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PHASE SPACE ELLIPSES FOR A MODEL AGS

G. Fiske

August 1969

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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AGS DIVISION TECHNICAL NOTE

No. 65

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August 26, 1969

PHASE SPACE ELLIPSES FOR A MODEL AGS

The accompanying graphs are plots of the AGS phase space ellipses for the horizontal (H) and vertical (V) directions, obtained on the CDC-6600 and the Calcomp 565 plotter.

The calculation assumes a model horizontal $\Psi(s)$ function[1] of the following form:

$$\Psi(s) = \Psi_L \left[\frac{S}{L} + \frac{\tau}{2\pi}\left(\cos\frac{2\pi}{L}S - 1\right) + \frac{\sigma}{2\pi}\left(\cos\frac{6\pi}{L}S - 1\right)\right] \text{ where:}$$

$L = \frac{2\pi R}{60}$; R = radius of machine = 5057.266 inches

$\Psi_L = \frac{2\pi\nu}{60}$; and for a form factor $\frac{\beta_{max}}{\beta_{av}} = 1.5$

$\tau = 3/8$

$\sigma = 1/72$

This phase function has a periodicity of four magnets (1/60 of the circumference), and agrees well with the results obtained from the BEAM program at intermediate field. The corresponding β function is of the form:

$$\beta(s) = \frac{R/\nu}{1 - \tau \sin\frac{2\pi}{L}S - 3\sigma \sin\frac{6\pi}{L}S}$$

Each set of eight graphs gives the phase space ellipses in either the horizontal or vertical direction for one particle at eight points within the super-superperiod (1/60 of the circumference). The following table lists the graphs shown:

Graph Numbers	ν	Y_{max} (inches)
1-8	8.0 (H)	2.00
9-16	8.0 (V)	1.00
17-24	8.5 (H)	2.00
25-32	8.5 (V)	1.00
33-40	9.0 (H)	2.00
41-48	9.0 (V)	1.00

In general, the program is designed to plot graphs for any ν value and any value of Y_{max}, where Y_{max} is the maximum value of the transverse excursion of the particle taken at the point where β itself is maximum. Thus, at no point in the machine is the greatest transverse excursion of the particle any larger than Y_{max}.

The phase invariant, W, is set at Y_{max}^2/β_{max}, and the equation of each ellipse is:

$$\gamma y^2 + 2\alpha y y' + \beta y'^2 = W$$

A diagram of a super-superperiod with the horizontal β function superimposed is given in Fig. 1.

The phase space ellipses are computed and plotted at the indicated points, A through H.

The graphs contain all parameters pertinent to the calculation, as well as the area (emittance) of the ellipses; that is:

$$A = \pi W \text{ rad-in.}$$

References

1. J.C. Herrera, BNL Accel. Dept. Int. Rept. AGS DIV 69-3 (1969).

Distr:

Department Administration
AGS Division Physicists

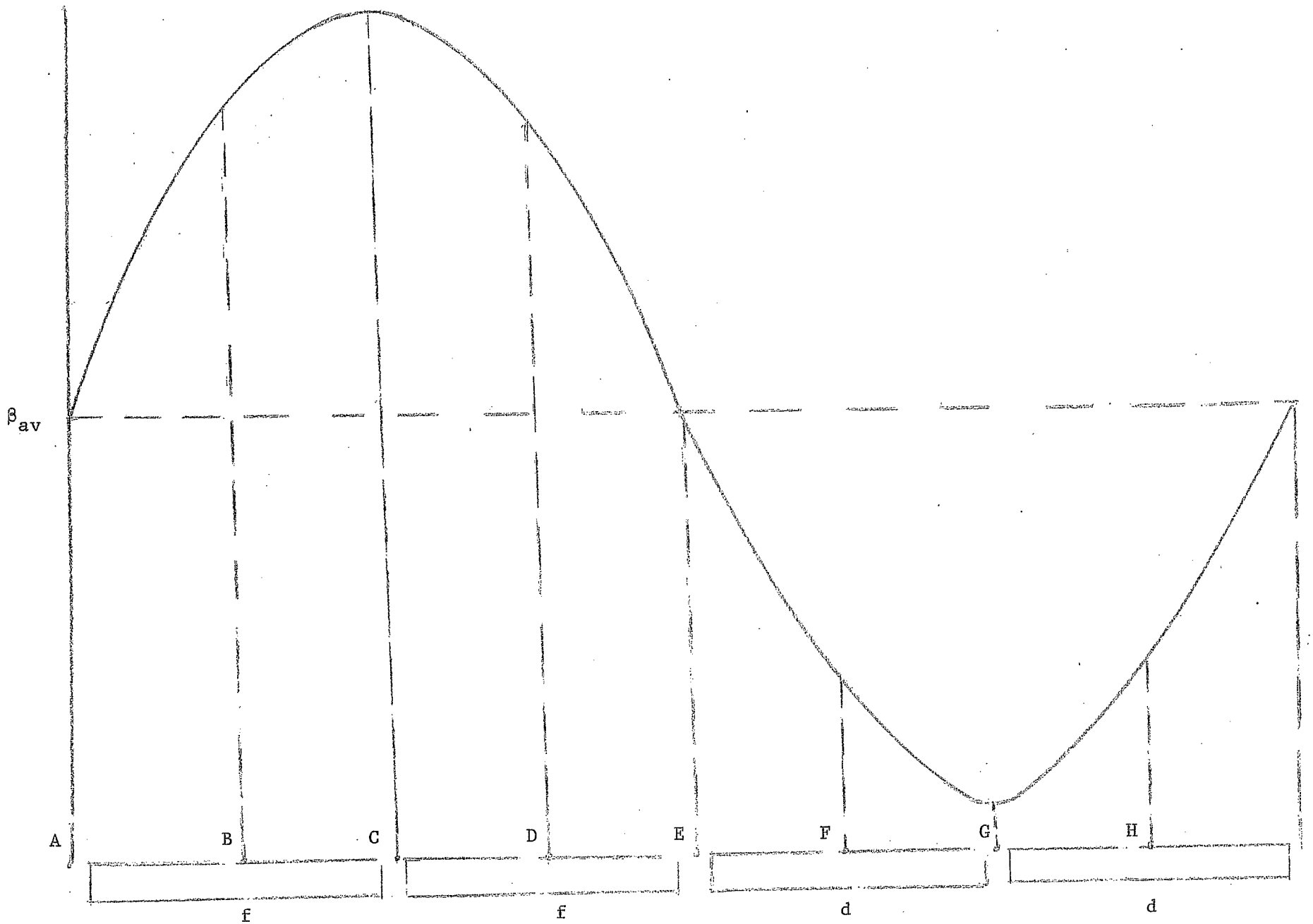
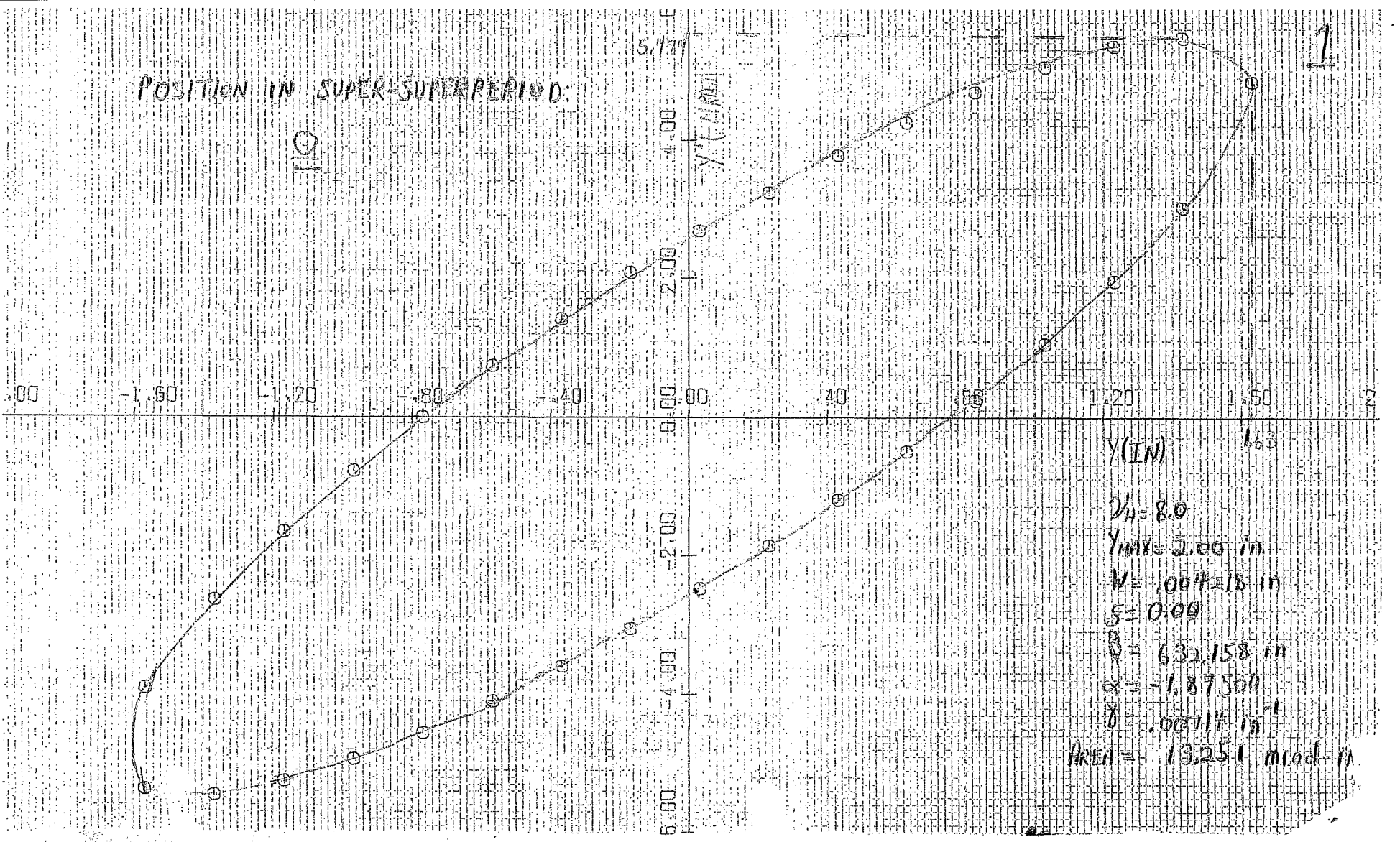


Fig. 1

POSITION IN SUPER-SUPERPERIOD:

01



POSITION IN SUPER-SUPERPERIOD:

1/8

$Y_0 = 8.0$

$Y_{max} = 2.00$ in.

$W = .004218$ in.

$S = 66.1995$ in.

$\beta = 896.205$ in.

$\alpha = 1.33235$

$\delta = .00810$ in^{-1}

Area = 018.251 mrad-in

POSITION IN SUPER-SUPER PERIOD:

1/4

$\nu_{11} = 8.0$

$Y_{max} = 2.00$ in

$W = .004218$ in

$S = 132.3987$ in

$R = 948.2371$ in

$\alpha = 0.00$ in^{-1}

$\delta = .00105$ in

AREA = 13251 in^2

POSITION IN SUPER-SUPERPERIOD:

