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The design of voltage control feedback loops for multi-phase rectifier systems

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The control of the current in charged particle accelerator magnets required the ultimate in stability. Current sampling and measurement plus digital command technology have advanced to the point that they seldom limit stability. Current stability is more commonly limited by the regulator's speed in response to external perturbation particularly those coming from the power line. To help the regulating system resist perturbations from the power line a voltage feedback loop is commonly used. The speed of this loop can be made much faster than the current control loop which must include as one of its transfer elements the load magnet which usually has a long time constant.

The purpose of this work is to examine the feedback criteria for this type of loop and determine the conditions which maximize the gain-speed product of its response.

The block diagram of the system to be analyzed is shown is Figure 1. The output of a phase controlled multi-phase rectifier is sampled and compared with a reference or command signal. This difference is amplified and fed to a system of amplifiers and networks which shape the closed loop response. This shaped signal is in turn used to control the firing of the rectifiers in the multiphase rectifier.

This feedback system contains four elements each of which has a transfer function. The product of these four transfer functions represents the closed loop response which must be tailored to produce the desired results. These four elements are as follows:

1. multi-phase rectifier
2. Voltage sampling attenuator
3. Network and amplifier system
4. Command comparison amplifier

For the purpose of this analysis the voltage sampling attenuator and the command comparison amplifier will be considered to be ideal elements, i.e. they have gain or attenuation but no phase shift. This leaves only two elements to be analyzed, the main rectifier and the network system.

The transfer function of a multi-phase rectifier was derived in 1968 and reported in Conversion Division Technical Note AGSCD-30. For completeness this derivation has been extracted, edited and attached to this work as Appendix A. The results are shown in Figure 3 of this appendix.

Normally the transfer function of an element is single valued, i.e. it can be represented by vector specifing a magnitude and an angle. In the case of a multi-phase rectifier this is not true. This transfer function has a third parameter namely the phase angle between the control or driving function and the rectifier firings which form the sampling events. This third parameter is defined as ϕ in the appended derivation. If the driving function is not integerly related to the sampling frequency the parameter ϕ will move through all values and the transfer function vector will move through a locus of points. These loci are those shown in Figure 3 of the Appendix.

The closed loop response of any feedback system is given by the following:

$$G = \frac{A}{1-AB} = \frac{1}{B}\left[\frac{AB}{1-AB}\right]$$

If 1/B is the desired response which in this case represents the comparison of the attenuated output with the command signal and is assumed to be ideal, then expression AB/(1-AB) represents the deviation from ideal. This vector expression has been analyzed in the literature and the results are shown in Figure 2. The circles shown on Figure 2 represent the loci of all values of the vector quantity AB which have the labeled transfer magnitude. For example,the circle labeled 120% shows the locus of the AB vector for which the closed loop response is 120% of the driving signal. There is a simular plot for the closed loop phase response not shown.

What is the desired closed loop response? At first one might suggest a response that has no overshoot, i.e. a response that does not exceed 100%; but by examining Figure 2 we see that this requirement implies that the AB vector lie to the right of the 100% line. The maximum phase angle of the AB vector is only slightly larger than 90°. If some overshoot is permitted a less restrictive limit can be placed on the phase angle of the AB vector. A compromise is needed trading overshoot for phase. Some designers allow a large phase angle when the AB vector is large and reduce this phase angle when the magnitude of the vector is 5 or less. Clearly this avoids the large overshoot loci, but I have had difficulty with this approach. During transient condition in high gain feedback system it is very easy to saturate one or many of the operational amplifiers used. Under this saturated condition the magnitude of the loop gain (AB vector) becomes very small sometimes almost zero. This collapse of the AB vector magnitude moves the response into the high overshoot region and produces a near oscillatory recovery from saturating transients. A better solution is to limit the phase angle of the AB vector to some chosen angle so that the saturated and unsaturated characteristic are simular. Overshoots between 120% and 150% are usually acceptable and the corresponding maximum phase angle can be read from Figure 2. For this exercise I will choose a permissible overshoot of 130% and use a maximum phase angle for the AB vector of 130°. Once the magnitude of the AB vector is less than ½ the phase angle can have any value and the close loop response will be less than 100%, see Figure 2. We can now proceed to design the amplifier and network system to meet these conditions.

Figure 3 shows the transfer characteristic and design parameters of a simple RCR type network. Network of this type can be cascaded together with buffer amplifiers and designed to produce an approximately uniform laging phase shift. Figure 3A shows an example of this cascading and the uniformity that results. Figure 4 shows the design relationship between the cascading parameters. The frequency spacing is defined in terms of a frequency ratio, R. For any α the lagging phase angle has a maximum value, Θ max, which occurs when $\omega/\omega_O = 1/(\alpha(\alpha+1))^{1/2}$. At some lower frequency, ω_L, the phase angle will be ½ of Θ max. Again at some higher frequency, ω_H, the phase angle will also be equal to Θ max/2. R is defined as the ratio ω_H/ω_L. If networks having the same α are cascaded and their respective ω_O are in the ratio R then the phase lag of the system is given by the lower curve of Figure 4 relating α to the system phase shift. The other curves are for closer spacing of the ω_O values expressed as a power of the ratio R. For any desired system phase shift one or more solutions can be found. For example, for 130° we could use any one of the following:

Freq. Spacing (Ratio)	α	ω/ω_1 High for $\frac{\Theta \max}{2}$	ω/ω_0 Low for $\frac{\Theta \max}{2}$	Ratio R	Freq. Spacing (factor)
$R^{1/2}$	0.09	17.84	.571	31.24	5.59
$R^{2/5}$	0.2	9.48	.440	21.55	3.41
$R^{1/3}$	0.325	6.52	.356	18.31	2.64

These three solutions produce identical results and any can be used. The attenuation characteristic of a cascaded set of these RCR networks is shown in Figure 5 and is a single valued function of the phase lag independent of the cascading scheme. For example, for 130° the attenuation is a factor of 27.9 per decade.

It is desirable to have this attenuation factor per decade large in this feedback system so that the gain can be large in the frequency region where regulation and response precision are required (ramp frequency) and small (less than ½) for frequencies where the characteristics of the rectifier are unmanageable. This is the same compromise that we examined before. The system phase lag wants to be large to improve the attenuation factor and small to minimize the overshoot. The following table will illustrate these competing considerations.

Phase angle	Overshoot	Attentuation per decade	Gain at ramp * fundamental	Gain at ramp 3^{rd} Harmonic
149°	200%	45.0	301	49
138°	150%	34.2	190	35
130%	130%	27.9	135	28
123°	120%	23.2	99	22
114°	110%	18.5	68	17
90°	100%	10.0	24	8

* gain at 360 H_Z = ½ the half frequency for a 12 phase rectifier.

ramp frequency = 7.5 H_Z

An overshoot of 130% is a reasonable compromise and yields a loop gain of 28 at the third harmonic of the ramp frequency.

For the Booster main magnet power supply there are only two possible choices for the rectifier type, 12 or 24 phase. For the numerical examples to follow I will assume a 12 phase rectifier, if 24 phase is ultimately chosen the frequency values can be doubled. In this case the sampling frequency is 720 H_Z and the rectifier response characteristic shown as Figure 3 in the appendix can be relabled replacing the k values with the frequency value of 720/k. The large circle labeled k=2 becomes the locus of the transfer function for the frequency 360 H_Z and the small half elipse labeled k=3.5 becomes the locus for the frequency 205.7 H_Z etc. If these loci are placed on a Nyquist diagram and constrained so that the maximum angle is 130° the plot would be as shown in Figure 6. For 360 H_Z the maximum, phase lag for the system external to the rectifier is 40°. The rectifier locus drawn around this point touches the 130° line. For lower frequencies higher values for the external phase lag can be permitted. Curve 3 on Figure 7 shows the maximum value for these external phase lags for all frequencies up to 360 H_Z. The ideal network set would be one that had a phase lag characteristic that matched curve 3. The cusp in this curve results from the k=2 characteristic of the rectifier and cannot be matched by simple networks. Also shown on Figure 7 is the phase characteristic of two network sets which represent the best approximation that I can find to curve 3. Curve 1 is the phase characteristic of a simple set of RCR networks having an α of 0.09 and nested in frequency to produce a system phase lag of 130° in the midband. The absolute value of ω_0 is chosen to yield a 40° phase lag at 360 H_Z. Curve 2 is similar except that the three higher frequency RCR networks have been adjusted by trial and error to tailor the high frequency shape of the phase lag characteristic to better match the optum shape. The amplitude characteristic of these two network sets are shown in Figure 8. The characteristic of network set No. 2 is better than that of set No. 1 by a factor of 1.86 and is therefore the recommended set.

The stability criterian for single valued transfer functions has been established by Nyquist and can be stated as follows. If the locus of the ends of all vectors which represent all frequencies encircle the point -1,0., then the closed loop operation is unstable. How is this to be applied when the transfer function for certain frequencies is no longer single value, but as in the case of a rectifier operating near the commutation half frequency, is itself a locus of values? The Nyquist plot enclosing a rectifier is no longer a line but has width see Figure 6 and in fact near the half frequency becomes very wide. A region is painted in this case. Is the response unstable, if the point -1,0 is touched by the painted region or must it be completely enclosed?

To answer these questions I constructed a compute model of the block diagram shown in Figure 1. This model was used to examine two cases. The first case using networks designed to make the Nyquist plot touch the -1,0 point with the derived rectifier transfer function and sufficient gain. The second case was designed using the same criteria described earlier in this note to produce a optimum response function. The networks used in these two cases are as follows:

Case 1 (two networks) α	W_o	Case 2 (one network) α	W_o
0.1	750	0.275	1339
0.1	750		

A Nyquist plot for the first case and showing only the loci of the 360 H_z vector is presented as Figure 9. The labels identify the amplifier gain for each locus presented. When the amplifier gain is small, less than 7.4, the locus is well away from the (-1,0) unstable point. However, when the gain is increased to 13 it passes very near this unstable point and for all higher gains the unstable point is encircled. The closed loop performance for these systems are shown in Figures 10A through 10G. The initial rectifier firing has been deliberately miss adjusted to produce a start-up transient. For the low gain systems this transient damps in a few commutations and the remaining output shows a stable recovery to an equilibrium value. For an amplifier gain of 13 this equilibrium is attained but only after a very long time. For all higher gains the system is unstable.

It is interesting to note the mode of this instability could be confused with high 360 H_z ripple. Figure 10G which shows the output for the highest amplifier gain is commutating very erratically and has almost degenerated to 6 phase operation.

Case 2 yields an entirely different story. The 360 H_z vector locus is shown in figure 11 and never comes near the (-1,0) unstable point regardless of the amplifier gain. For very high gain the 360 vector locu is a circle of large radius with a center of equal value out along the 40° line. Hence, that part of this locui that falls on the Figure 11 field is approximately a straight line down the 130° line. The closed loop performance is stable for all values of gain and the starting transient recovers in only one commutation, see Figure 12A through 12E.

I believe these computer models confirm the derivation in the attached appendix and the design application used in this note. If the unstable point (-1,0) is touched by any part of the Nyquist painted region the system is unstable!

