.}$
 $r = .39552$
 $i = .00237 \text{ in.}$
 $Area = 16251 \text{ mod-in}$

POSITION IN SUPER-SUPERPERIOD:

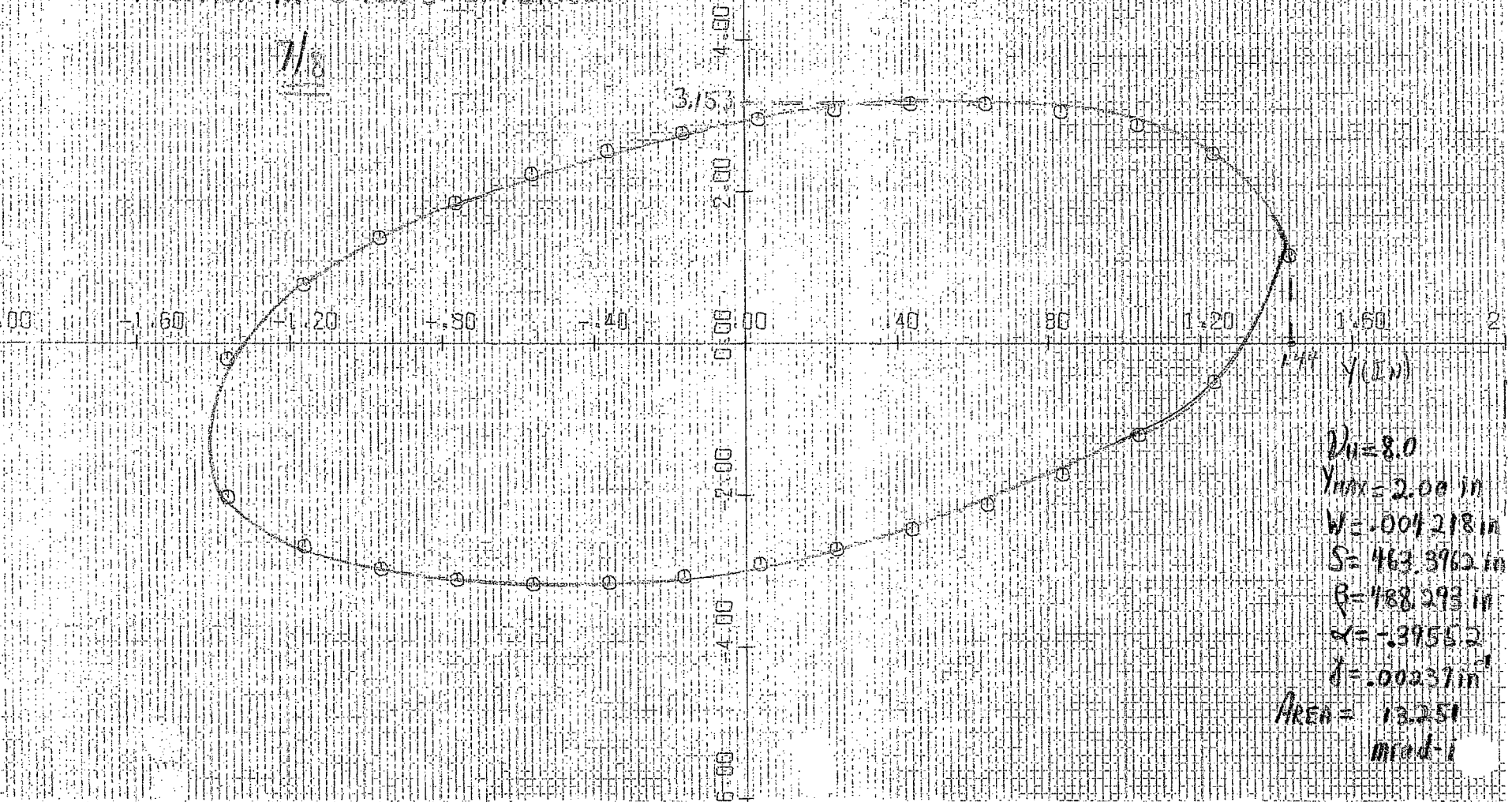
3/4



$D_H = 8.0$
 $Y_{MAX} = 2.00$
 $W = 1.004218$
 $S = 397.1967$
 $B = 177.119$
 $\alpha = 0.00$
 $\lambda = 0.00211$
 Area = 132.51
 mod = 11

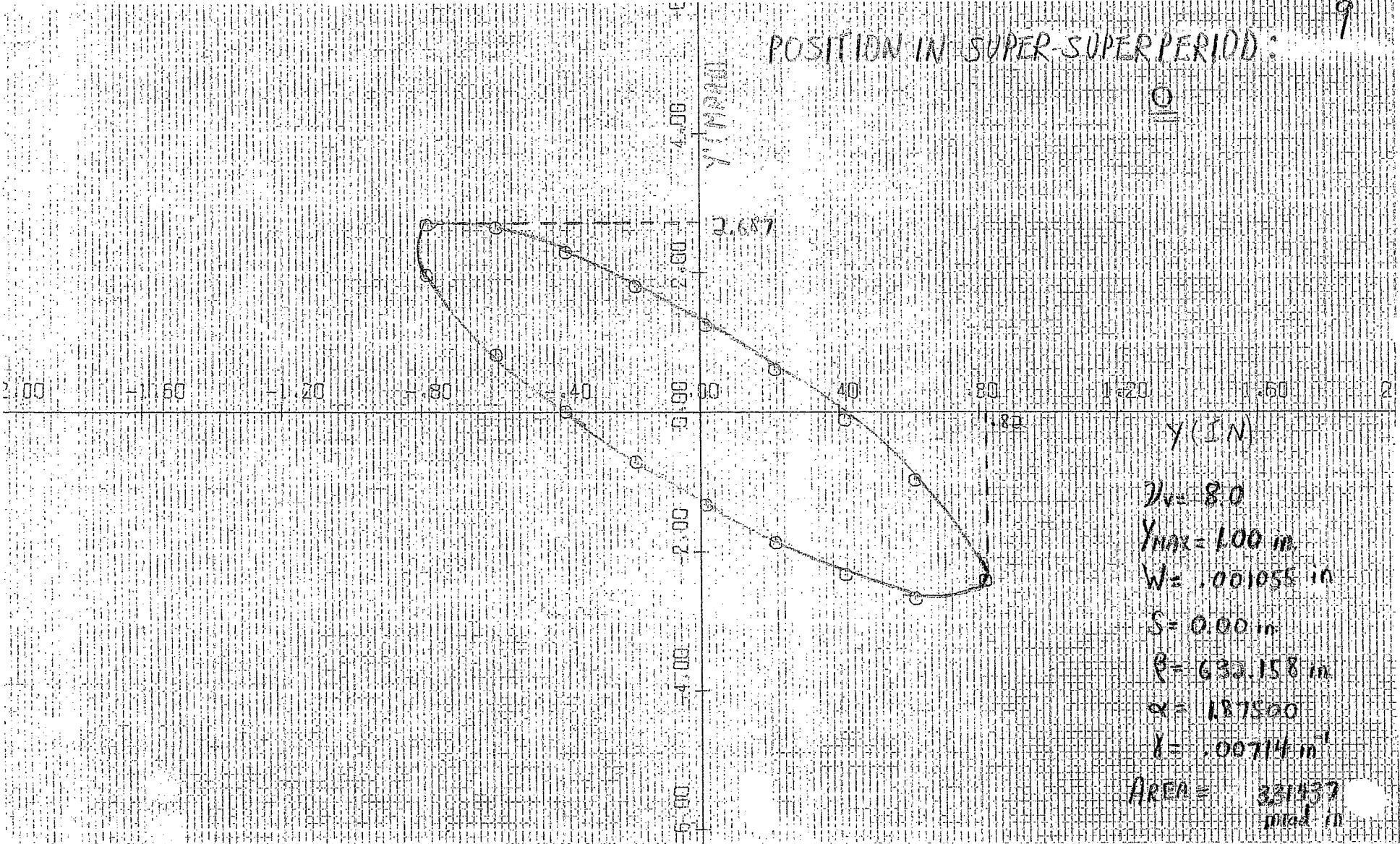
POSITION IN SUPER-SUPERPERIODS

7/8



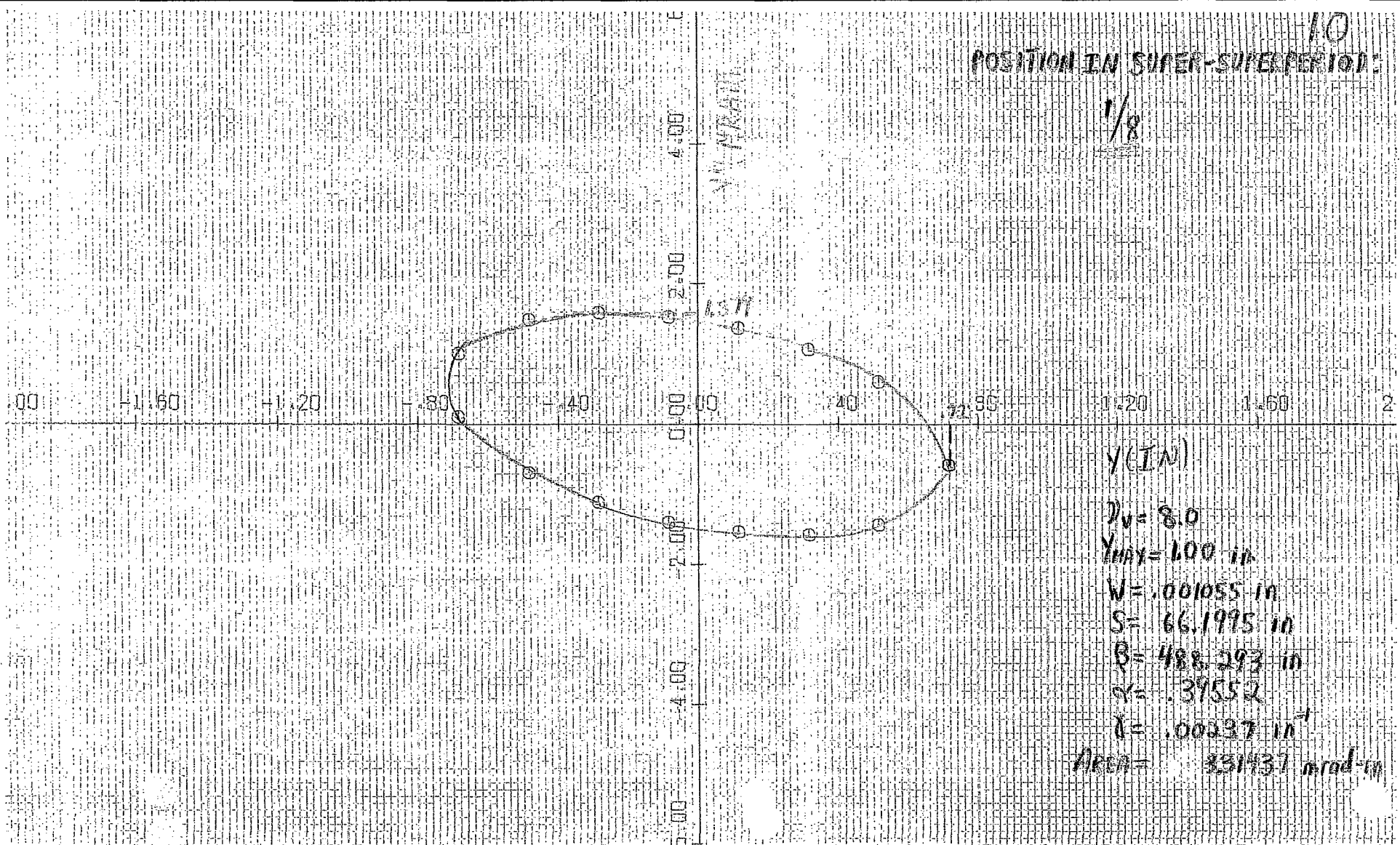
$D_{ii} = 8.0$
 $Y_{max} = 2.00 \text{ in}$
 $W = .004218 \text{ in}$
 $S = 463.3962 \text{ in}$
 $B = 488.293 \text{ in}$
 $\alpha = -.39552$
 $\delta = .00237 \text{ in}^{-1}$
Area = 13.251
med-i

POSITION IN SUPER-SUPER PERIOD: 9



10
POSITION IN SUPER-SUPERPERIOD:

1/8



Y (IN)

$D_v = 8.0$

$Y_{MAX} = 1.00 \text{ in}$

$W = .001055 \text{ in}$

$S = 66.1995 \text{ in}$

$B = 488.293 \text{ in}$

$\alpha = .39552$

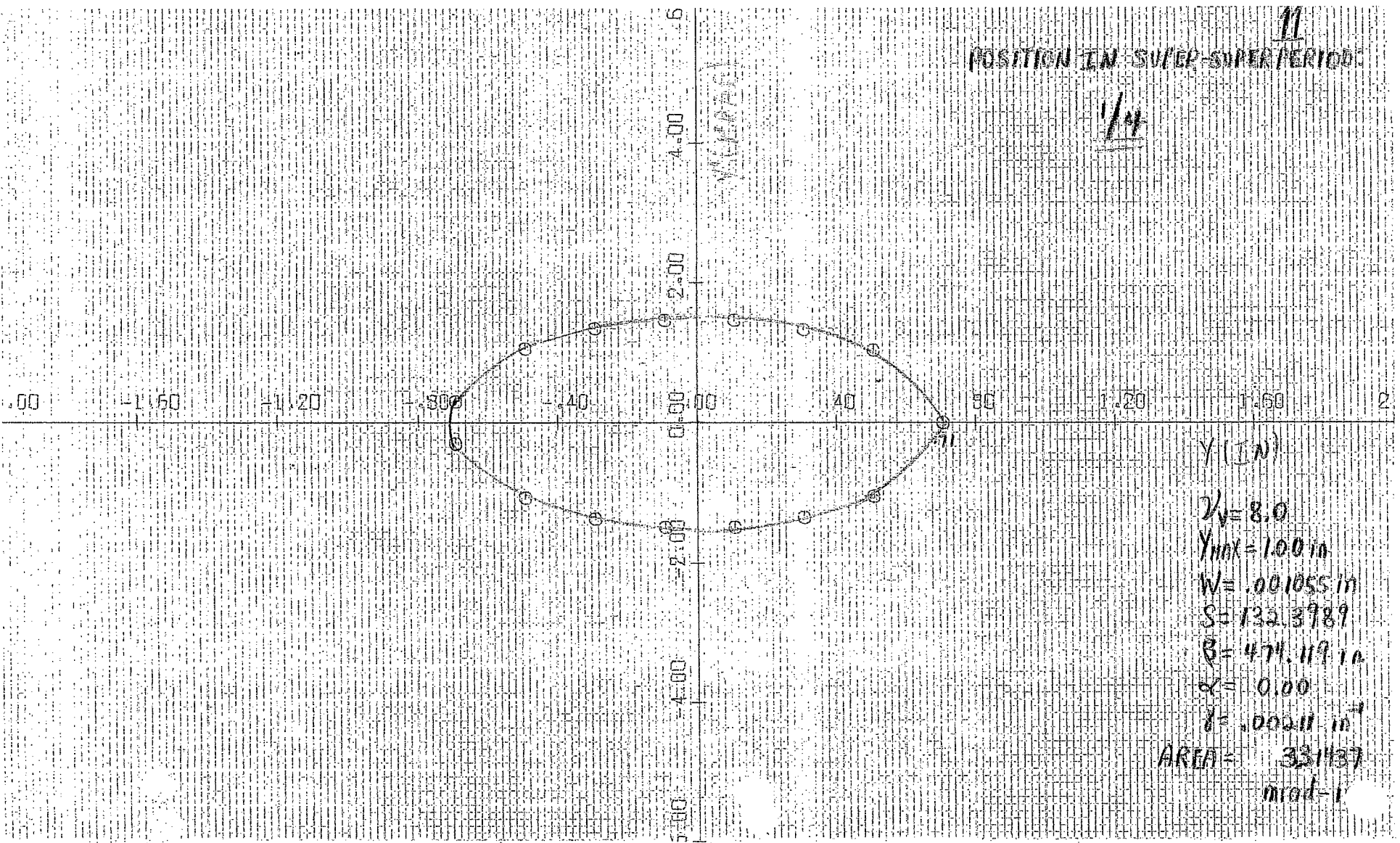
$\delta = .00337 \text{ in}^2$

AREA = 331437 in²

11

POSITION IN SUPER-SUPERPERIOD:

1/4



Y (IN)