3/8

$2\beta = 8.0$

$Y_{max} = 2.00$ in

$W = .004218$ in

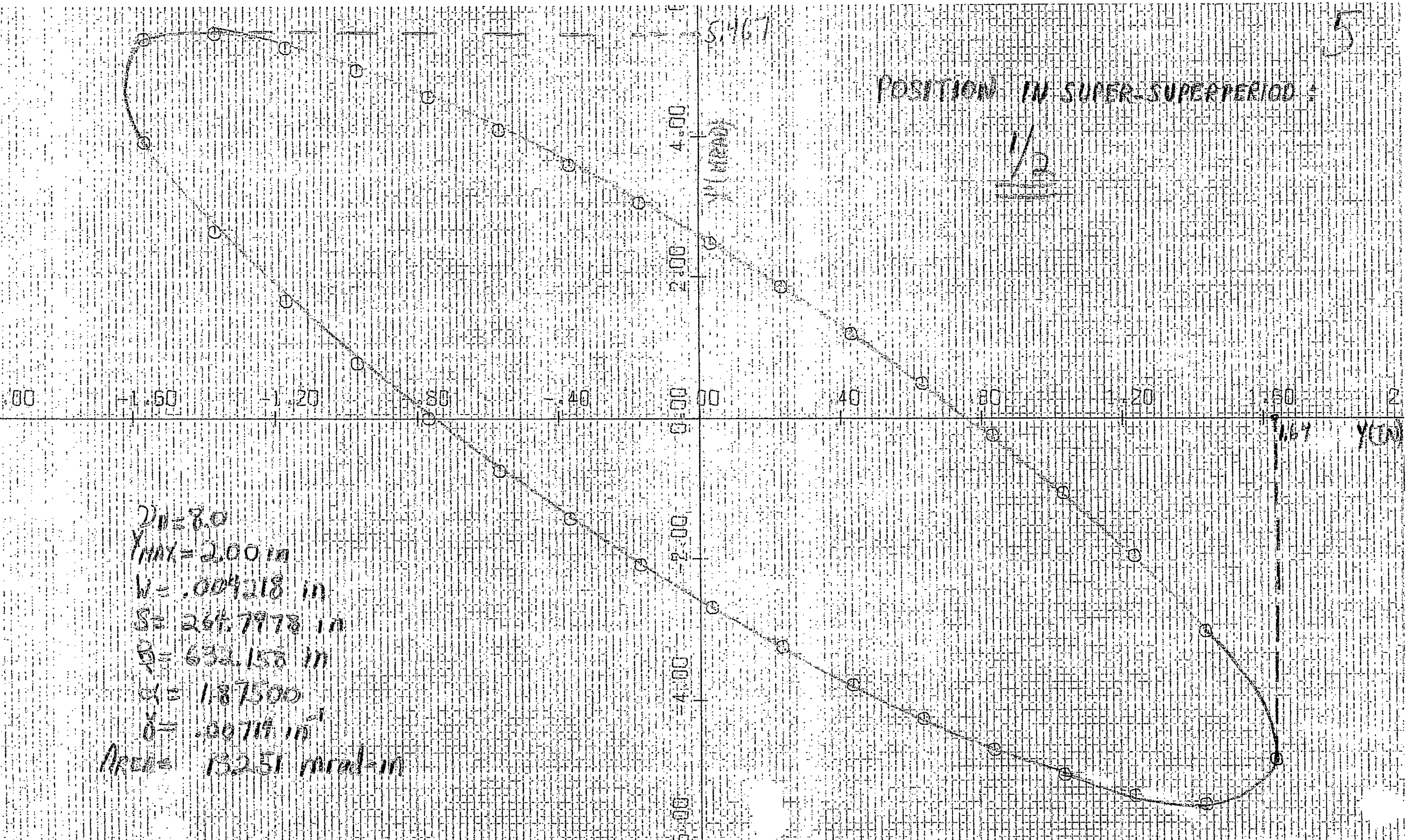
$S = 198.5984$ in

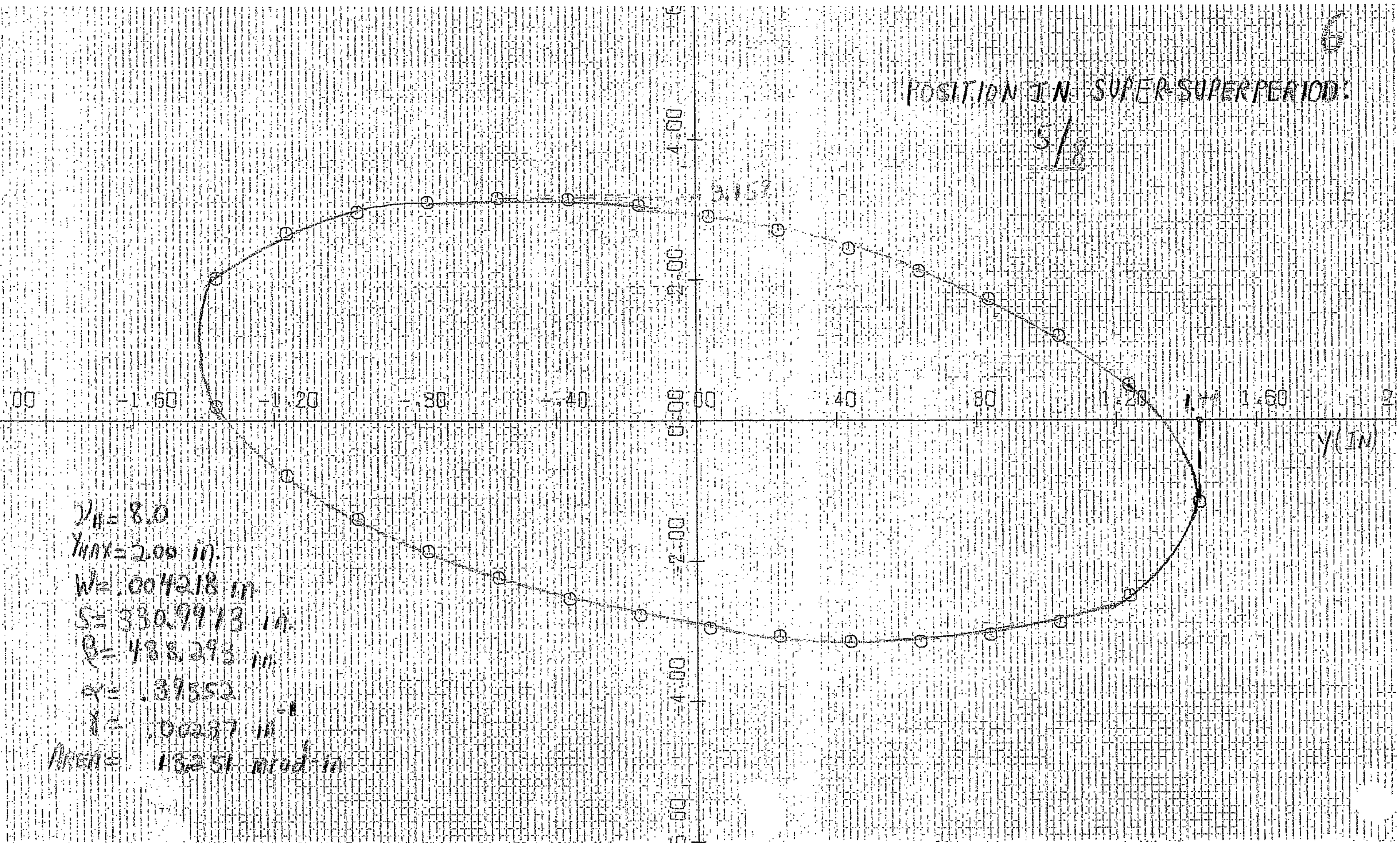
$\theta = 896.205$ in

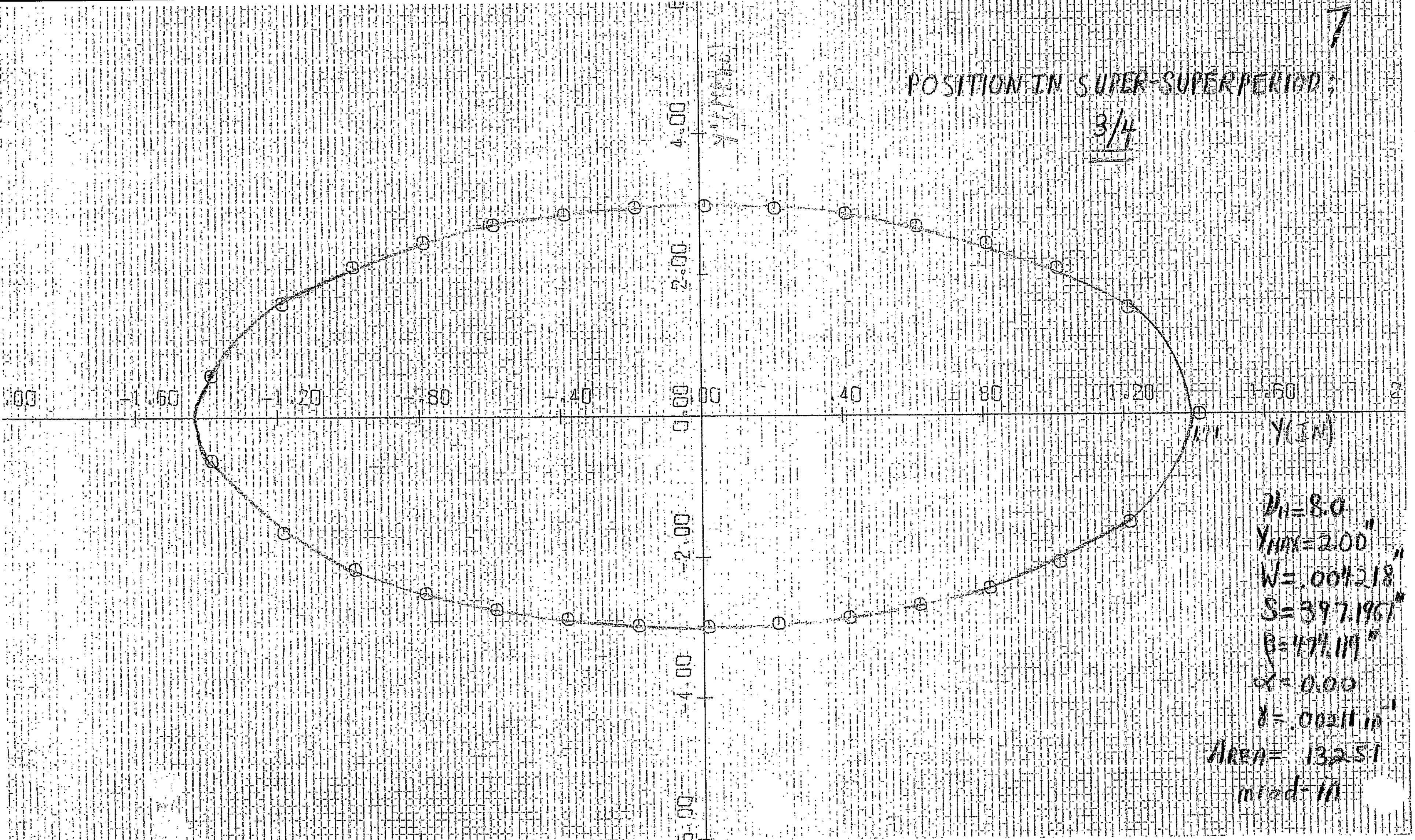
$\alpha = 1.33235$

$\beta = .00310$ in^{-1}

$Area = 1.9251$ mrad-in


5
POSITION IN SUPER-SUPERPERIOD :
1/2
5.1467
Y(IN)
1.64
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
4.00
2.00
-2.00
-4.00
2n=8.0
Y_MAX=2.00 in
W=.004218 in
S=264.7778 in
634.158 in
α=1.87500
γ=.00714 in⁻¹
Area= 13.251 mrad-in

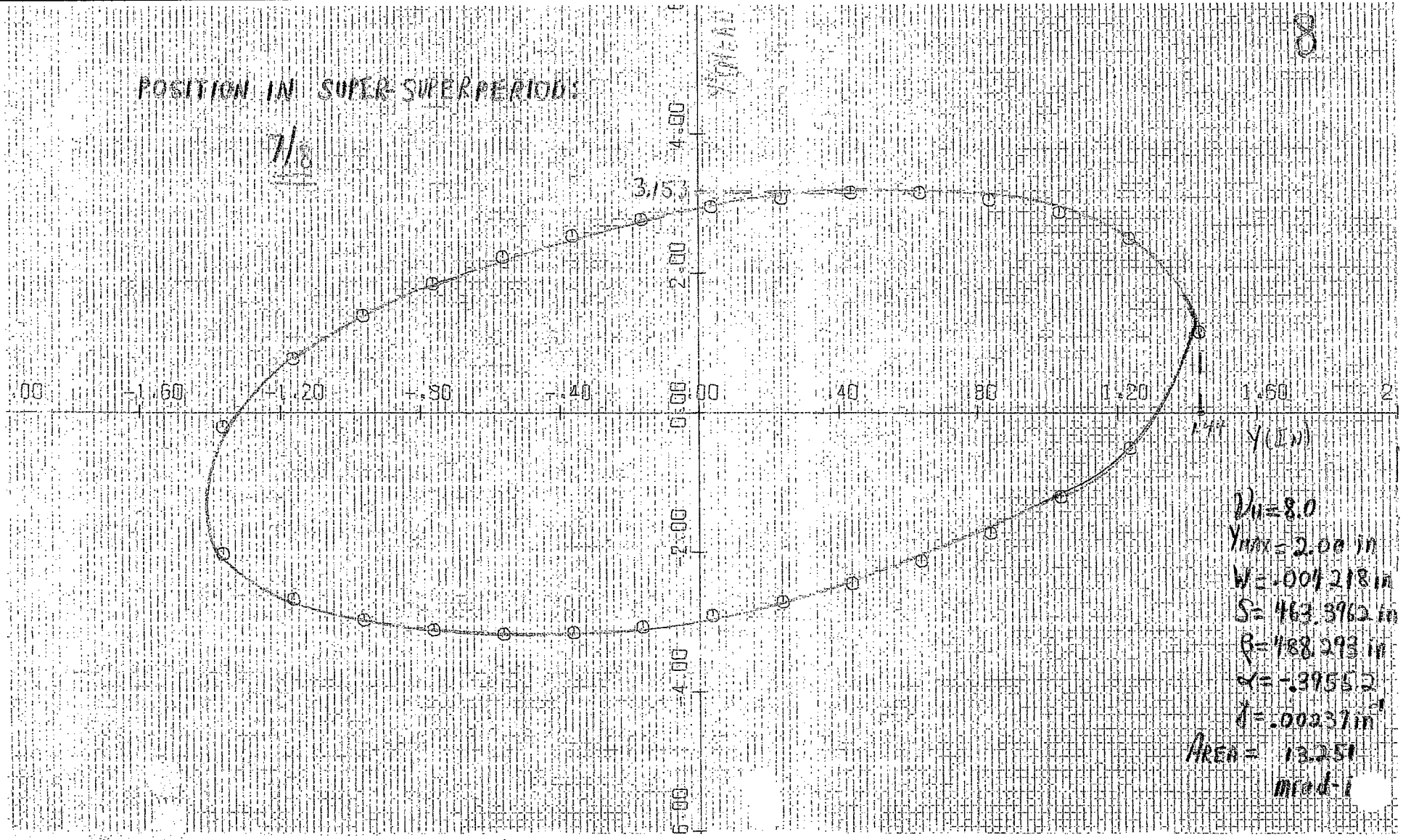

6
POSITION IN SUPER-SUPERPERIOD:
5/8
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.40
.80
1.20
1.60
Y(IN)
ν_H = 8.0
Y_{MAX} = 2.00 in.
W = .004218 in
S = 330.9973 in.
β = 488.298 in
α = .39552
γ = .00237 in^{-1}
Area = 1.6251 mrad-in



7
POSITION IN SUPER-SUPERPERIOD:
3/4
Y'(MRAD)
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
Y(IN)
ν_H = 8.0
Y_{MAX} = 2.00"
W = .004218"
S = 397.1967"
β = 474.119"
α = 0.00
γ = .00211 in^{-1}
AREA = 13.251
mrad-in

POSITION IN SUPER-SUPERPERIOD

T/2



$\lambda_{11} = 8.0$

$Y_{max} = 2.00$ in

$W = .004218$ in

$S = 463.3763$ in

$\beta = 1688.293$ in

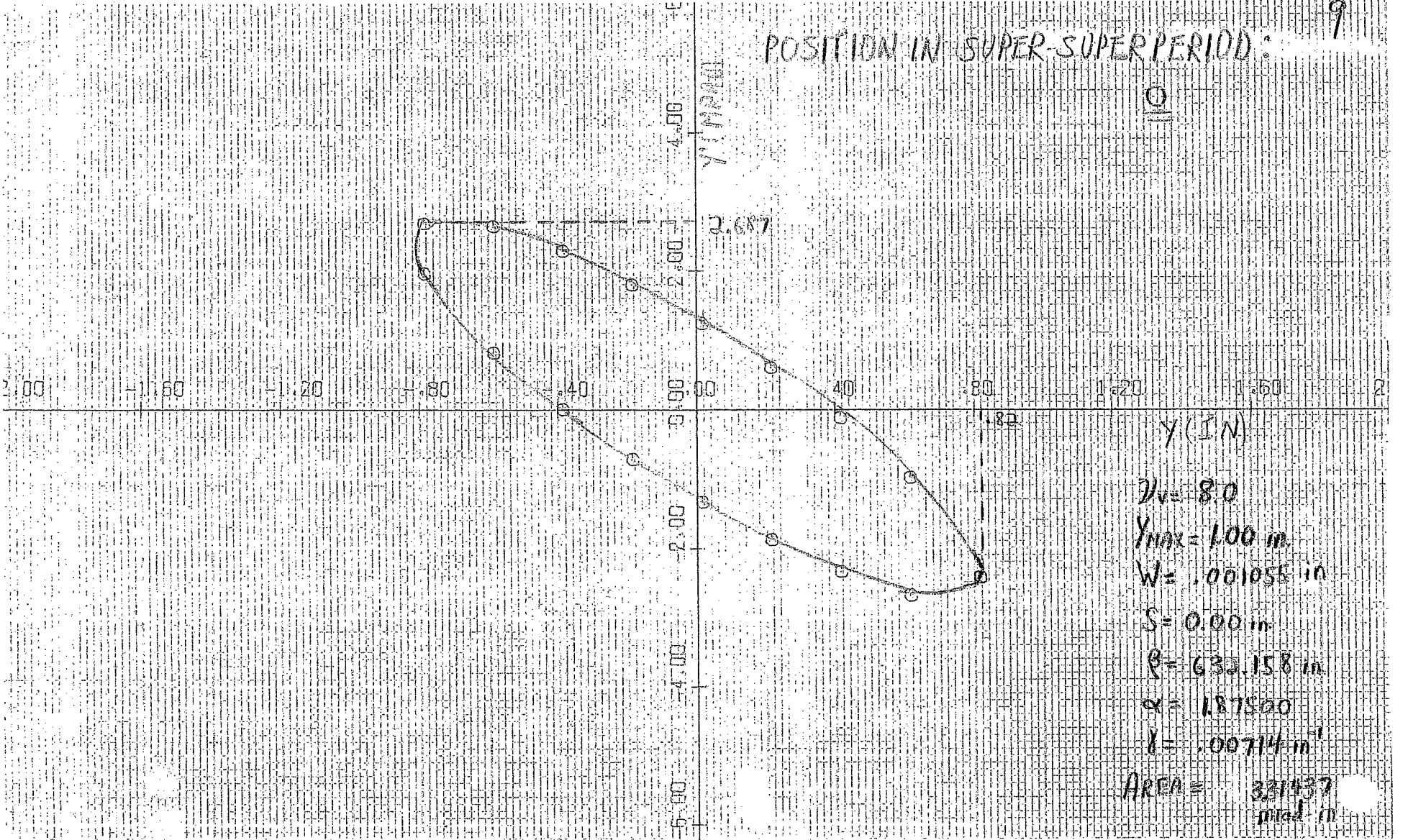
$\alpha = -.39552$

$\delta = .00237$ in^{-1}

AREA = 13.251

mid-i

POSITION IN SUPER-SUPER PERIOD: 0



Y(IN)

$\mathcal{D}_V = 8.0$

$Y_{MAX} = 1.00$ in.

$W = .001055$ in

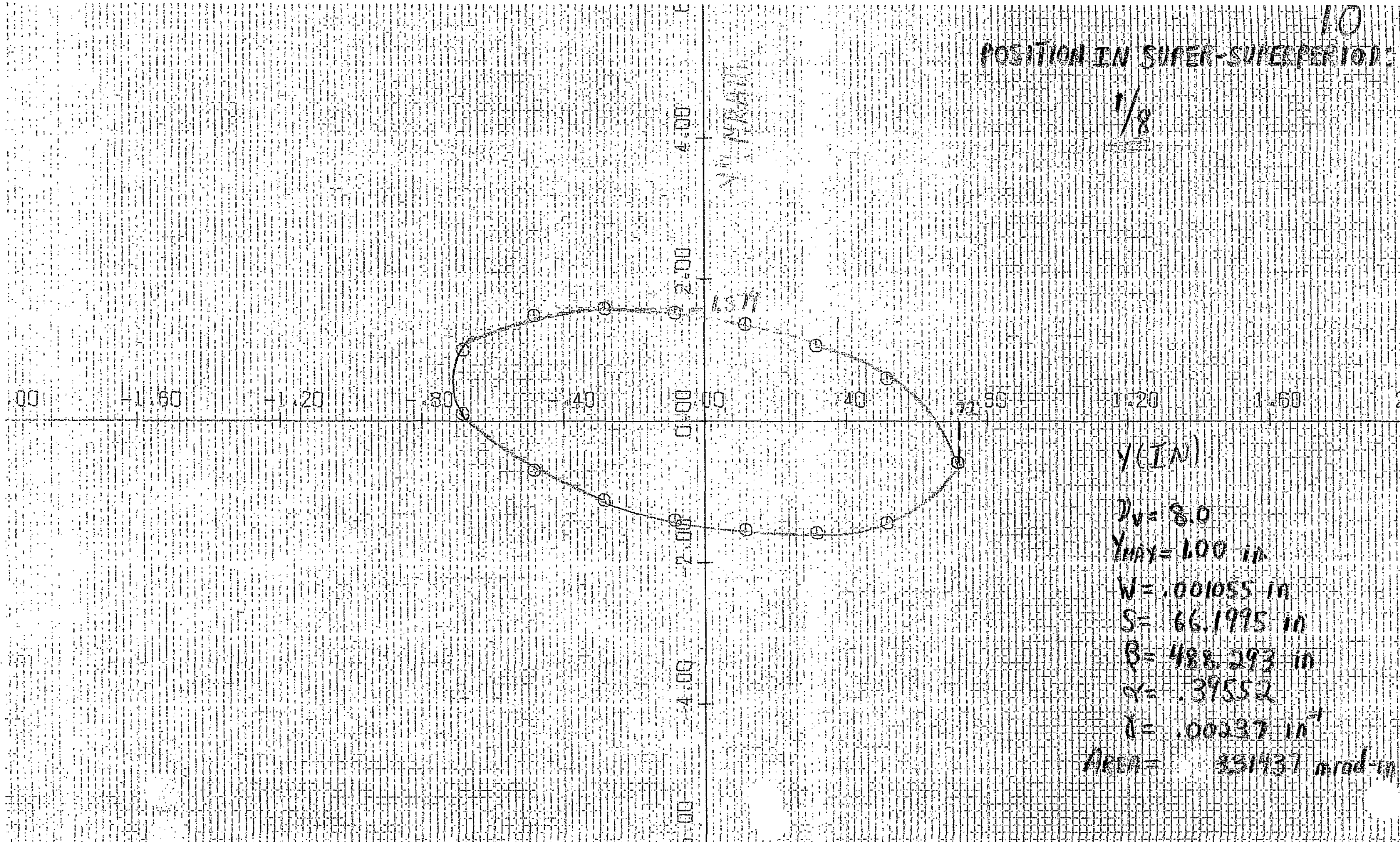
$S = 0.00$ in

$\rho = 633.158$ in

$\alpha = 1.87500$

$\beta = .00714$ in^{-1}

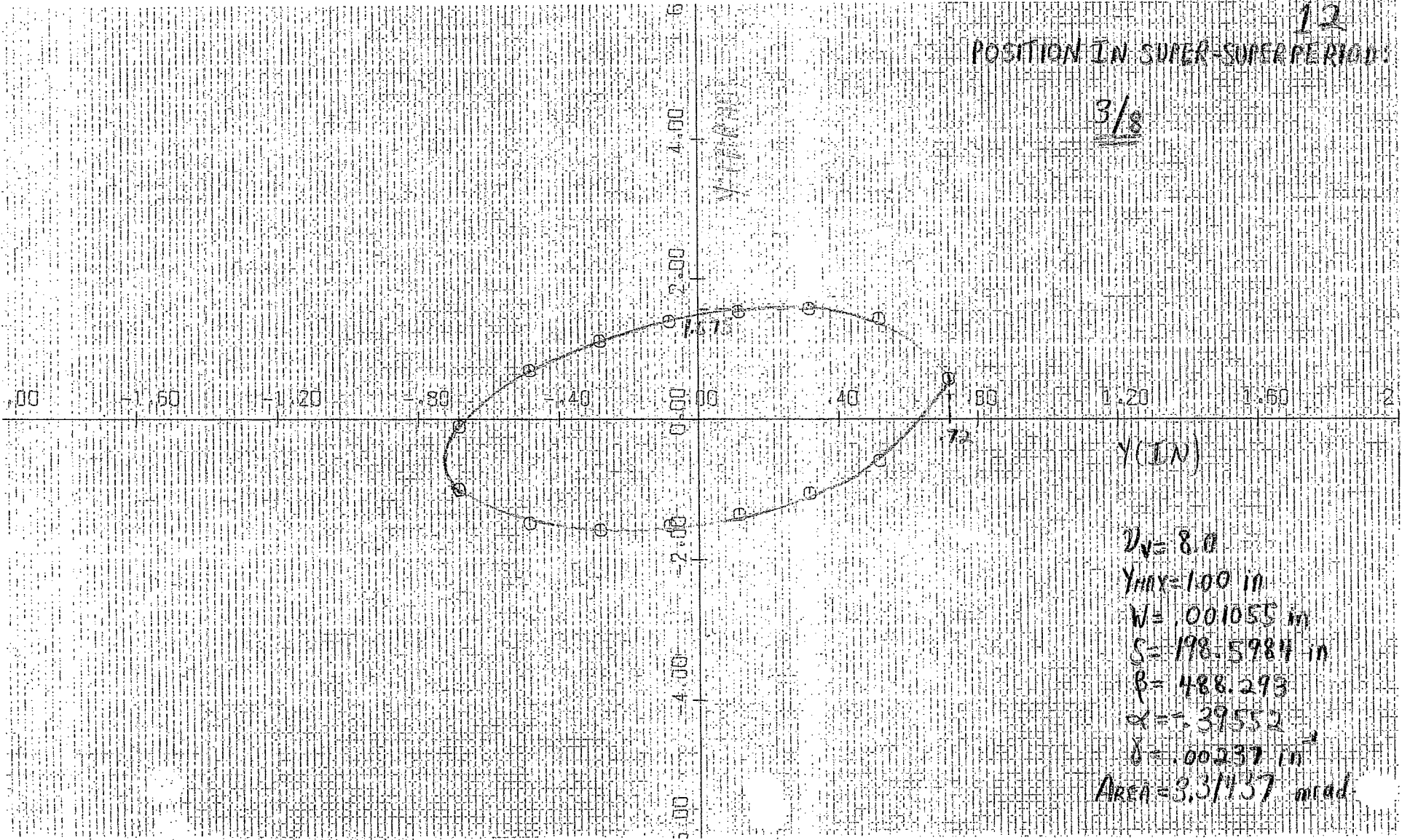
AREA = 331437 mod in


POSITION IN SUPER-SUPERPERIOD:
1/8
4.00
2.00
0.00
-2.00
-4.00
MRAD
1.3 M
-2.00
-1.60
-1.20
-.80
-.40
0.00
.40
.72
.80
1.20
1.60
2
Y(IN)
ν_v = 8.0
Y_MAX = 1.00 in
W = .001055 in
S = 66.1995 in
β = 488.293 in
α = .39552
γ = .00237 in⁻¹
AREA = 331437 mrad-in