System Block Diagram

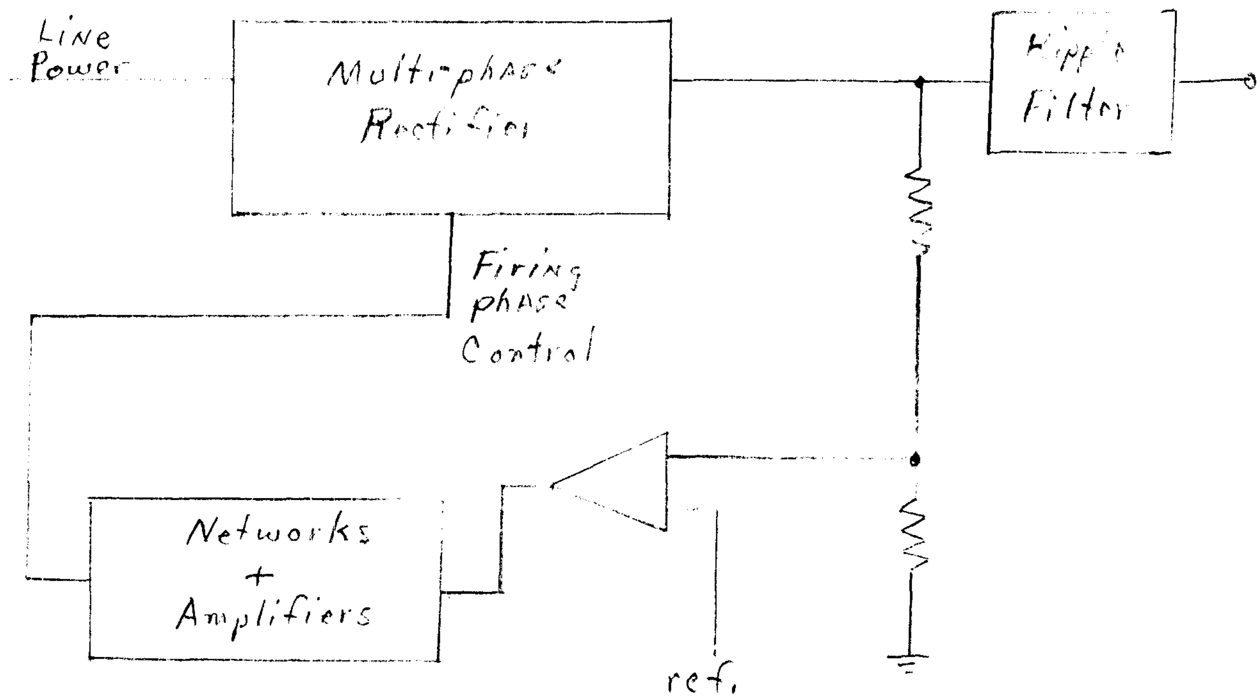
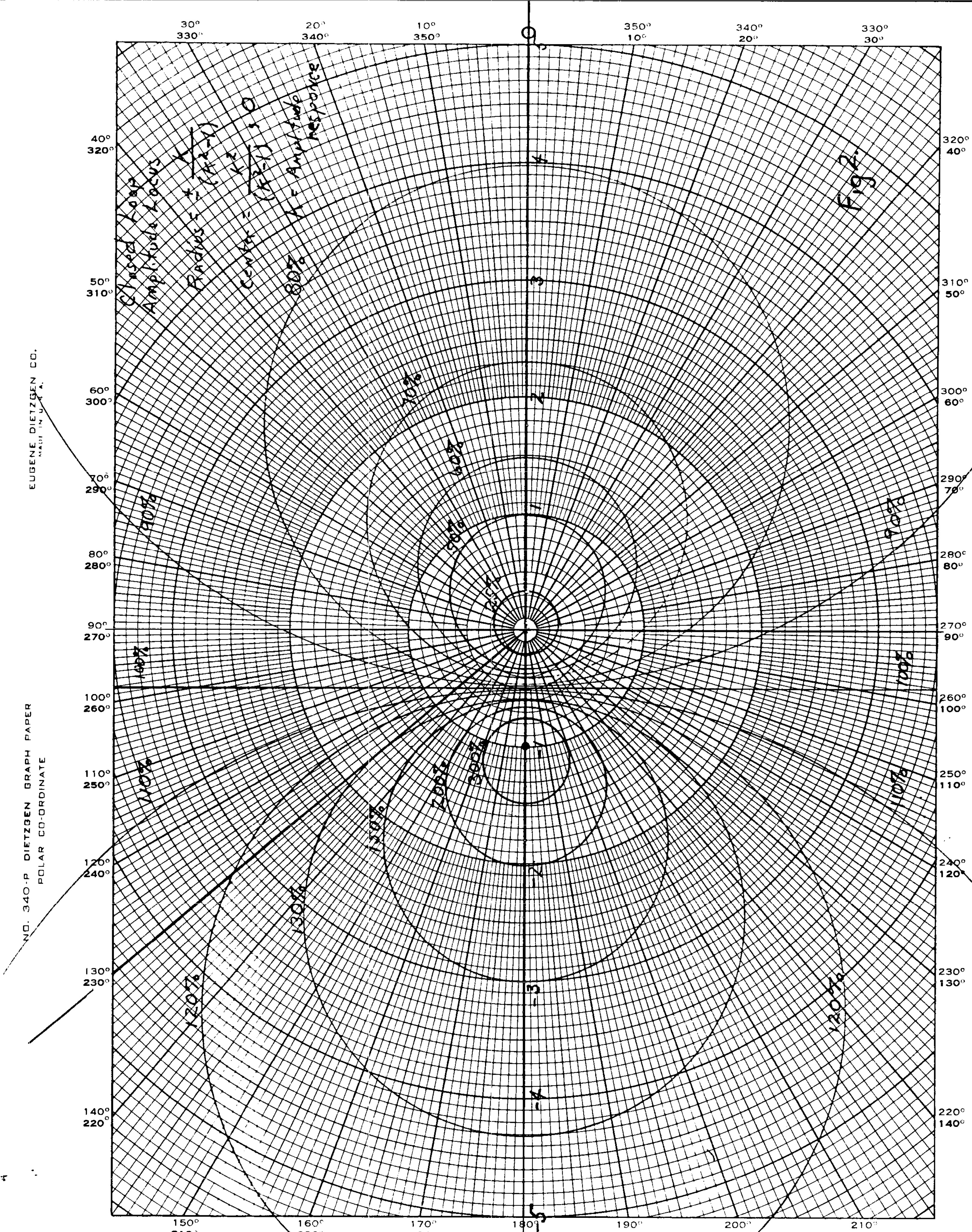
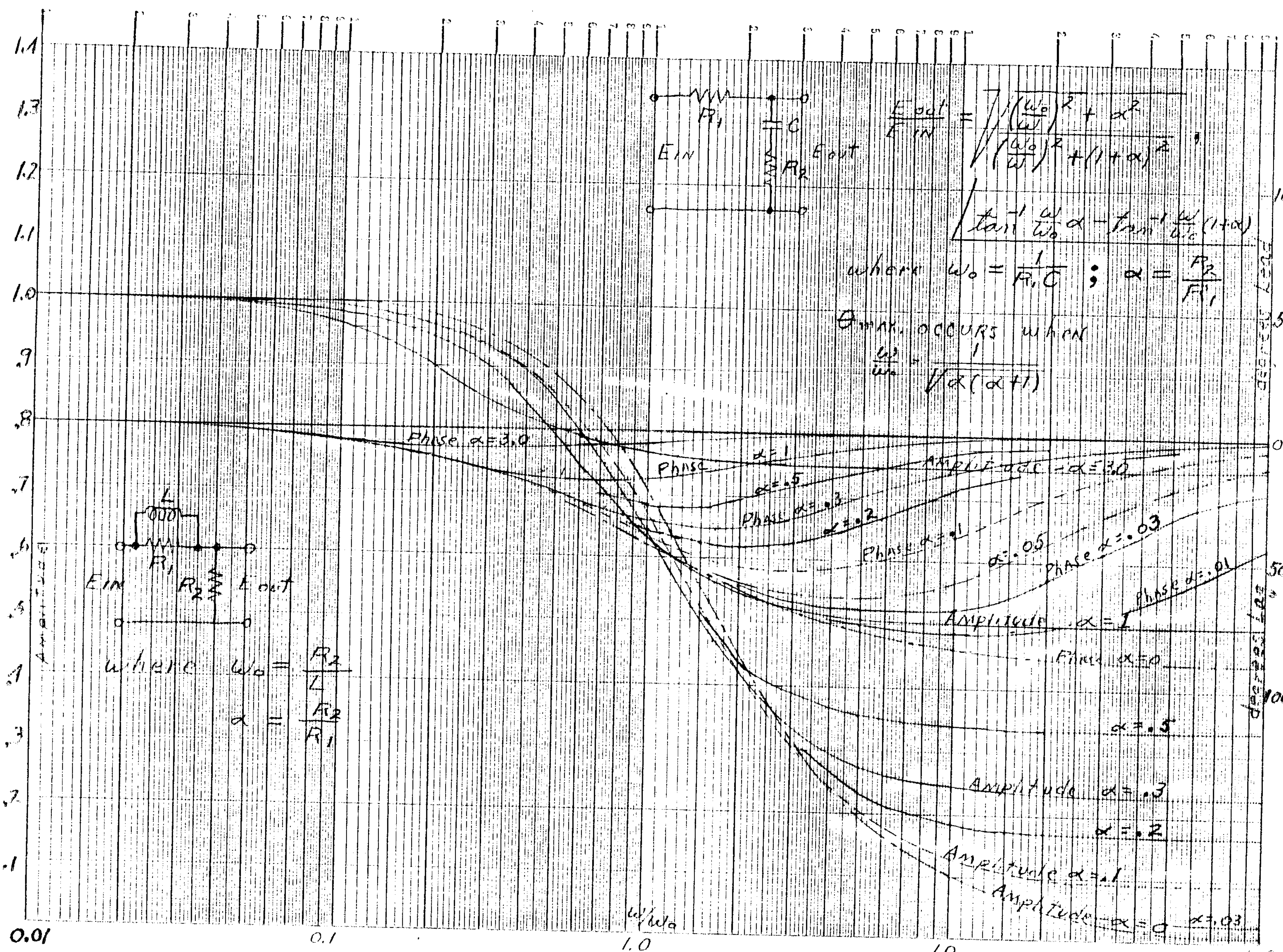


Fig 1


Closed Loop
Amplitude Locus
Radius = K/(K²-1)
Center = K²/(K²-1), 0
K = Amplitude Response
Fig 2
80%
70%
60%
50%
20%
90%
100%
110%
120%
130%
200%
300%
EUGENE DIETZGEN CO.
MADE IN U.S.A.
NO. 340-P DIETZGEN GRAPH PAPER
POLAR CO-ORDINATE



E out / E IN = √(((ω₀/ω)² + α²) / ((ω₀/ω)² + (1+α)²)) ∠ tan⁻¹ (ω/ω₀) α − tan⁻¹ (ω/ω₀)(1+α)
where ω₀ = 1/R₁C ; α = R₂/R₁
θ max. occurs when ω/ω₀ = 1/√(α(α+1))
where ω₀ = R₂/L
α = R₂/R₁
E IN
E out
R₁
R₂
C
L
Phase α=3.0
Phase α=1
α=.5
Phase α=.3
α=.2
Phase α=.1
α=.05
Phase α=.03
Phase α=.01
Phase α=0
Amplitude α=3.0
Amplitude α=1
α=.5
Amplitude α=.3
α=.2
Amplitude α=.1
Amplitude α=0
α=.03
ω/ω₀
0.01
0.1
1.0
10
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
50°
100°
degrees lag

