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Evaluation of the Booster resonance lines

J. Kats

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Collider Accelerator Department

Brookhaven National Laboratory

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J. KATS

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ACCELERATOR DEVELOPMENT DEPARTMENT

Brookhaven National Laboratory

Upton, N.Y. 11973

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ABSTRACT:

Stopband widths for three types of Booster resonance lines are calculated.

The first are the resonance lines due to random sextupole components at the end of each dipole.

The second are the lines due to random skew sextupoles superimposed into each chromaticity sextupole.

The third are the systematic resonances.

INTRODUCTION:

In a course of Booster operation its working point (horizontal tune, vertical tune) on a tune diagram can approach or cross several resonance lines (Figure 1).

Some of them have a special interest: the third and fourth integer resonance lines. Both are determined on tune diagram by equation

$$n_x \nu_x + n_y \nu_y = p, \quad (1)$$

where ν_x, ν_y are horizontal and vertical tune, n_x, n_y, p are integers and

$$|n_x| + |n_y| = N, \quad (2)$$

where integer N is an order of resonance.

If n_x, n_y have the same sign then this is the case of sum resonance.

If n_x, n_y have opposite signs then this is the case of difference resonance.

We are interested in lines with N = 3 and N = 4.

These two types of line have a very different nature. The third integer lines are due to the magnet imperfections. Particularly the sextupole components at the end of each dipole can drive the third integer resonance. Also, the chromaticity sextupoles due to the misalignment could create a small skewed sextupole which can drive the third integer resonance.

The fourth integer resonance can be driven by any kind of sextupoles if they compose a certain periodic structure along the machine circumference.

To see the roots which lead to resonance in both cases we should notice [1,2] that the point sextupole field

$$\Delta B = \frac{B''}{2} x^2 \quad (3)$$

kicks a particle with azimuthal angle θ, betatron phase νθ, inducing increment in amplitude a proportional to

$$\Delta a \sim \frac{B''}{2} a^2 \operatorname{Cos}^2 \nu\theta \operatorname{Sin} \nu\theta = \frac{a^2}{8} B'' (\operatorname{Sin} 3\nu\theta + \operatorname{Sin} \nu\theta). \quad (4)$$

This is a single kick at the point sextupole.

Now let's consider all sextupoles with strength distribution

$$B'' = B''(\theta) \qquad (5)$$

and its Fourier expansion

$$B''(\theta) = \sum_n (A_n \operatorname{Cos} n\theta + B_n \operatorname{Sin} n\theta). \qquad (6)$$

Substitution (6) into (4) yields with subsequent integration over m turns particle undergo

$$\Delta a \sim \frac{1}{8} \sum_n \int_0^{2\pi m} a^2 \,(\operatorname{Sin} 3\nu\theta + \operatorname{Sin} \nu\theta)(A_n \operatorname{Cos}\, n\theta + B_n \operatorname{Sin} n\theta)\, d\theta. \qquad (7)$$

If $3\nu = p$ is integer then the most significant contribution to Δa from the sum (7) will be made by term*

$a^2 \operatorname{Sin} 3\nu\theta \; B_p \operatorname{Sin}\, p\theta$ <u>during each turn.</u>

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* we will not consider the strongest case ν = integer, assuming that the case will be avoided by all means.

This case (m=1) can occur if there is a magnet imperfection with randomly distributed sextupoles. Then the (6) represents that distribution and particles will picks out the sine component with n = p increasing the amplitude by Δa every other turn. This is a third integer resonance due to magnet imperfection.

Let us consider another case. Because the Booster possess a superperiodicity of 6 order the sextupole distribution with no imperfection can be rewritten from (6) to

$$B''(\theta) = \sum_n (A_n \text{Cos}6n\theta + B_n \text{Sin}6n\theta). \tag{8}$$

Introducing new variables $t = \theta/m$, $L = nm$ and substituting (8) to (7) we'll get

$$\Delta a \sim m \sum_L \int_0^{2\pi} a^2 (\text{Sin}3\nu mt + \text{Sin}\nu mt)(A_L \text{Cos}6Lt + B_L \text{Sin}6Lt)dt. \tag{9}$$

If $\nu m = p$ is integer then the most significant contribution to Δa from the sum (9) is to be made by term $a^2 \sin\nu mt B_L \text{Sin } 6Lt$

$$\text{at} \quad \nu m = 6L. \tag{10}$$

This contribution is made when t changes from 0 to 2π and θ changes accordingly from 0- to $2\pi m$. In other words, this contribution is made by particle <u>after m turns</u>. For example, if $\nu = 4.5$ then equation (10) admits the solution m = 4, 1 = 3. That's why we consider the line $4\nu = 18$ as of fourth not half ($2\nu = 9$) integer resonance. And that resonance results from

6 superperiodicity of Booster sextupoles. Such type of resonance is often referred as systematic or structure resonance.