11
POSITION IN SUPER-SUPERPERIOD
1/4
4.00
2.00
0.00
-2.00
-4.00
-1.60
-1.20
-.80
-.40
.40
.80
1.20
1.60
2
Y (IN)
ν_V = 8.0
Y_{MAX} = 1.001 in
W = .001055 in
S = 132.3787
β = 471.1916
α = 0.00
δ = .0022 in^{-1}
AREA = 33.1137
mrad-in

POSITION IN SUPER-SUPERPERIOD

3/8



$D_V = 8.0$

$Y_{MAX} = 1.00$ in

$W = .001055$ in

$\zeta = 198.5984$ in

$\beta = 488.293$

$\alpha = -.39552$

$\delta = .00237$ in

AREA = 3.31437 mrad

POSITION IN SUPER-SUPERPERIOD

1/5

$Y(IN)$

$N_V = 8.0$

$Y_{MAX} = 1.00$ in

$W = .001055$ in

$S = 261.7778$ in

$\theta = 633.158$ in

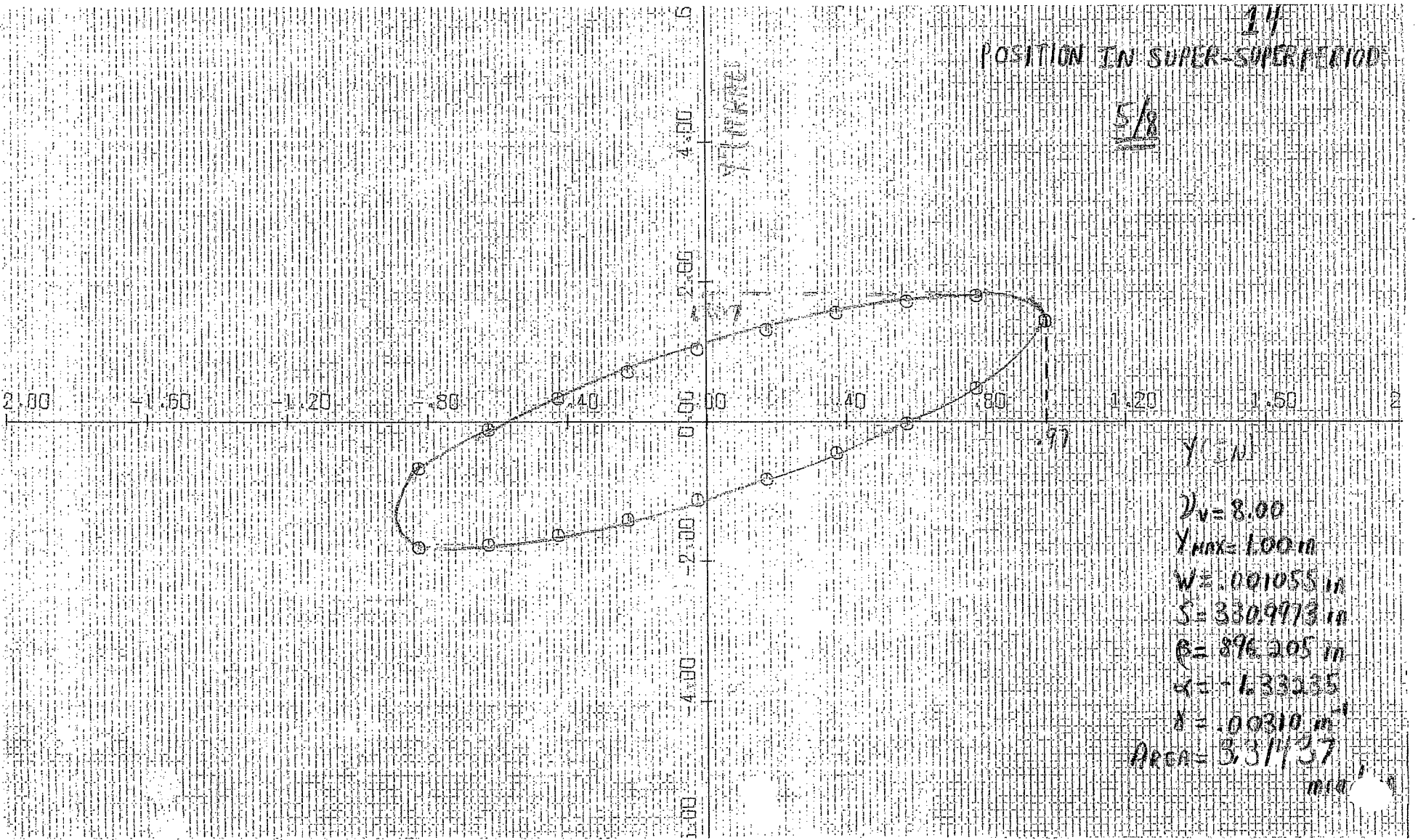
$\alpha = -1.87500$

$\chi = .0071^{4}$ in^{-1}

AREA $= 3.31737$ mm^{2}

POSITION IN SUPER-SUPER PERIOD

5/8



$\nu_v = 8.00$

$Y_{MAX} = 1.00$ in

$W = .001055$ in

$S = 3309773$ in

$\beta = 896.205$ in

$\alpha = -1.33235$

$\delta = .00310$ in^{-1}

$A_{RCA} = 3.31737$ in^2

POSITION IN SUPER-SUPERPERIOD:

3/4

$N_c = 8.0$

$Y_{MAX} = 1.00$ in

$W = .001055$ in

$S = 397.1967$ in

$\theta = 946.237$ in

$\alpha = 0.00$

$\gamma = .001055$ in^{-1}

AREA = 3.3437 mrad-m

POSITION IN SUPER-SUPERPERIOD:

7/8

$\nu_d = 8.0$

$Y_{MAX} = 1.00$ m

$W = .001055$ in

$S_1 = 463.3962$ in

$\beta = 876.205$ in

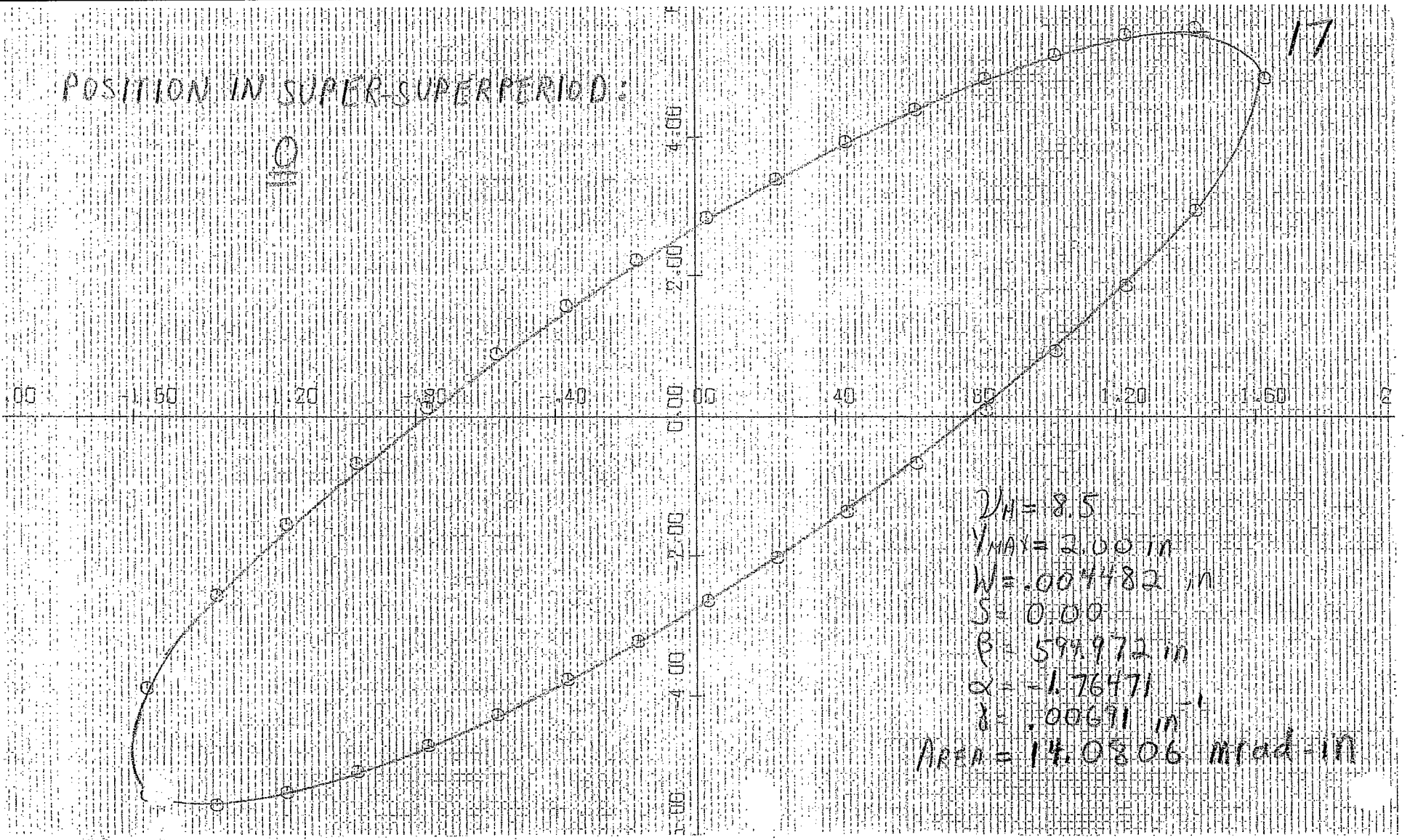
$\alpha = 1.33235$

$\gamma = .00310$ in^{-1}

AREA = 3.31737 mm^2

POSITION IN SUPER-SUPERPERIOD:

$\frac{Q}{11}$



$\nu_H = 8.5$

$\nu_{MAX} = 2.00$ in

$W = .0074482$ in

$S = 0.00$

$\beta = 599.972$ in

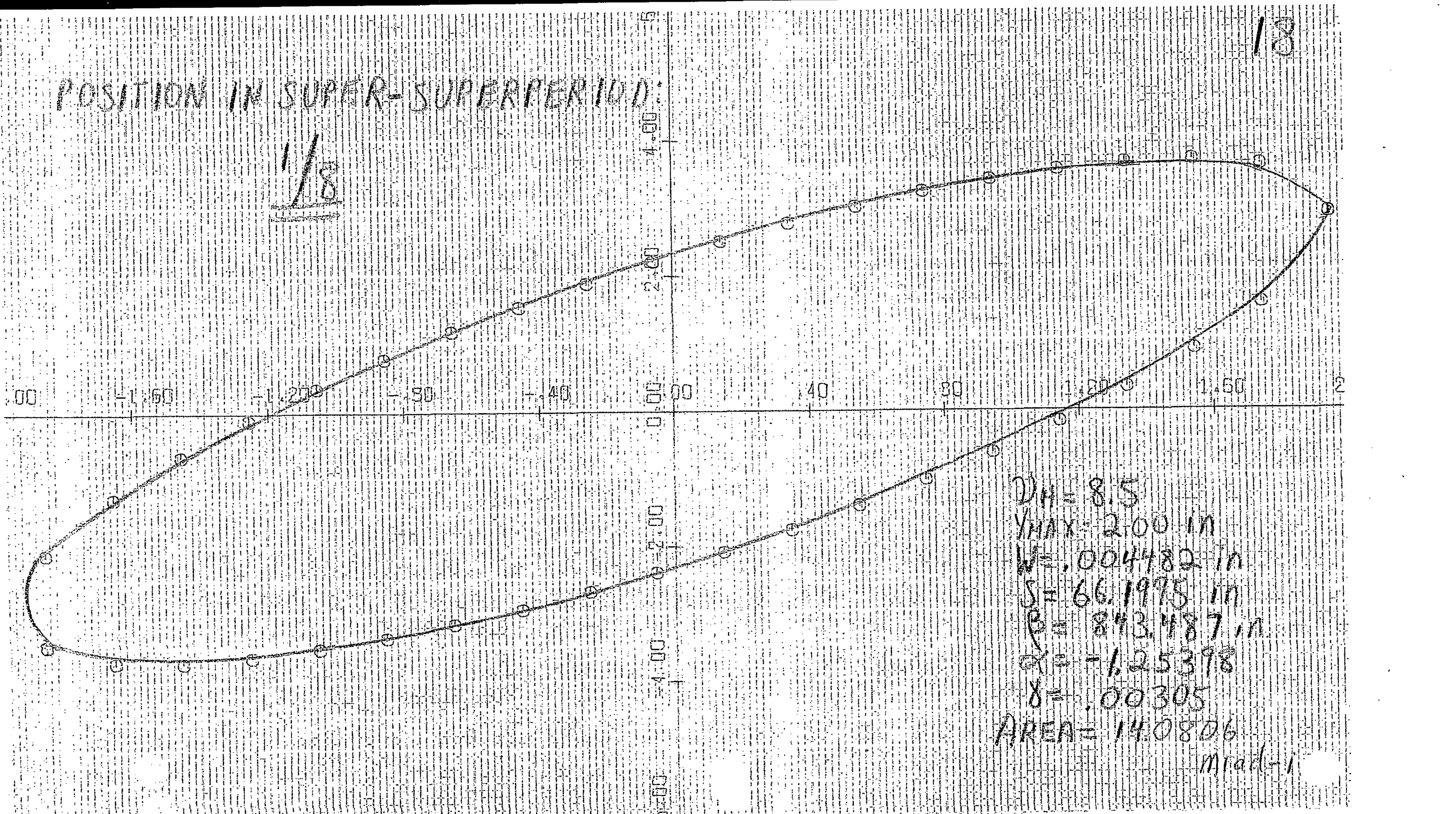
$Q = 1.76471$

$\delta = .00691$ in^{-1}

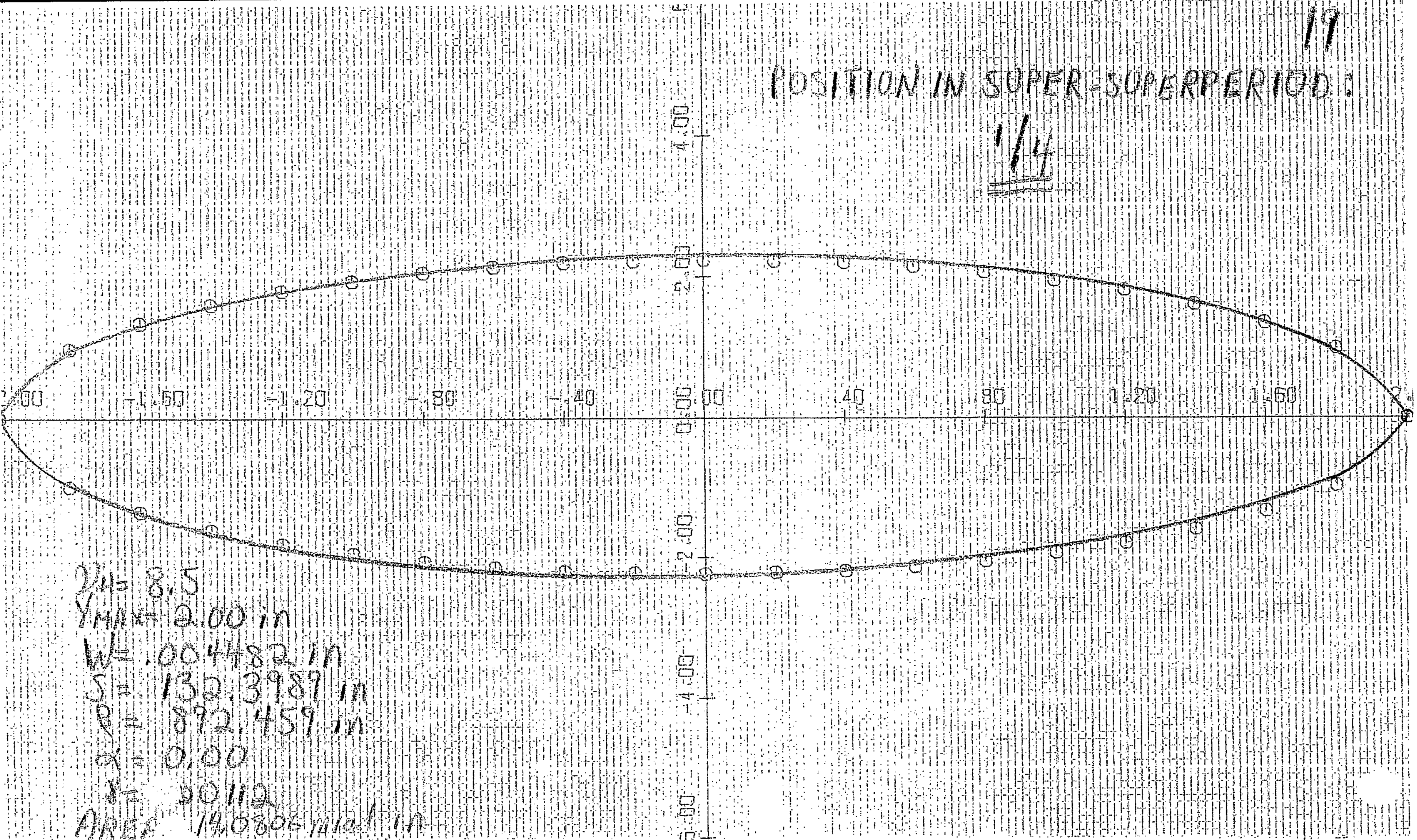
$A_{REA} = 14.0306$ mrad-in

POSITION IN SUPER-SUPERPERIOD:

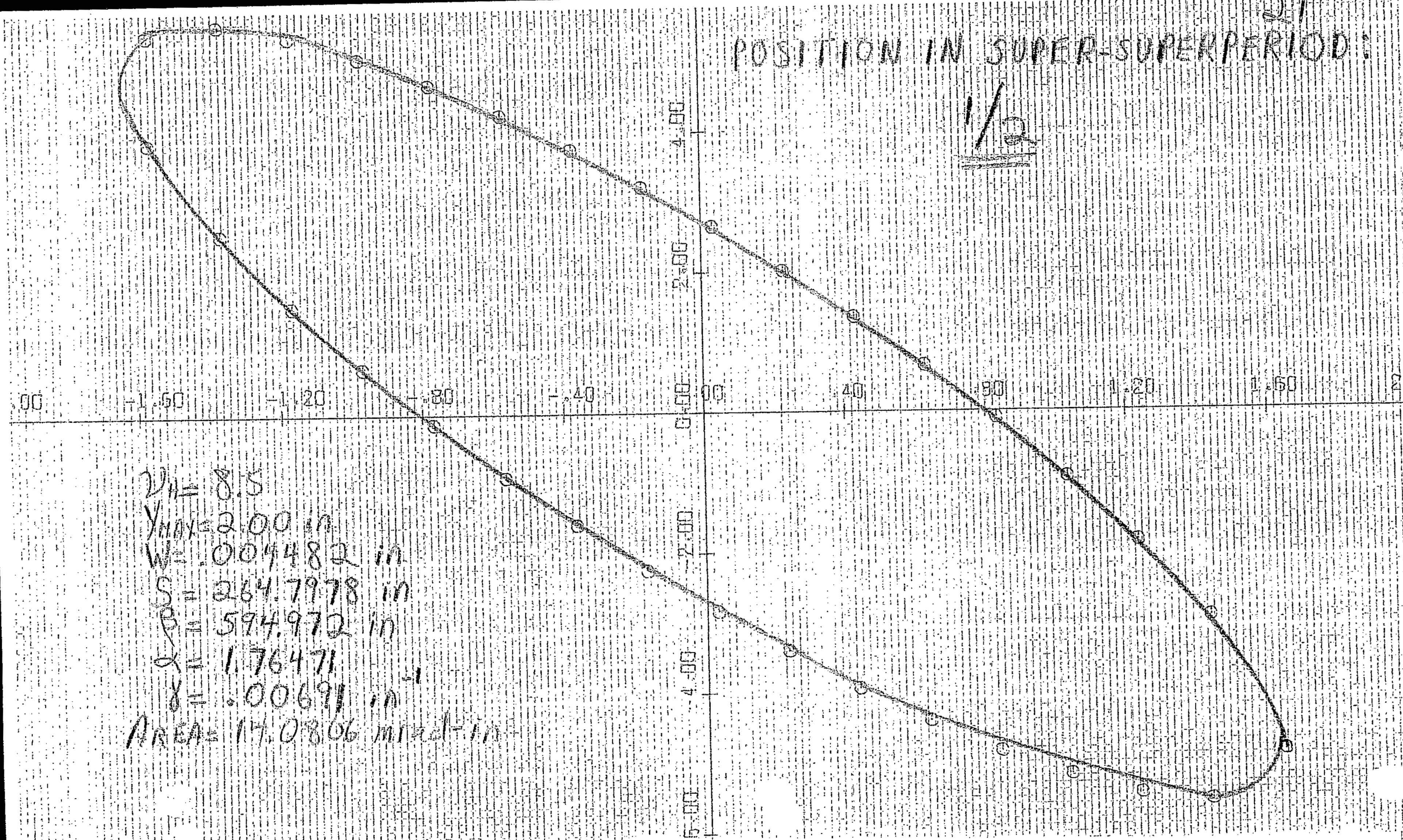
1/8



D_H = 8.5
Y_{MAX} = 2.00 in
W = .004482 in
S = 66.1975 in
ρ = 843.487 in
α = -1.25378
δ = .00305
AREA = 14.0806
mrad-1

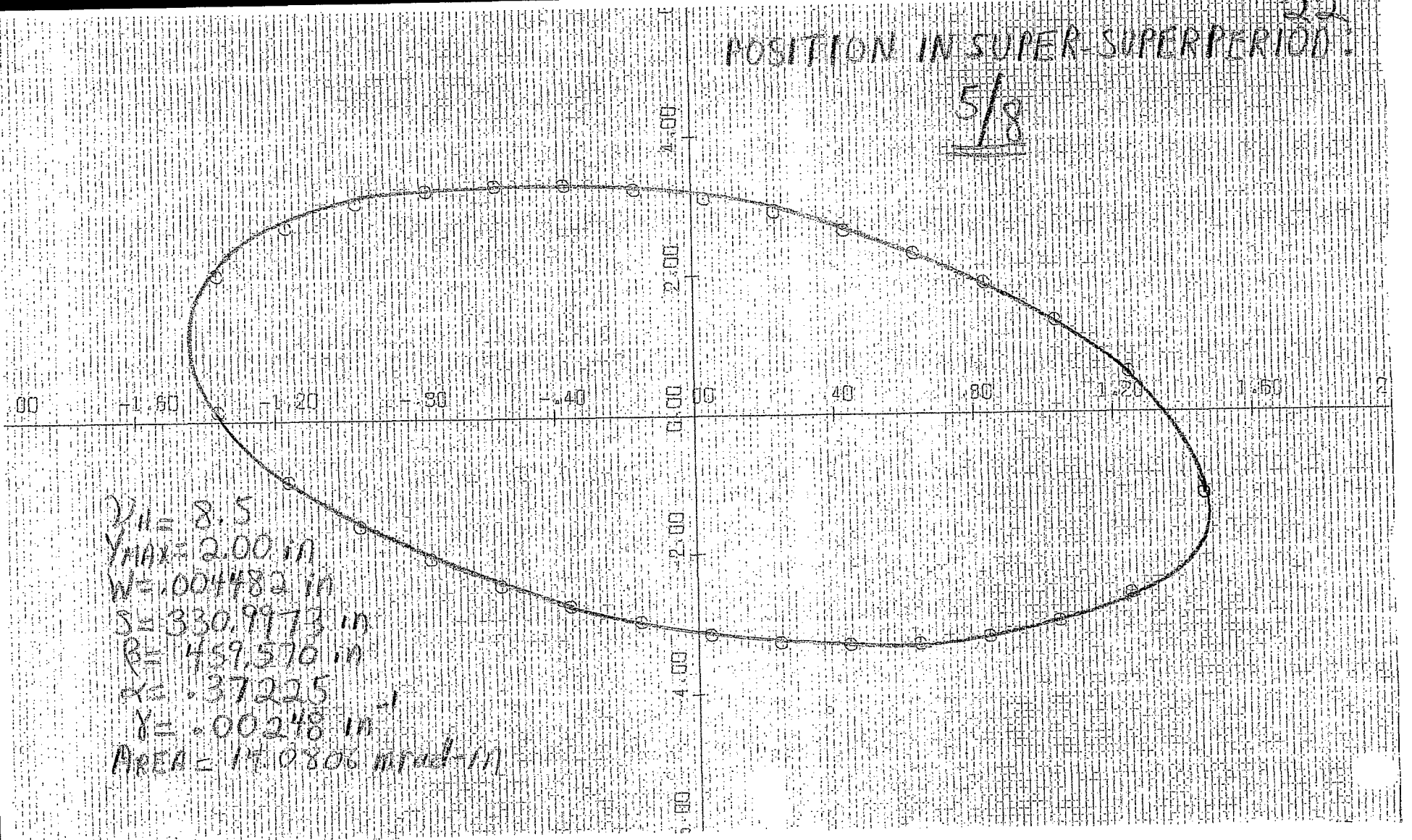

POSITION IN SUPER-SUPERPERIOD:
1/4
4.00
2.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
-2.00
-4.00
-6.00
2/4 = 8.5
YMAX = 2.00 in
W = .004482 in
S = 132.3987 in
B = 872.459 in
α = 0.00
γ = .00119
AREA 1408.06 mil. in

-20
POSITION IN SUPER-SUPERPERIOD:
3/8
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00
4.40
2H = 8.5
Ymax = 2.00 in
W = .004483 in
S = 198.5984 in
R = 843.487 in
α = 1.25398
γ = .00305
AREA = 11.0806 mrad-in


POSITION IN SUPER-SUPERPERIOD:
1/2
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
4.00
2.00
0.00
-2.00
-4.00
-6.00
νH = 8.5
Ymax = 2.00 in
W = .004482 in
S = 264.7978 in
β = 594.972 in
α = 1.76471
γ = .00691 in^-1
AREA = 14.0806 mrad-in

POSITION IN SUPER-SUPERPERIOD:

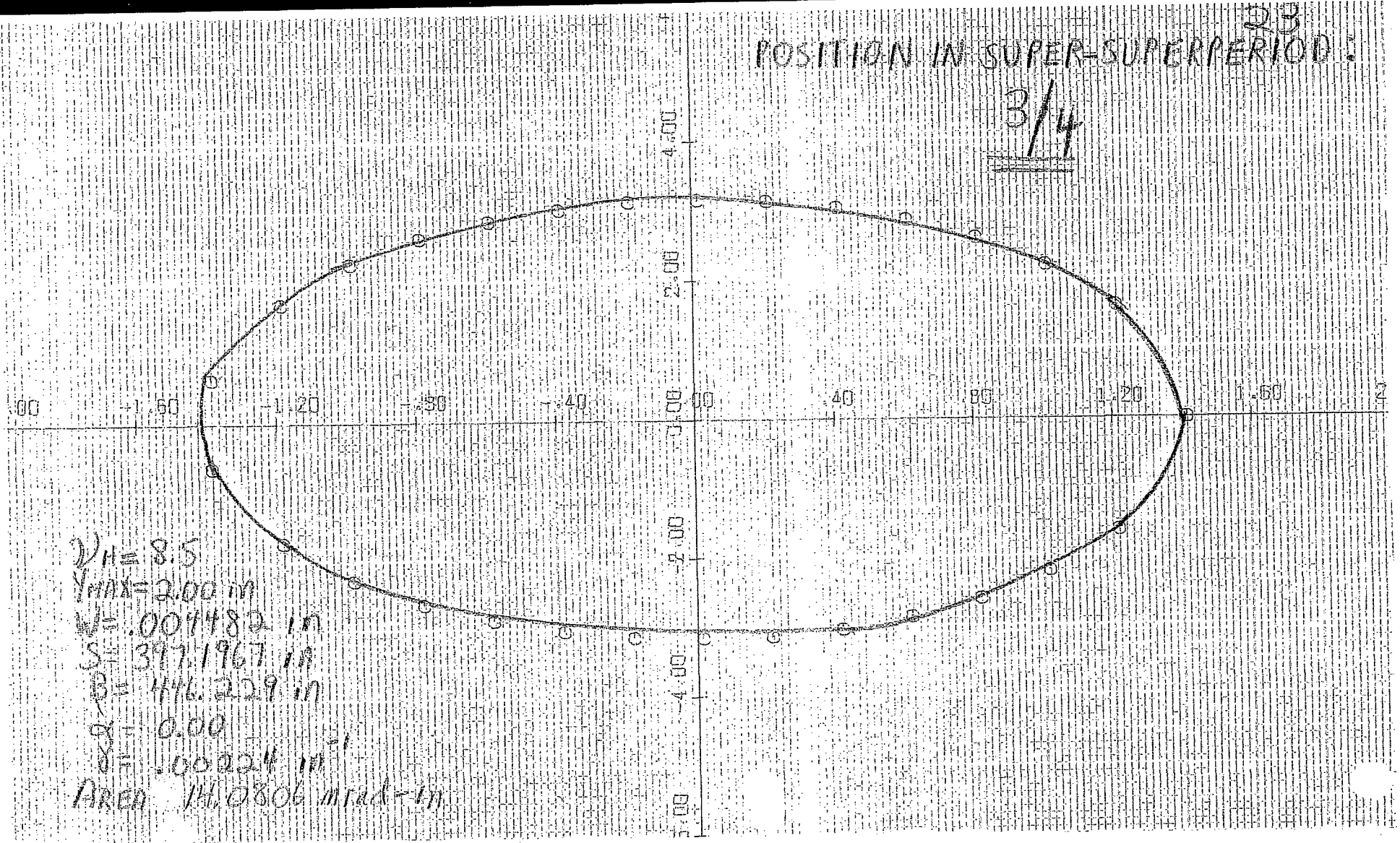
5/8



$\nu_H = 8.5$
$Y_{max} = 2.00$ in
$W = .004482$ in
$S = 330.9773$ in
$R = 459.570$ in
$\chi_2 = .37225$
$\gamma = .00248$ in^{-1}
AREA $= 11.0806$ mrad-in

POSITION IN SUPER-SUPERPERIOD:

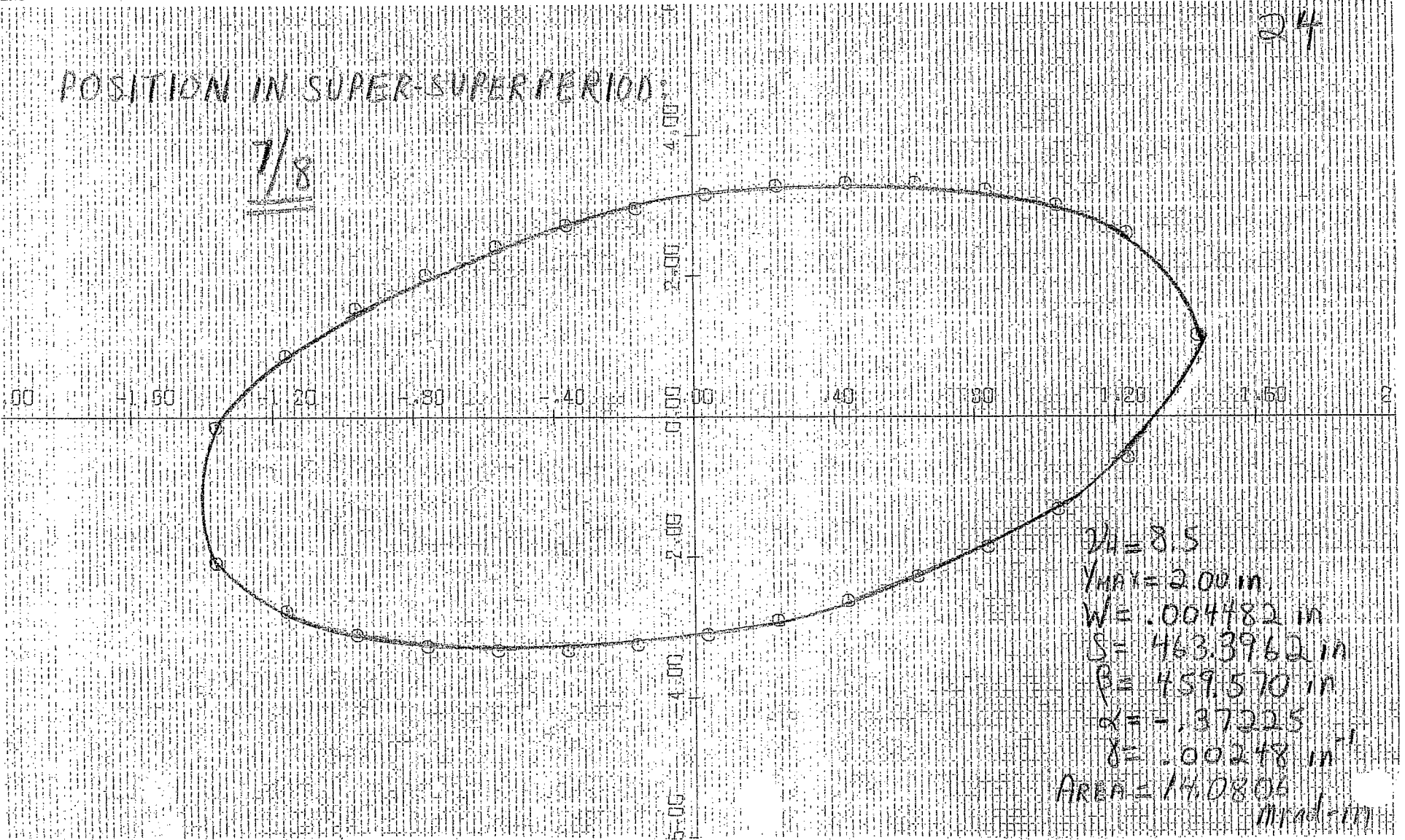
3/4



$\nu_H = 8.5$
$y_{max} = 2.00$ m
$W = .0074822$ m
$S = 397.1967$ m
$B = 476.3229$ m
$\alpha = 0.00$
$\delta = .00224$ m^{-1}
AREA 111.0806 mrad-m

POSITION IN SUPER-SUPERPERIOD

7/8



$2/4 = 8.5$

$Y_{MAX} = 2.00$ in.

$W = .004882$ in

$S = 463.3960$ in

$\rho = 459.570$ in

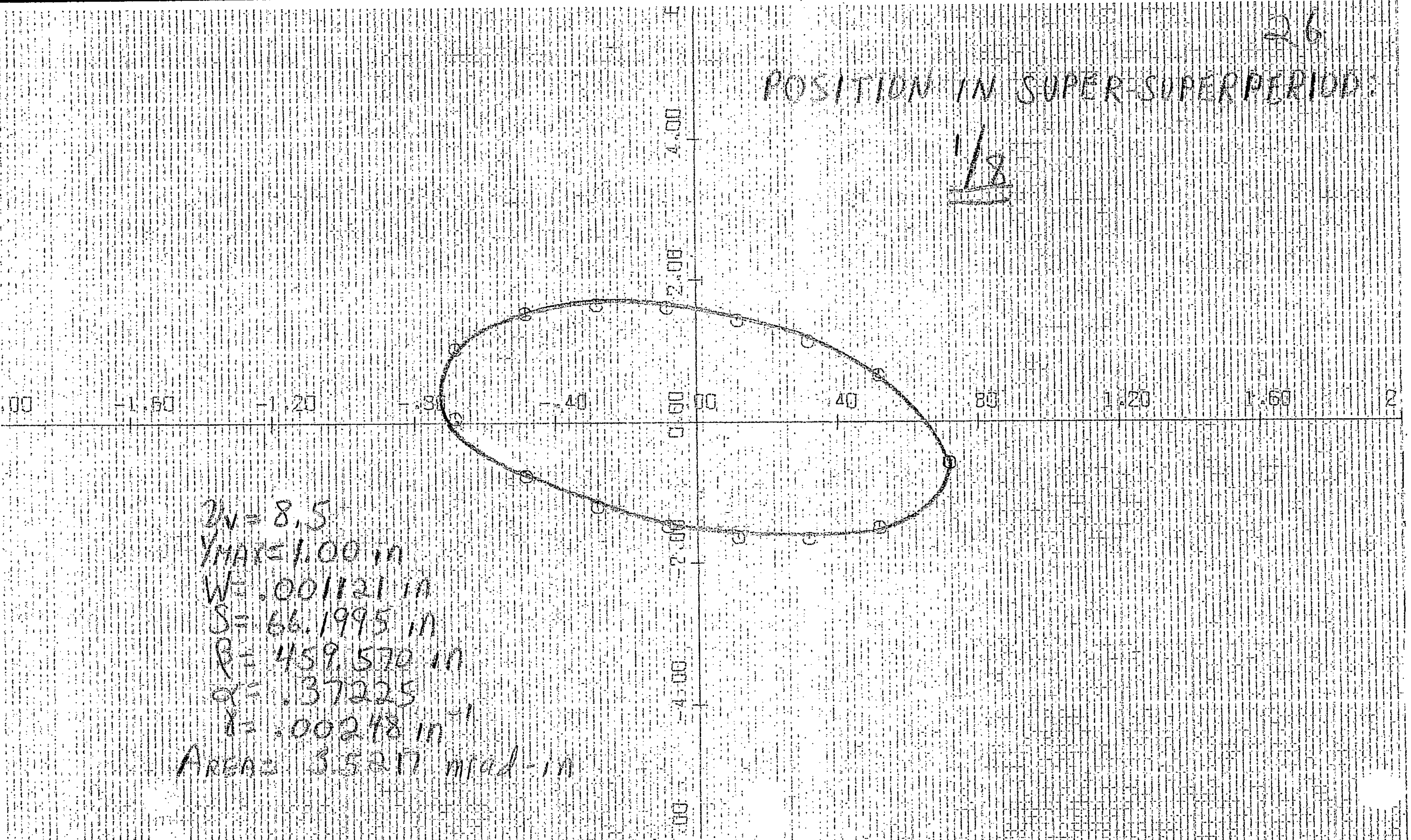
$\alpha = -.37225$

$\delta = .00248$ in

AREA $= 14.0804$ mm(-in)

