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Beam loading compensation and Robinson instability limit

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DISCLAIMER

BEAM LOADING COMPENSATION AND ROBINSON INSTABILITY LIMIT

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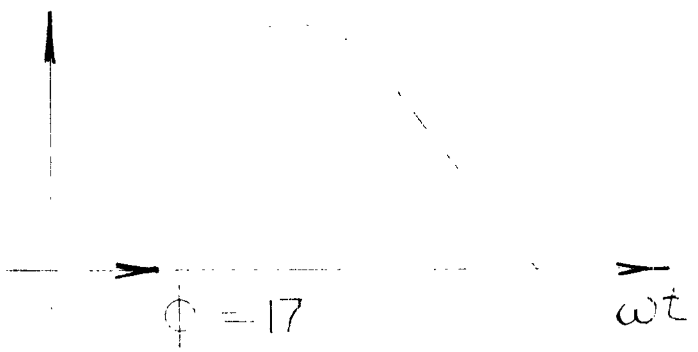
For treating the Robinson instability it is important to define a reference frame for the voltages indulged at the gap by the beam and the driving amplifier.

In the previous technical notes the total gap voltage has been assumed as a sine wave and the phase of the beam is measured from the origin.

Consequently, the reference frame is as shown in the figure.

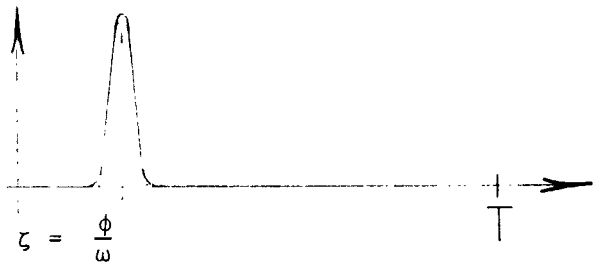
1) Reference for phase: The Gap Voltage



$$V = V_0 \text{ Sin } \omega t; \quad \bar{V} = V_o e^{-j\pi/2}$$

2) Beam Current.

In the ideal case of a series of delta functions:



$$I(t) = \Sigma\, q\, \delta\, [t-(\zeta+nT)]$$

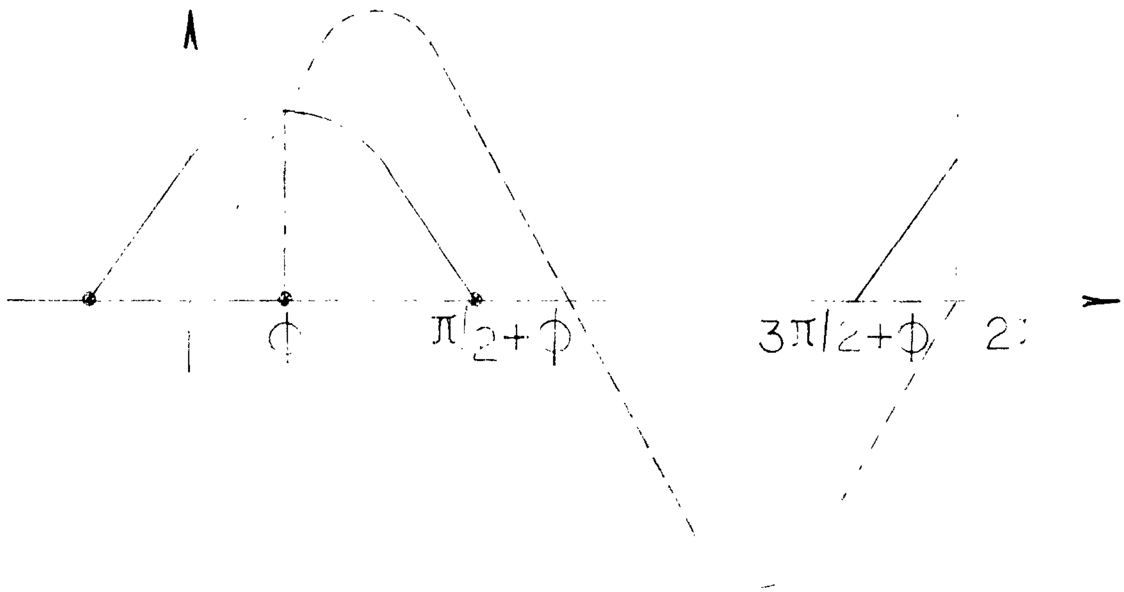
and $q = ne$ is the charge per bunch.