$V = 8.0$

$Y_{max} = 1.00 \text{ in}$

$W = .001055 \text{ in}$

$S = 132.3989$

$B = 474.119 \text{ in}$

$\alpha = 0.00$

$\delta = .00211 \text{ in}^2$

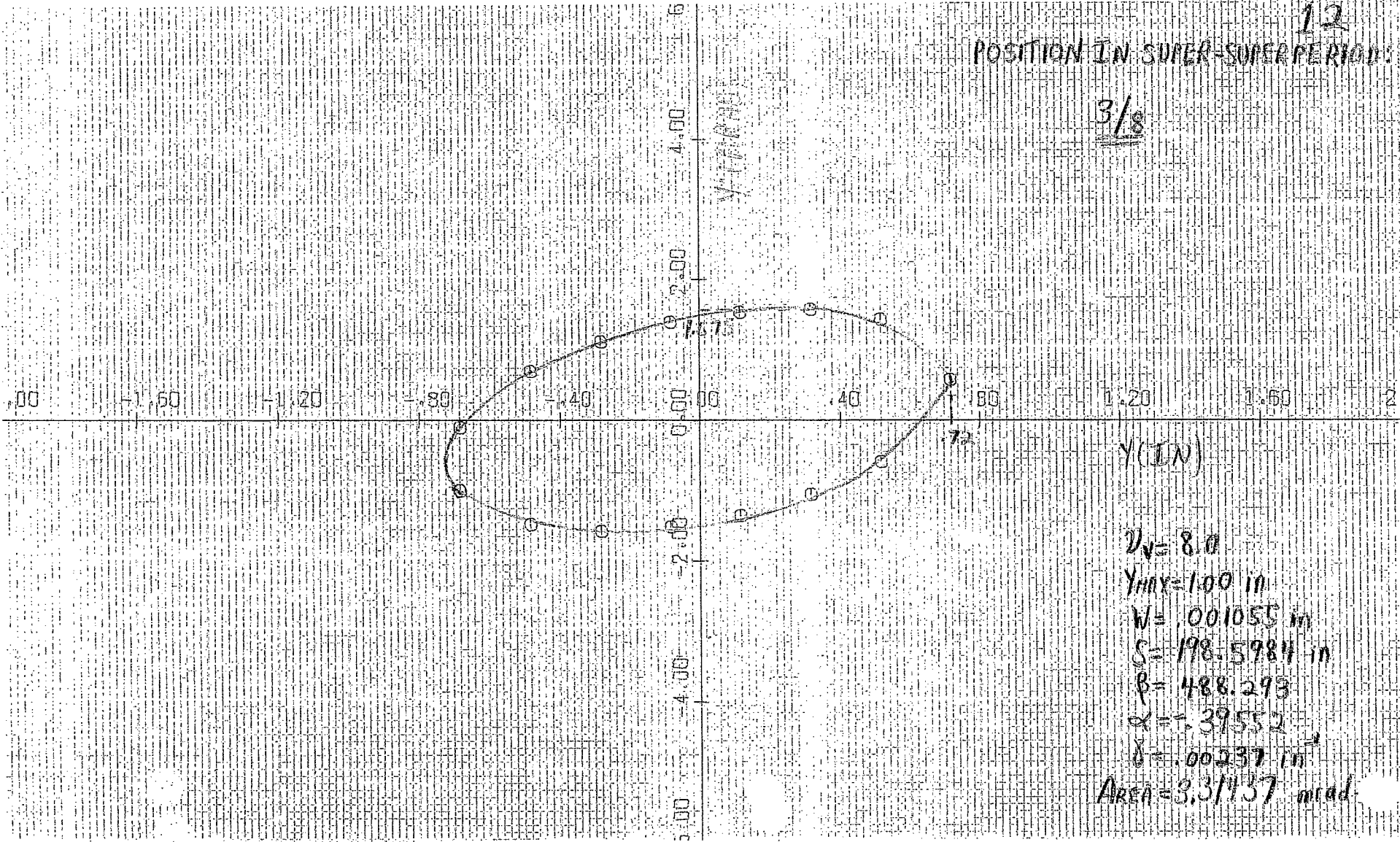
AREA = 32.1437

mod = 1

12

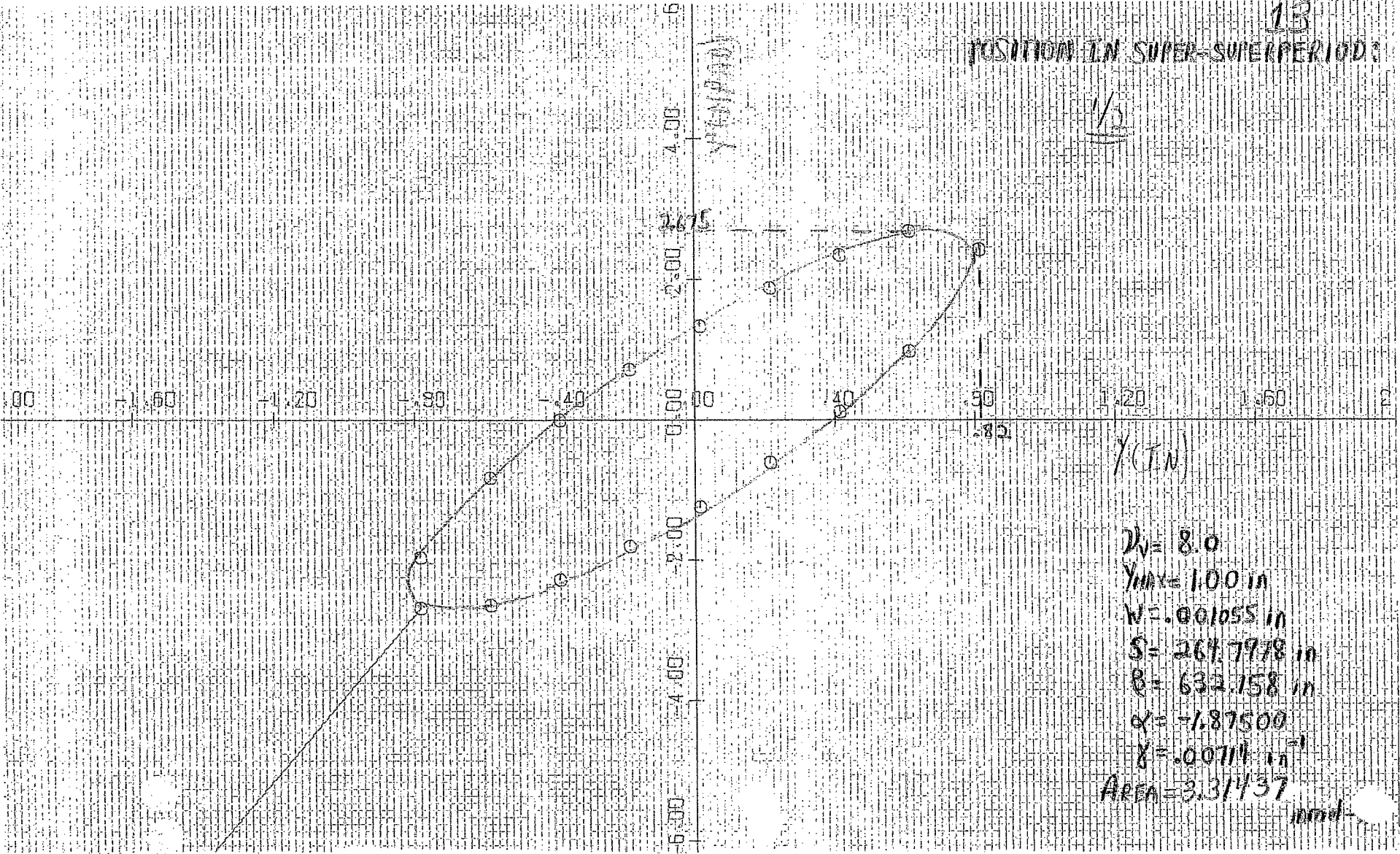
POSITION IN SUPER-SUPER PERIOD:

3/8



POSITION IN SUPER-SUPERPERIOD 3

1/3



X (IN)

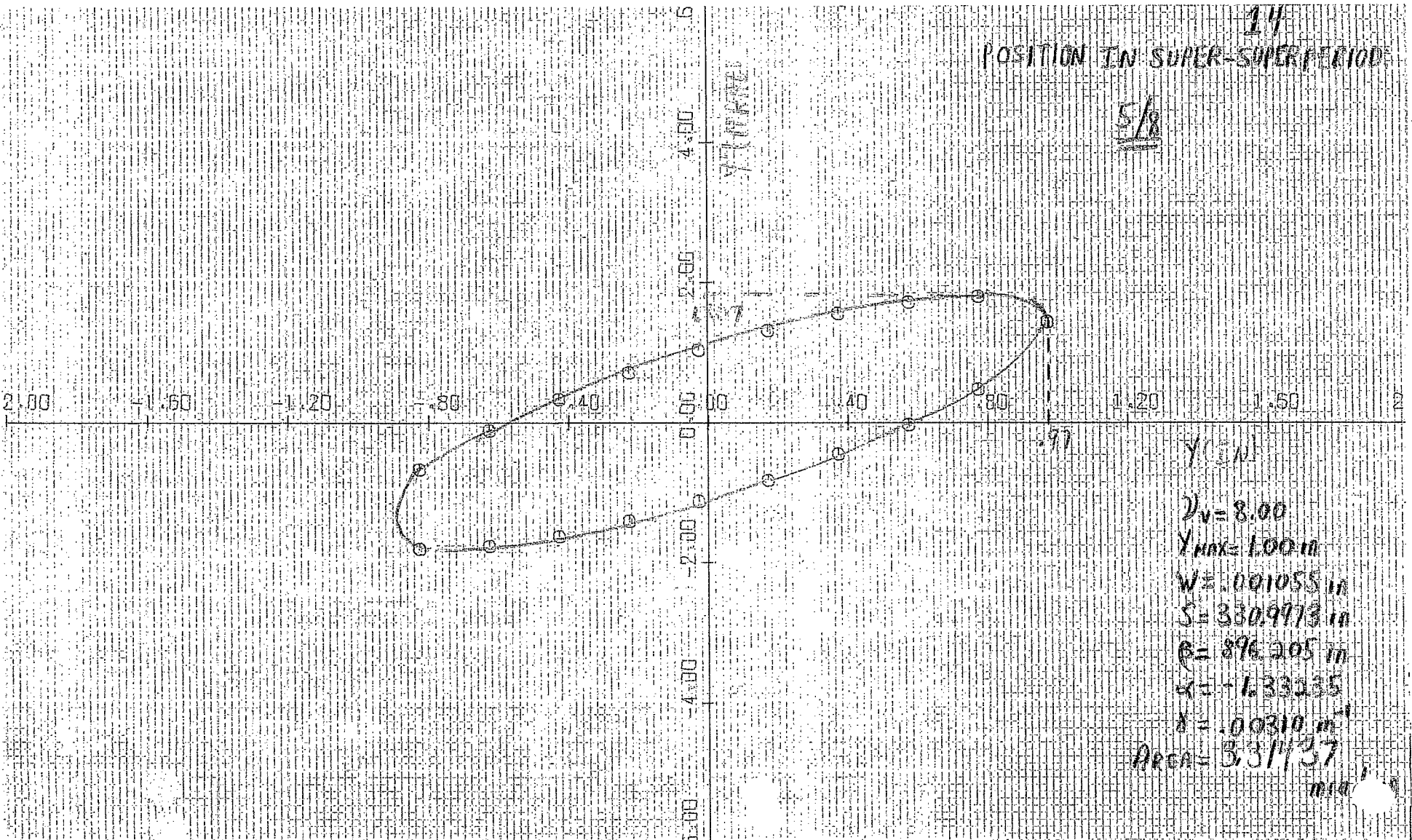
$W = 8.0$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = .001055 \text{ in}$
 $S = 264.7978 \text{ in}$
 $B = 632.158 \text{ in}$
 $\alpha = -1.87500$
 $\gamma = .00714 \text{ in}^{-1}$
 $Area = 3.31437$

mmol

14

POSITION IN SUPER-SUPERPERIOD

5/8



Y (CM)

$D_v = 8.00$

$Y_{max} = 1.00 \text{ in}$

$W = .001055 \text{ in}$

$S = 330.9773 \text{ in}$

$\rho = 896.205 \text{ in}$

$\alpha = -1.33035$

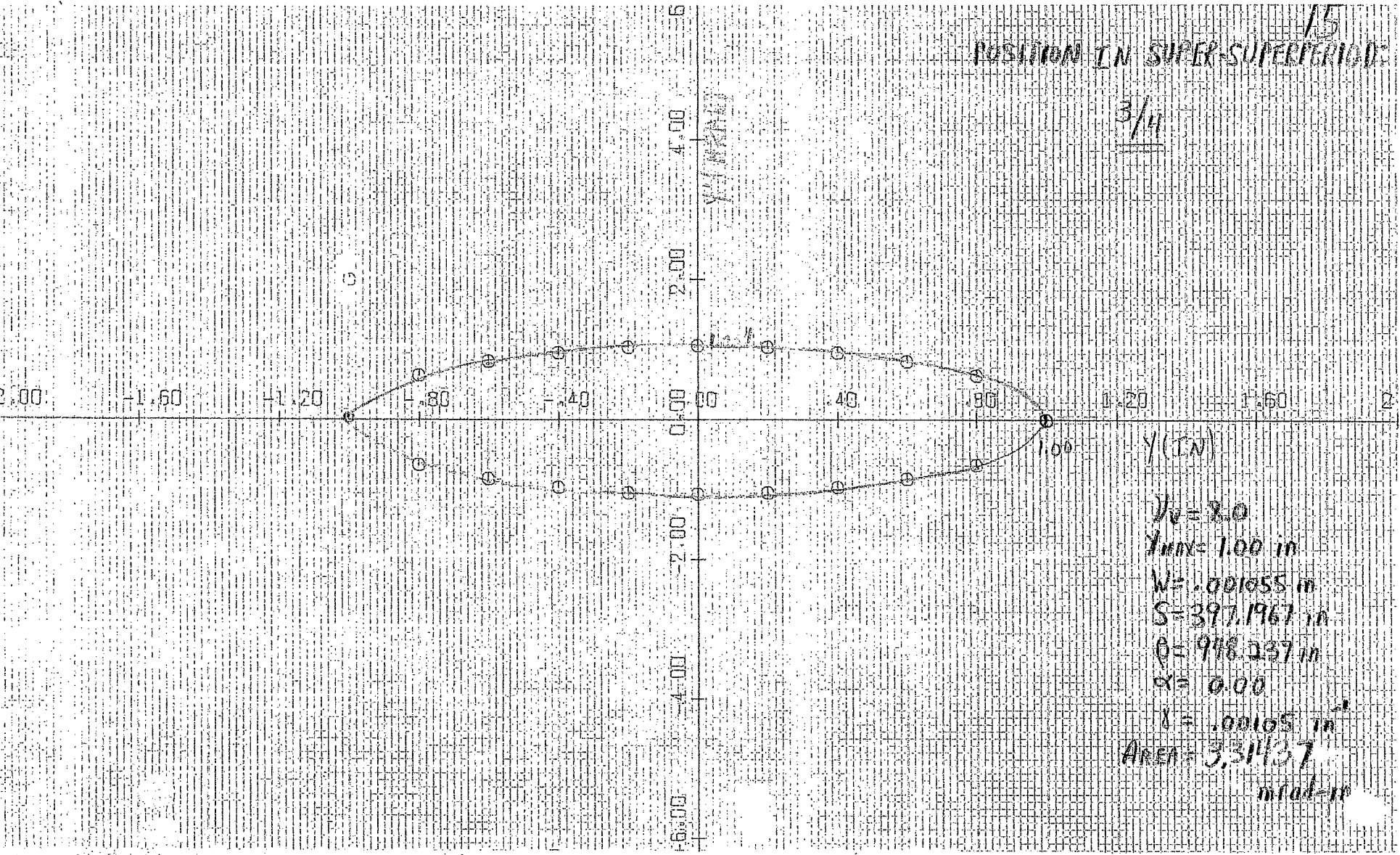
$\delta = .00310 \text{ m}^2$

Area = 3.31737

mic

15
POSITION IN SUPER-SUPERPERIOD:

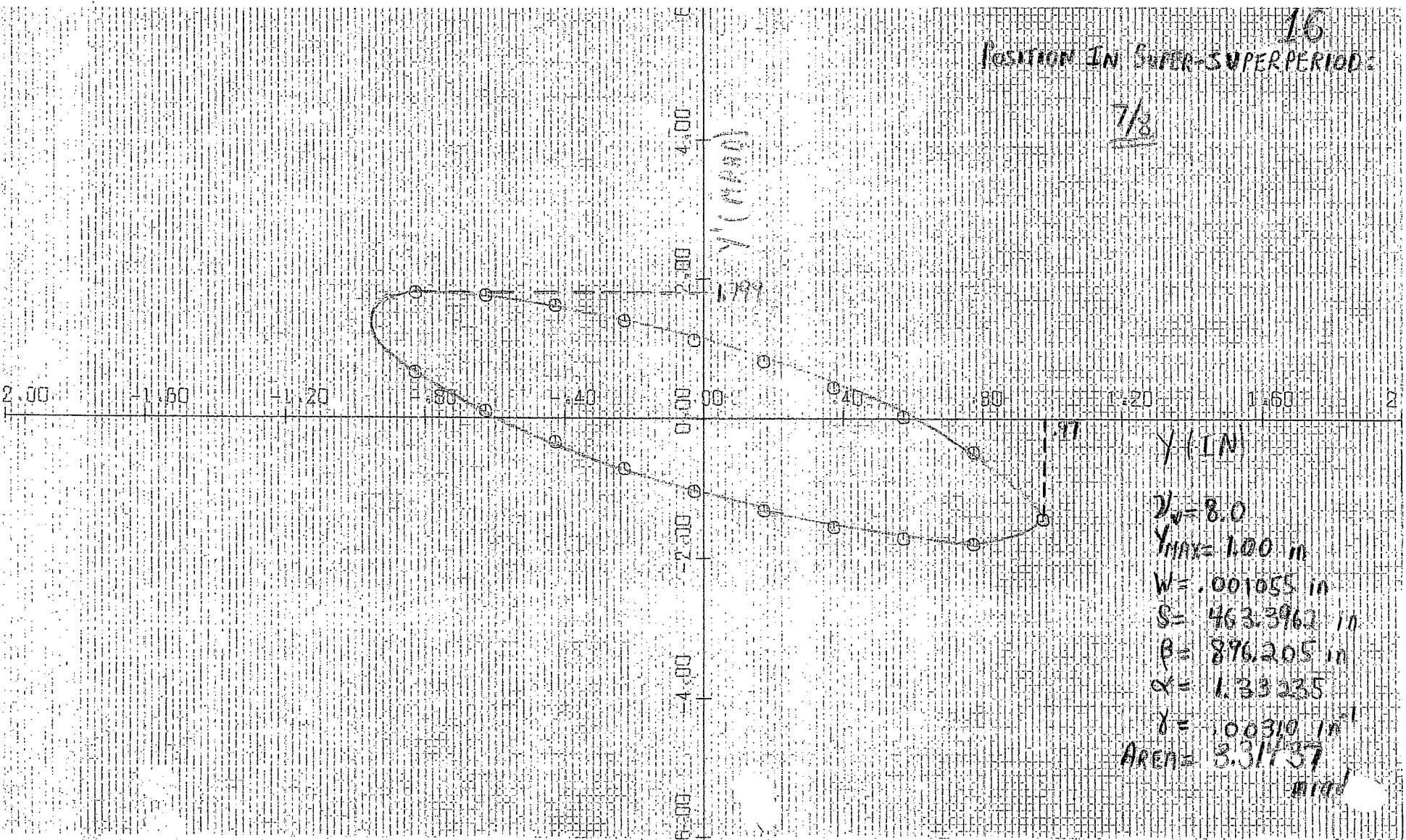
3/4



$V_0 = 2.0$
 $T_{max} = 1.00 \text{ in}$
 $W = .001055 \text{ in}$
 $S = 397.1967 \text{ in}$
 $\theta = 948.237 \text{ in}$
 $\alpha = 0.00$
 $\lambda = .00105 \text{ in}^2$
 $\text{AREA} = 3.31437$
msd-in