Fig 3

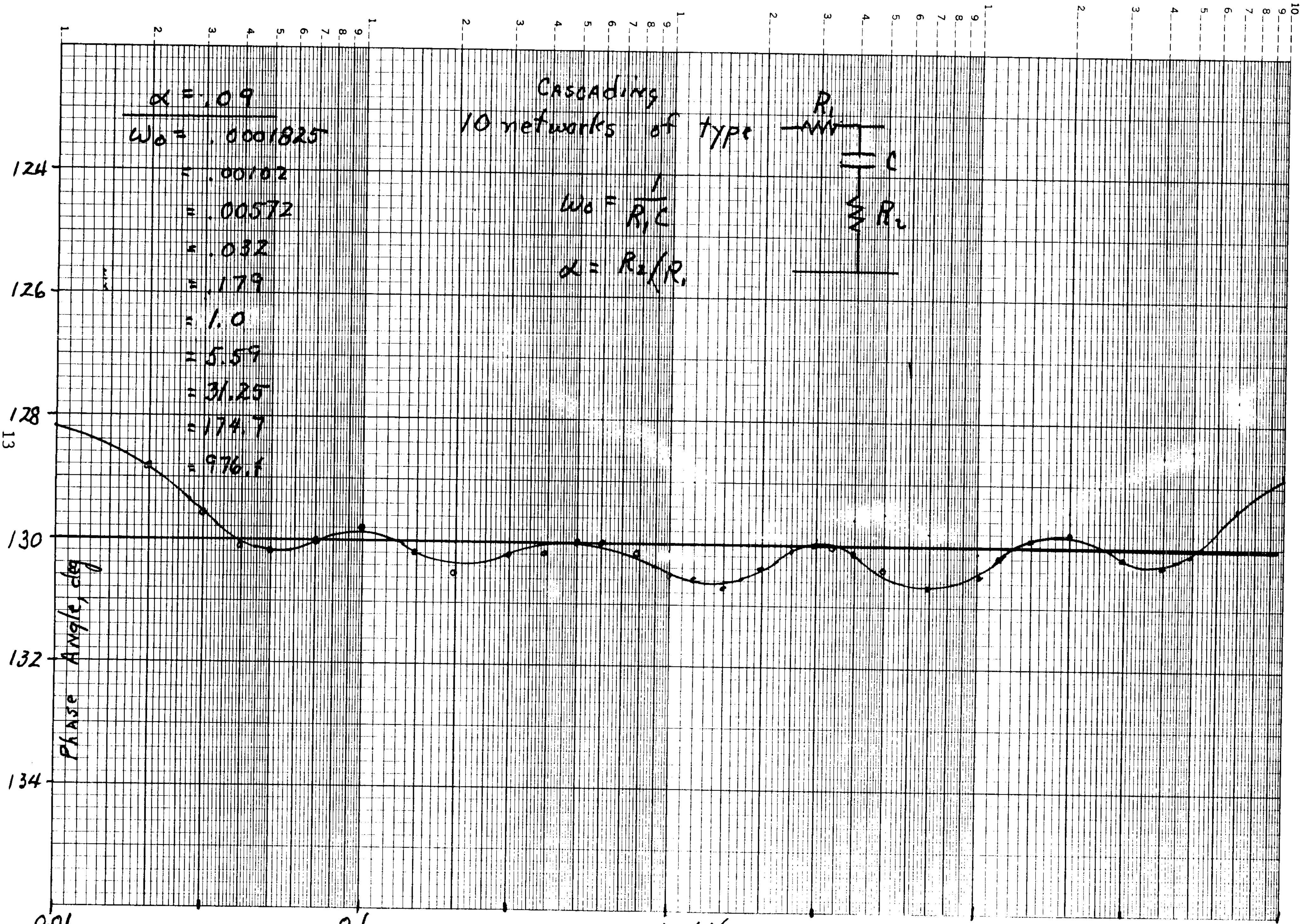
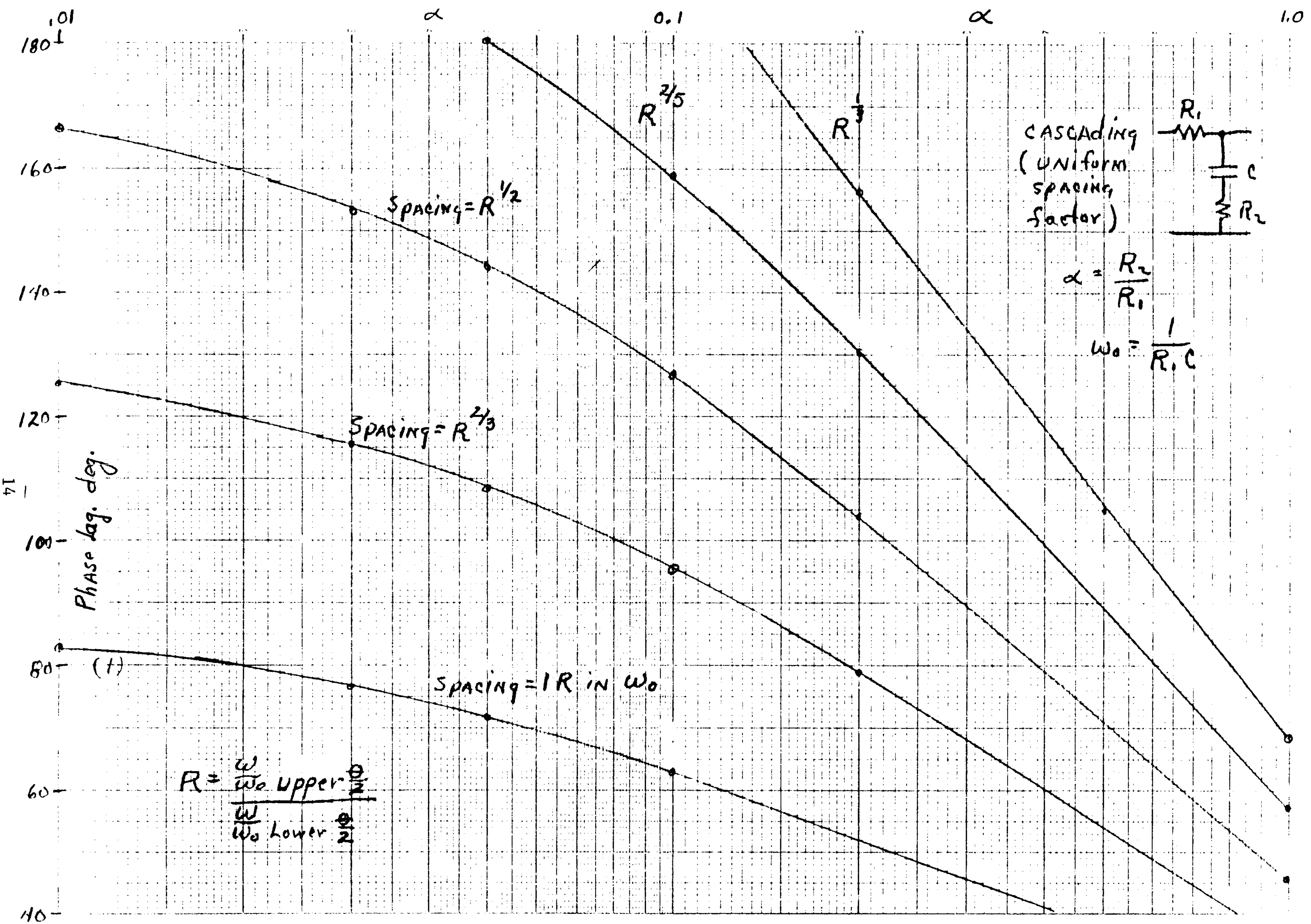

α = .09
Wo = .0001825
= .00102
= .00572
= .032
= .179
= 1.0
= 5.59
= 31.25
= 174.7
= 976.1
Cascading
10 networks of type
R1
C
R2
Wo = 1/(R1 C)
α = R2/R1
124
126
128
130
132
134
Phase Angle, deg
0.01
0.1
1
ω/ωo
10
100

Fig 3A



.01
α
0.1
α
1.0
180
160
140
120
100
80
60
40
Phase lag. deg.
(t)
$R^{2/5}$
$R^{1/3}$
Spacing = $R^{1/2}$
Spacing = $R^{2/3}$
Spacing = 1 R in ω_o
CASCADING (UNIFORM SPACING FACTOR)
R_1
C
R_2
$\alpha = \frac{R_2}{R_1}$
$\omega_o = \frac{1}{R_1 C}$
$R = \frac{\frac{\omega}{\omega_o} \text{ upper } \frac{\theta}{2}}{\frac{\omega}{\omega_o} \text{ Lower } \frac{\theta}{2}}$

Fig 4

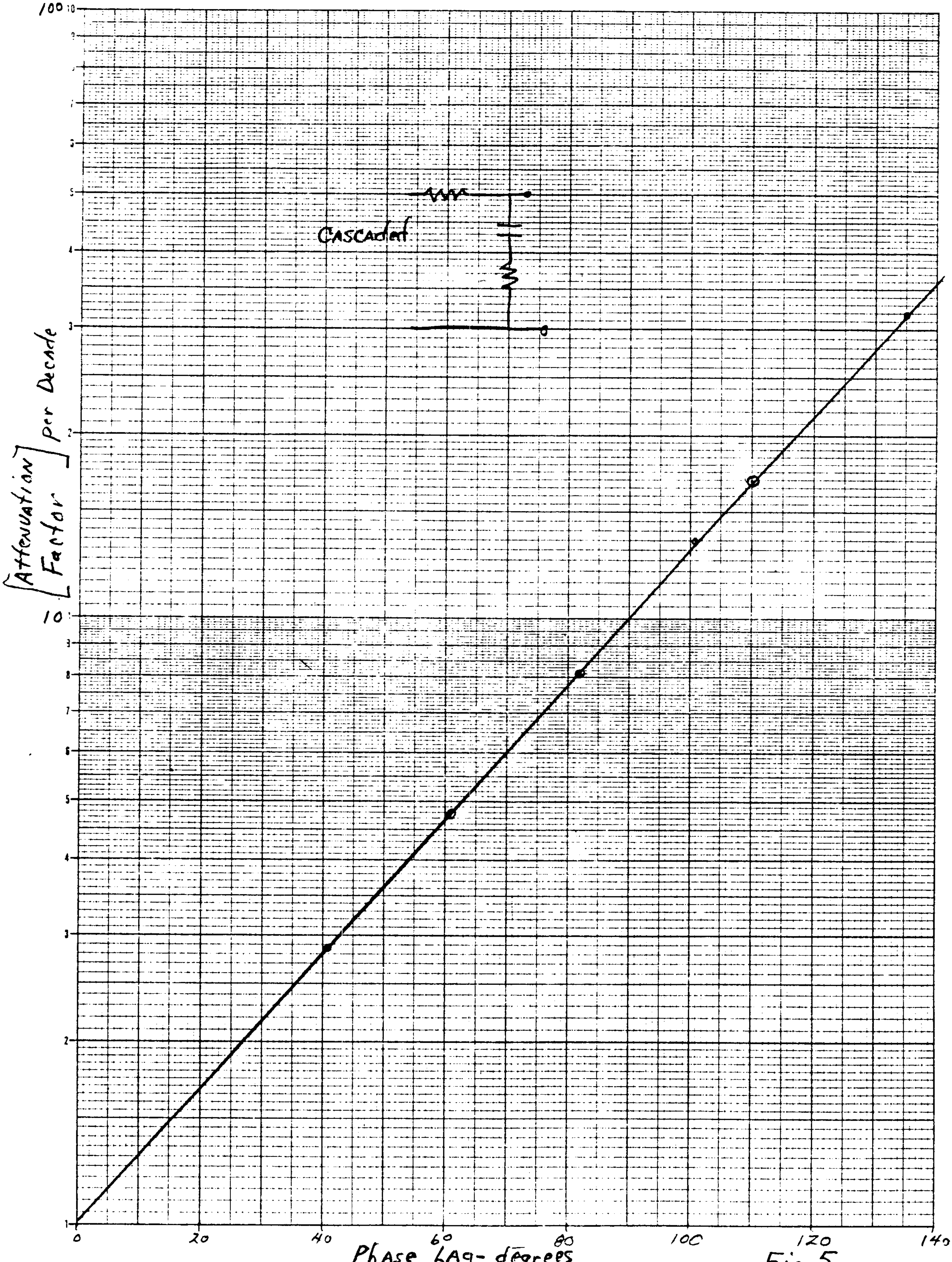

CASCADED
[Attenuation Factor] per Decade
Phase LAG - degrees
100
10
1
0
20
40
60
80
100
120
140