The strength of the resonance line $n_x\nu_x + n_y\nu_y = p$ can be evaluated in term of its stopband width Δe which bound a tune area between the lines $n_x\nu_x + n_y\nu_y = p \pm \frac{\Delta e}{2}$ in such a way that the particle with tune inside that area can build up growing or beating amplitude [3].

In this note we will evaluate a stopband width for the Booster third and fourth integer resonance lines.

THEORY.

Following G. Guignard's treatment of resonance theory [3] we will use for a stopband width formulae

$$\frac{\Delta e}{2} = |\kappa| \left(\frac{R}{2\pi}\right)^{\frac{M}{2}-1} E_x^{\frac{|n_x|}{2}} E_y^{\frac{|n_y|}{2}} \left(|n_x|^S/E_x + |n_y|^S/E_y\right) . \qquad (11)$$

Here R is the machine radius, for Booster R = 32.1143m; M is the multiple order, M = 3 for sextupoles; S = 2 for the sum resonances, S = 1 for the difference resonances; E_x, E_y are horizontal and vertical emittances.

The most important part of (11) is excitation coefficient κ which is determined by the integral over all machine lattice

$$\kappa = \frac{1}{2\pi\,(2R)^{M/2}\,|n_x|!\,|n_y|!} \int_0^{2\pi} d\Theta\ \beta_x^{\frac{|n_x|}{2}} \beta_y^{\frac{|n_y|}{2}} \quad \times$$

$$\exp\left\{i[n_x\mu_x(\theta)+n_y\mu_y(\theta)-(n_x\nu_x+n_y\nu_y-p)\theta]\right\} \quad \times \begin{cases} (-1)^{\frac{|n_y|}{2}+1} K_z^{(M-1)}(\theta) & \text{for } n_y \text{ even} \\ (-1)^{\frac{|n_y|}{2}-1} K_x^{(M-1)}(\theta) & \text{for } n_y \text{ odd,} \end{cases}$$

where β_x, β_y are beta functions for linear part of machine, μ_x, μ_y are corresponding phase advances $K_x^{(M-1)}$, $K_y^{(M-1)}$ are multipole (sextupole if M=3) strengths:

$$K_y^{(M-1)}(\theta) = \frac{R^2}{|B_o\rho|} \frac{\partial^{M-1} B_y}{\partial x^{M-1}} , \tag{13}$$

$$K_x^{(M-1)}(\theta) = \frac{R^2}{|B_o\rho|} \frac{\partial^{M-1} B_x}{\partial x^{M-1}} ,$$

$B_x(\theta)$, $B_y(\theta)$ - magnetic field, B_o - magnetic rigidity

Following further the Guignard's treatment a criterion giving the distance δe of the working point (ν_x, ν_y) from the resonance line, which has to be maintained to prevent the relative amplitude growth $\Lambda = \delta a/a$ above one is

$$\delta e \geq \frac{\Delta e}{2} \left(1 + \frac{1}{\Lambda} \cdot \frac{n_x E_y + n_y E_x}{n_x^2 E_y + n_y^2 E_x} \right) \tag{14}$$

for a sum resonance and

$$\delta e \geq \frac{\Delta e}{2} \cdot \frac{1}{\Lambda} \tag{15}$$

for a difference resonance.

In the next section we present computed values of Δe and δe for all lines shown in Fig. 1.

THE COMPUTED RESULTS.

The following tables represent the results $\Delta e/2$ and δe for three type of resonances:

1. Table I - resonance due to random sextupole components at the end of each dipole;
2. Table II - resonance due to random skew sextupole component superimposed into each of the chromaticity sextupole.
3. Table III - systematic (structure) resonances.

All calculations were performed with emittances

$$E_x = 100\pi \text{ mm.mrad}, \; E_y = 50\pi \text{ mm.mrad}.$$

The strength of sextupole components at the end of each dipole was chosen as K2 (1+rq), where K2 is the strength of eddy current sextupole at the dipole end, q=5% or 10% and r is the random number uniformly distributed between -1 and +1.