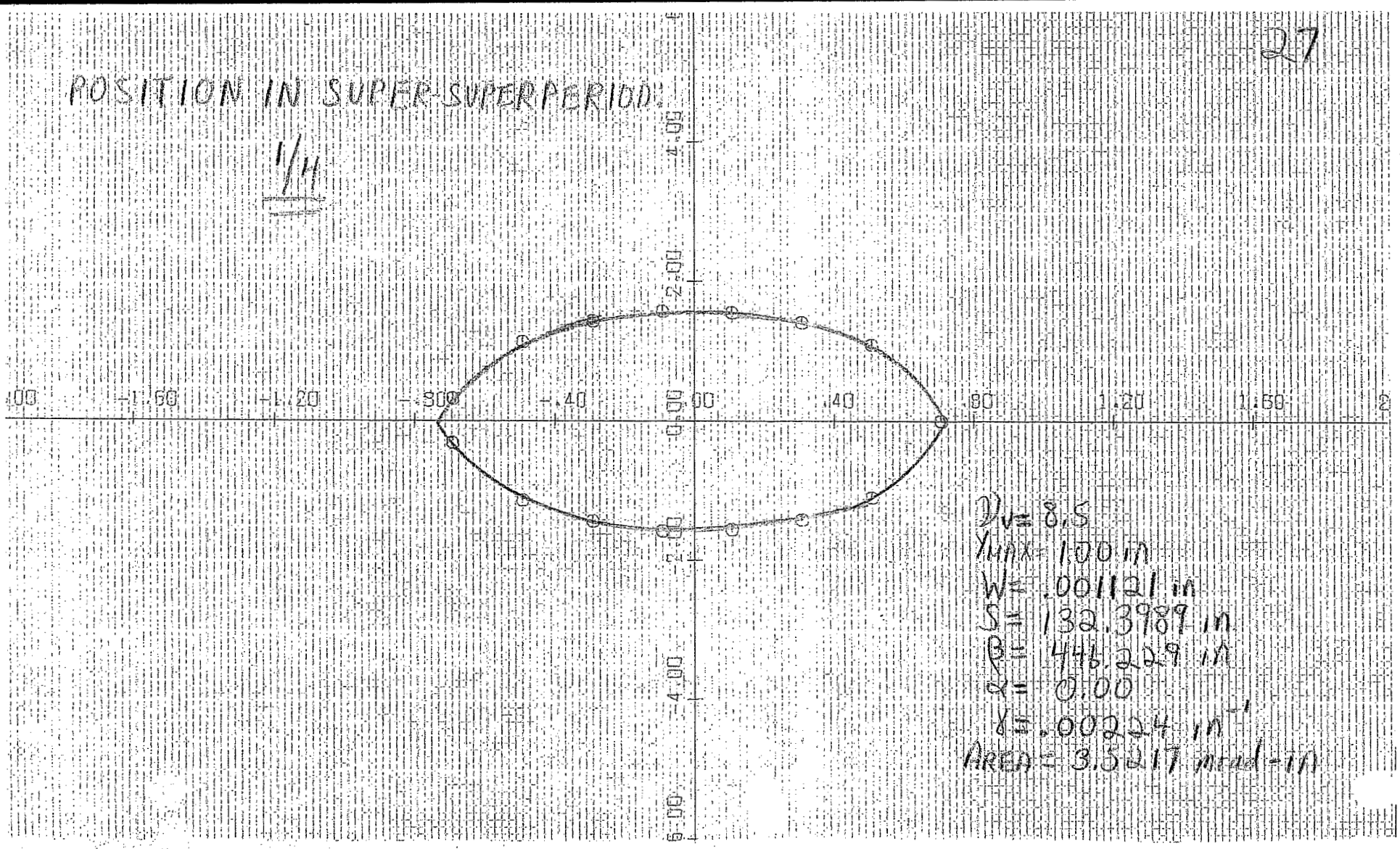
POSITION IN SUPER-SUPERPERIOD: 0

$2V = 8.5$
$Y_{MAX} = 1.00$ in
$WE = .001121$ in
$S = 0.00$
$R = 594.973$ in
$\alpha = 1.76471$
$\beta = .00691$ in^{-1}
AREA = 3.5217 mrad-in


POSITION IN SUPER-SUPERPERIOD:
1/8
-2.00
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
2
4.00
2.00
-2.00
-4.00
$2\nu = 8.5$
$Y_{MAX} = 1.00$ in
$W = .001121$ in
$S = 66.1995$ in
$\beta = 459.570$ in
$\alpha = .37225$
$\gamma = .00248$ in^{-1}
AREA = 3.5217 mrad-in

POSITION IN SUPER-SUPERPERIOD:

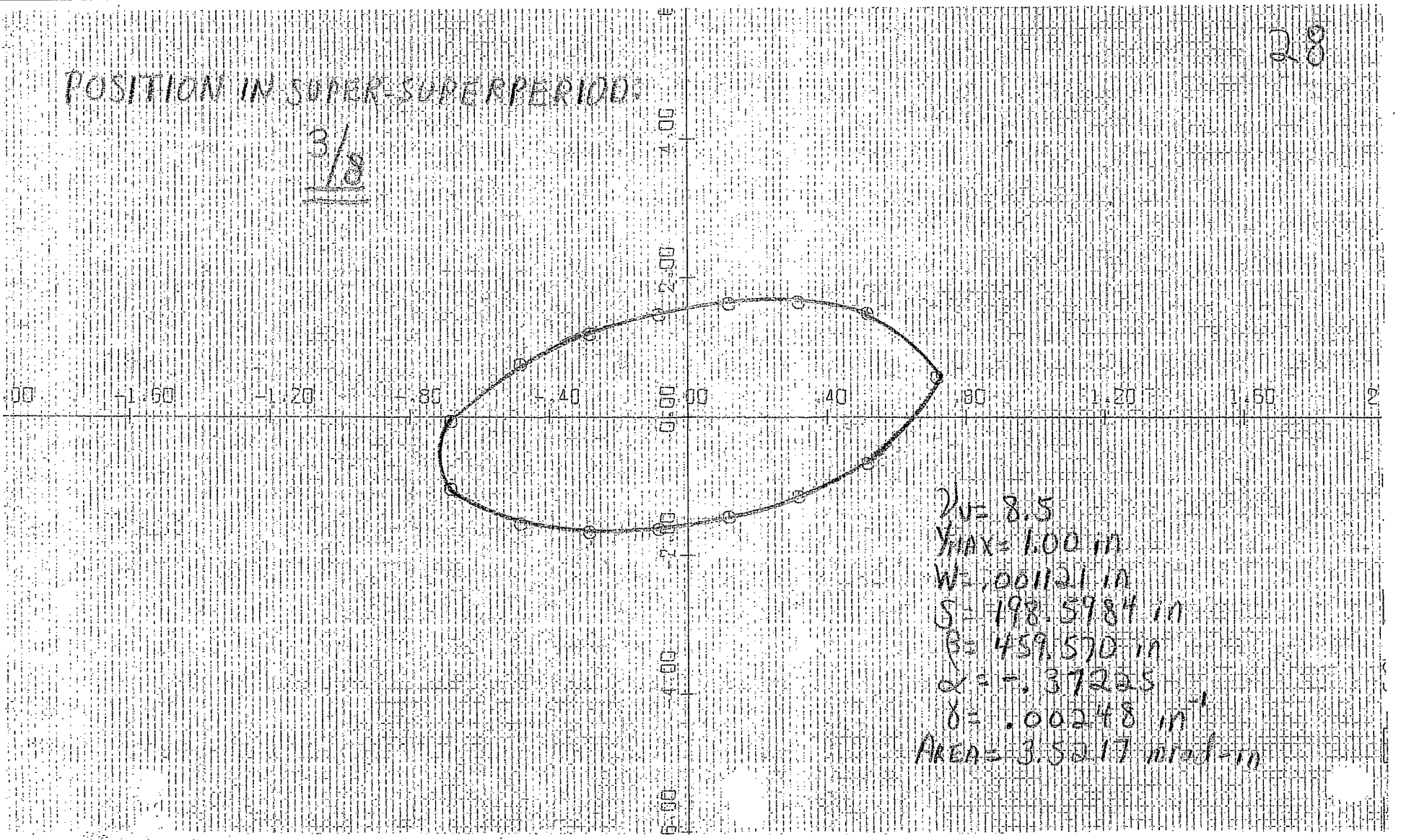
1/4



$2\nu = 8.5$
$x_{max} = 1.00\ m$
$W = .001121\ m$
$S = 132.3909\ m$
$\beta = 44.2029\ m$
$\alpha = 0.00$
$\gamma = .0029\ m^{-1}$
$AREA = 3.5217\ mrad\text{-}m$

POSITION IN SUPER-SUPERPERIOD:

3/8

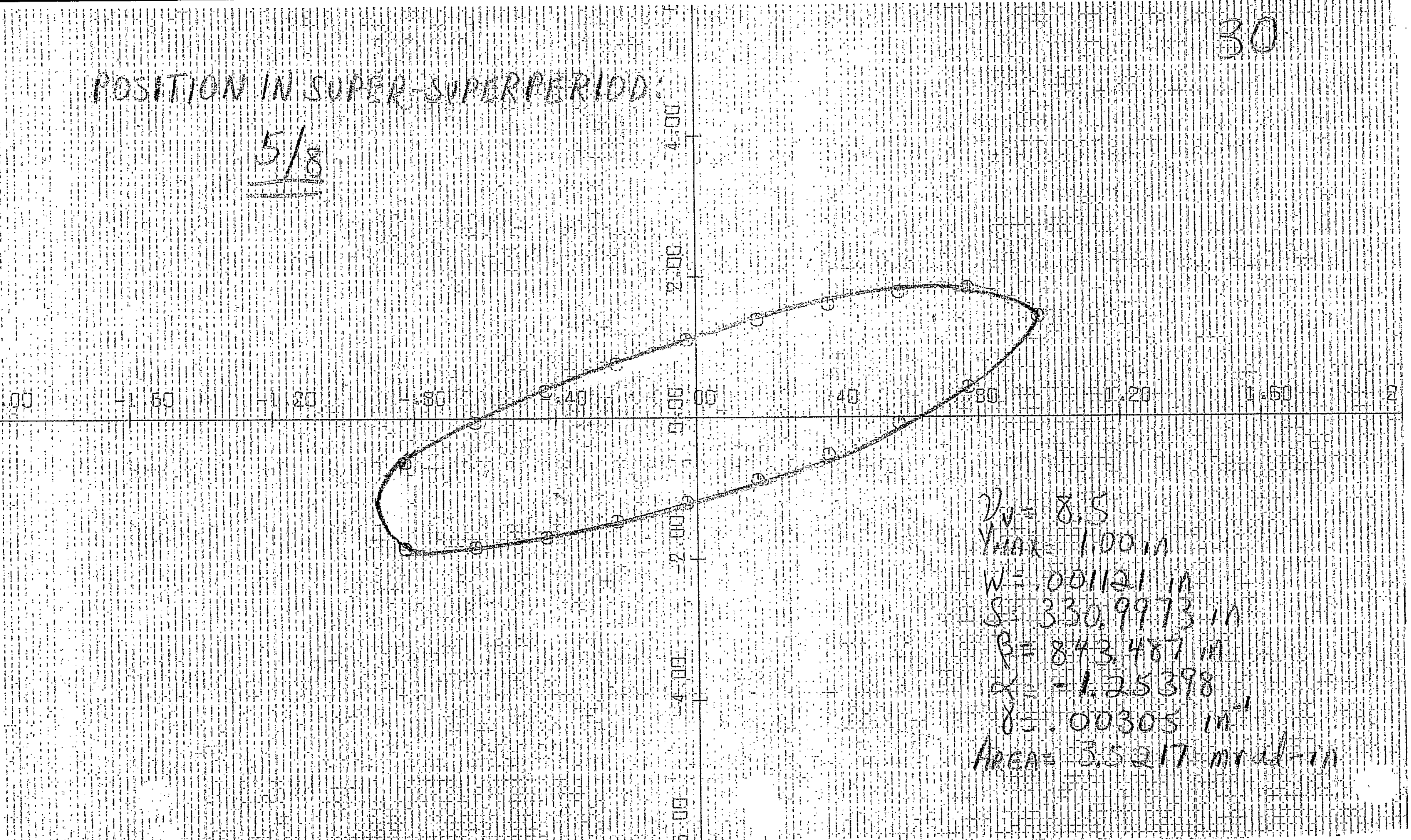


$2N = 8.5$
$Y_{MAX} = 1.00$ in
$W = .00121$ in
$S = 198.5984$ in
$\beta = 459.520$ in
$\alpha = -1.37295$
$\delta = .00248$ in^{-1}
AREA $= 3.5017$ in-deg-in

POSITION IN SUPER-SUPERPERIOD:

1/2

ν^2 = .5
Y_{max} = 1.00 in
W = .00121
S = 264.7978 in
B = 594.972 in
α = -1.76471
γ = .0061 in^{-1}
AREA = 3.5217 rad·in


POSITION IN SUPER-SUPERPERIOD:
5/8
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
4.00
2.00
0.00
-2.00
-4.00
-6.00
ν_v = 8.5
Y_{MAX} = 1.00 in
W = .001121 in
S = 330.9973 in
β = 843.487 in
α = -1.25398
γ = .00305 in^{-1}
AREA = 3.5217 mrad-in

POSITION IN SUPER-SUPERPERIOD:

3/4

$2\nu_x = 8.5$

$Y_{MAX} = 1.00$ in

$W = .00112$ in

$S = 397.1967$ in

$\beta = 892.459$ in

$\alpha = 0.00$

$\gamma = .0012$

AREA = 3.5 3.17 mrad-in

8.2

POSITION IN SUPER-SUPERPERIOD:

1/8

$D_V = 8.5$

$V_{MAX} = 1.00$ in

$W = .001121$ in

$S = 463.3962$ in

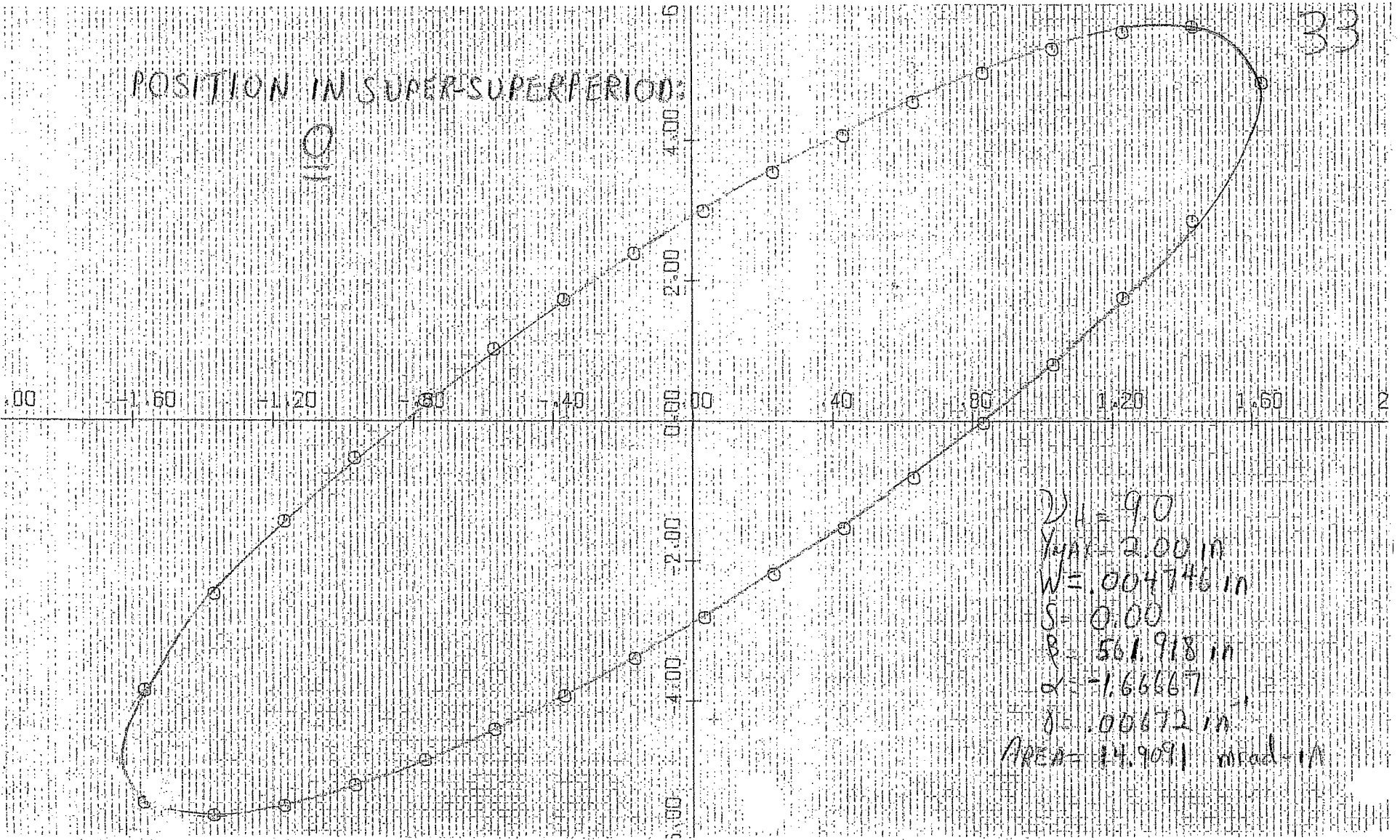
$B = 843.4871$ in

$Q = 1.25398$

$\delta = .009305$ in^{-1}

AREA = 3.0217 mrad-in

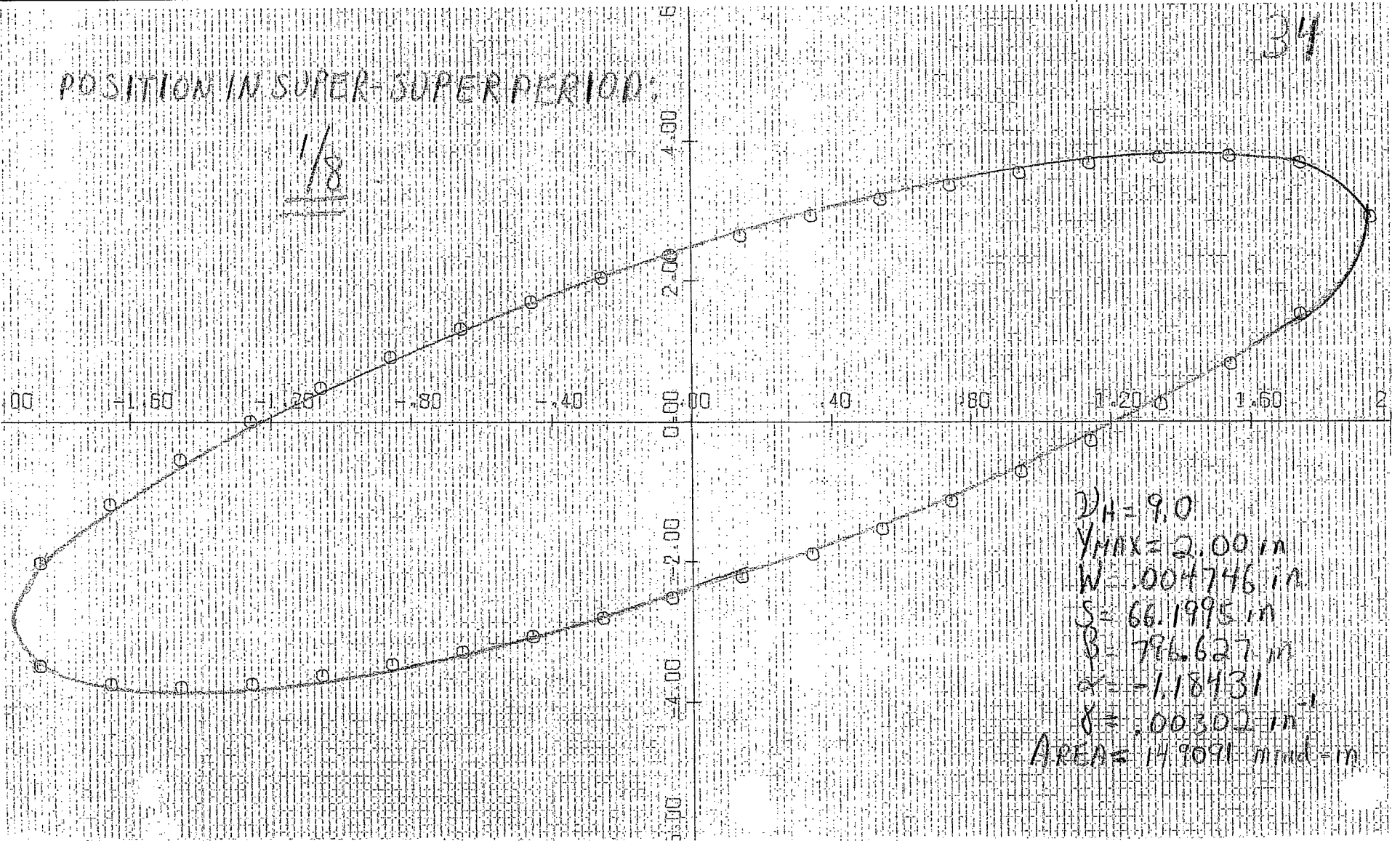
POSITION IN SUPER-SUPERPERIOD:
0



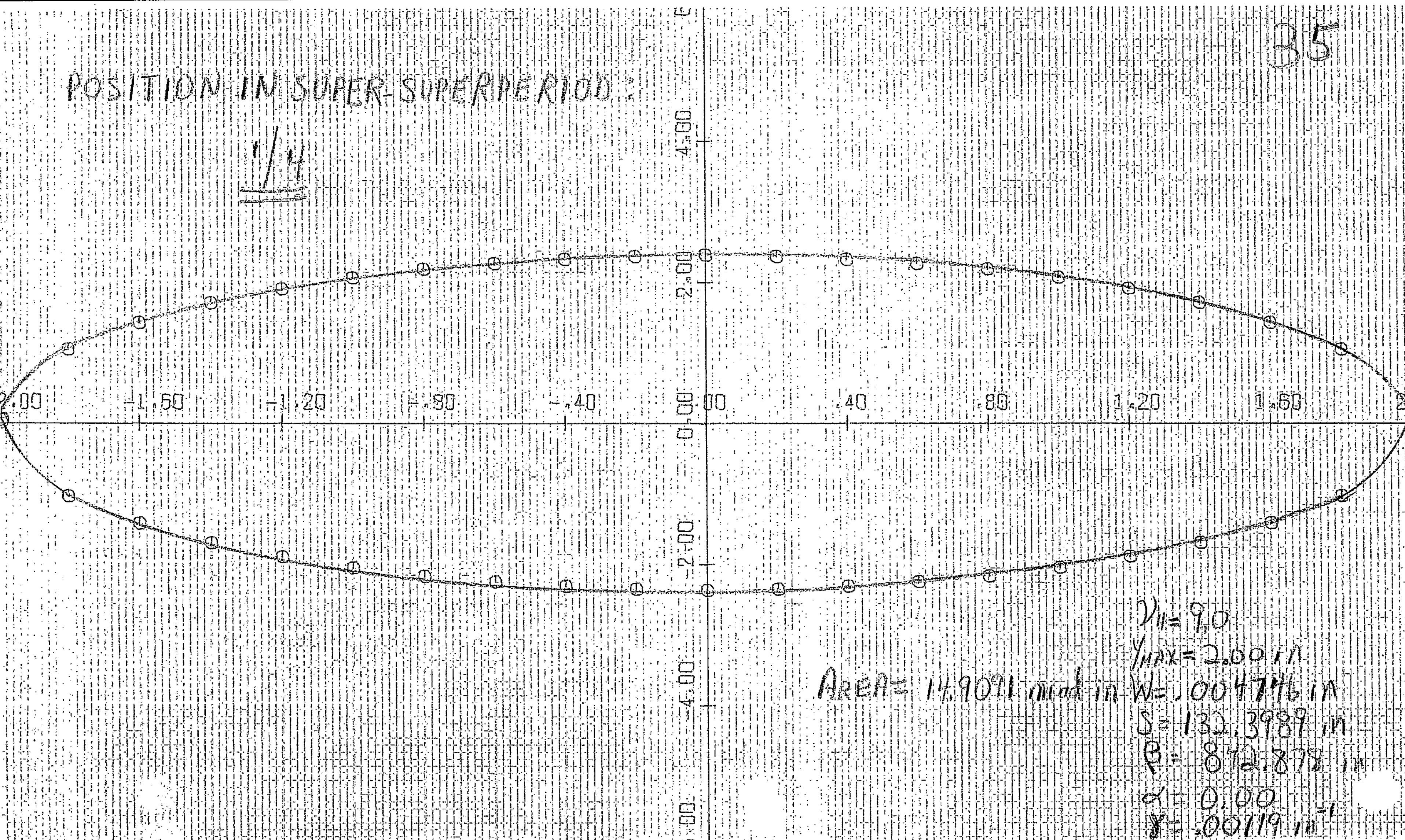
$\nu_H = 9.0$
$h_{max} = 2.00$ in
$W = .004746$ in
$S = 0.00$
$R = 561.918$ in
$\alpha = -1.66667$
$\delta = .00672$ in^{-1}
$\beta_{mean} = 14.9091$ mrad^{-1}

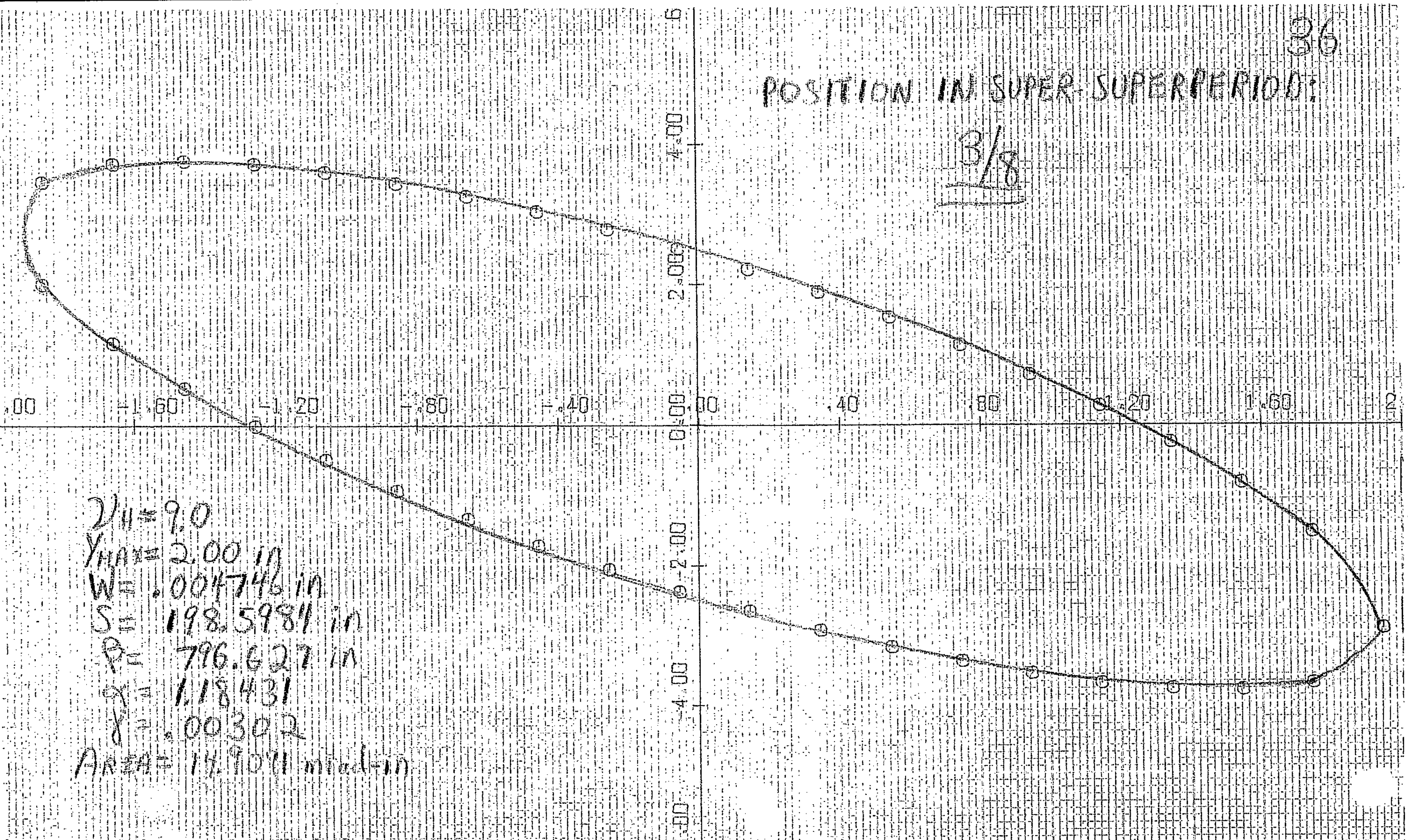
POSITION IN SUPER-SUPERPERIOD:

1/8



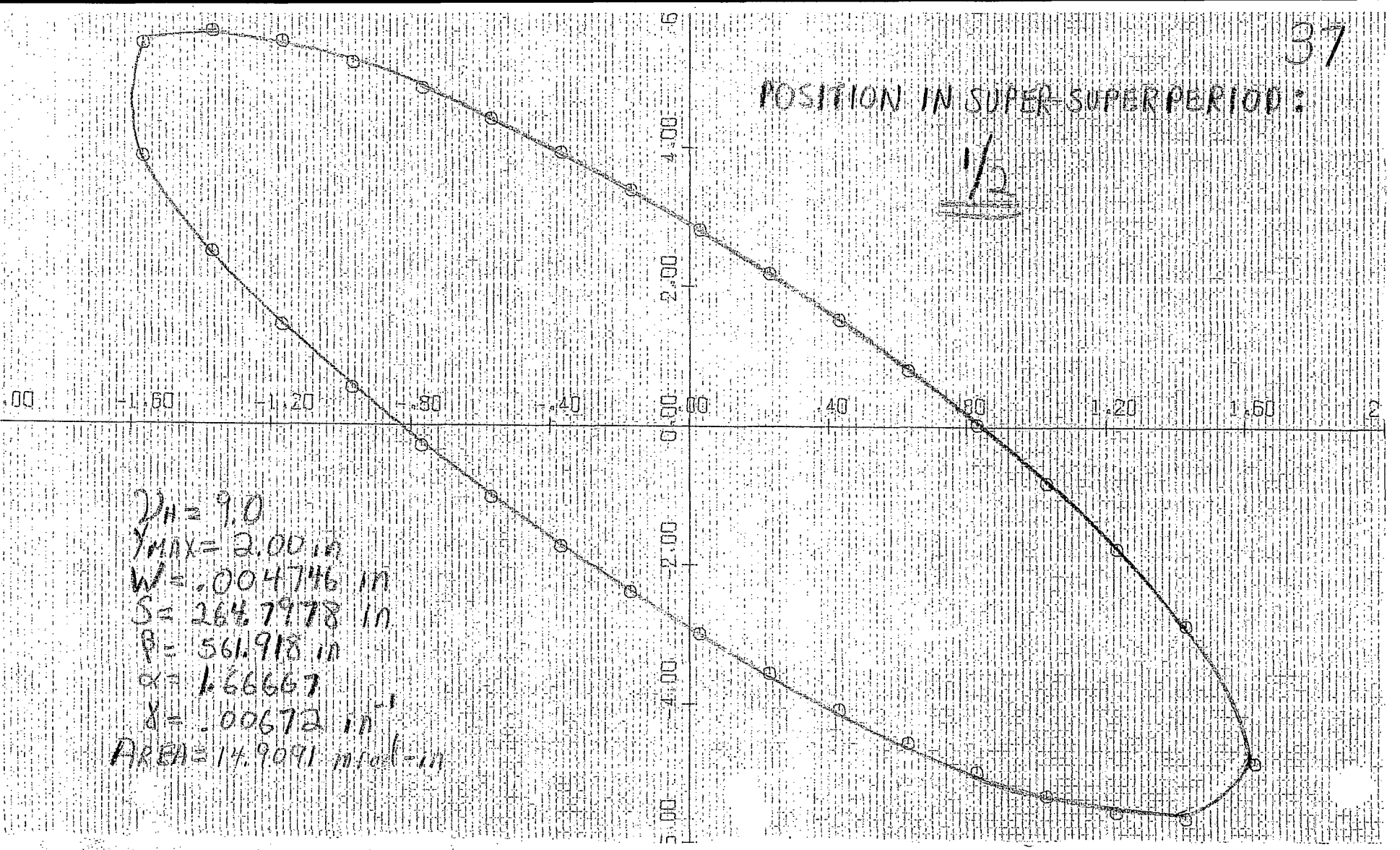
$\mathcal{D}_H = 9.0$
$y_{MAX} = 2.00$ in
$W = .0044746$ in
$S = 66.1995$ in
$\beta = 796.627$ in
$\alpha = -1.18431$
$\gamma = .003002$ in^{-1}
AREA $= 14.9091$ mod-in


POSITION IN SUPER-SUPERPERIOD:
1/4
4.00
2.00
0.00
-2.00
-4.00
-2.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
1.60
2.00
ν_{ll} = 9.0
y_{max} = 2.00 in
AREA = 14.9071 mrad in
W = .004746 in
S = 130.3989 in
β = 842.878 in
α = 0.00
γ = .00119 in^{-1}


POSITION IN SUPER-SUPERPERIOD:
3/8
ν_H= 9.0
Y_{MAX}= 2.00 in
W= .004746 in
S= 198.5981 in
ρ= 796.627 in
α= 1.18431
γ= .00322
AREA= 14.901 mrad-in
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
4.00
2.00
-2.00
-4.00

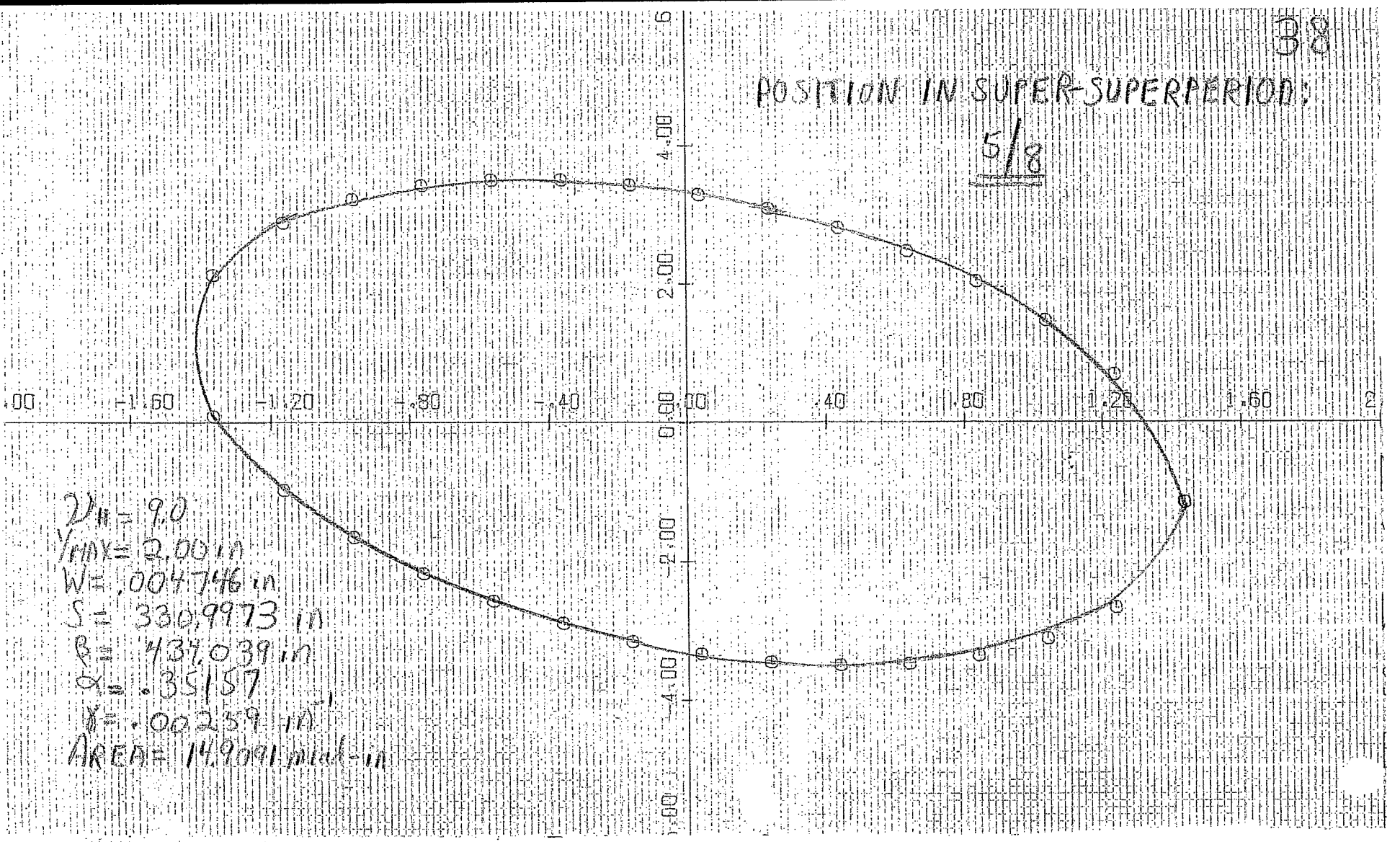
POSITION IN SUPER-SUPER PERIOD:

1/2



$2_H = 9.0$
$Y_{max} = 2.00$ in
$W = .004746$ in
$S = 264.7978$ in
$P = 561.918$ in
$\alpha = 1.66667$
$\lambda = .00672$ in^{-1}
AREA $= 14.9091$ rad-in