Fig. 5

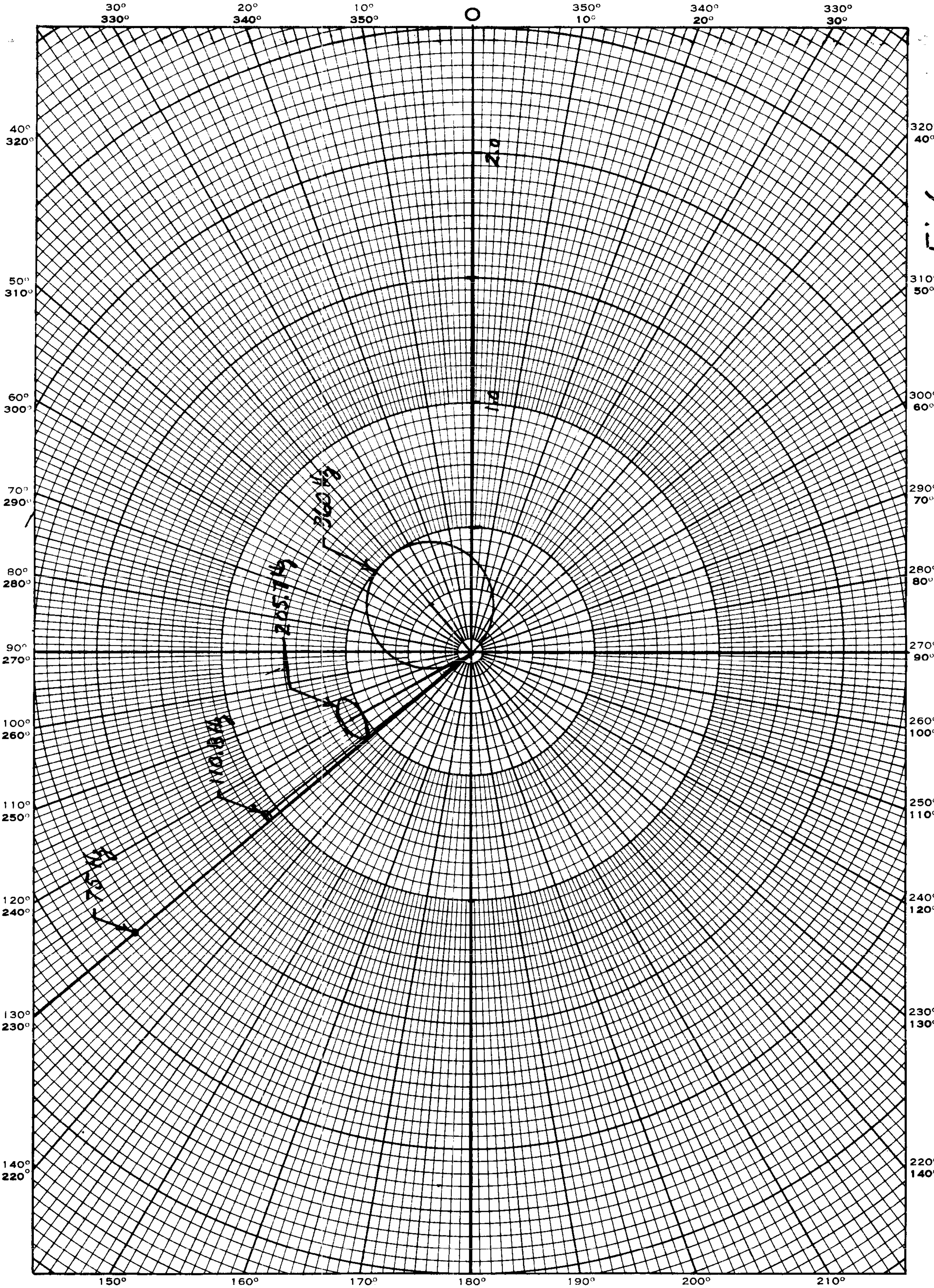


Fig 6

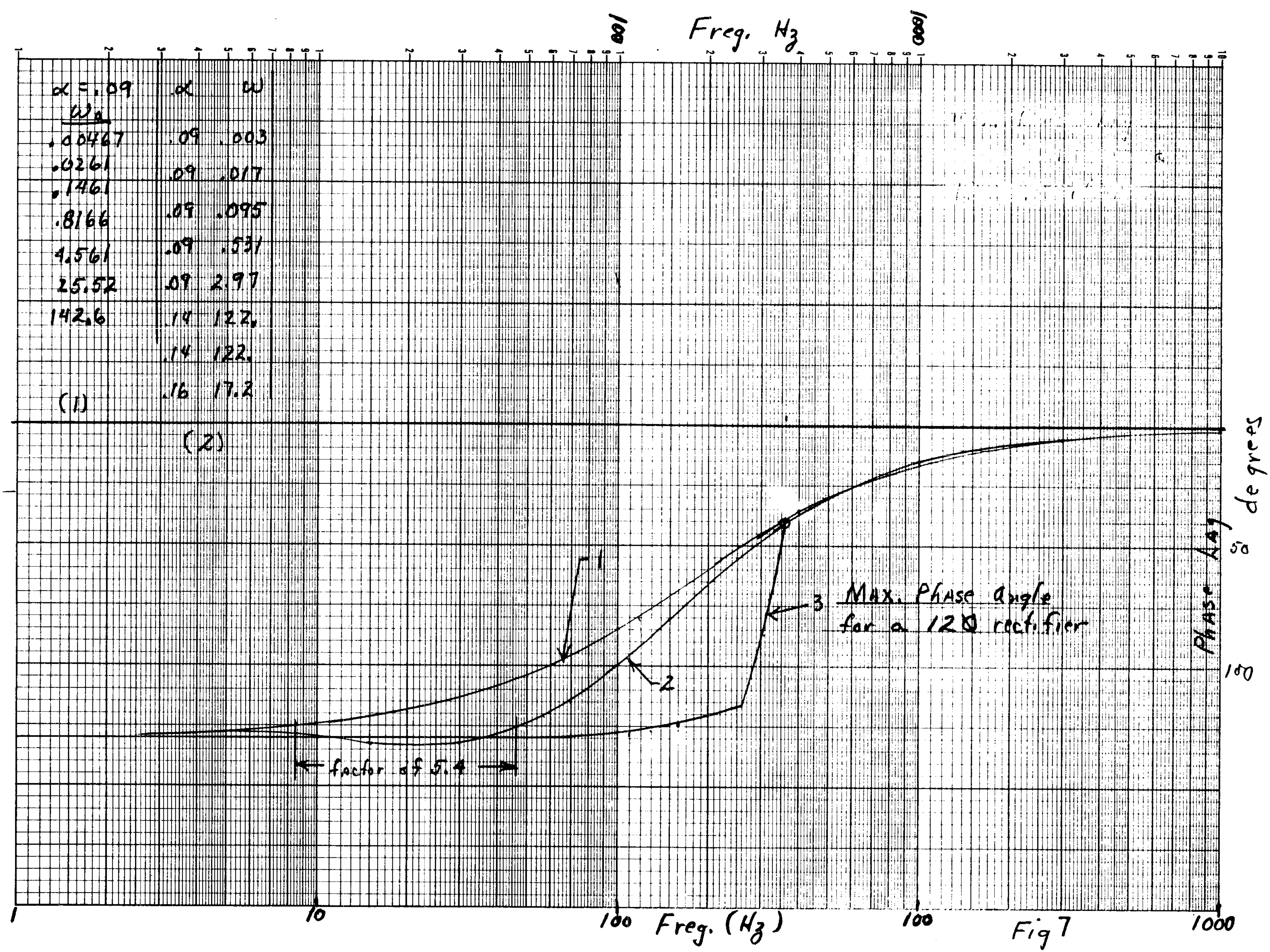
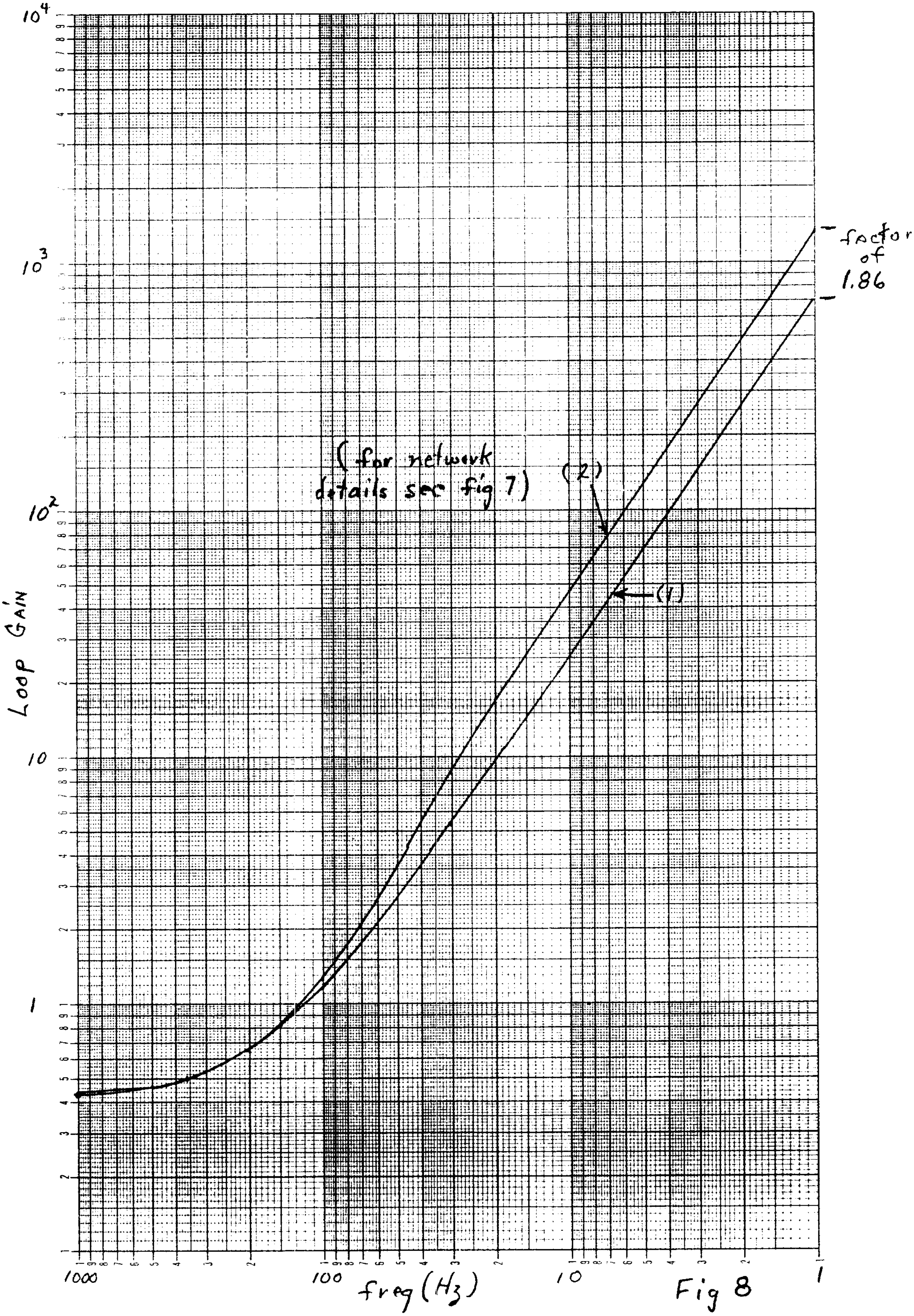
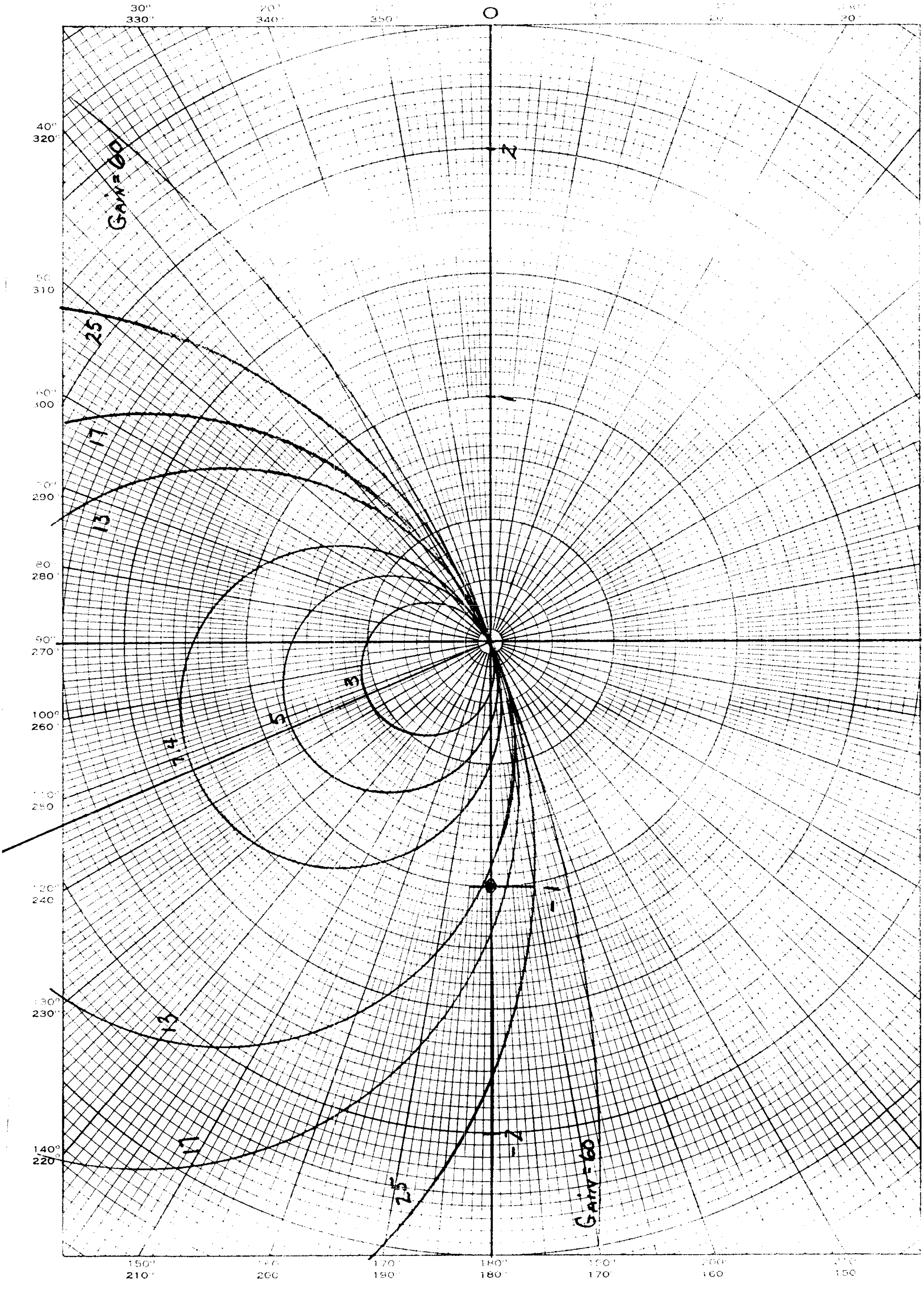