16
POSITION IN SUPER-SUPERPERIOD:

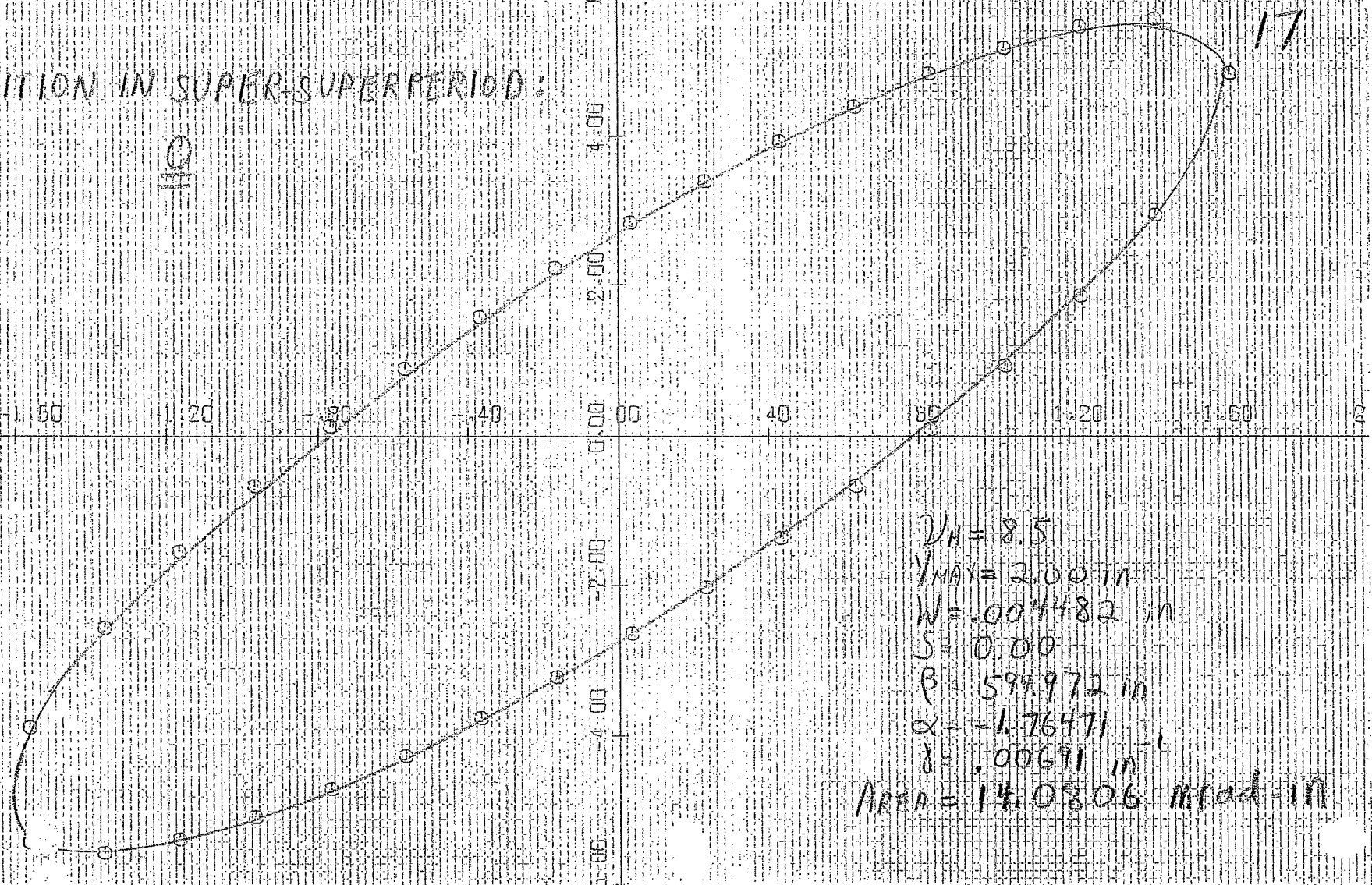
$\frac{7}{8}$



POSITION IN SUPER-SUPERPERIOD:

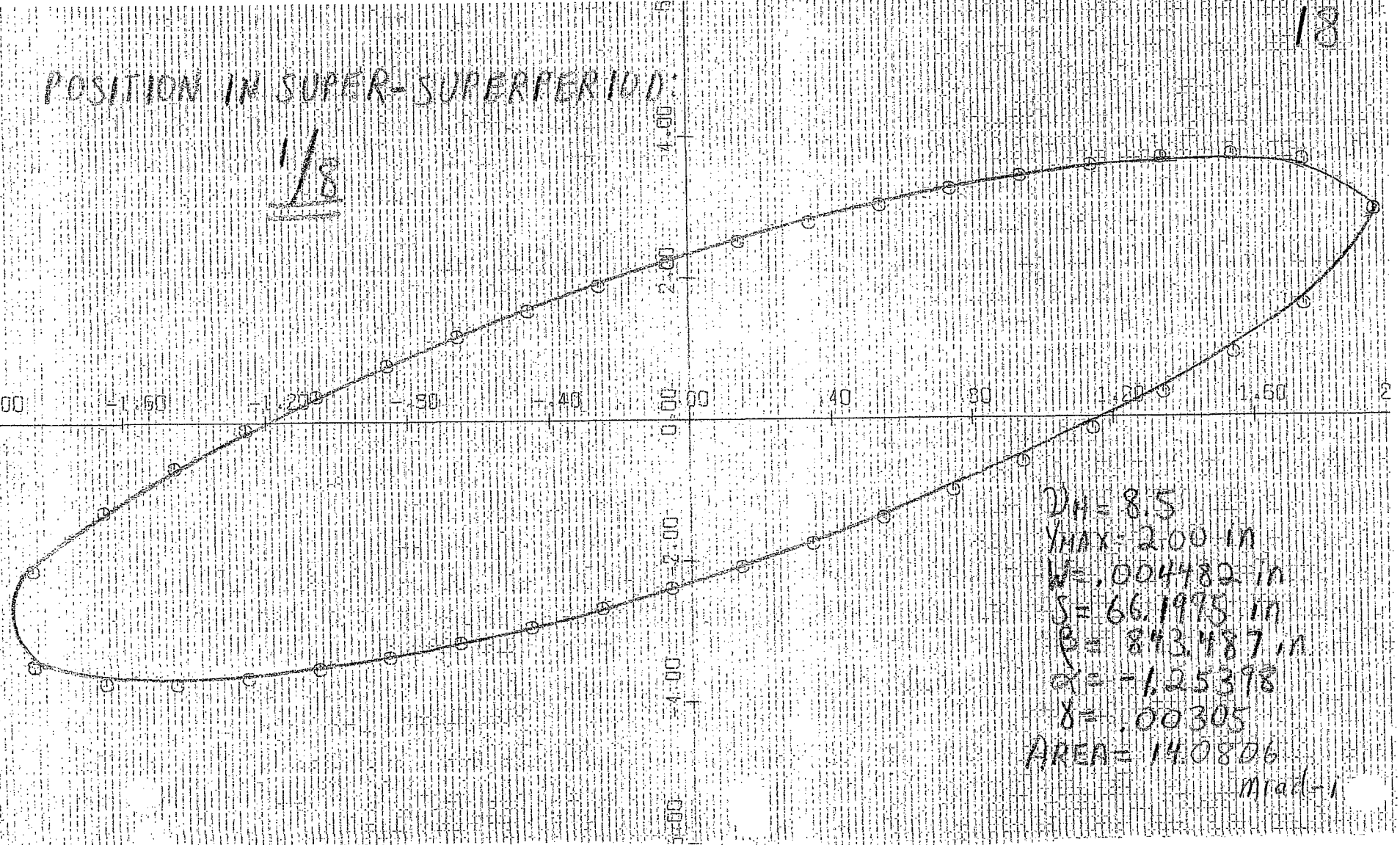
0

17



POSITION IN SUPER-SUPERPERIOD:

1/8



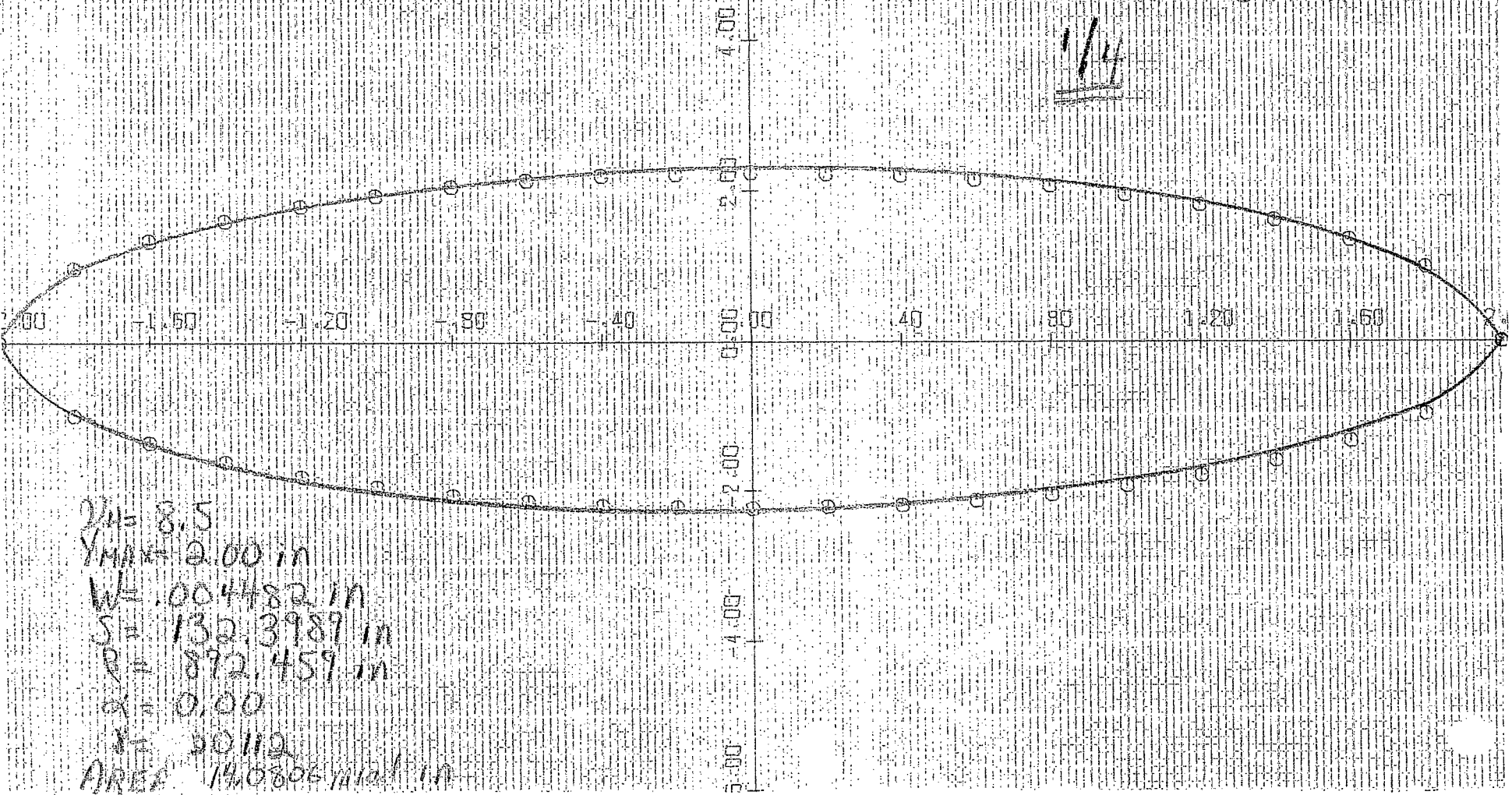
$DH = 8.5$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = .004482 \text{ in}$
 $S = 66.1975 \text{ in}$
 $\beta = 843.487 \text{ in}$
 $\alpha = -1.25398$
 $\delta = .00305$
 $AREA = 14.0806$

Mrad-1

19

POSITION IN SUPER-SUPERPERIOD:

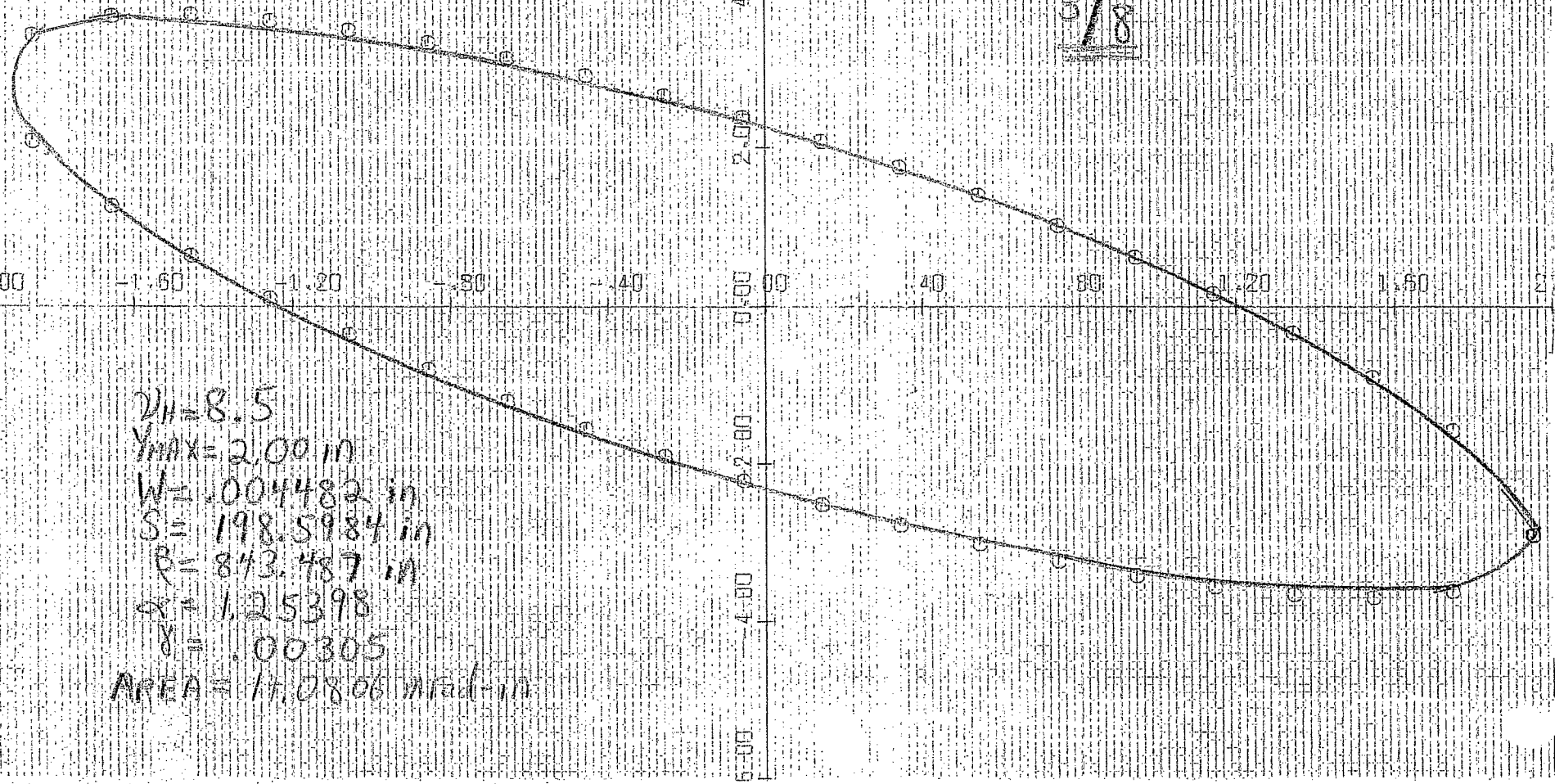
1/4



$D/H = 8.5$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = .004482 \text{ in}$
 $S = 132.3989 \text{ in}$
 $R = 892.459 \text{ in}$
 $\alpha = 0.00$
 $\beta = 30112$
 $AREA = 14.0806 \text{ in}^2 \text{ in}$

POSITION IN SUPER-SUPERPERIOD:

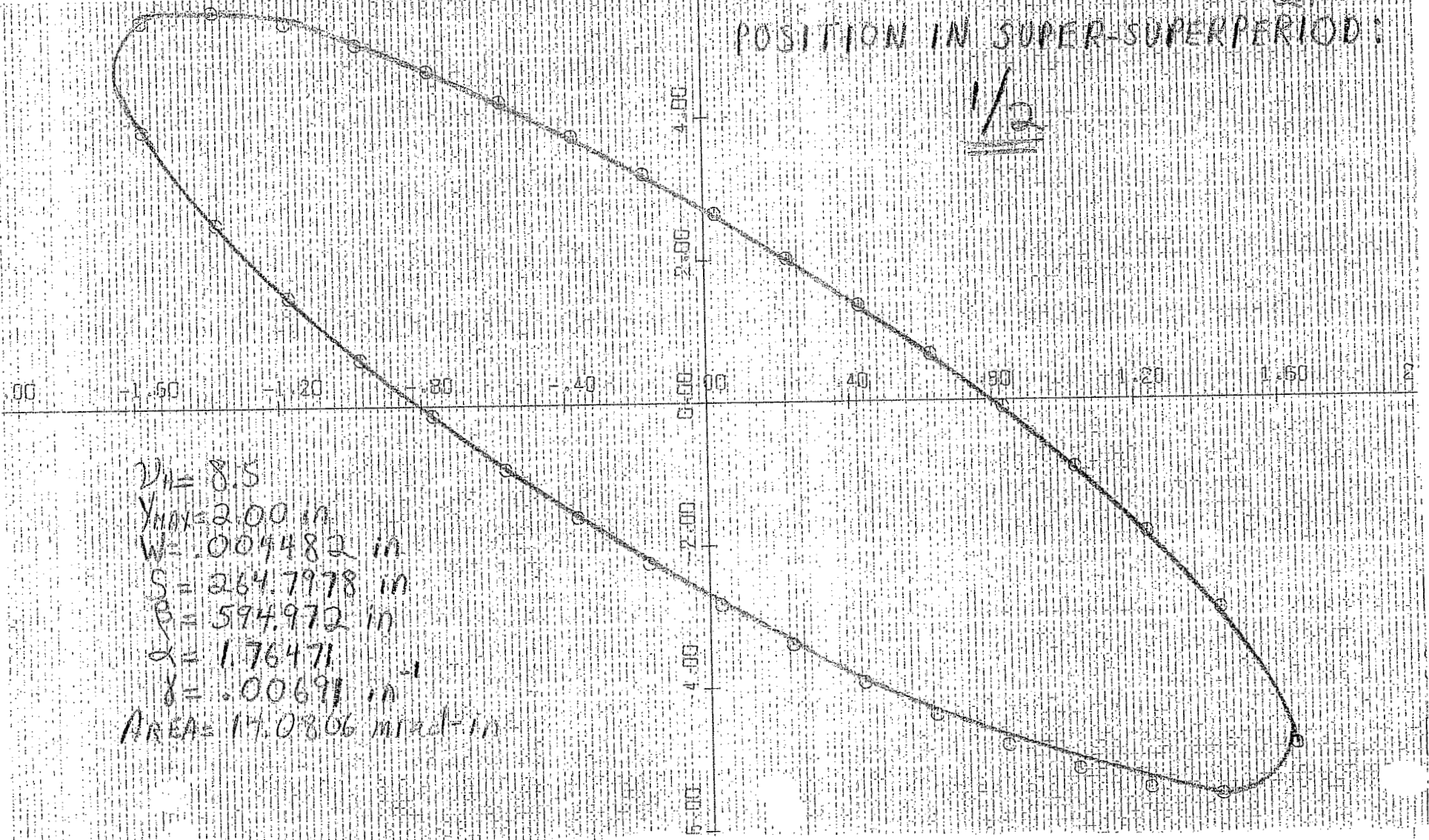
3/8



$D_H = 8.5$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = .004482 \text{ in}$
 $S = 198.5984 \text{ in}$
 $B = 843.487 \text{ in}$
 $\alpha = 1.25398$
 $\gamma = .00305$
 $AREA = 17.0806 \text{ in}^2$