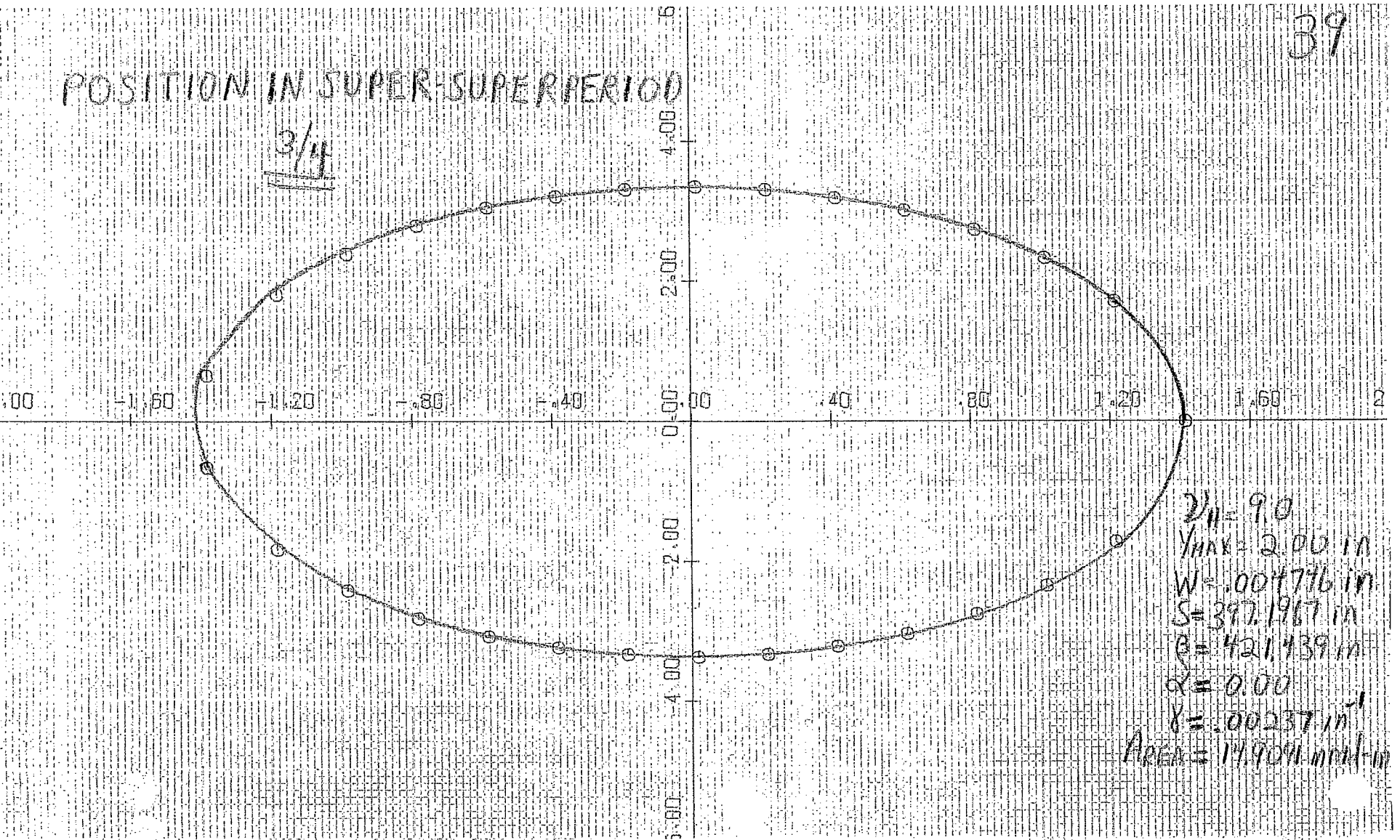
POSITION IN SUPER-SUPERPERIOD:
5/8



$\nu_H = 9.0$
$Y_{MAX} = 2.00$ in
$W = .004746$ in
$S = 330.9973$ in
$R = 437.039$ in
$\alpha = .35157$
$\beta = .00259$ in^{-1}
AREA = 14.9091 unit-in

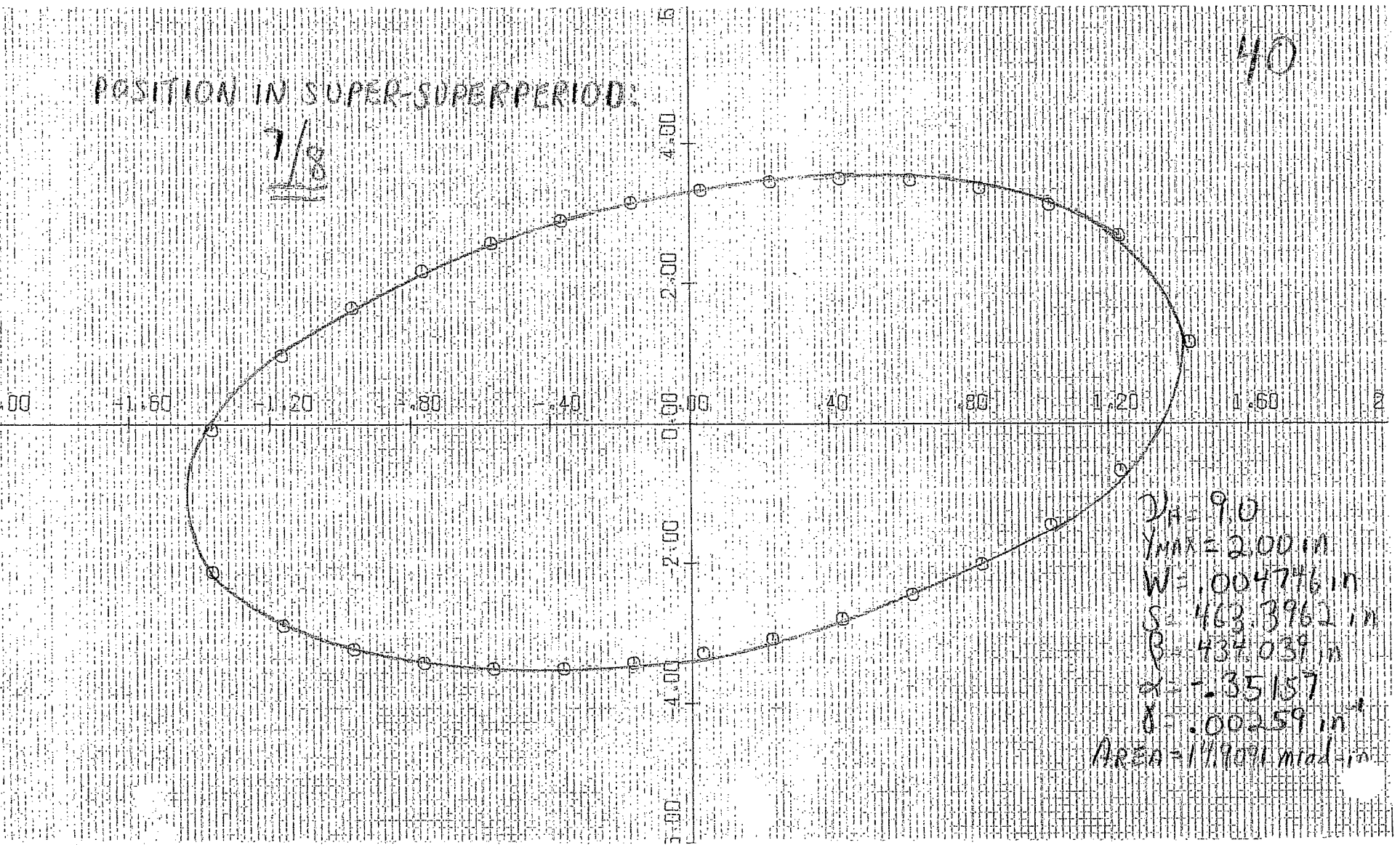
POSITION IN SUPER-SUPERPERIOD

3/4



POSITION IN SUPER-SUPERPERIOD:

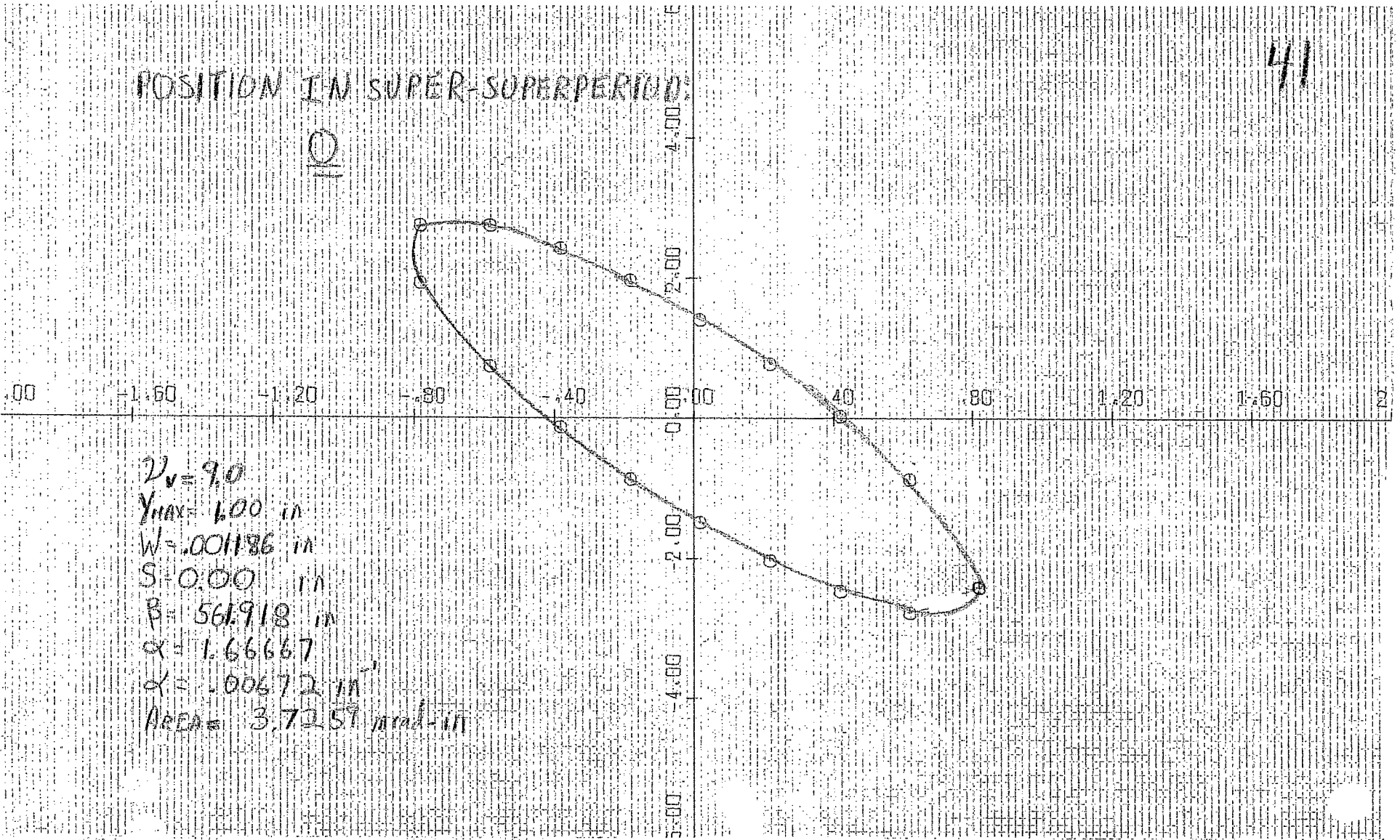
1/18



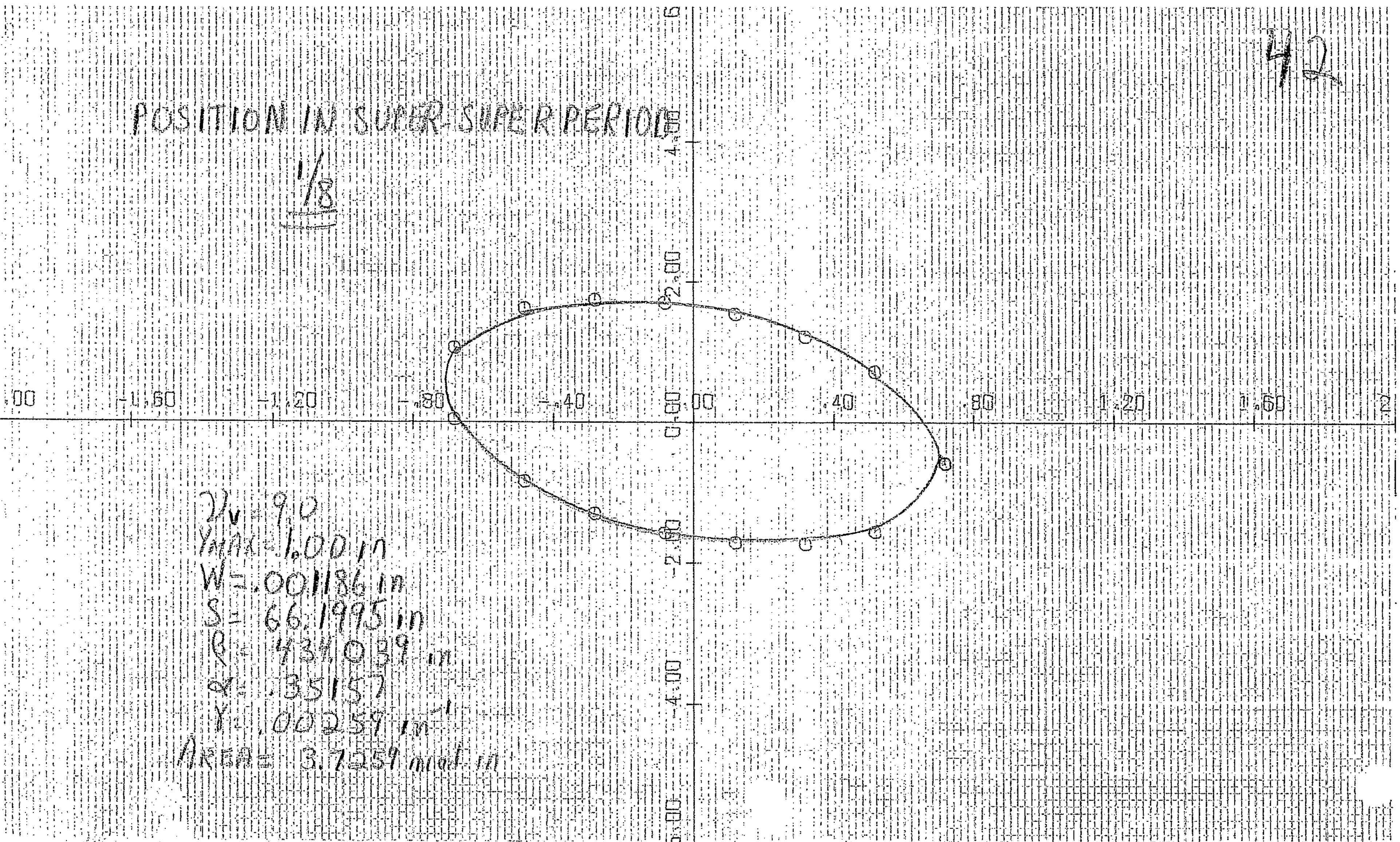
D_H = 90
Y_{MAX} = 2.00 in
W = .004776 in
S = 463.3962 in
B = 434.039 in
α = 1.35157
δ = .00259 in^{-1}
AREA = 1.11901 mid-in^2

POSITION IN SUPER-SUPERPERIOD:

01

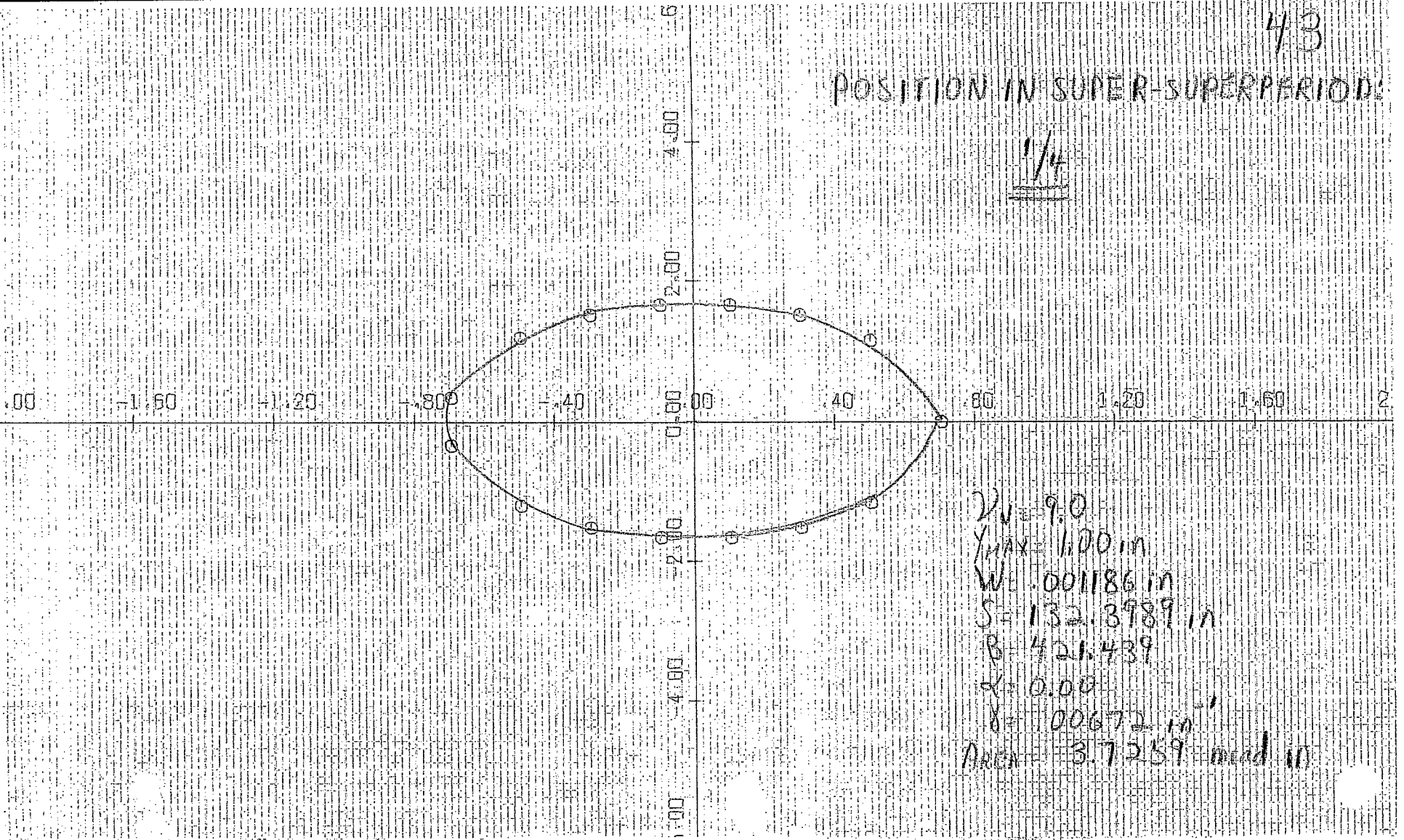


$\nu_v = 9.0$
$y_{max} = 4.00$ in
$W = .001186$ in
$S = 0.00$ in
$\beta = 561.918$ in
$\alpha = 1.66667$
$\gamma = .006702$ in^{-1}
AREA $= 3.7257$ mrad-in


42
POSITION IN SUPER-SUPER PERIOD
1/8
ν_v = 9.0
Y_MAX = 1.00 in
W = .001186 in
S = 66.1995 in
β = 434.039 in
α = .35157
γ = .00259 in⁻¹
AREA = 3.7259 mrad-in
-1.60
-1.20
-.80
-.40
0.00
.40
.80
1.20
1.60
4.00
2.00
-2.00
-4.00

POSITION IN SUPER-SUPERPERIOD:

1/4



2ν = 9.0

Y_{MAX} = 1.20 in

W = .001186 in

S = 132.3989 in

β = 421.439

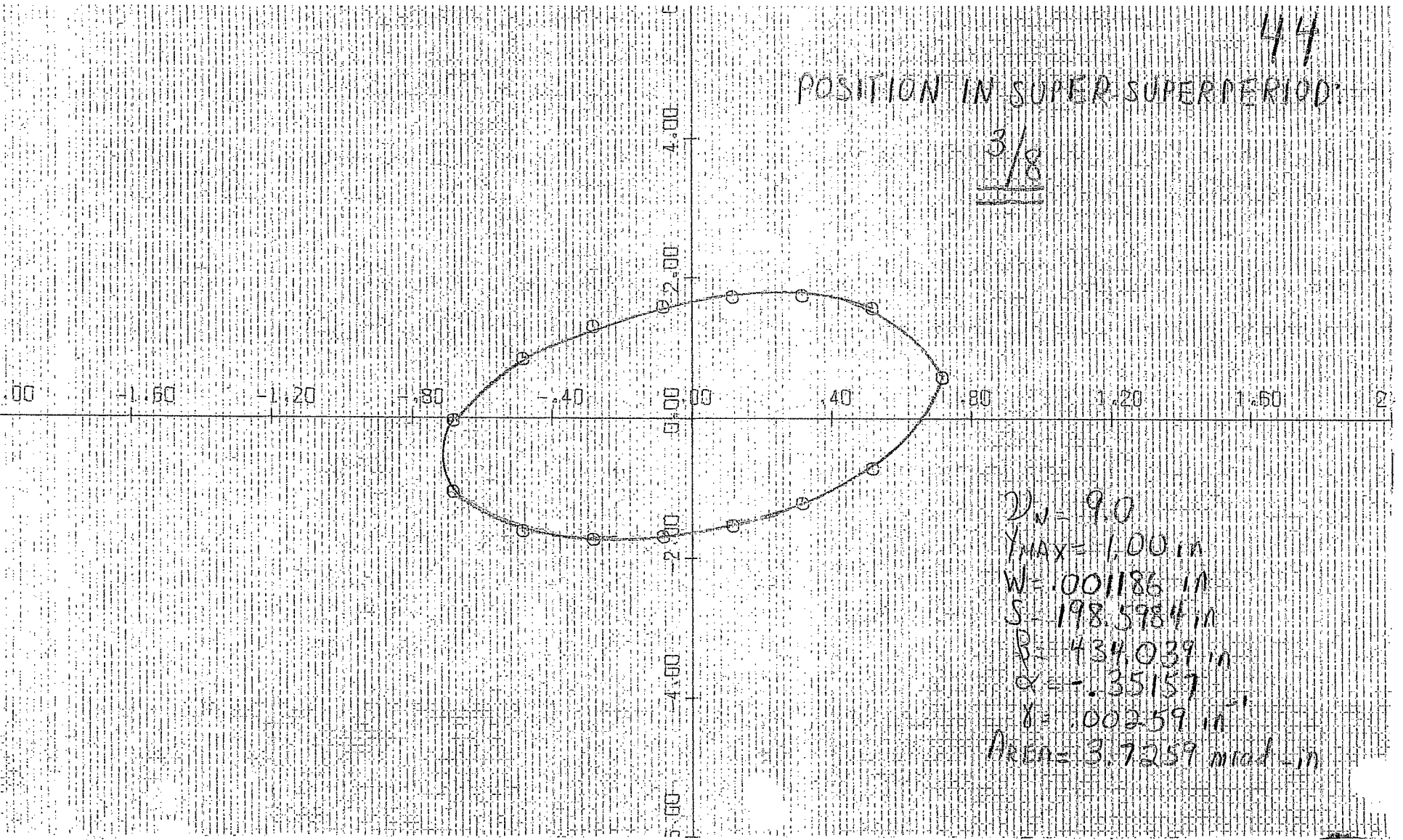
α = 0.00

γ = .00672 in^{-1}

A_{MIN} = 3.7259 mm-mr

POSITION IN SUPER SUPERPERIOD:

3/8



$2\nu = 9.0$

$Y_{MAX} = 1.00$ in

$W = .0011186$ in

$S = 198.5984$ in

$\beta = 434.039$ in

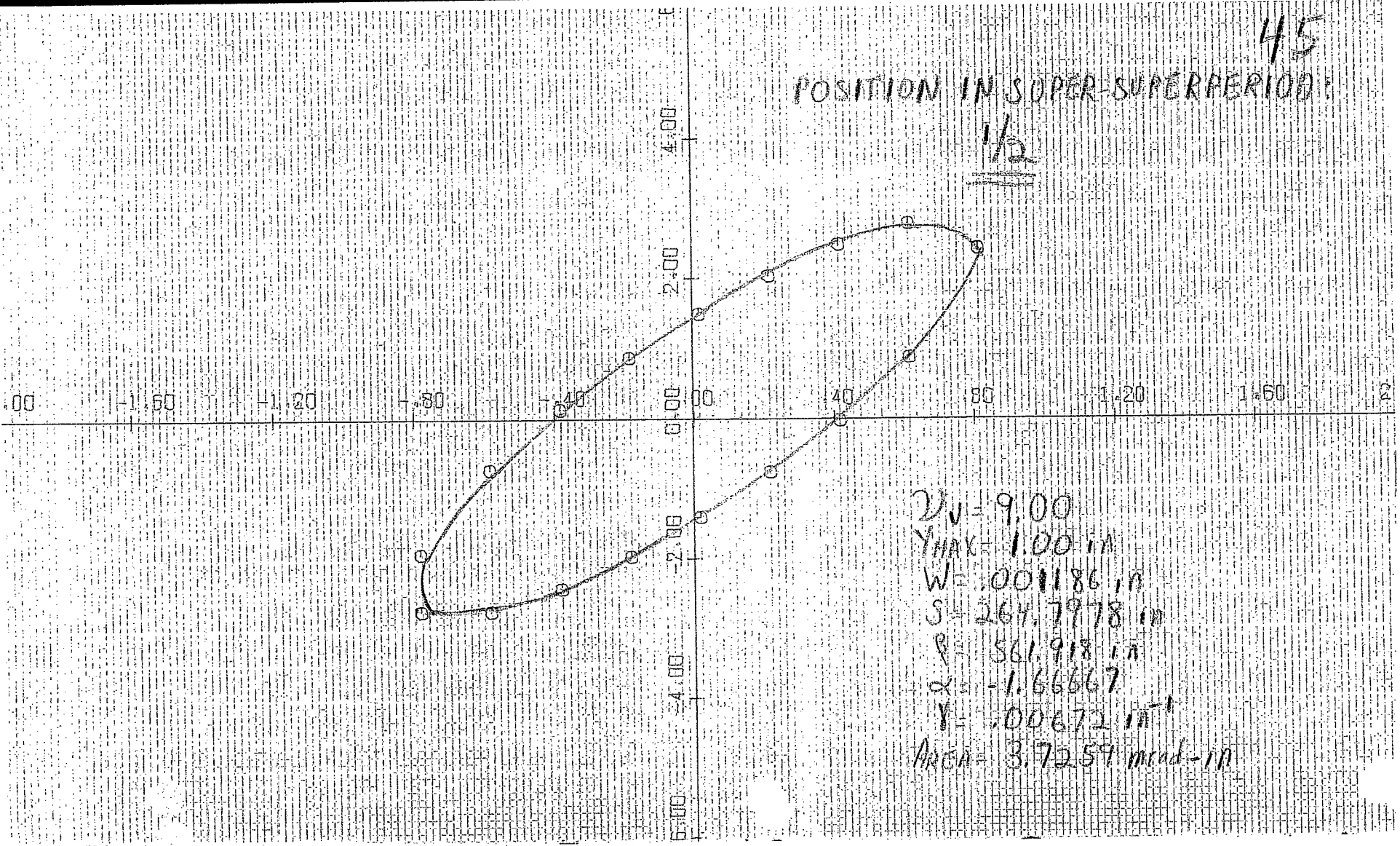
$\alpha = -.35157$

$\gamma = .002259$ in^{-1}

$Area = 3.7259$ mrad-in

POSITION IN SUPER-SUPERPERIOD:

1/2



$2\nu = 9.00$

$Y_{MAX} = 1.00$ in

$W = .001186$ in

$S = 264.7778$ in

$R = 561.918$ in

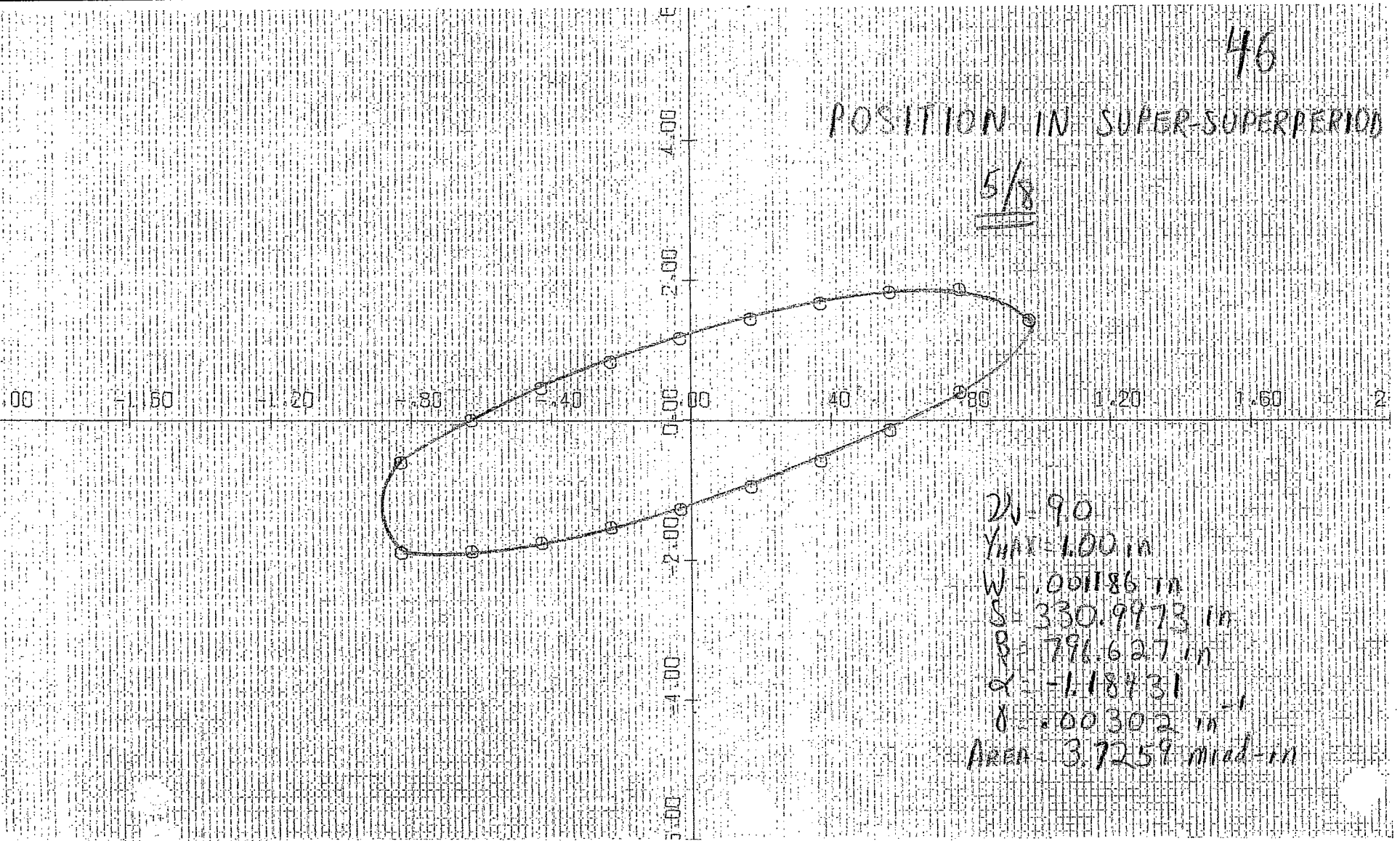
$\alpha = -1.66667$

$\gamma = .00672$ in^{-1}

$A_{MCH} = 3.7259$ mrad-in

POSITION IN SUPER-SUPERPERIOD

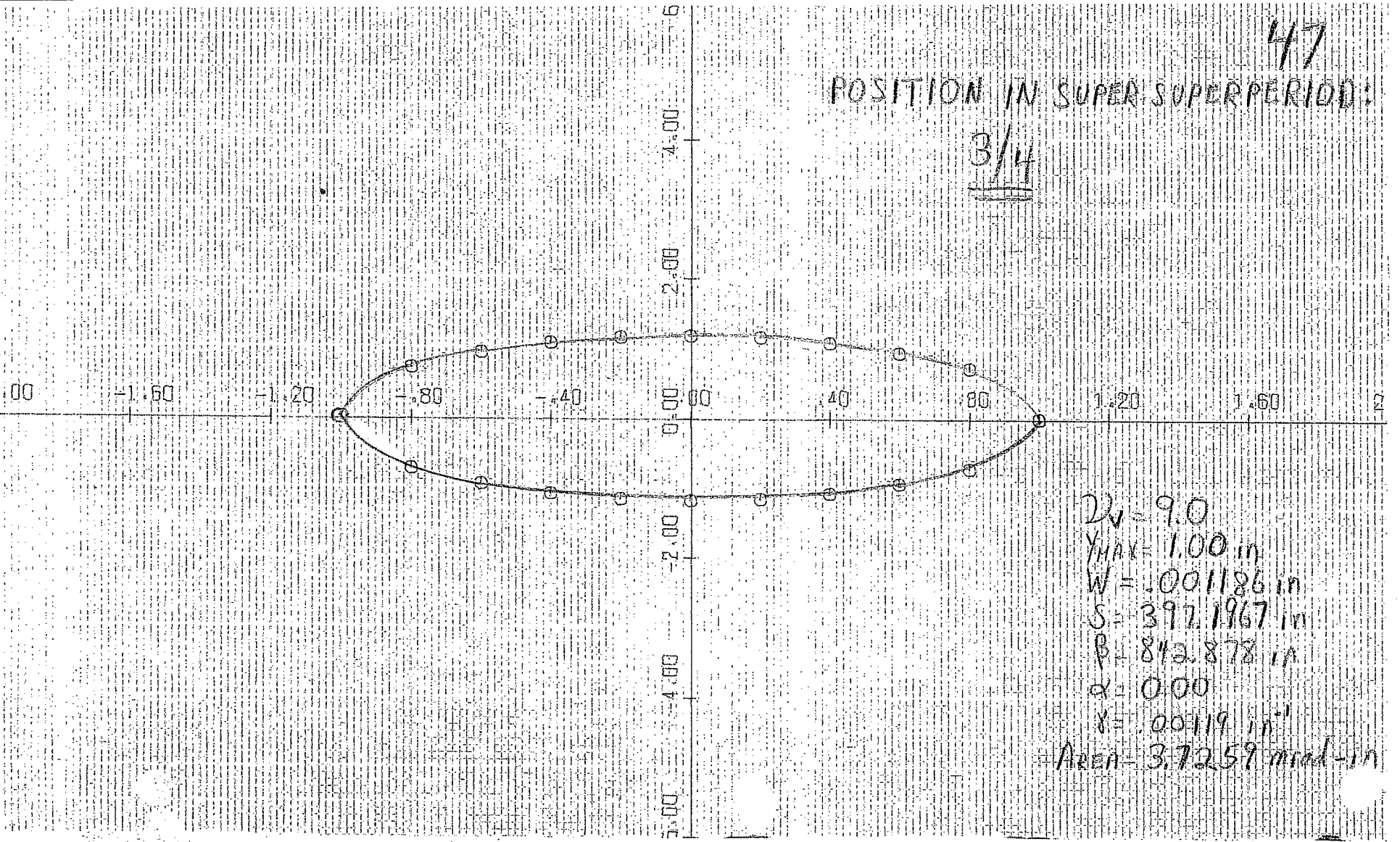
5/8



$D_N = 9.0$
$Y_{MAX} = 1.00$ in
$W = .00186$ in
$S = 330.9973$ in
$R = 796.627$ in
$\alpha = -1.187731$
$\delta = .00302$ in^{-1}
AREA $= 3.7259$ mid-in

POSITION IN SUPER SUPERPERIOD:

3/4



$2\nu = 9.0$

$Y_{MAX} = 1.00$ in

$W = .001186$ in

$S = 372.1967$ in

$\beta = 842.878$ in

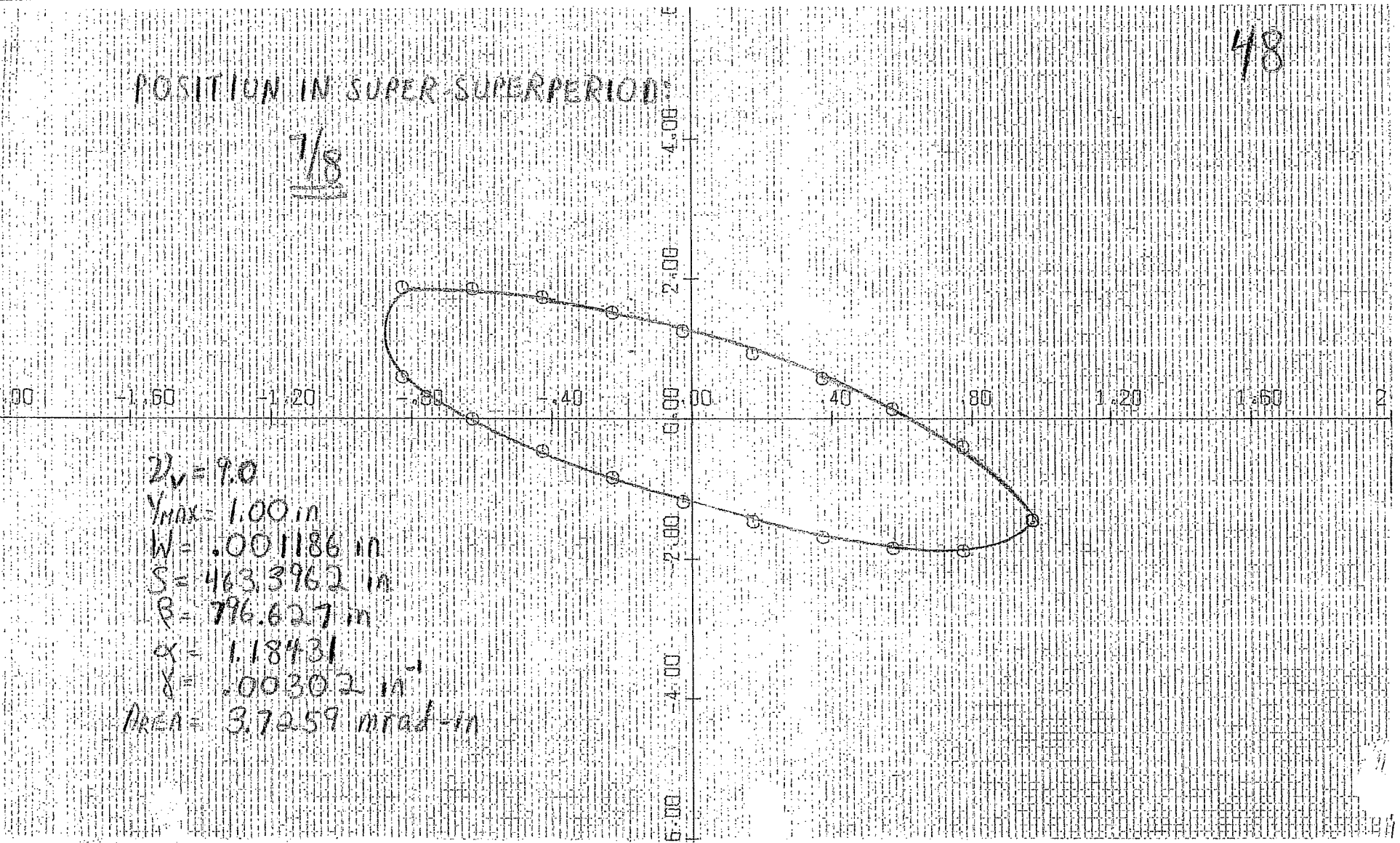
$\alpha = 0.00$

$\gamma = .00119$ in^{-1}

$\Delta_{REF} = 3.7257$ mrad-in

POSITION IN SUPER-SUPERPERIOD:

7/8



$2\nu = 9.0$
$Y_{MAX} = 1.00$ in
$W = .0011186$ in
$S = 463.3962$ in
$\beta = 796.627$ in
$\alpha = 1.18431$
$\gamma = .00302$ in^{-1}
AREA $= 3.7259$ mrad-in