Freq. Hz
α = .09
ωa
.00467
.0261
.1461
.8166
4.561
25.52
142.6
(1)
α ω
.09 .003
.09 .017
.09 .095
.09 .531
.09 2.97
.14 122.
.14 122.
.16 17.2
(2)
1
2
3
MAX. Phase angle
for a 12Ø rectifier
factor of 5.4
Phase lag degrees
50
100
1
10
100
1000
Freq. (Hz)

Fig 7



Loop Gain
freq (Hz)
1000
100
10
1
10⁴
10³
10²
10
1
(for network details see fig 7)
(2)
(1)
factor of 1.86

Fig 8



0
Gain = 60
25
17
13
5
3
14
2
1
-1
-2
Gain = 60
30° 330°
20° 340°
350°
40° 320°
310°
300°
290°
80° 280°
90° 270°
100° 260°
250°
240°
230°
140° 220°
150° 210°
200
190°
180° 180°
170°
160
150
40
50
70
80
270 90
260 100
120
230° 130°
220° 140°

Fig 9

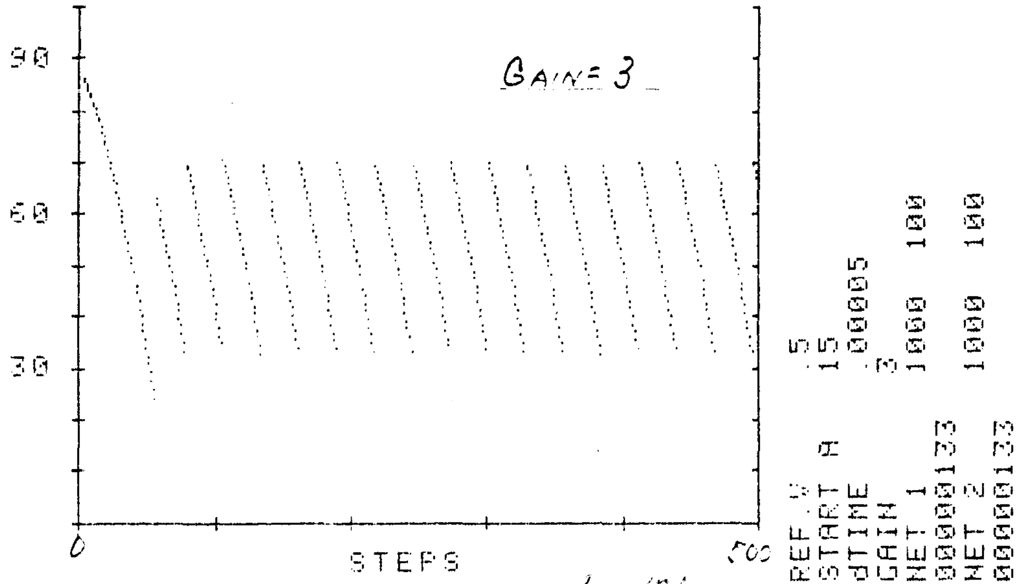


Fig 10A

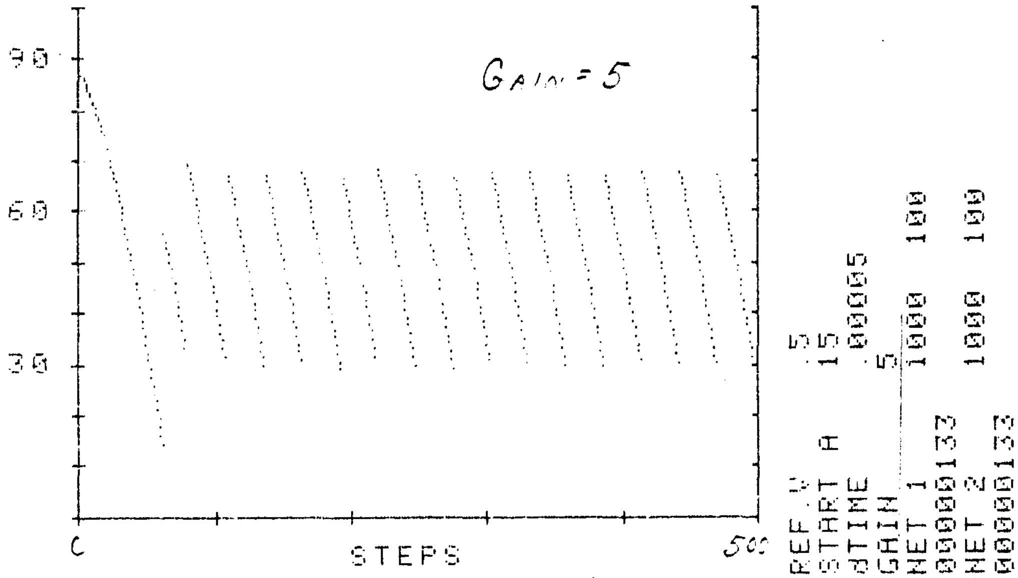


Fig 10B

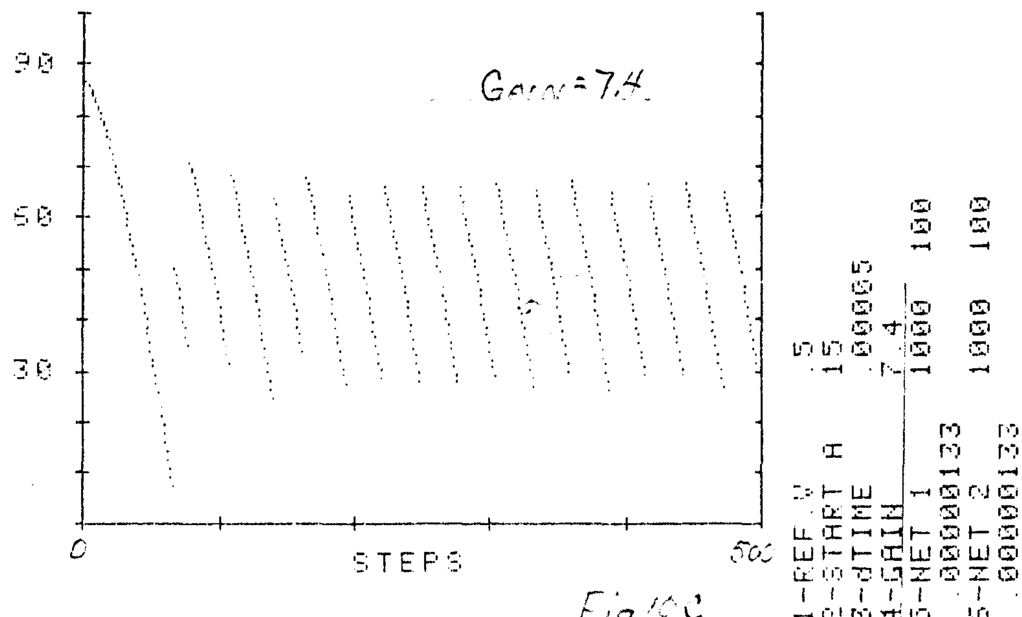


Fig 10C

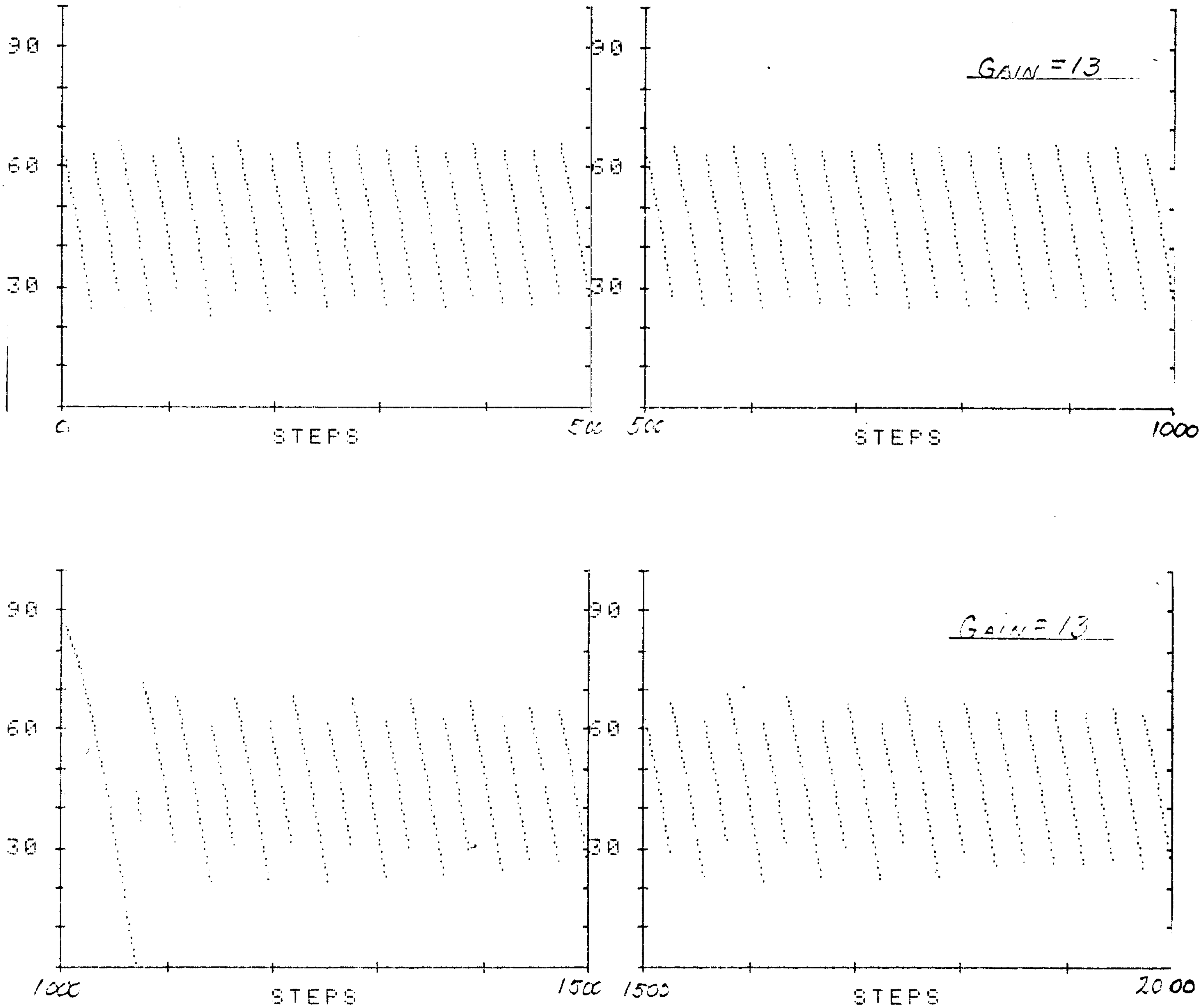


Fig 10D

```
1-REF.W        .5
2-START A      15
3-dTIME        .00005
4-GAIN         13
5-NET 1        1000   100
 .00000133
6-NET 2        1000   100
 .00000133
```