21
POSITION IN SUPER-SUPERPERIOD:

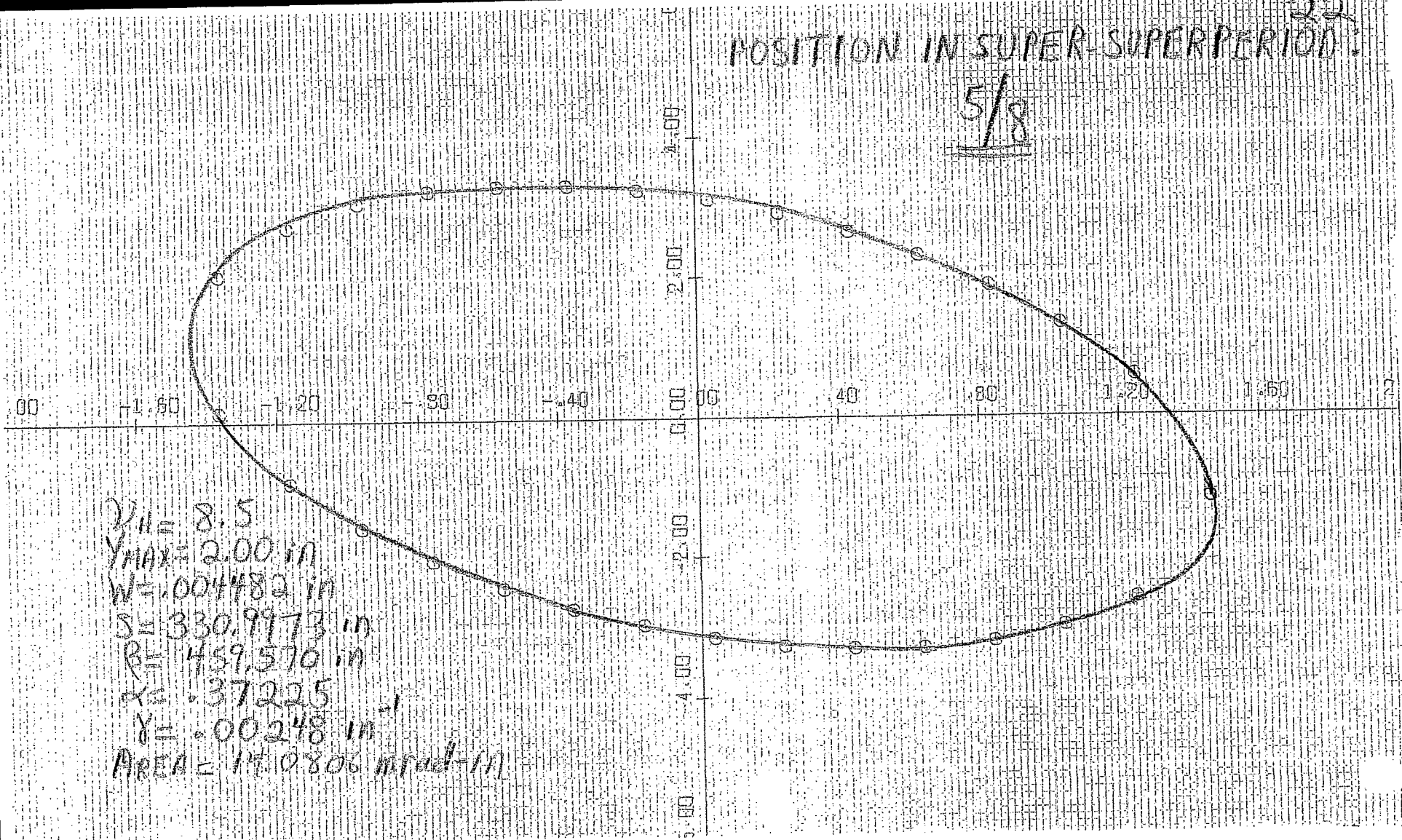
1/2



$\omega = 8.5$
 $Y_{max} = 2.00 \text{ in}$
 $W = 0.004482 \text{ in}$
 $S = 264.7978 \text{ in}$
 $\beta = 594.972 \text{ in}$
 $\alpha = 1.76471$
 $\delta = 0.00691 \text{ in}^{-1}$
AREA = 17.0806 $\text{mm}^2 \cdot \text{in}$

POSITION IN SUPER-SUPERPERIOD:

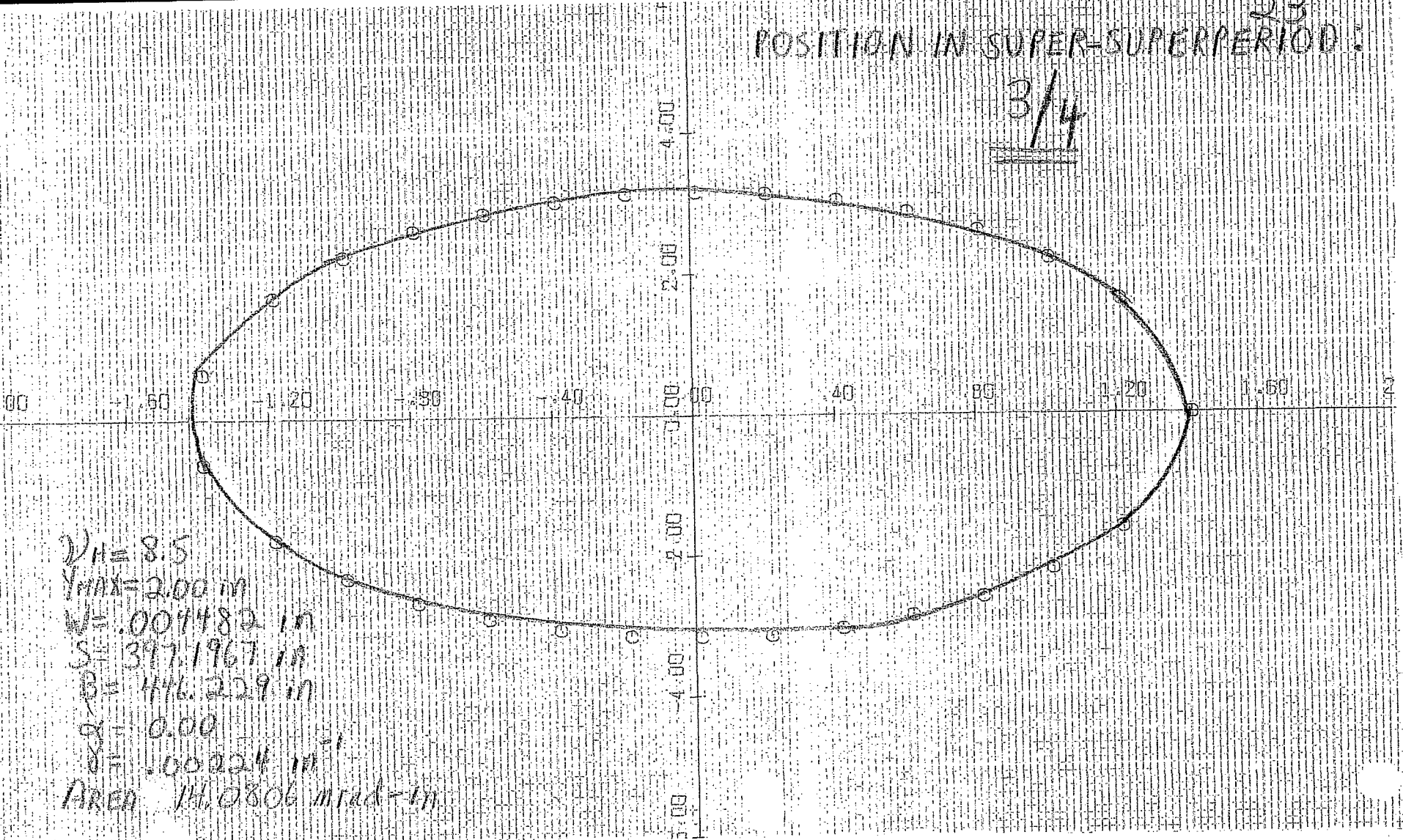
5/8



$D_H = 8.5$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = 1.004482 \text{ in}$
 $S = 330.9973 \text{ in}$
 $R = 1489.570 \text{ in}$
 $X_C = .37225$
 $Y_C = .00248 \text{ in}$
 $AREA = 14.0806 \text{ in}^2$

23
POSITION IN SUPER-SUPERPERIOD:

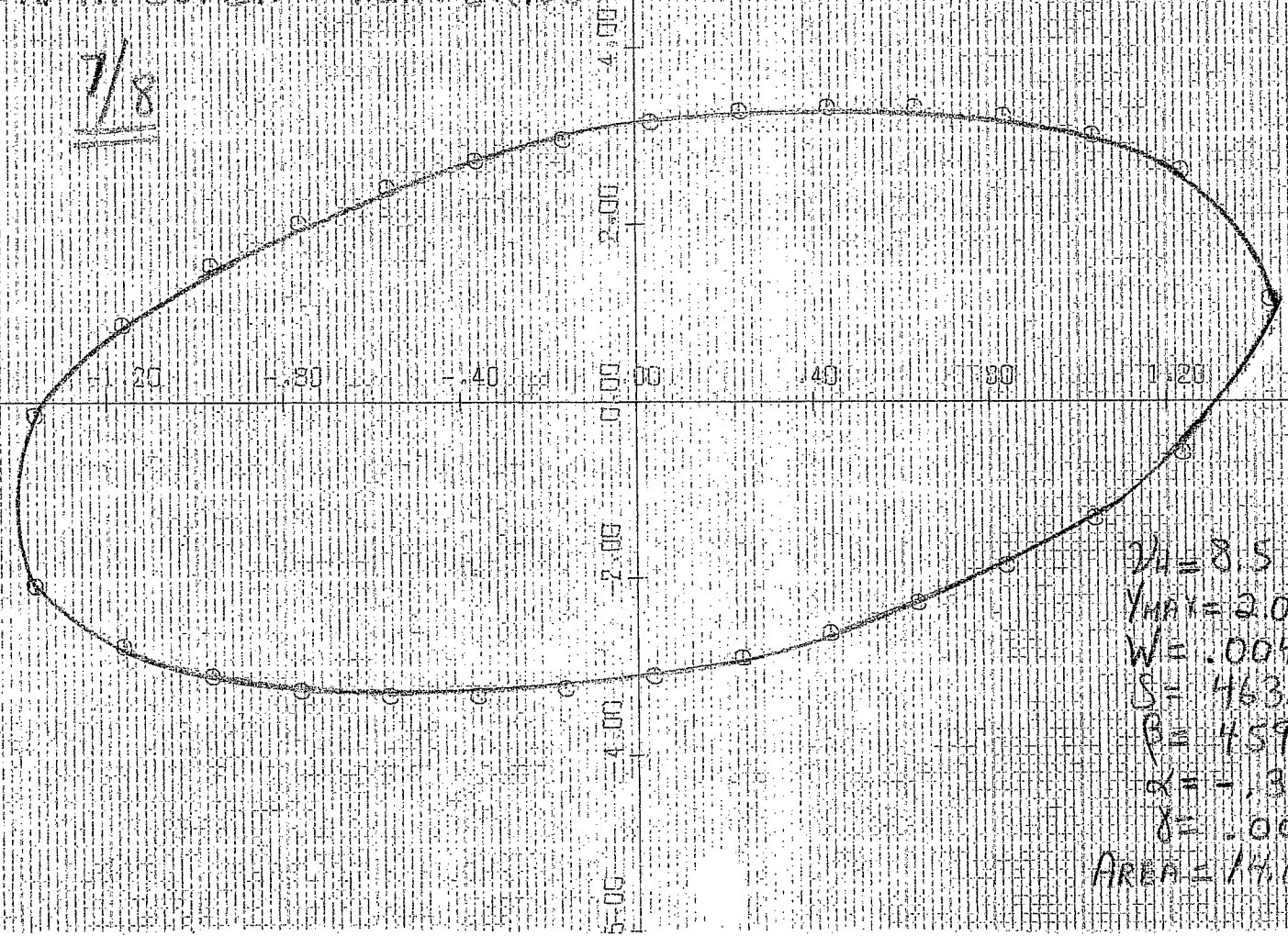
3/4



$\omega = 8.5$
 $Y_{MAX} = 2.00 \text{ m}$
 $W = .001482 \text{ m}$
 $S = 397.1967 \text{ m}$
 $B = 446.229 \text{ m}$
 $\alpha = 0.00$
 $\beta = .00224 \text{ m}^{-1}$
Area 14.0806 mrad-m

POSITION IN SUPER-SUPER PERIOD

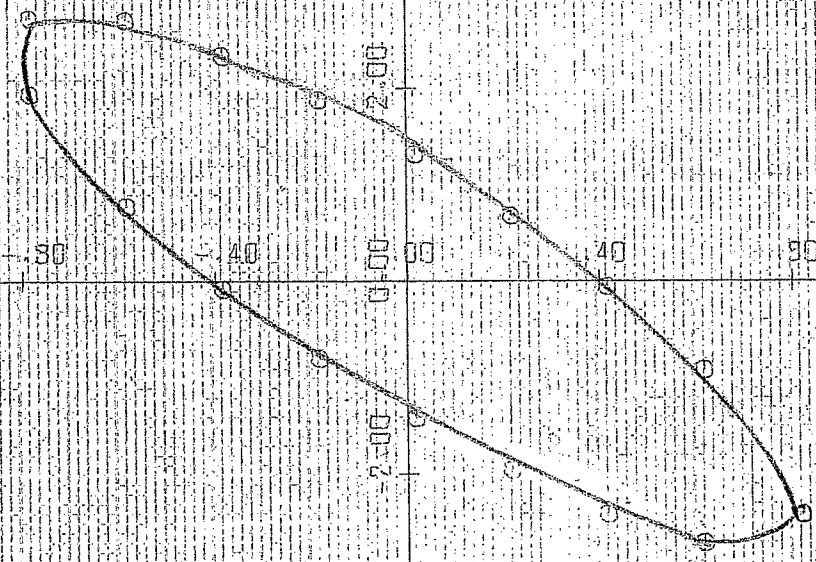
7/8



$2u = 8.5$
 $Y_{max} = 2.00 \text{ in}$
 $W = .004482 \text{ in}$
 $S = 463.3960 \text{ in}$
 $B = 459.570 \text{ in}$
 $\alpha = -.37225$
 $\delta = .00248 \text{ in}^{-1}$
 $\text{AREA} = 14.0806$
 $\text{Mrad} = 177$

POSITION IN SUPER-SUPERPERIOD:

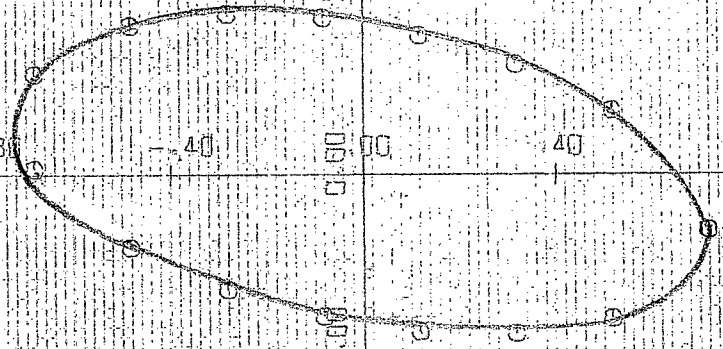
Q



$D_v = 8.5$
 $Y_{max} = 6.00 \text{ m}$
 $W = 0.001121 \text{ m}$
 $S = 0.00$
 $B = 594.972 \text{ m}$
 $\alpha = 1.76471$
 $b = 0.00691 \text{ m}^{-1}$
 $AREA = 3.5217 \text{ mrad-m}$

POSITION IN SUPER-SUPERPERIODS

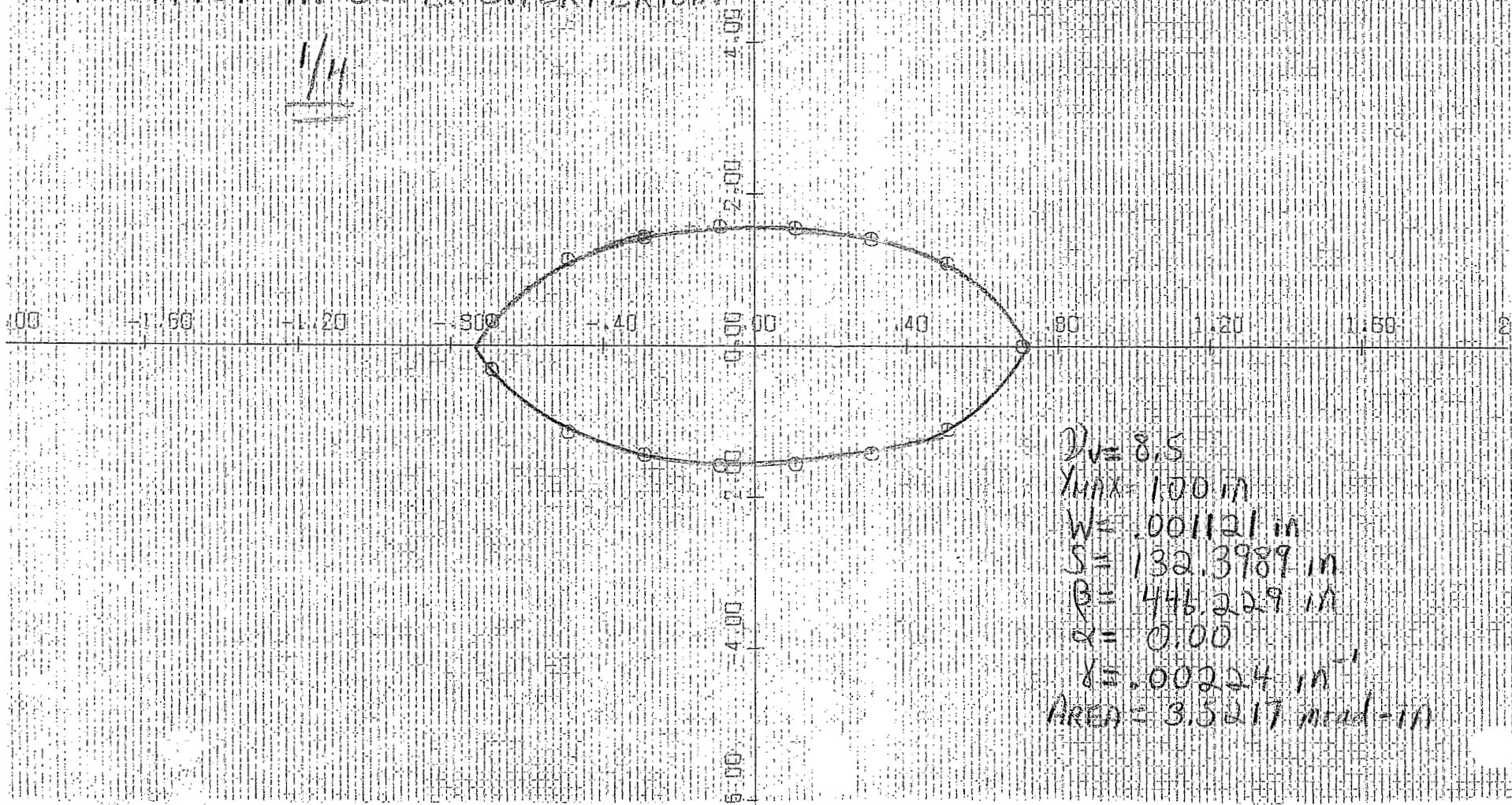
1/8



$2N = 8.5$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = .001121 \text{ in}$
 $S = 66.1995 \text{ in}$
 $\beta = 459.570 \text{ in}$
 $\alpha = .37225$
 $\gamma = .00218 \text{ in}^{-1}$
 $\text{Area} = 3.527 \text{ mad-in}$

POSITION IN SUPER-SUPER PERIOD:

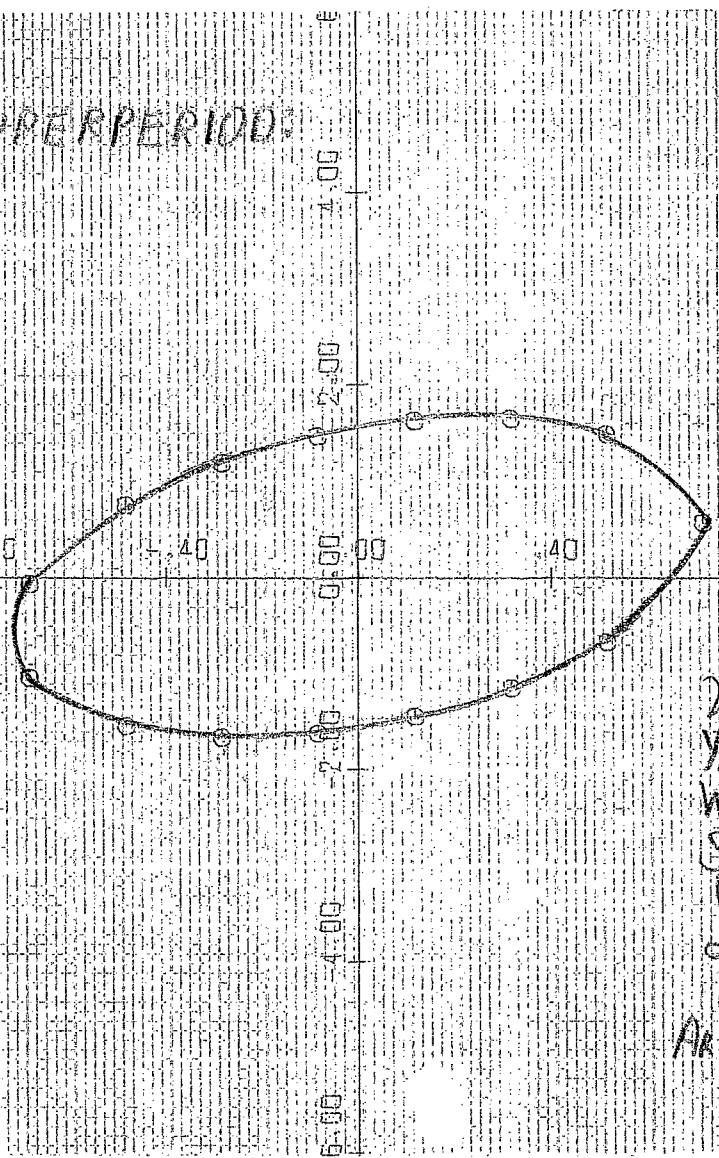
1/4



$D_v = 8.5$
 $Y_{max} = 1.00 \text{ in}$
 $W = .001121 \text{ in}$
 $S = 132.3989 \text{ in}$
 $B = 446.229 \text{ in}$
 $\alpha = 0.00$
 $\gamma = .00224 \text{ in}^{-1}$
 $\text{AREA} = 3.5217 \text{ mead-in}$

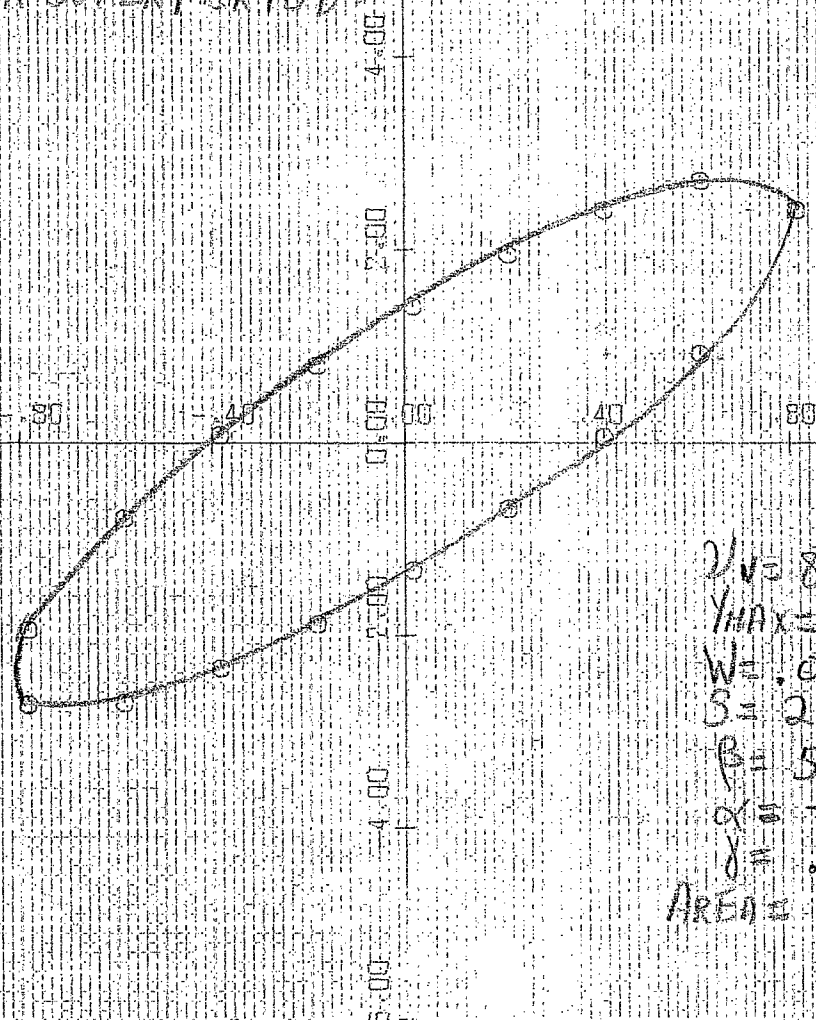
POSITION IN SUPER-SUPERPERIOD:

3/8



$\Delta V = 8.5$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = .001121 \text{ in}$
 $S = 798.5984 \text{ in}$
 $\beta = 459.570 \text{ in}$
 $\alpha = -.37225$
 $\delta = .00248 \text{ in}^{-1}$
 $AREA = 3.5217 \text{ in}^2$

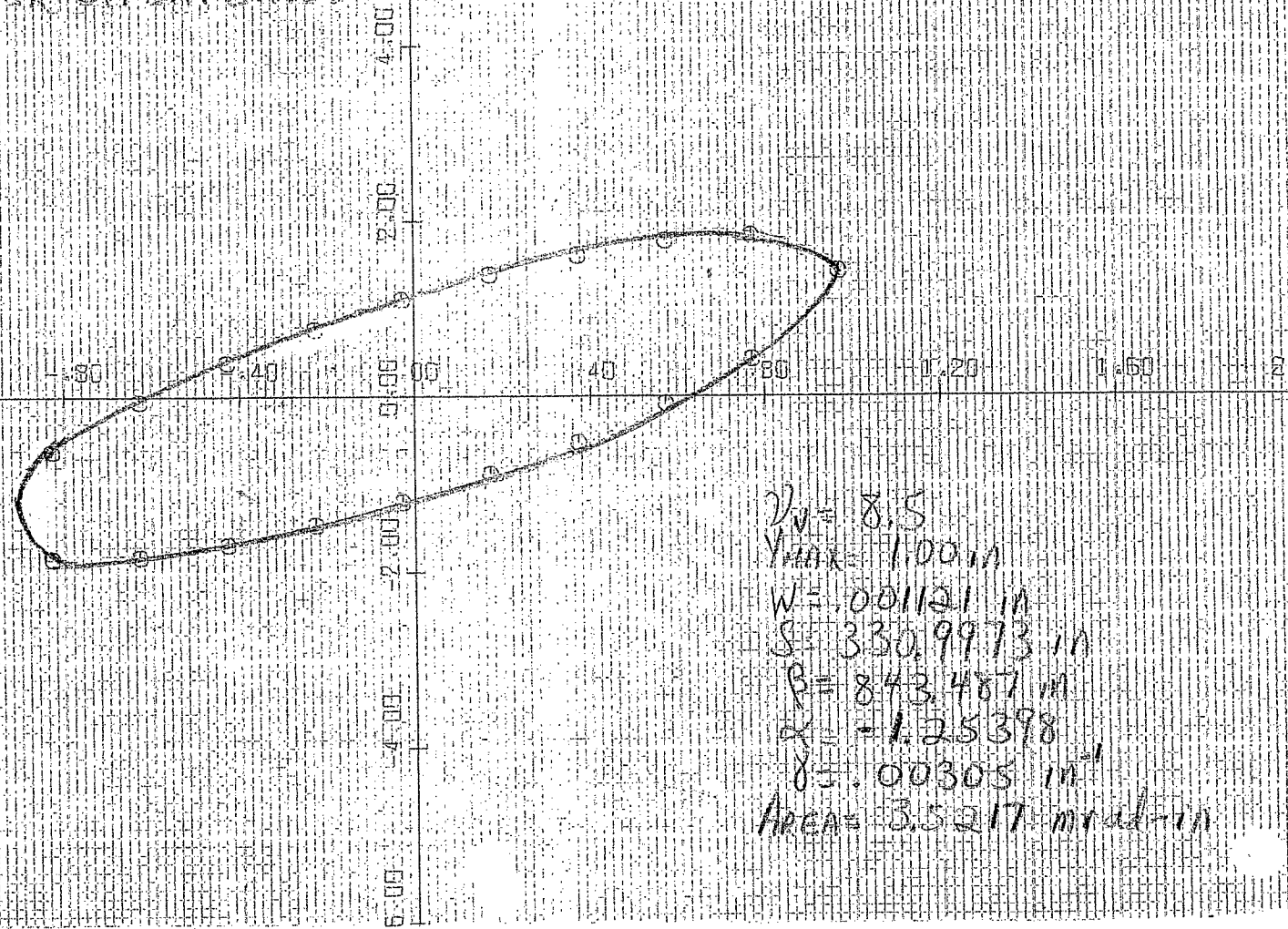
POSITION IN SUPER-SUPERPERIOD:

1/2

$$\begin{aligned}
 2V &= 8.5 \\
 Y_{MAX} &= 1.00 \text{ in} \\
 W &= .001121 \\
 S &= 264.7978 \text{ in} \\
 \beta &= 594.972 \text{ in} \\
 \alpha &= -1.76771 \text{ in} \\
 \gamma &= .00691 \text{ in} \\
 \text{AREA} &= 3.5217 \text{ in}^2 \text{ in}
 \end{aligned}$$

POSITION IN SUPER-SUPERPERIODS:

5/8

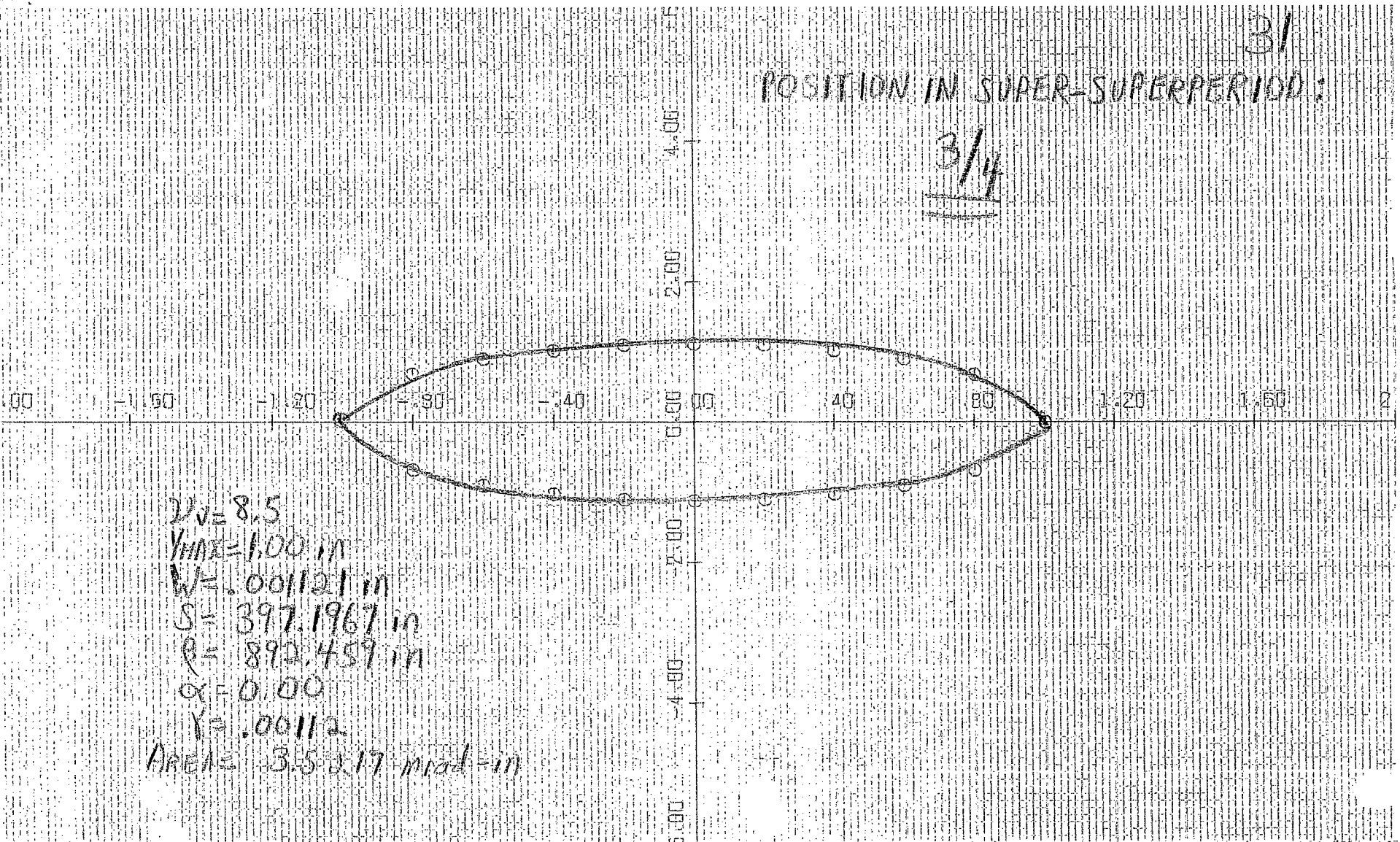


$\nu_w = 8.5$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = 0.01121 \text{ in}$
 $S = 330.9973 \text{ in}$
 $\beta = 843.487 \text{ in}$
 $\alpha = -1.25398$
 $\delta = .00305 \text{ in}^2$
 $AREA = 3.5217 \text{ mrad-in}$

3/

POSITION IN SUPER-SUPERPERIOD:

3/4

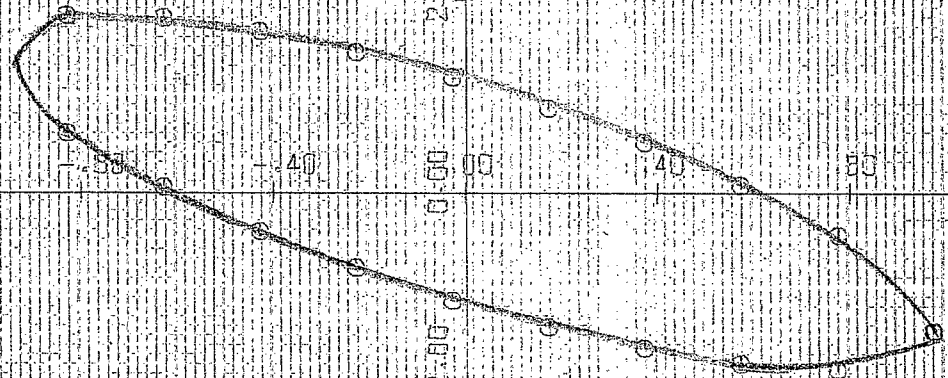


$DV = 8.5$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = 1.001121 \text{ in}$
 $S = 397.1967 \text{ in}$
 $R = 892.459 \text{ in}$
 $\alpha = 0.00$
 $Y = .00112$
 $AREA = 3.5217 \text{ in}^2 = \text{in}$

32

POSITION IN SUPER-SUPERPERIOD:

7/8

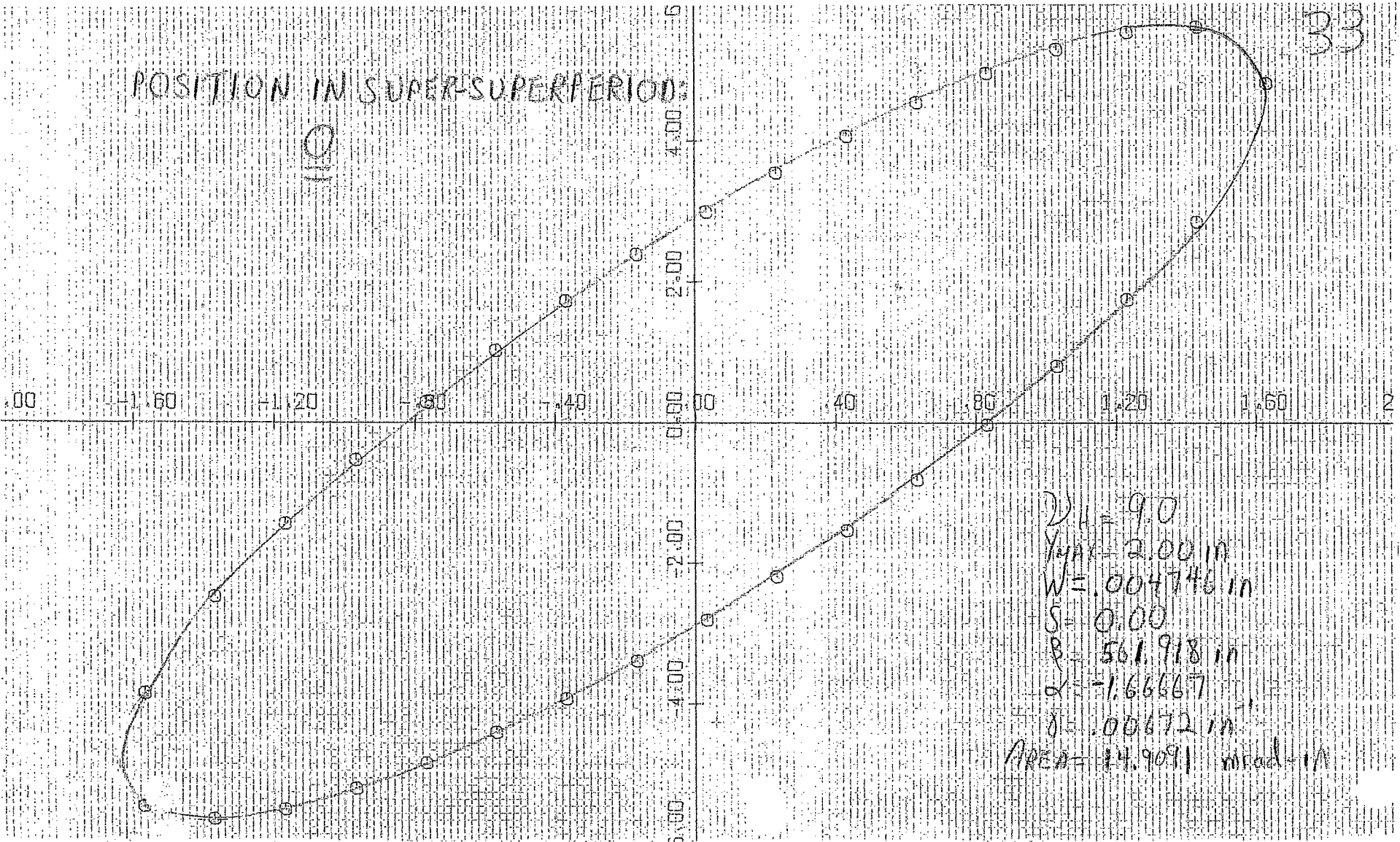


$D_v = 8.5$
 $Y_{MAX} = 6.00 \text{ in}$
 $W = 1.001121 \text{ in}$
 $S = 463.3962 \text{ in}$
 $\beta = 843.487 \text{ in}$
 $\alpha = 1.25398$
 $\delta = .00305 \text{ in}^{-1}$
 $AREA = 3.5217 \text{ in}^2$