$$\begin{cases} a_n = \dfrac{2}{T}\displaystyle\int_0^T q\,\delta\,(t-\zeta) \text{ Cos } 2\pi n \frac{t}{T}\, dt = \frac{2q}{T} \text{ Cos } \pi \frac{\zeta}{T} \\[2ex] b_n = \dfrac{2}{T}\displaystyle\int_0^T q\,\delta\,(t-\zeta) \text{ Sin } 2\pi n \frac{t}{T}\, dt = \frac{2q}{T} \text{ Sin } 2\pi \frac{\zeta}{T} \end{cases}$$

$$I_n = \frac{2q}{T} \{\text{Cos}\ \phi\ \text{Cos}\ n\ \omega t + \text{Sin}\ \phi\ \text{Sin}\ n\ \omega t\} =$$

$$= \frac{2q}{T}\ \text{Cos}\ [n\ \omega t - \phi]$$

For the more physical case: beam angular length = π (radians)



$$I_t = I_p\ \text{Cos}\ (\omega t - \phi) \qquad 0 \leq \omega t \leq \pi/2 + \phi$$

$$I_t = 0.0 \qquad \pi/2+\phi \leq \omega t \leq 3\pi/2 - \phi$$

$$I_t = I_p\ \text{Cos}\ (\omega t - \phi) \qquad 3\pi/2+\phi \leq \omega t \leq 2\pi$$

$$a_1 = \frac{2}{T} \int_0^T I_p\ \text{Cos}\ (\omega t - \phi)\ \text{Cos}\ \omega t\ dt$$

$$b_1 = \frac{2}{T} \int_0^T I_p\ \text{Cos}\ (\omega t - \phi)\ \text{Sin}\ \omega t\ dt$$

expanding integrands we obtain:

$$\mathrm{Cos}\ (\omega t - \phi)\ \mathrm{Cos}\ \omega t = \tfrac{1}{2}\left\{\cos \phi + \mathrm{Cos}\ (2\omega t - \phi)\right\}$$

$$\mathrm{Cos}\ (\omega t - \phi)\ \mathrm{Sin}\ \omega t = \tfrac{1}{2}\left\{\mathrm{Sin}\ \phi + \mathrm{Sin}\ (2\omega t - \phi)\right\}$$

integrating:

$$a_1 = \frac{2}{T} * \tfrac{1}{2} * \frac{I_p}{w} \left\{\left(\frac{\pi}{2} + \phi\right) + \left[2\pi - \left(\frac{3\pi}{2} - \phi\right)\right]\right\} \mathrm{Cos}\ \phi$$

$$b_1 = \frac{2}{T} * \tfrac{1}{2} * \frac{I_p}{\omega} \left\{\left(\frac{\pi}{2} + \phi\right) + \left[2\pi - \left(\frac{3\pi}{2} - \phi\right)\right]\right\} \mathrm{Sin}\ \phi$$

because $wt = 2\pi$ we obtain:

$$a_1 = \frac{I_p}{2} \mathrm{Cos}\ \phi; \quad b_1 = \frac{I_p}{2} \mathrm{Sin}\ \phi$$

and the first harmonic is as follows:

$$I_1 = \frac{I_p}{2} \left[\mathrm{Cos}\ \phi\ /\cos \omega t + \mathrm{Sin}\ \phi\ \mathrm{Sin}\ \omega t\right] \text{ or}$$

$$I_1 = \frac{I_p}{2}\ \mathrm{Cos}\ (\omega t - \phi)$$

the peak value I_p can be calculated:

$$q = ne = \int_{-T/4}^{+/T/4} I_p \ \mathrm{Cos}\ 2\pi \frac{t}{T}\, dt = \frac{1}{\pi} I_p T$$