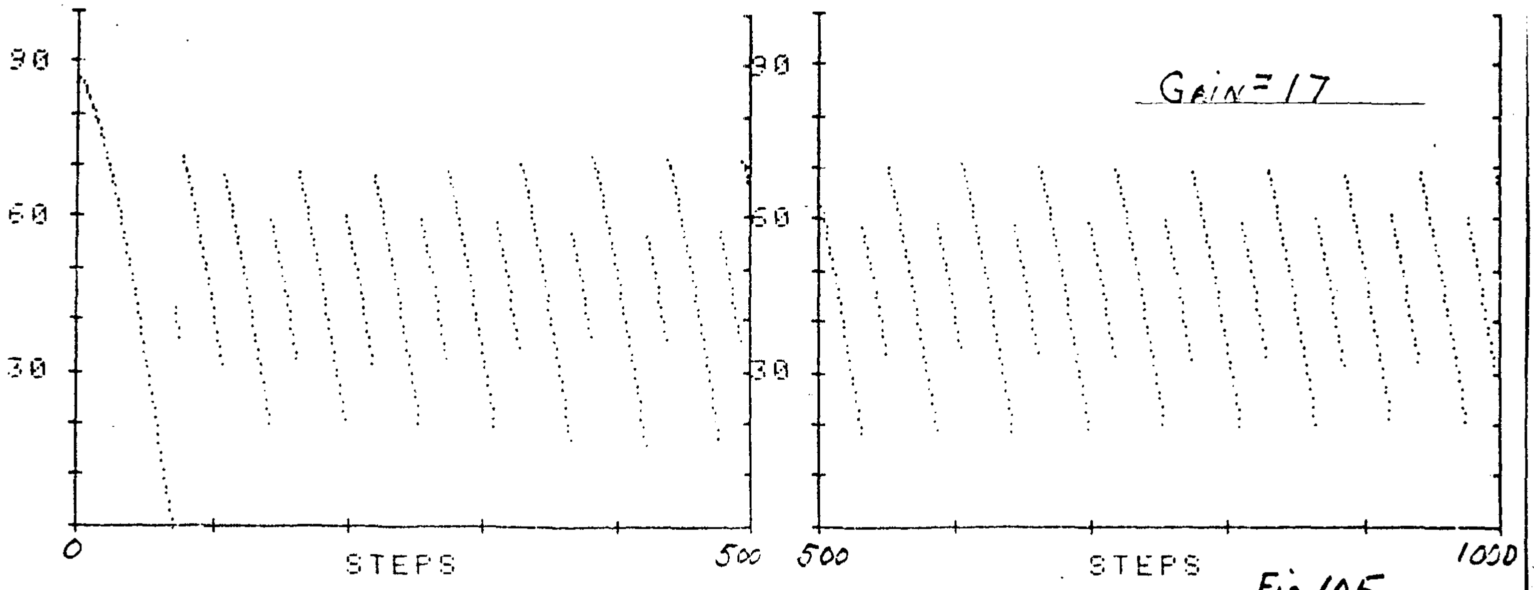


Fig 10E

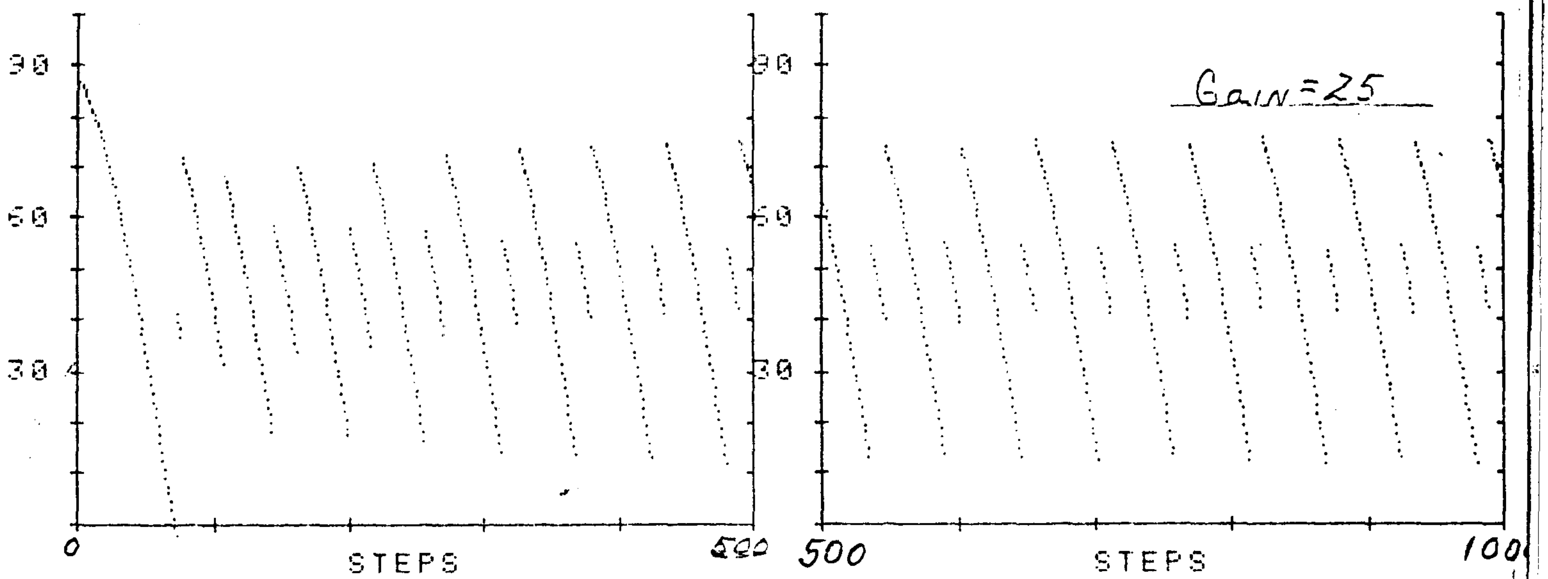


Fig 10F

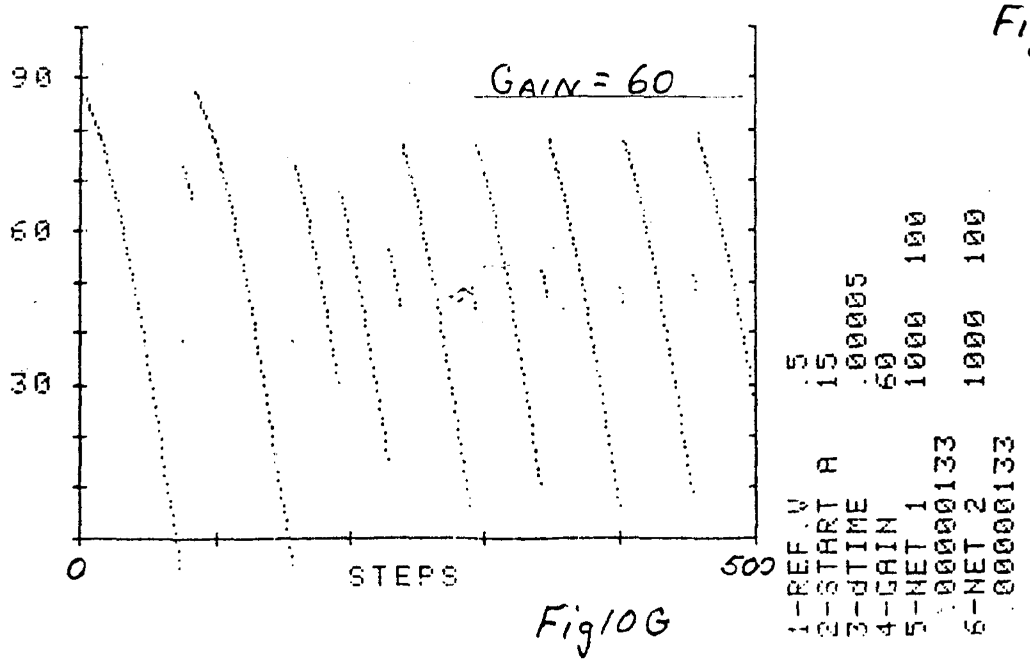


Fig 10G

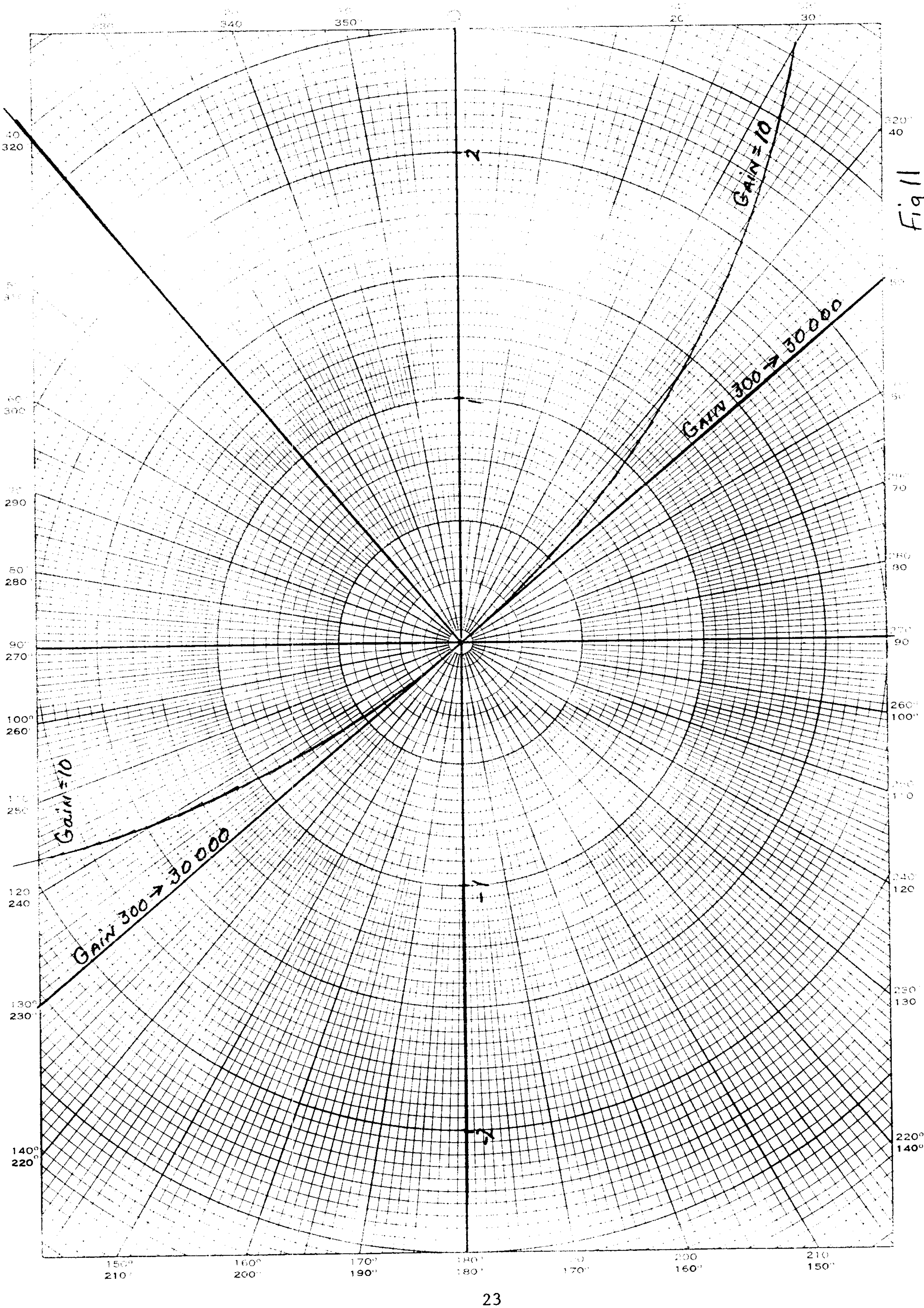


Fig 11
GAIN = 10
GAIN 300 → 30000
Gain = 10
Gain 300 → 30000
2
1
-1
-2

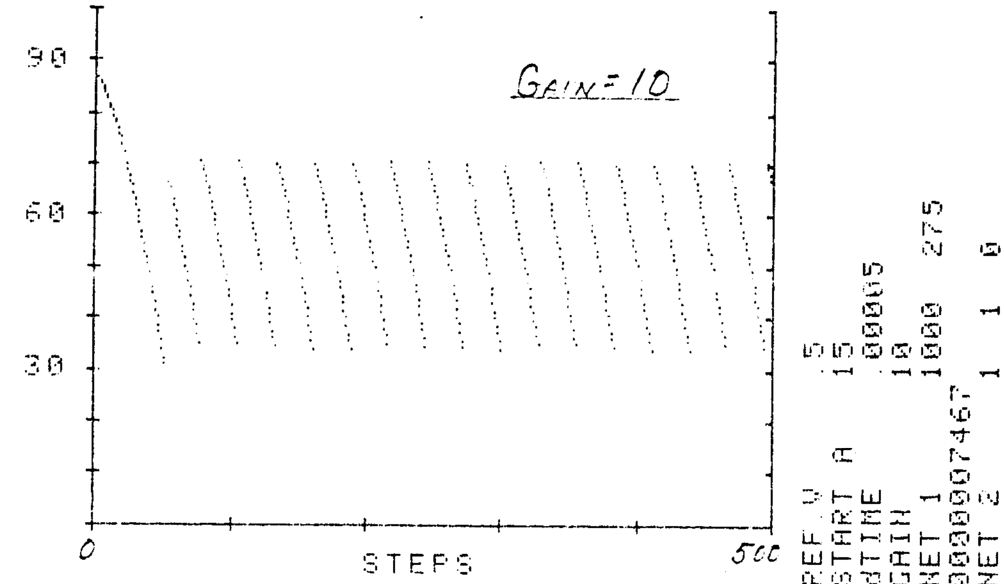


Fig 12A

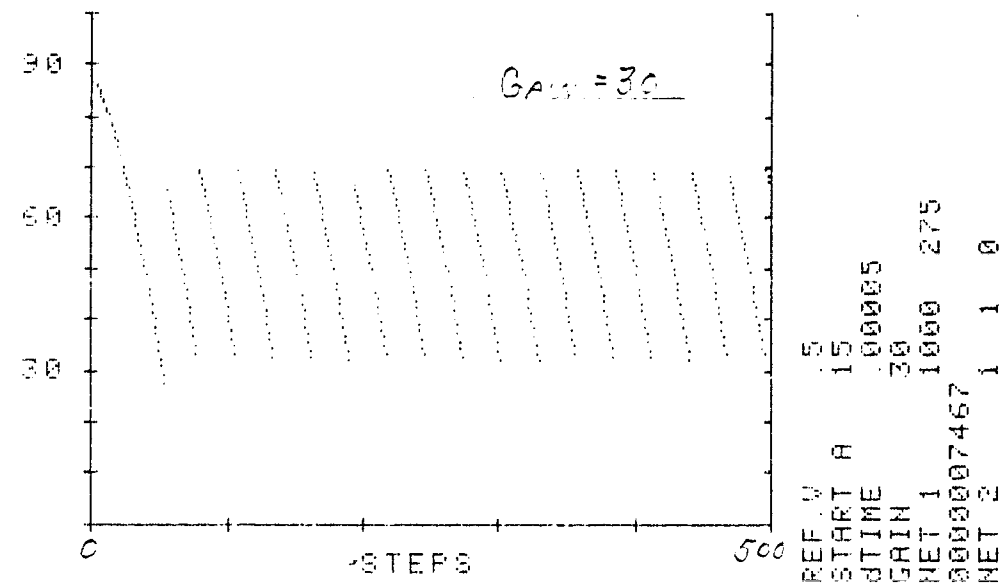


Fig 12B

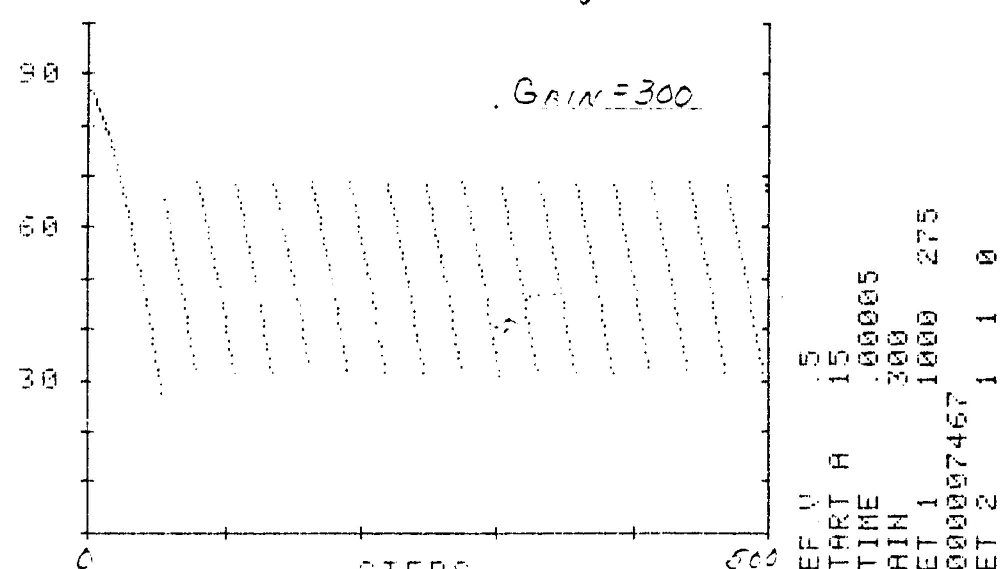


Fig 12C

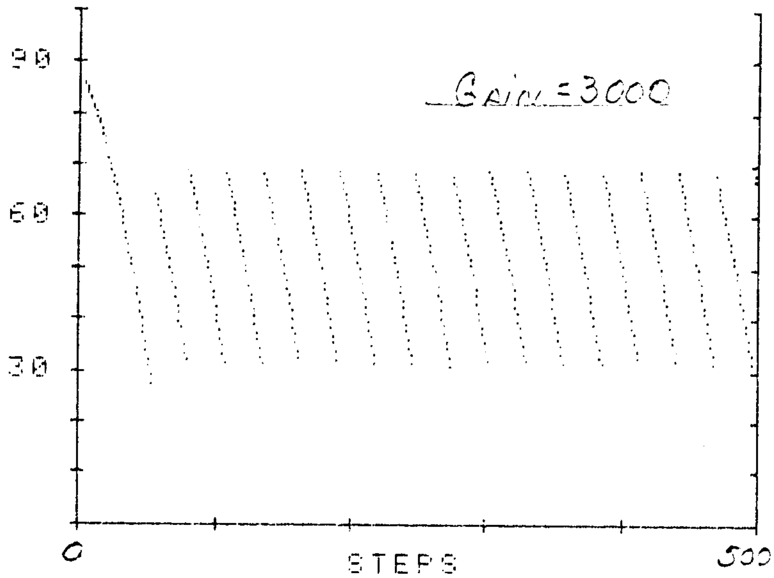


Fig 12D

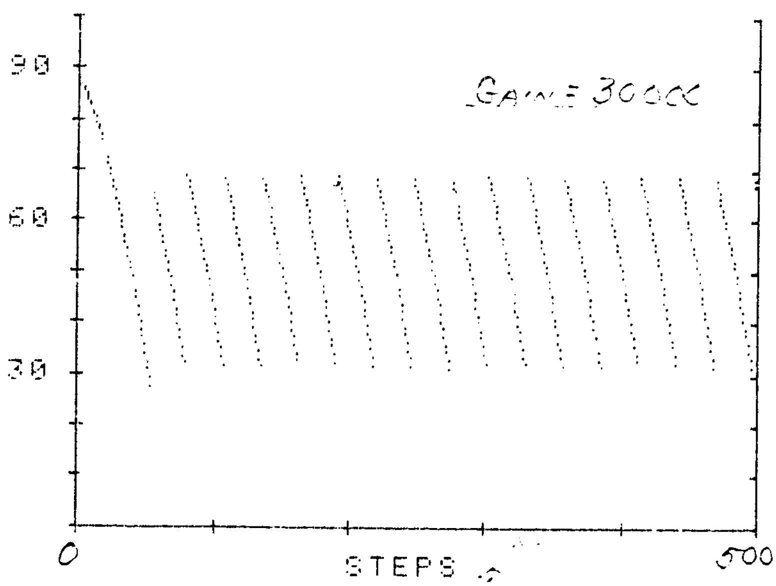


Fig 12E

APPENDIX A

Transfer Function of a Phase Control Rectifier

Linear feedback systems are today well understood and great progress has been made in the understanding of feedback systems involving sampling systems. A "grid-controlled" rectifier is a sampling system in that the controlling information affects the output only at certain instants in time. Figure 1 displays this sampling process. Waveform A is the rectifier output, with the solid line representing the output with zero control voltage. Waveform B represents the change in output that results from the control voltage C. If the control voltage is limited to small signals, then the change in output can be represented by an impulse whose magnitude is proportional to the instantaneous values of the control voltage. The sampling intervals are approximately uniform in time.

Using this impulse approximation the rectifier small signal transfer function can be determined. This transfer function is defined as the amplitude and phase relation between the change in rectifier output resulting from the action of a sinusoidal control voltage. Only the output components having the same frequency as the control voltage is considered.

The analysis of this model is basically straightforward and has an analytical solution. The resulting transfer function is a function of frequency and the parameter ϕ, the electrical angle between the control voltage zero crossing and the preceeding sampling time (see Figure 2).