POSITION IN SUPER-SUPERPERIOD:

10

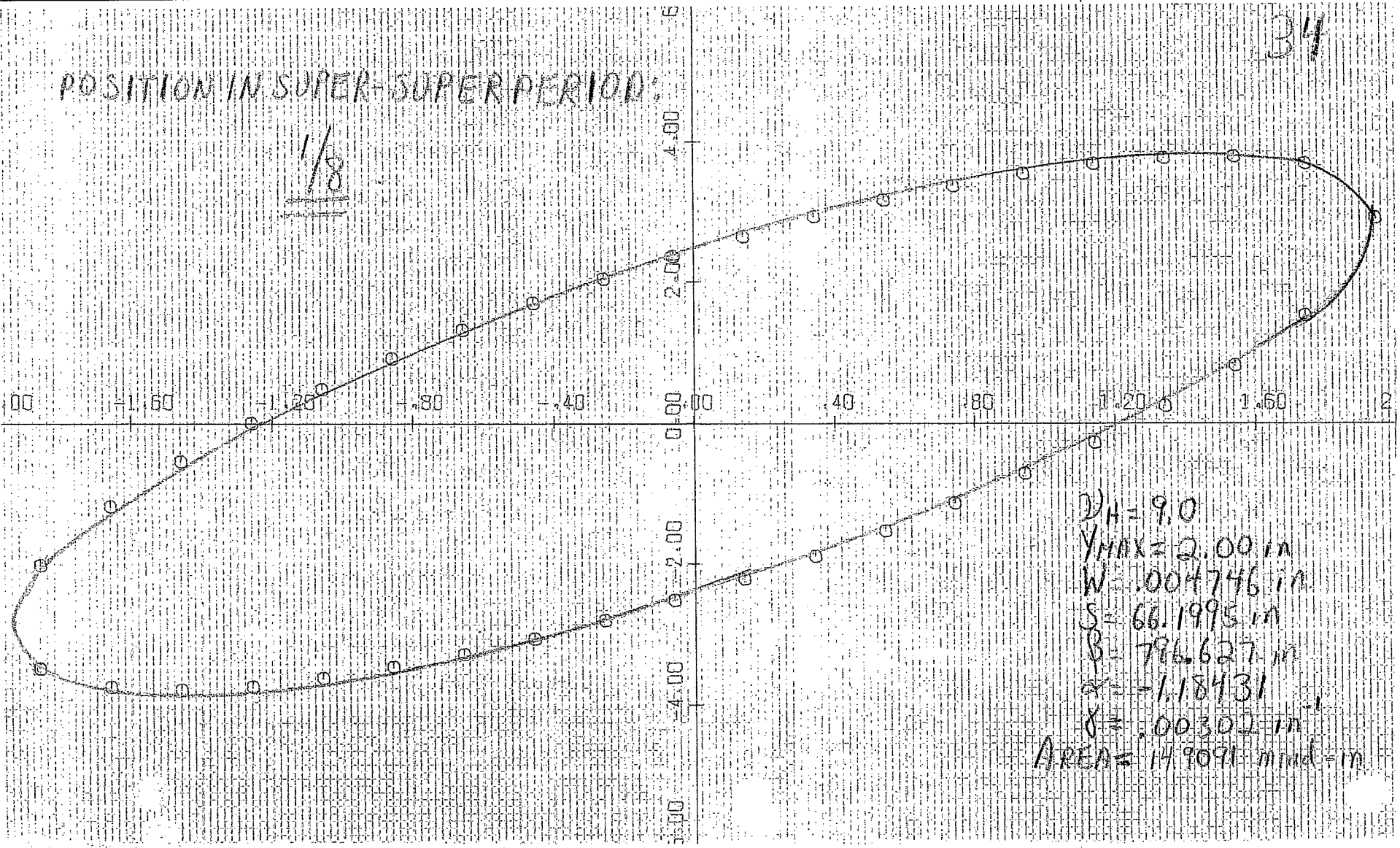
33



$D_1 = 9.0$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = 0.004746 \text{ in}$
 $S = 0.00$
 $R = 561.918 \text{ in}$
 $\alpha = -1.66667$
 $\delta = 0.00672 \text{ in}$
 $AREA = 14.9091 \text{ msrad-in}$

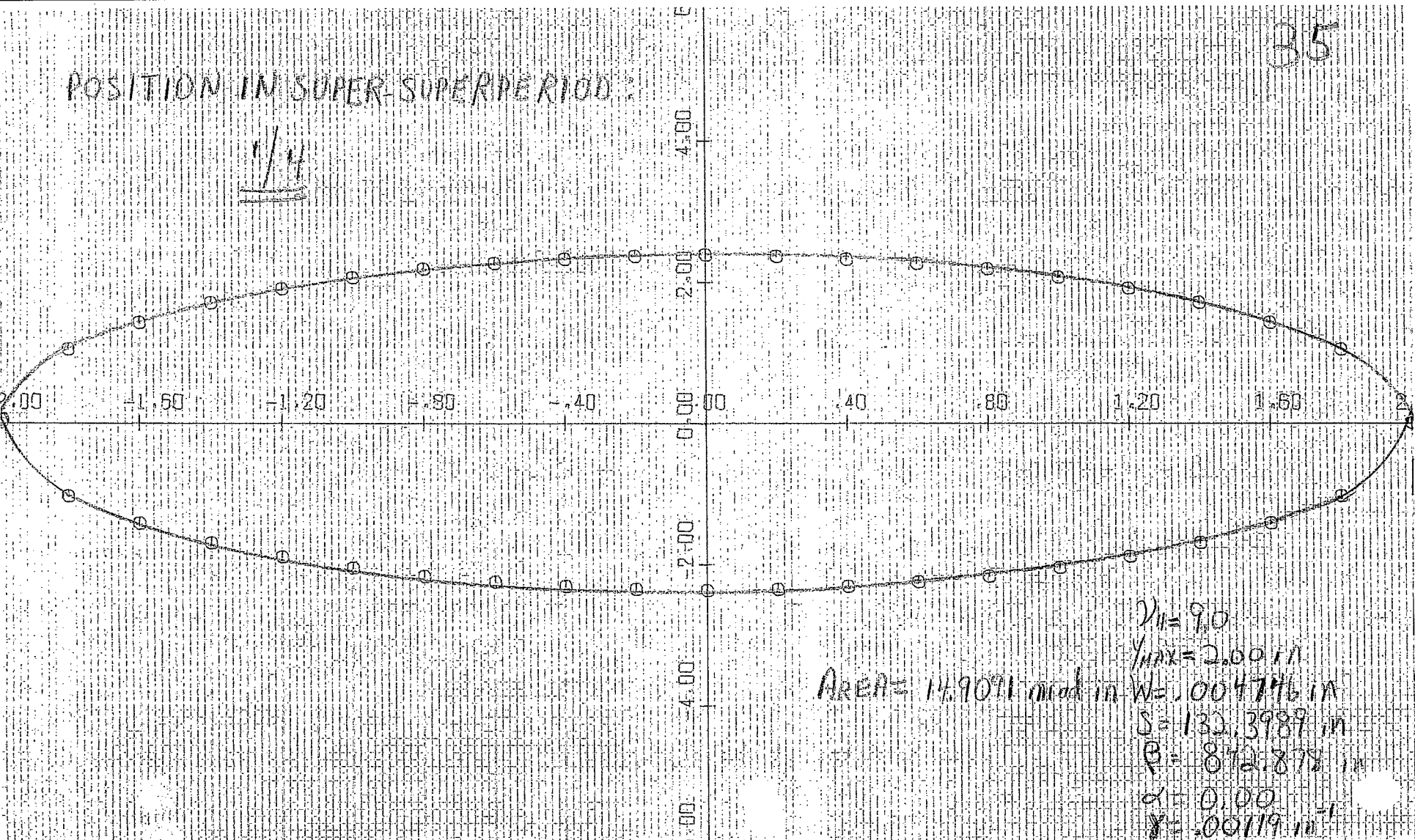
POSITION IN SUPER-SUPER PERIOD:

1/8



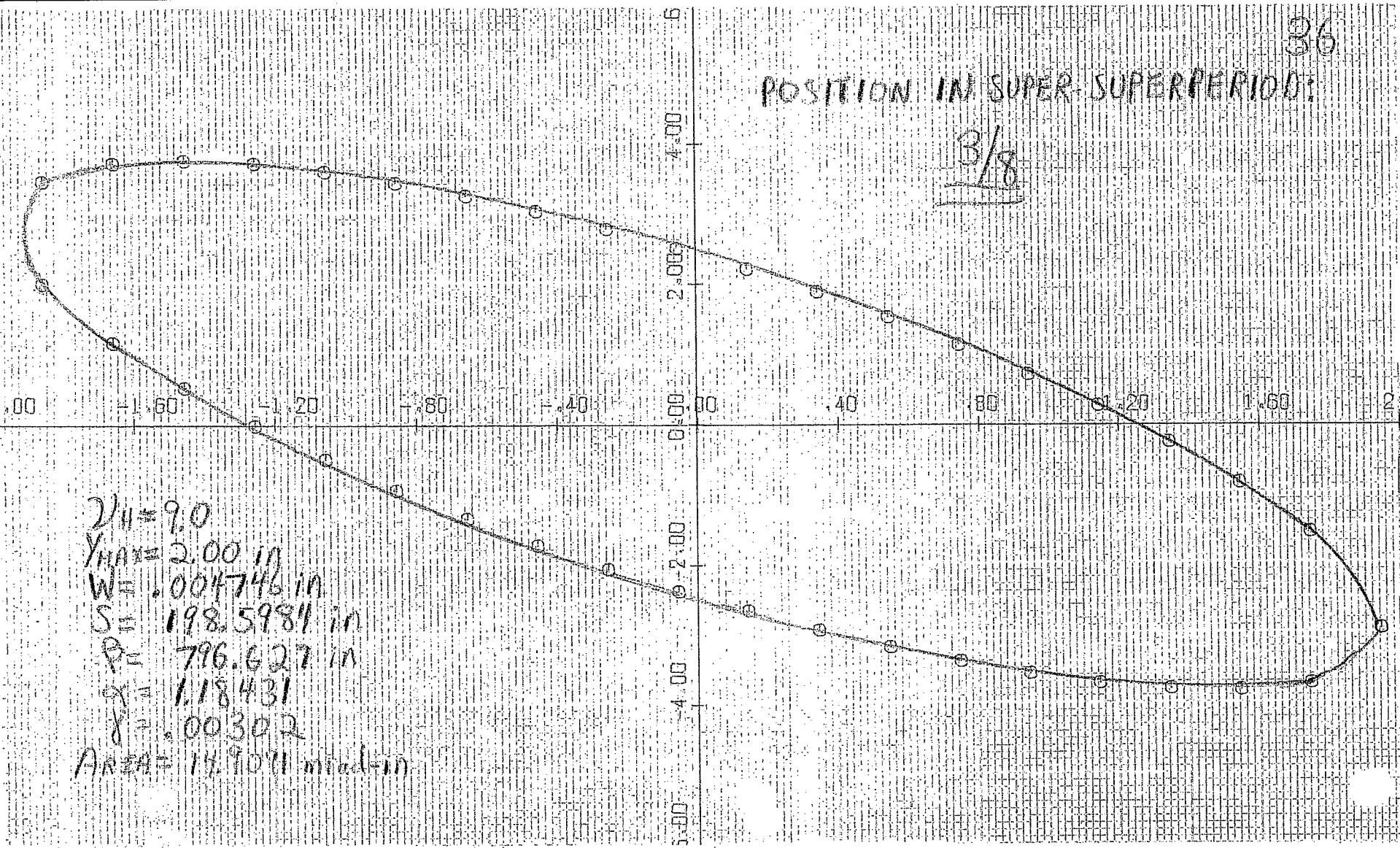
POSITION IN SUPER-SUPERPERIOD:

1/4



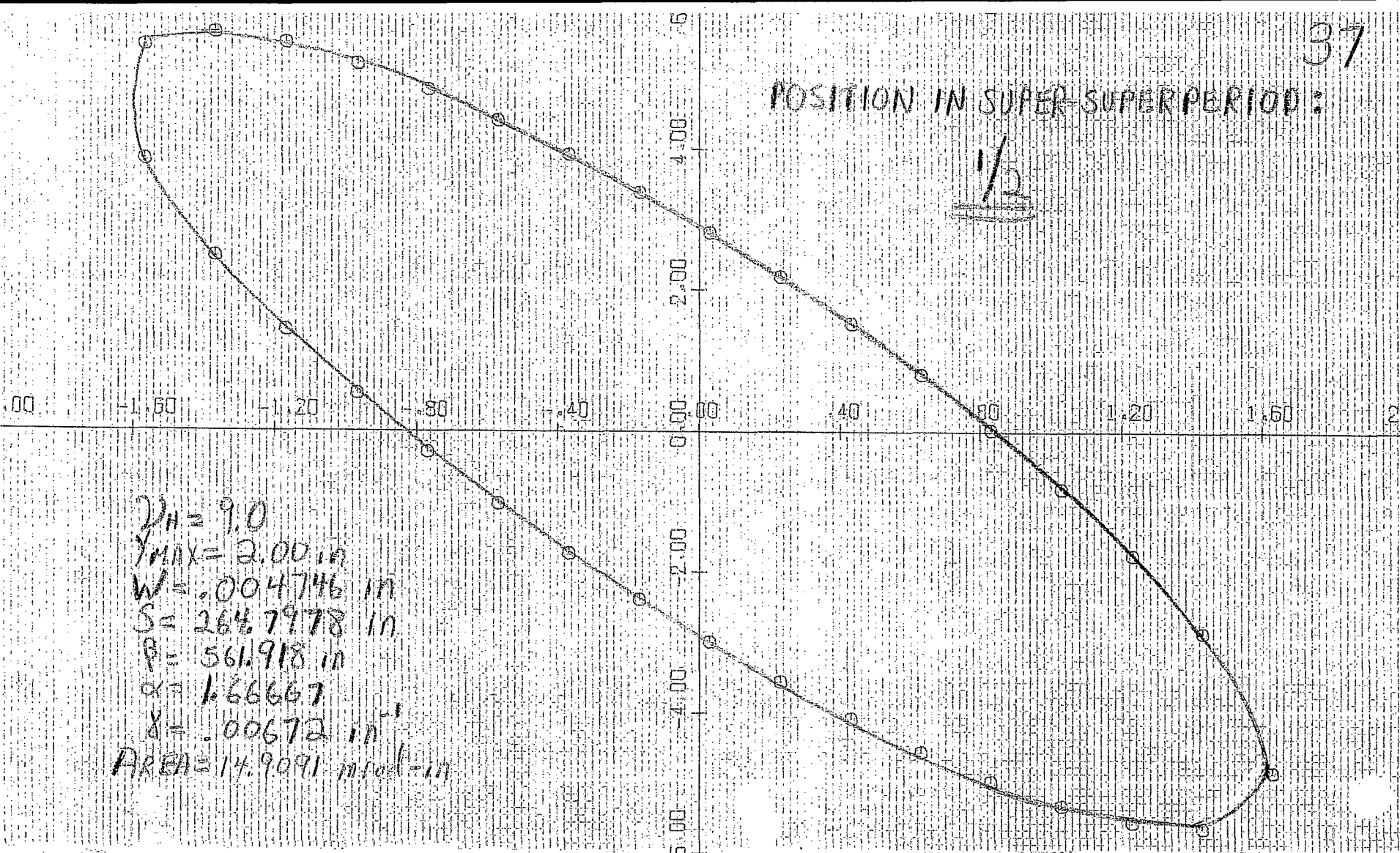
POSITION IN SUPER-SUPERPERIOD:

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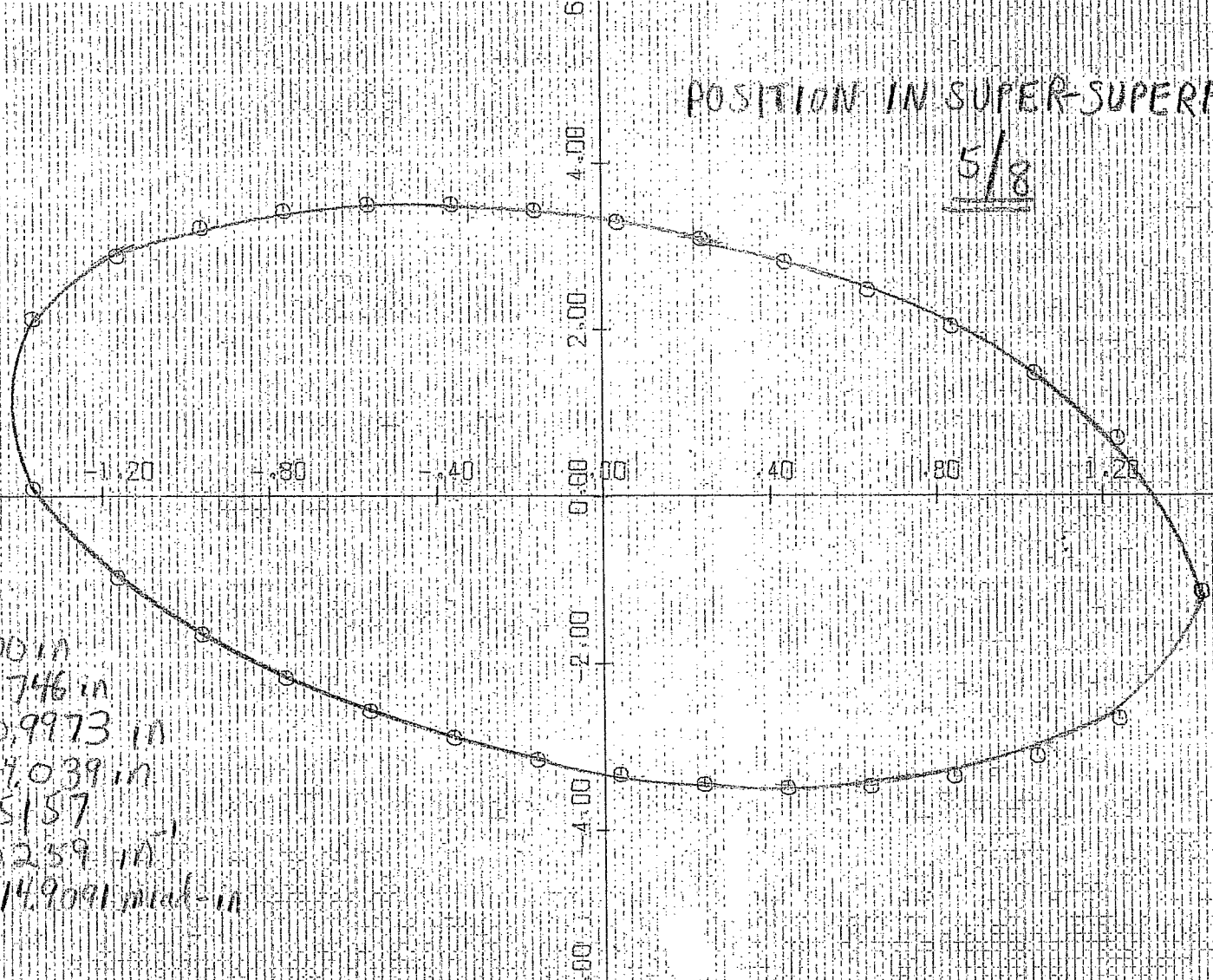
$2H = 9.0$
 $Y_{max} = 2.00 \text{ in}$
 $W = .004746 \text{ in}$
 $S = 198.5981 \text{ in}$
 $P = 796.627 \text{ in}$
 $\alpha = 1.18431$
 $\gamma = .00302$
 $AREA = 14.9011 \text{ mod-in}$