The strength of (random) skew sextupoles placed at the same position as chromaticity sextupoles was chosen as .0005K2r, where K2 is the strength of chromaticity sextupole and $-1 \leq r \leq 1$ is uniformly distributed random number.

TABLE I. Third integer resonaces due to random sextupoles.

Random ERROR	10%			5%		
Resonance LINE	$\Delta e/2$	δe 5%	δe 10%	$\Delta e/2$	δe 5%	δe 10%
$3\nu_x = 12$	$1.665.10^{-2}$	$1.277.10^{-1}$	$7.2.10^{-2}$	$1.66.10^{-2}$	$1.273.10^{-1}$	$7.2.10^{-2}$
$3\nu_x = 13$	$4.3.10^{-4}$	$3.3.10^{-3}$	$1.9.10^{-4}$	$2.1.10^{-4}$	$1.6.10^{-3}$	$9.3.10^{-4}$
$3\nu_x = 14$	$6.8.10^{-4}$	$5.2.10^{-3}$	$2.9.10^{-3}$	$3.4.10^{-4}$	$2.6.10^{-3}$	$1.5.10^{-3}$
$\nu_x+2\nu_y = 14$	$4.1.10^{-4}$	$5.0.10^{-3}$	$2.7.10^{-3}$	$2.1.10^{-4}$	$2.5.10^{-3}$	$1.4.10^{-3}$
$\nu_x+2\nu_y = 13$	$1.1.10^{-4}$	$1.4.10^{-3}$	$7.5.10^{-4}$	$5.7.10^{-5}$	$6.9.10^{-4}$	$3.7.10^{-4}$
$2\nu_y-\nu_x = 5$	$7.3.10^{-5}$	$1.5.10^{-3}$	$7.3.10^{-4}$	$3.7.10^{-5}$	$7.3.10^{-4}$	$3.4.10^{-4}$
$2\nu_y-\nu_x = 4$	$6.0.10^{-4}$	$1.2.10^{-2}$	$6.0.10^{-3}$	$3.0.10^{-4}$	$6.0.10^{-3}$	$3.0.10^{-3}$

TABLE II. Resonances due to random skew sextupoles.

Resonance LINE	$\Delta e/2$	δe	
		5%	10%
$\nu_x - \nu_y = 0$	$4.7.10^{-5}$	$9.3.10^{-4}$	$4.7.10^{-4}$
$2\nu_x + \nu_y = 14$	$7.3.10^{-6}$	$1.1.10^{-4}$	$5.6.10^{-5}$
$2\nu_x + \nu_y = 13$	$8.3.10^{-6}$	$1.2.10^{-4}$	$6.3.10^{-5}$
$\nu_x + \nu_y = 9$	$5.9.10^{-4}$	$1.2.10^{-2}$	$6.4.10^{-3}$
$2\nu_x - \nu_y = 4$	$4.3.10^{-6}$	$8.6.10^{-5}$	$4.3.10^{-5}$
$2\nu_x - \nu_y = 5$	$3.3.10^{-6}$	$6.7.10^{-5}$	$3.3.10^{-5}$
$3\nu_y = 14$	$3.7.10^{-5}$	$2.8.10^{-4}$	$1.6.10^{-4}$
$3\nu_y = 13$	$2.9.10^{-5}$	$2.2.10^{-4}$	$1.34.10^{-4}$

TABLE III. Systematic resonances.

Resonance LINE	$\Delta e/2$	δe	
		5%	10%
$2\nu_x - 2\nu_y = 0$	$5.8.10^{-4}$	$1.2.10^{-2}$	$5.8.10^{-3}$
$2\nu_x + 2\nu_y = 18$	$2.8.10^{-3}$	$3.0.10^{-2}$	$1.7.10^{-2}$
$4\nu_x = 18$	$1.2.10^{-3}$	$6.9.10^{-3}$	$4.0.10^{-3}$
$4\nu_y = 18$	$2.2.10^{-3}$	$1.3.10^{-2}$	$7.8.10^{-3}$
$3\nu_x = 12$	$1.7.10^{-2}$	$1.3.10^{-1}$	$7.2.10^{-2}$