The output change impulse train amplitude can be represented by

$$C\left[\sin \frac{2\pi\ \omega\ n}{\omega_o}\right]_{n=1}^{n=n}$$

where

C = amplitude constant (volt-seconds)

ω = radial frequency of the control function

ω_o= radial sampling frequency

n = positive integer

ϕ = as stated above

The Fourier component of the radial frequency ω for this impulse train can be obtained by the normal integration. Since the impulse train is zero everywhere except at the impulses, the integral becomes a finite summation:

$$A_n = \frac{1}{\pi} \sum_{0}^{2\pi} C\omega \sin\left[\frac{2\pi\ \omega\ n}{\omega_o} - \phi\right] \cos\left[\frac{2\pi\ \omega\ n}{\omega_o} - \phi\right] d\Theta$$

$$B_n = \frac{1}{\pi} \sum_{0}^{2\pi} C\omega \sin\left[\frac{2\pi\ \omega\ n}{\omega_o} - \phi\right] \sin\left[\frac{2\pi\ \omega\ n}{\omega_o} - \phi\right] d\Theta$$

where A_n and B_n are the coefficients of the cosine and sine terms respectively. The transfer function vector has a magnitude of

$$M = \sqrt{A_n^2 + B_n^2}$$

and an angle of

$$\phi = \tan^{-1} \frac{A_n}{B_n} .$$

With the following substitutions:

$$z = \frac{2\pi\ \omega}{\omega_o} + k = \frac{\omega_o}{\omega} ,$$

and some trigonometric algebra, we obtain

$$\frac{\pi A_n}{\omega C} = (\cos^2 \phi - \sin^2 \phi)\left[\sum_{n=1}^{k} \sin zn \cos zn\right] + \sin \phi \cos \phi\left[\sum_{n=1}^{k} \sin^2 zn - \cos^2 zn\right]$$

$$\frac{\pi B_n}{\omega C} = \cos^2 \phi\left[\sum_{n=1}^{k} \sin^2 zn\right] - 2\sin \phi \cos \phi\left[\sum_{n=1}^{k} \sin zn \cos zn\right] + \sin^2 \phi \sum_{n=1}^{k} \cos^2 zn$$

The above are relatively easy to evaluate since the summations are independent of ϕ. If k is not an integer some care must be exercised to determine that limit of summation. The proper integer is the number of samplings in the control function period which may change by one count for certain values of ϕ producing cusps in the transfer function.