POSITION IN SUPER-SUPERPERIOD:

1/2

POSITION IN SUPER-SUPERPERIOD:

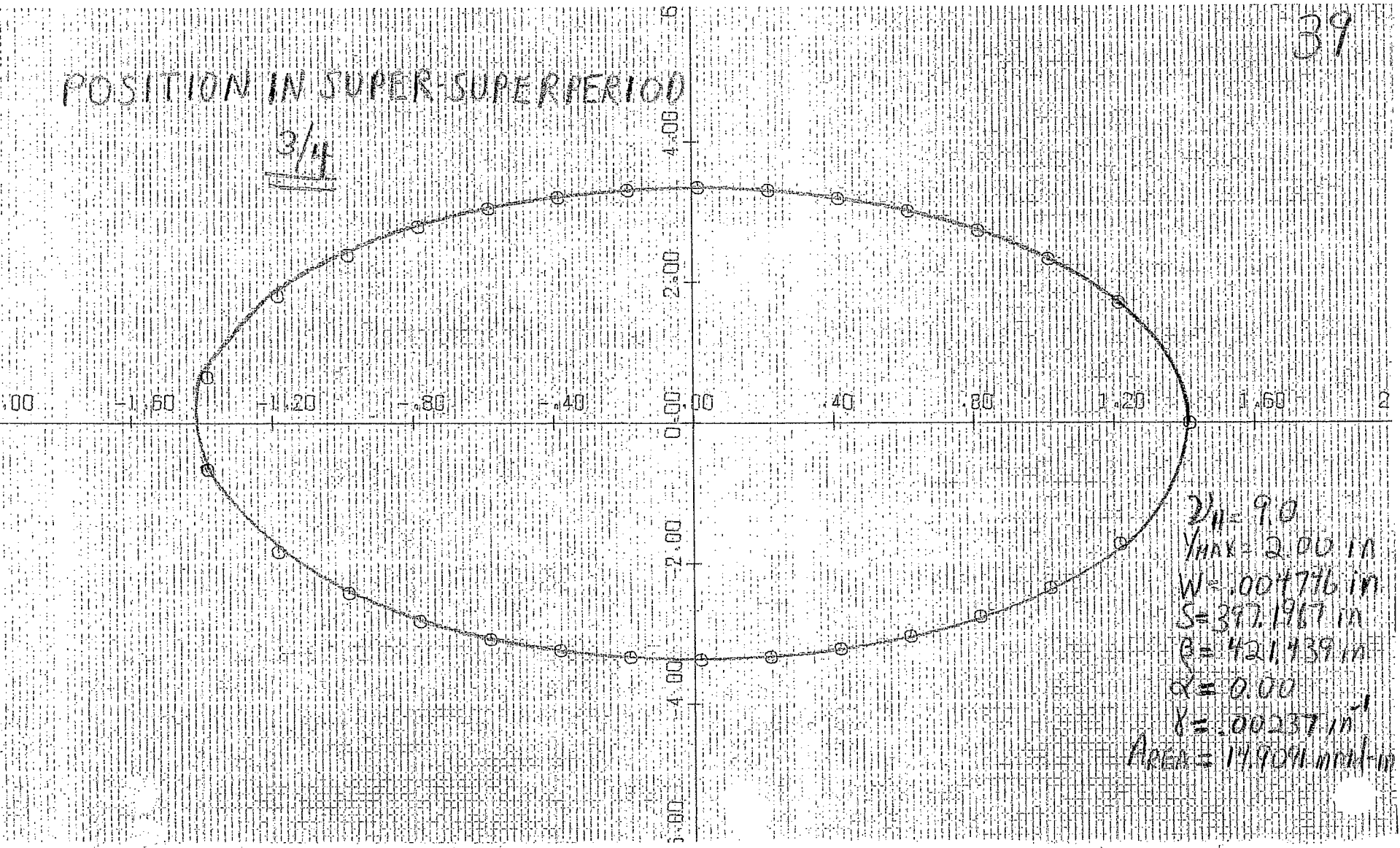
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$D_H = 9.0$
 $V_{MAX} = 2.00 \text{ in}$
 $W = .004746 \text{ in}$
 $S = 330.9973 \text{ in}$
 $B = 437.039 \text{ in}$
 $\alpha = .35187$
 $\gamma = .00259 \text{ in}$
 $AREA = 14.9091 \text{ in}^2$

POSITION IN SUPER-SUPER PERIOD

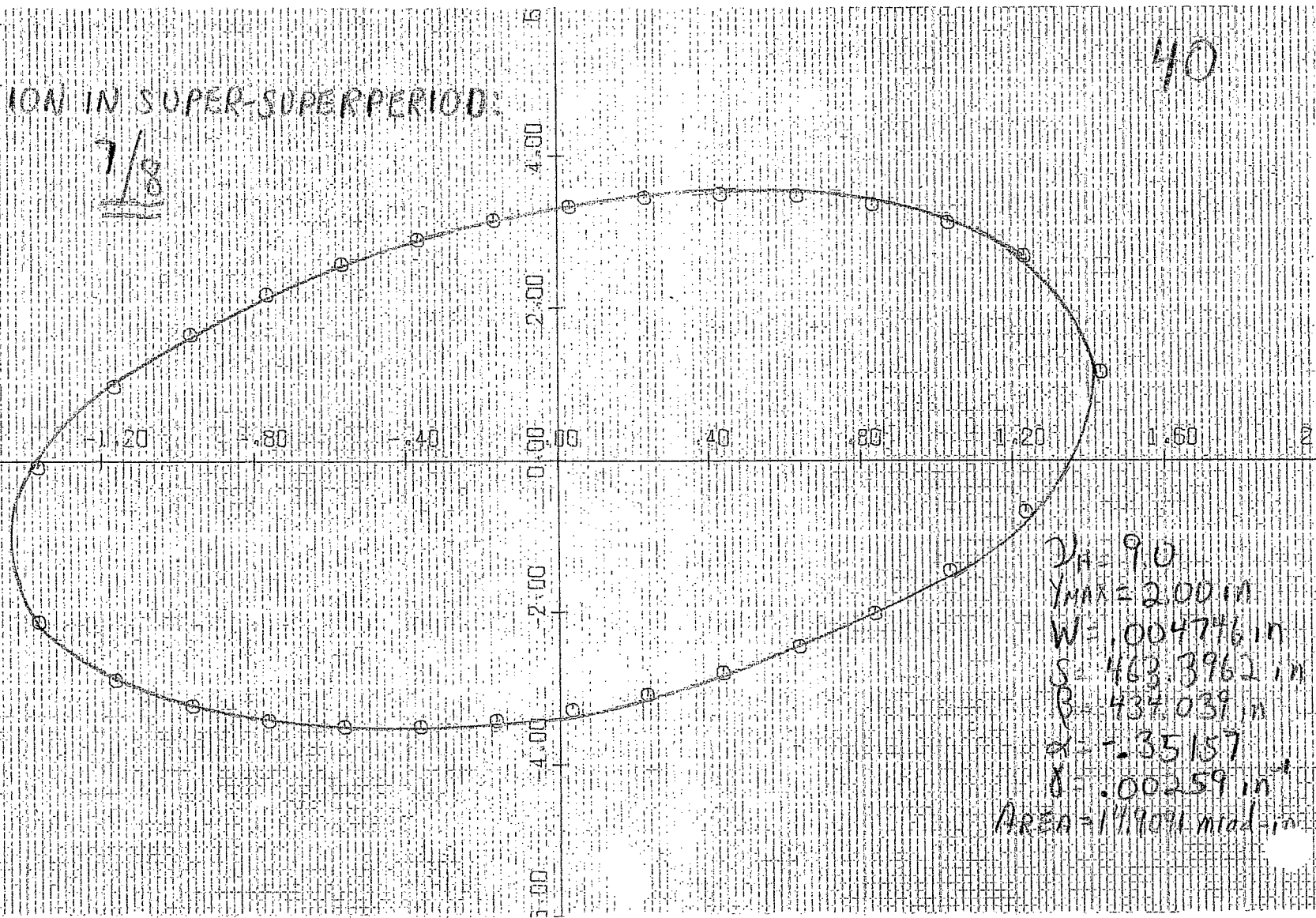
3/4



$2r = 9.0$
 $Y_{MAX} = 2.00 \text{ in}$
 $W = .004776 \text{ in}$
 $S = 397.1967 \text{ in}$
 $B = 421.439 \text{ in}$
 $\alpha = 0.00$
 $\delta = .00237 \text{ in}$
 $Area = 14.9091 \text{ in}^2$

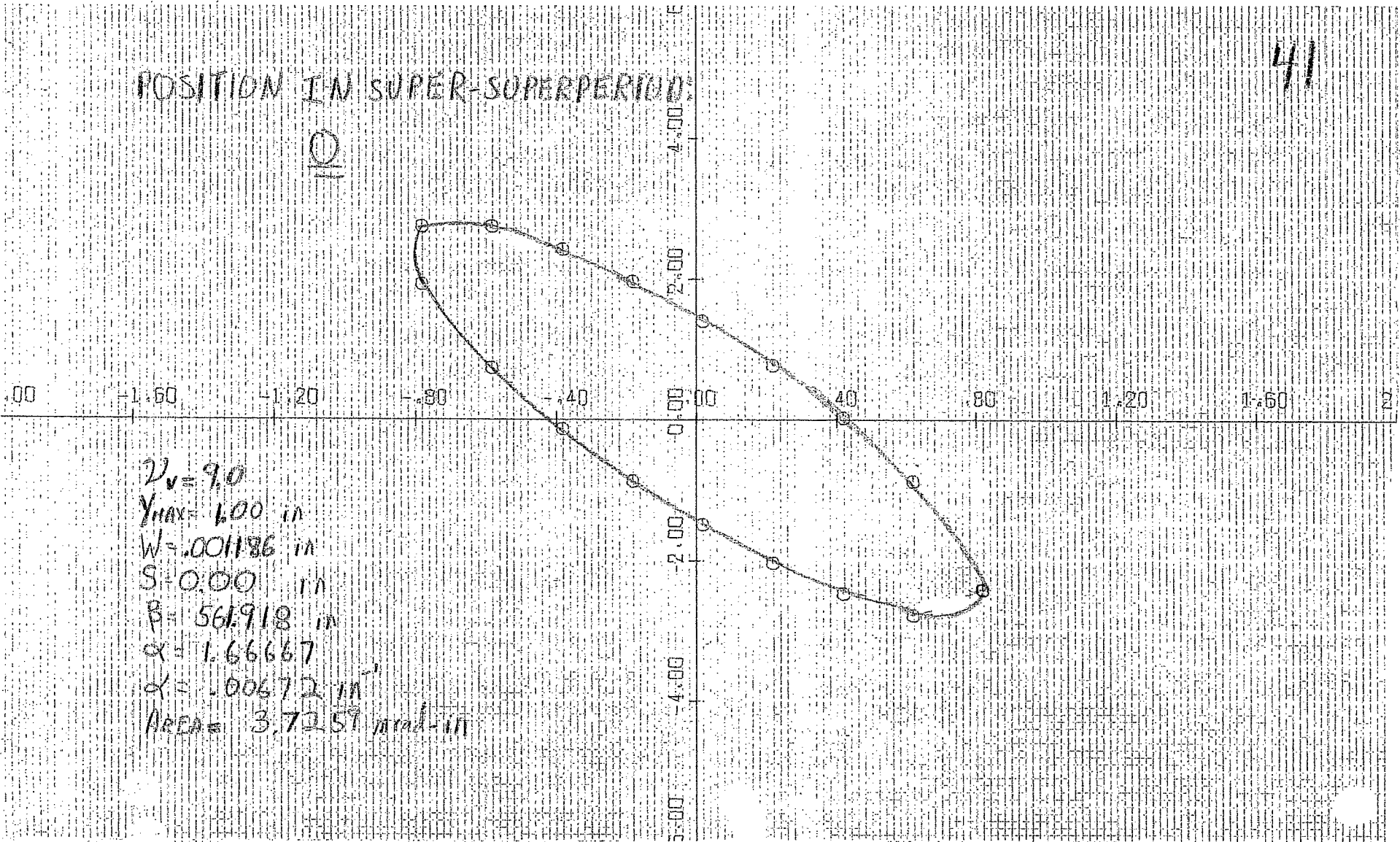
POSITION IN SUPER-SUPERPERIOD:

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POSITION IN SUPER-SUPERPERIOD

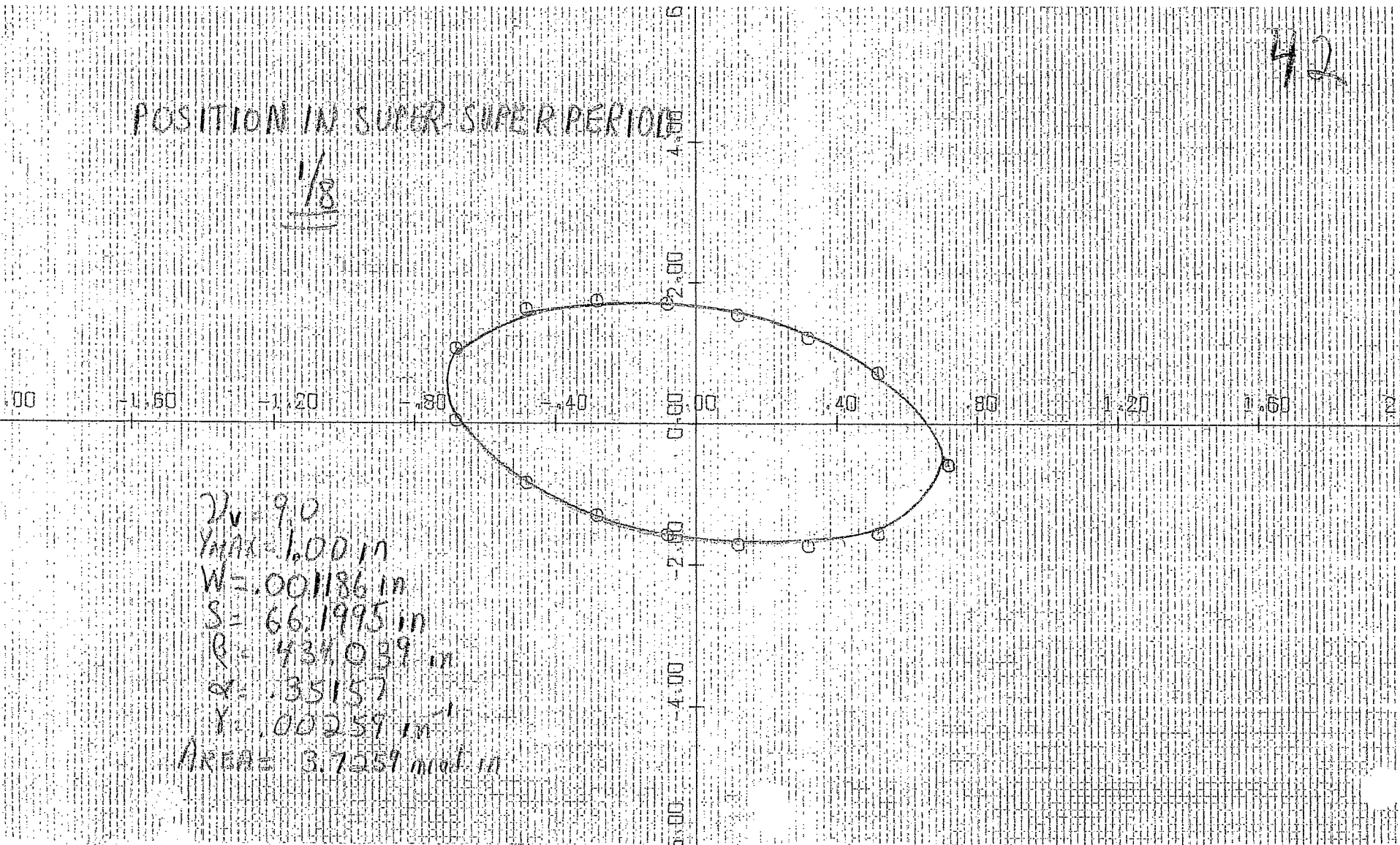
Q



$\nu = 9.0$
 $Y_{max} = 6.00 \text{ in}$
 $W = .001186 \text{ in}$
 $S = 0.00 \text{ in}$
 $\beta = 56.918 \text{ in}$
 $\alpha = 1.66667$
 $\alpha = .00672 \text{ in}^{-1}$
 $\text{AREA} = 3.7259 \text{ mm}^2\text{-in}$