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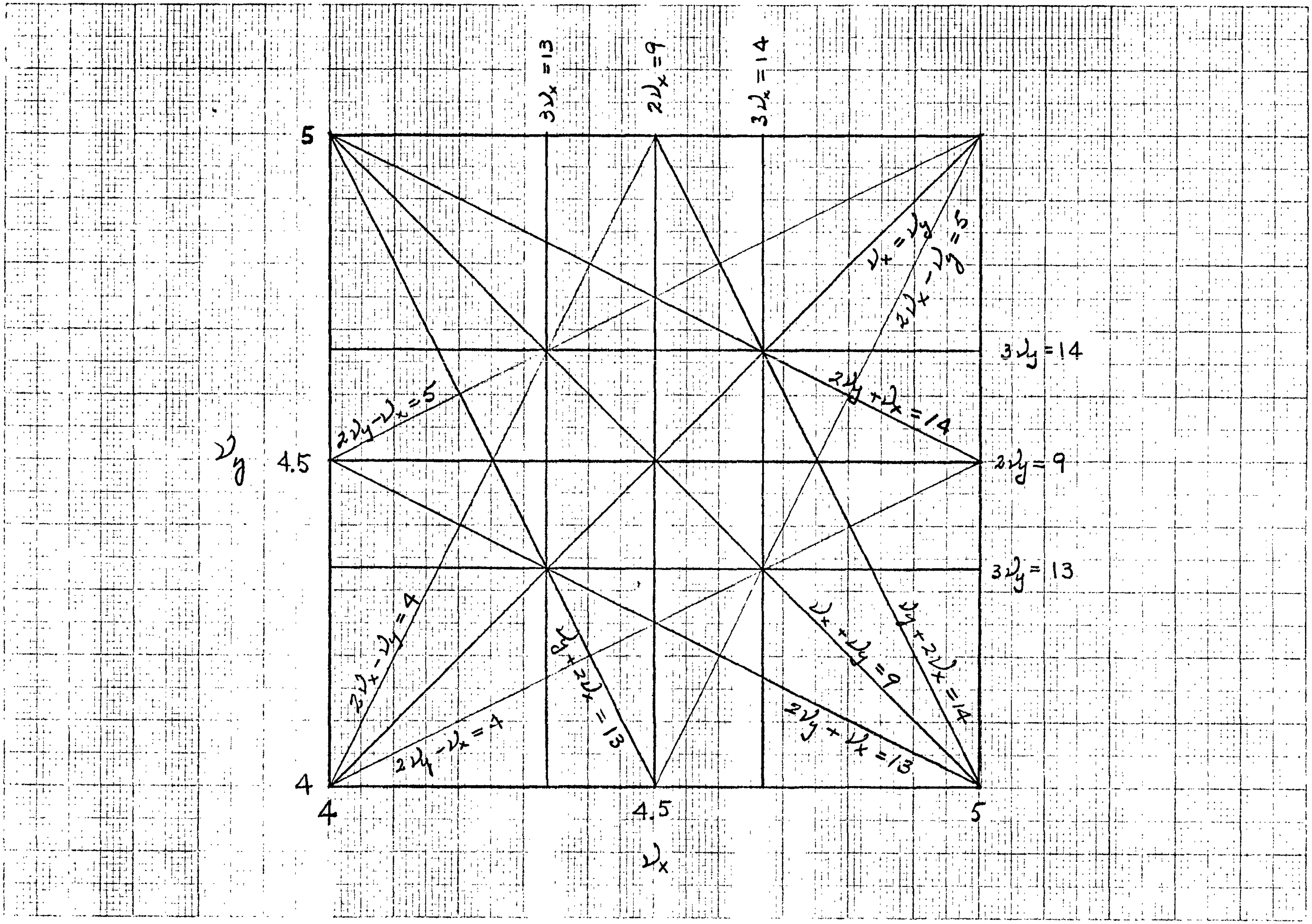

3ν_x = 13
2ν_x = 9
3ν_x = 14
3ν_y = 14
2ν_y = 9
3ν_y = 13
ν_x = ν_y
2ν_x − ν_y = 5
2ν_y + ν_x = 14
2ν_y − ν_x = 5
2ν_x − ν_y = 4
2ν_y − ν_x = 4
ν_y + 2ν_x = 13
ν_x + ν_y = 9
ν_y + 2ν_x = 14
2ν_y + ν_x = 13
5
4.5
4
4
4.5
5
ν_y
ν_x

FIGURE 1