The transfer function plots shown in Figure 3 have been normalized to unity to remove the constant C.

The resulting transfer function is not single valued but is a locus of values for each frequency with ϕ as an independent variable. If the control or driving frequency is small (k large) then the transfer function locus becomes small approaching a point or single value. The curves labeled with the k values represents the locus of the transfer function for that frequency ratio for all values of ϕ. Figure 3 shows only the first quadrant results; the function is symetrical about the horizontal axis.

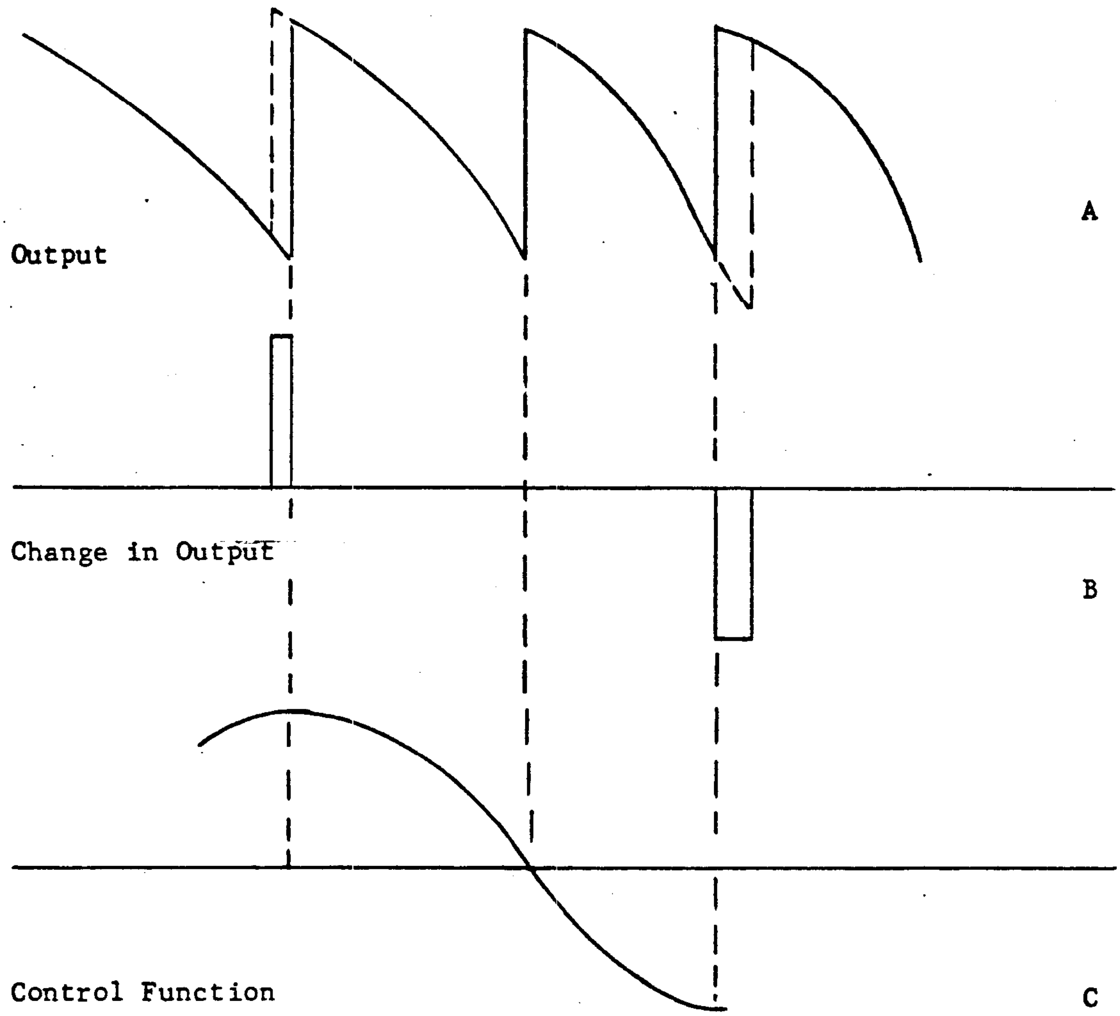


Fig. 1

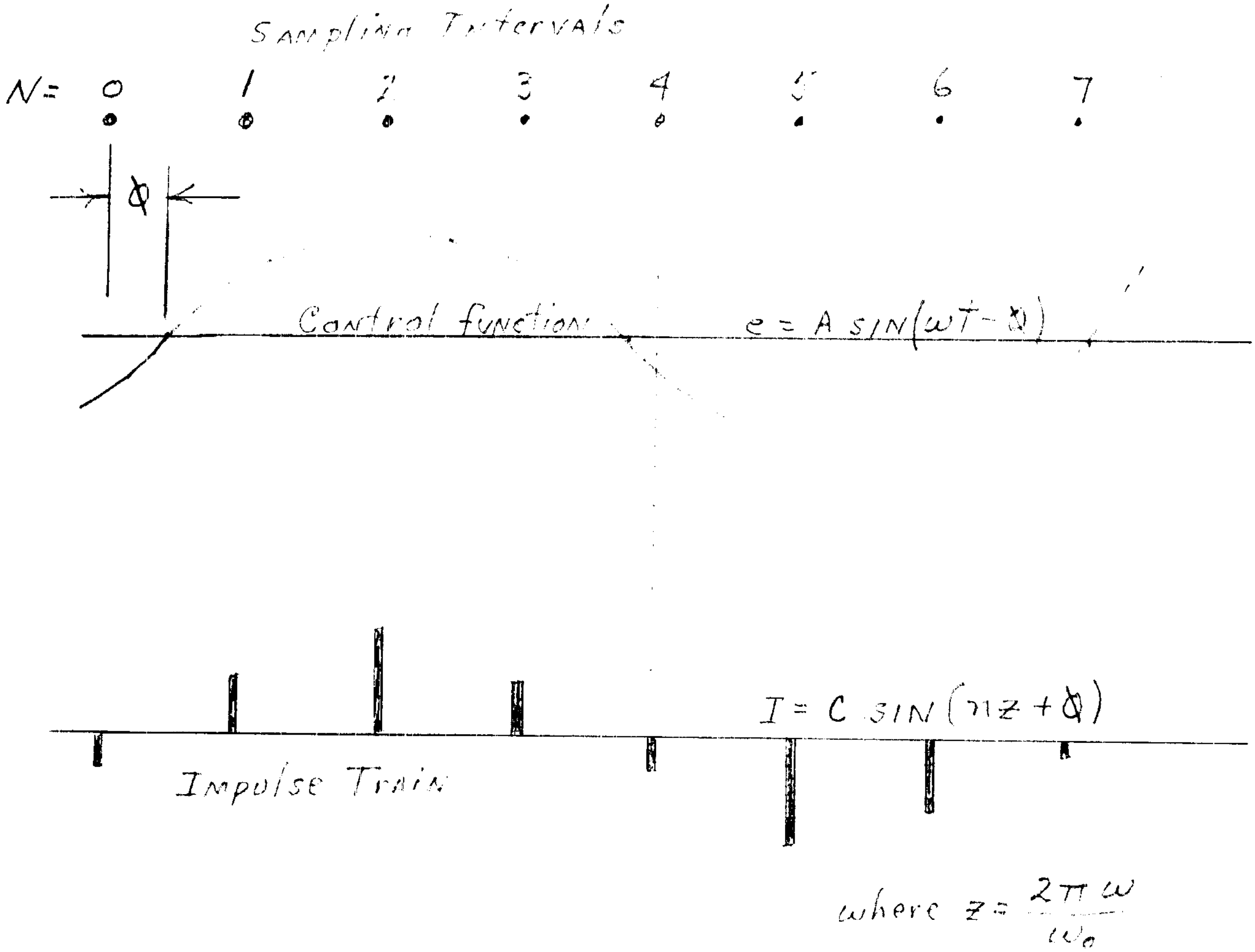

Sampling Intervals
N= 0 1 2 3 4 5 6 7
Φ
Control function
e = A SIN(ωT - Φ)
I = C SIN(nz + Φ)
Impulse Train
where z = 2π ω / ω₀

Fig 2

Locus of Transfer function - Controlled Rectifier - First Quad only

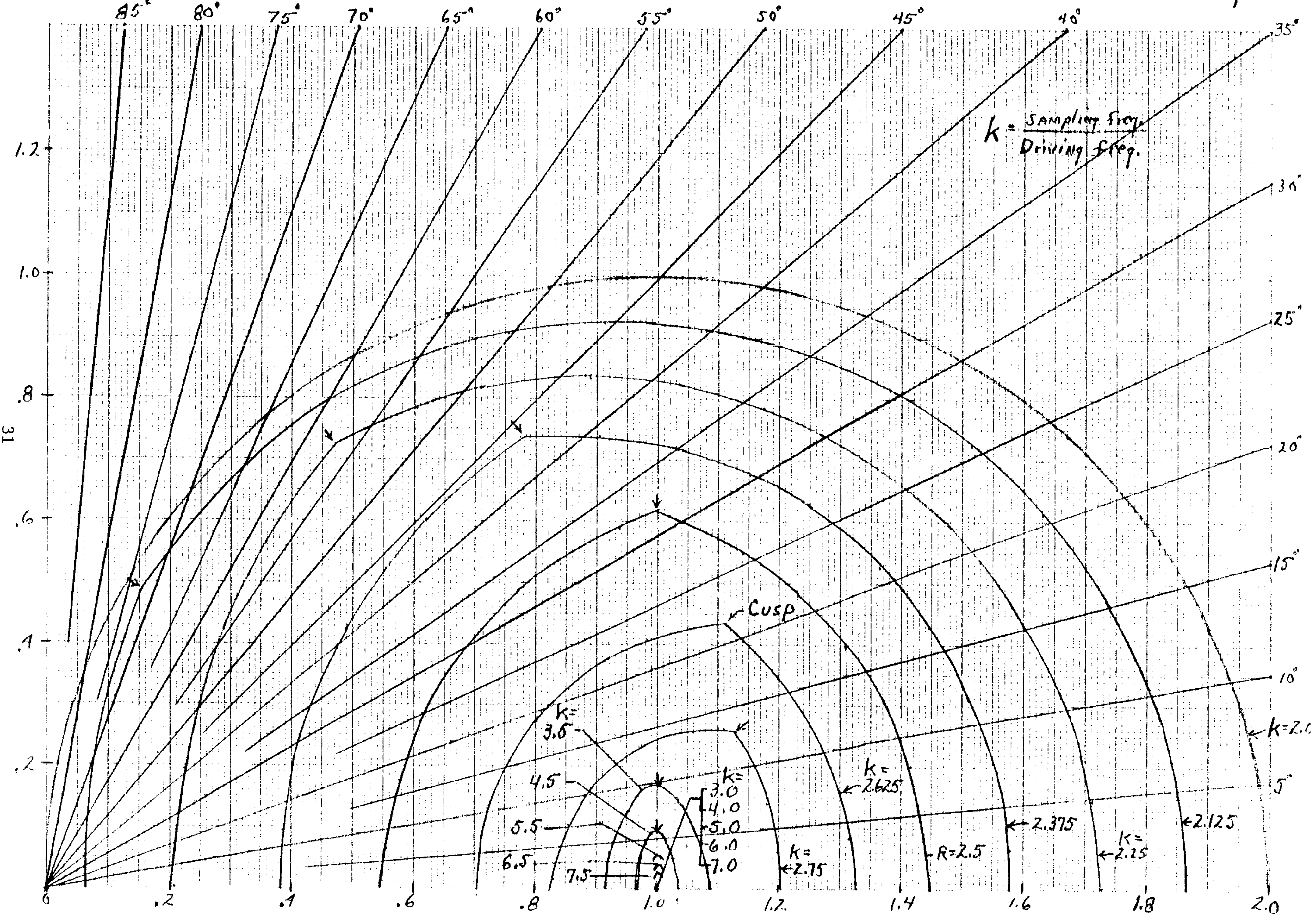


Fig 3