POSITION IN SUPER-SUPER PERIOD

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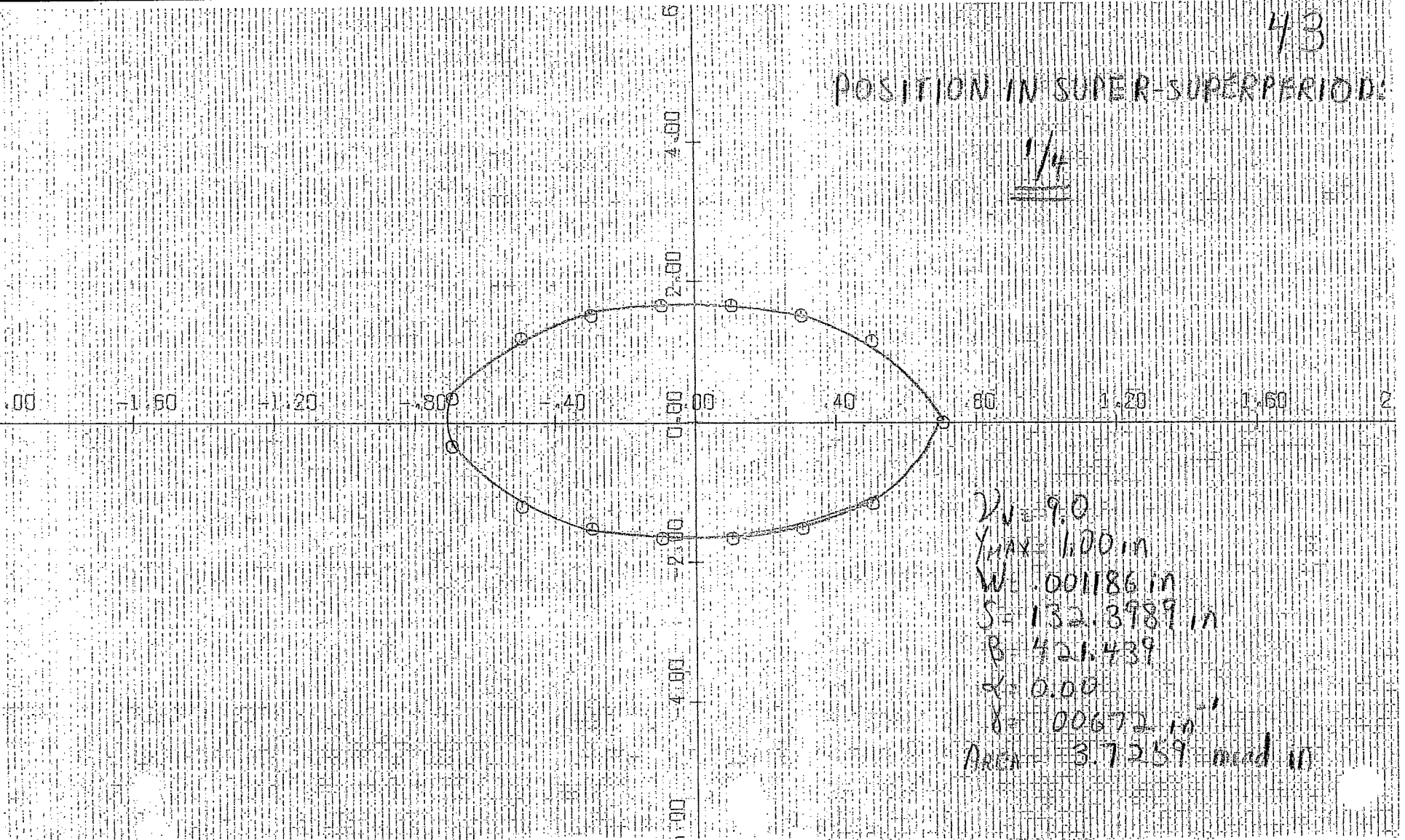


$W = 9.0$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = .001186 \text{ in}$
 $S = 66.1995 \text{ in}$
 $R = 434.039 \text{ in}$
 $\alpha = .35157$
 $Y = .00259 \text{ in}^2$
 $AREA = 3.7259 \text{ in}^2$

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POSITION IN SUPER-SUPERPERIOD:

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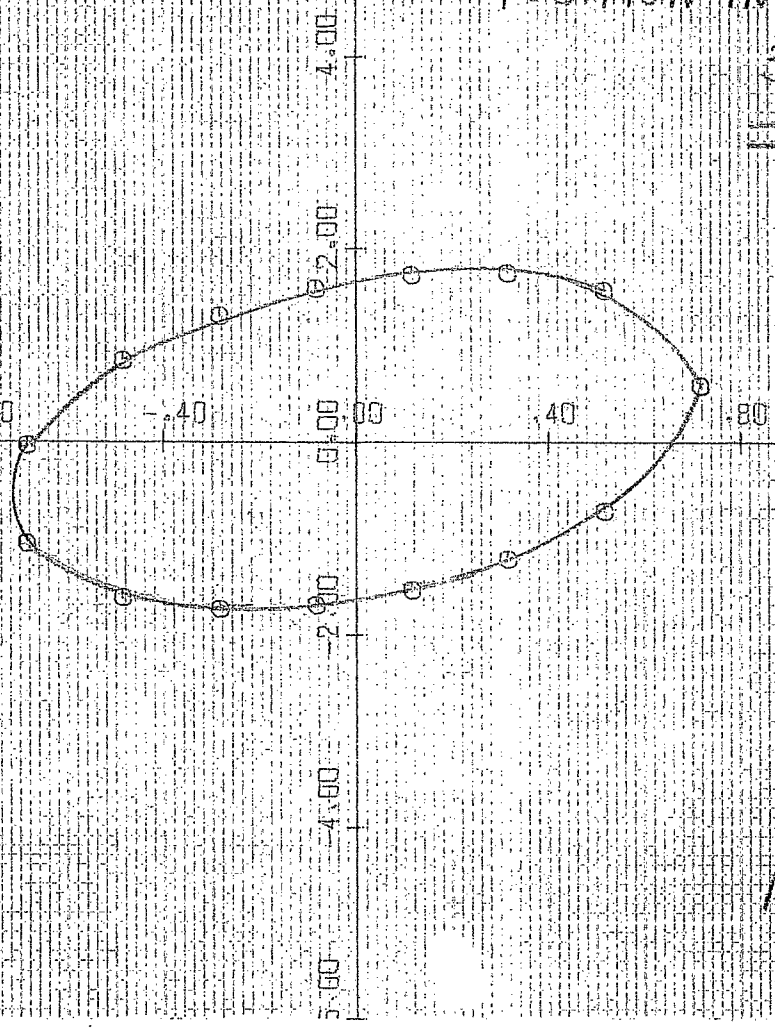


$W = 9.0$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = .001186 \text{ in}$
 $S = 132.3989 \text{ in}$
 $B = 421.439$
 $\alpha = 0.00$
 $\delta = .00672 \text{ in}$
Area = 3.7259 quad in

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POSITION IN SUPER-SUPERPERIOD:

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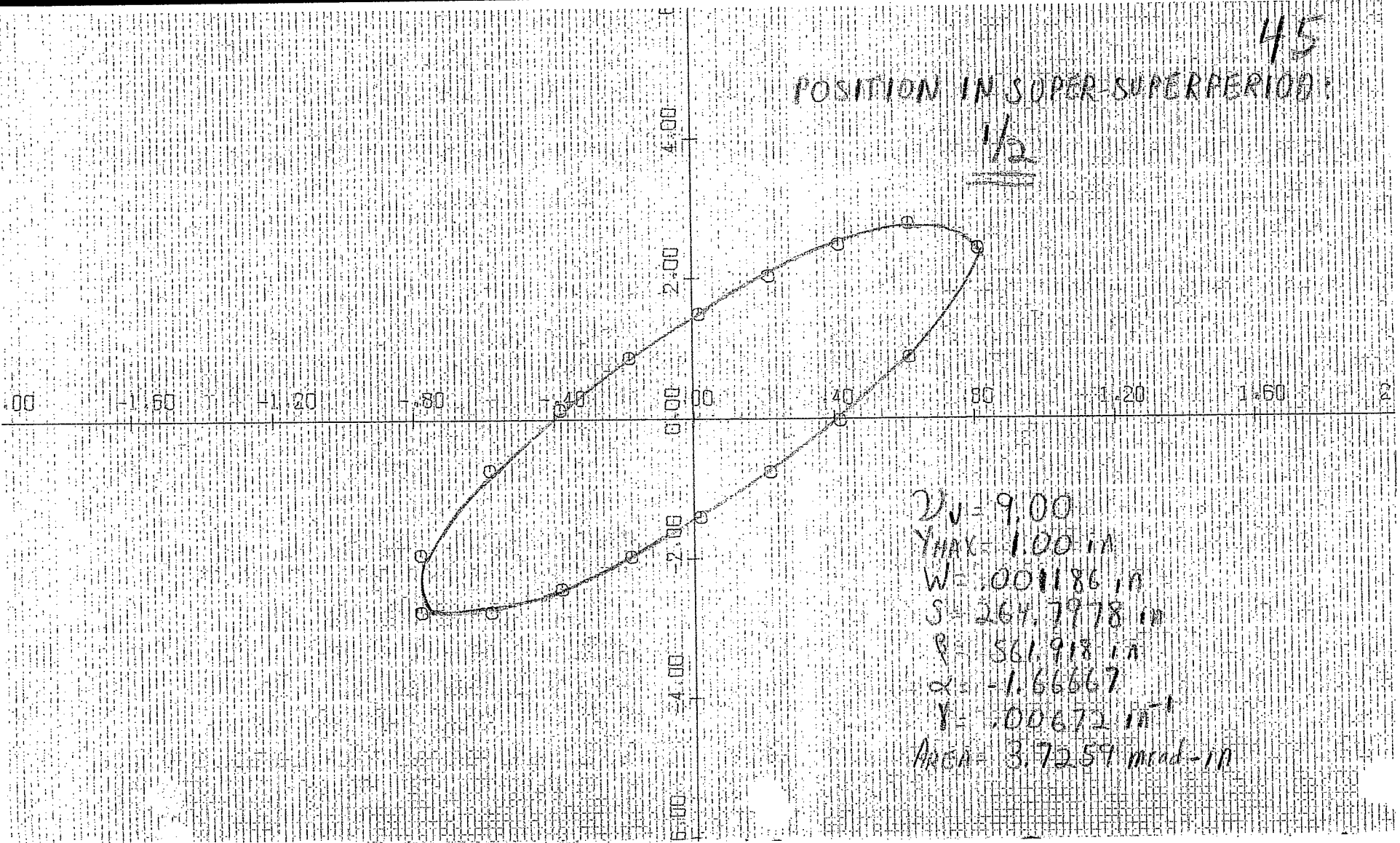


$D_w = 9.0$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = 1.001186 \text{ in}$
 $S = 198.5984 \text{ in}$
 $R = 434.039 \text{ in}$
 $\alpha = -0.35157$
 $\gamma = 0.00259 \text{ in}$
 $Area = 3.7259 \text{ mrad-in}$

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POSITION IN SUPER-SUPERPERIOD:

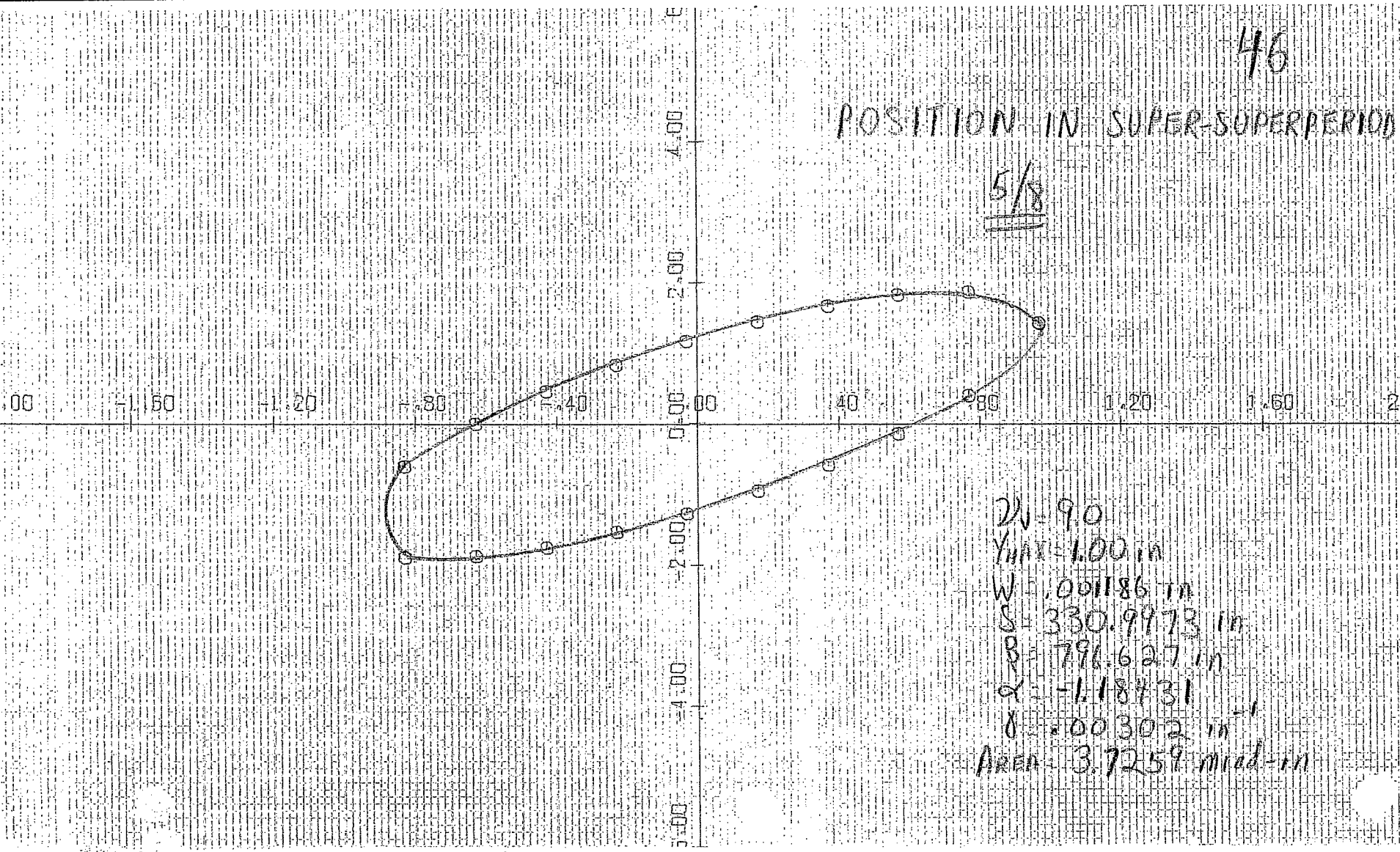
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POSITION IN SUPER-SUPERPERIOD

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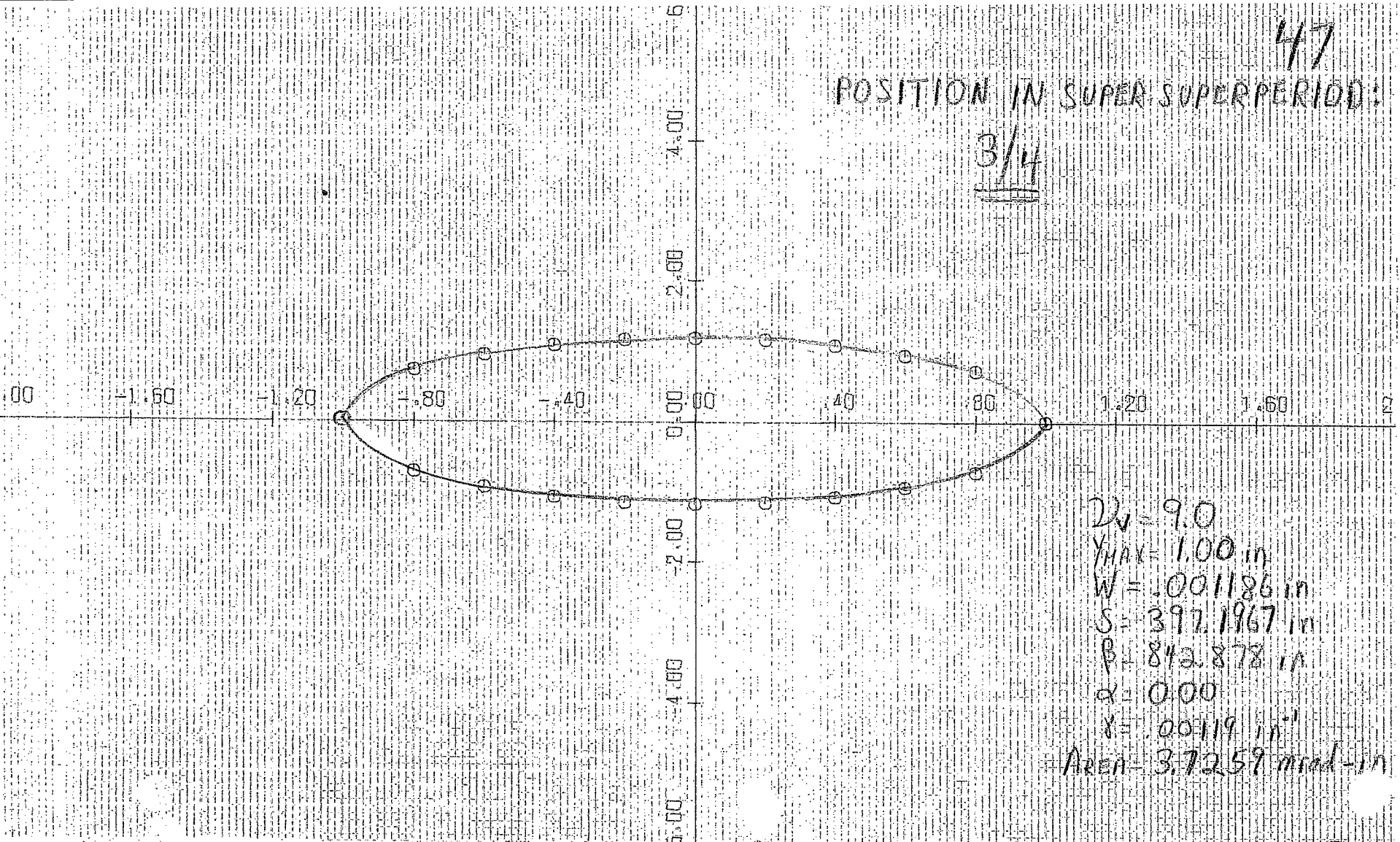


$2\lambda = 9.0$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = 1.00186 \text{ in}$
 $S = 330.9973 \text{ in}$
 $\beta = 796.627 \text{ in}$
 $\alpha = -1.18431$
 $\delta = 0.00302 \text{ in}^{-1}$
 $\text{AREA} = 3.7259 \text{ in}^2$

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POSITION IN SUPER PERIOD:

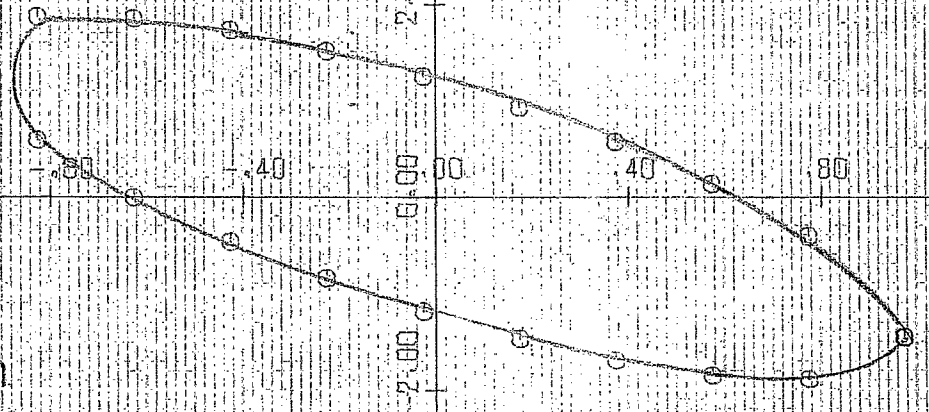
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$D_v = 9.0$
 $Y_{MAX} = 1.00 \text{ in}$
 $W = .001186 \text{ in}$
 $S = 392.1967 \text{ in}$
 $\beta = 812.878 \text{ in}$
 $\alpha = 0.00$
 $\gamma = .00119 \text{ in}^2$
 $\text{Area} = 3.7259 \text{ msd-in}$

POSITION IN SUPER-SUPERPERIOD:

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$2v = 9.0$
 $y_{max} = 1.00 \text{ in}$
 $W = .001186 \text{ in}$
 $S = 463.3962 \text{ in}$
 $R = 796.627 \text{ in}$
 $\alpha = 1.18431$
 $\delta = .00302 \text{ in}^{-1}$
 $Area = 3.7259 \text{ mrad-in}$