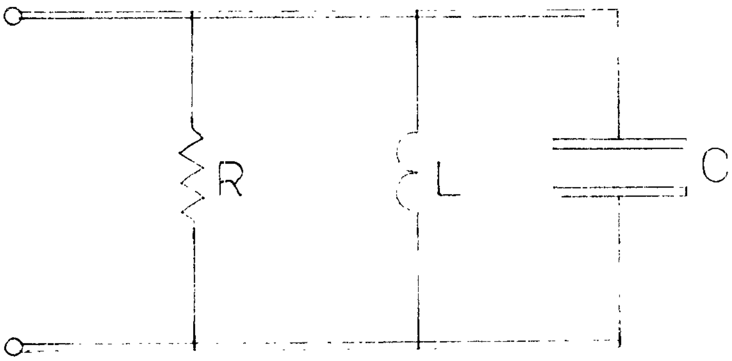
and we obtain: $I_p = \pi n e \nu$

Collecting the results for $n = 0.75\ 10^{13}$ and $\nu = 4.2\ 10^{6}$ we obtain:

q	$= ne$	$= 1.2\ 10^{-6}$	coulomb
I_{ave}	$= ne\nu$	$= 5.04$	amp.
I_{peak}	$= \pi n e \nu$	$=15.83$	amp.
$I_1 = I_b$	$= \frac{\pi}{2} n e \nu$	$= 7.91$	amp.

3) The cavity coupling system.

it is possible to demonstrate that the equivalent circuit of the cavity, seen from the gap, is shown in the figure.



Here R is the equivalent shunt resistance of the cavity. In other words, R is the parameter that accounts for the power losses of the cavity in the considered mode.

$$R = \frac{V^2}{2P_c}$$

We define: $\omega_o^2 = \frac{1}{LC}$ = Resonant frequency

$Q_o = \omega_o RC$ = Cavity Quality Factor.

The meaning of the quality factor is as follows:

$$Q_o = \omega_o RC = \frac{2\pi}{T} \frac{\frac{1}{2} V^2 C}{\frac{1}{2} \frac{V^2}{R}} = 2\pi \frac{\frac{1}{2} CV^2}{\frac{1}{2} \frac{V^2}{R} T}$$

and it is the ratio, multiplied by 2π, between the stored energy and the energy wasted per cycle.

* * *

The quality factor Q is a very important parameter because while L and C can be very difficult to define and to measure in a cavity it is very simple to measure the power losses, the resonant frequency and the quality factor.

Where P_c = power.

$$\frac{V^2}{2P_c} = R$$

$$\omega_o^2 = \frac{1}{LC}$$

$$Q_o = \omega_o RC$$

Solving we obtain:

$$C = \frac{Q_o}{\omega_o R}$$

$$L = \frac{R}{\omega_o Q_o}$$

From the above formula we obtain:

$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{R}{1 + jQ_o \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Normally the function $D(\omega) = \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}$ = dissonance is expanded around $\omega=\omega_o$ and we obtain

$$D(\omega) = 2\,\frac{\Delta\omega}{\omega_o} + \ldots$$

Where $\Delta\omega$ is the deviation around $\omega=\omega_o$ and can be either positive or negative.

For $\pm\,\Delta w \cdot Q_o = \pm\,0.5$ we obtain $|Z| = R\sqrt{2}$ and consequently $BW = \frac{\nu_o}{Q_o}$ is the conventional band width of the mode. If we define a tuning angle Ψ as

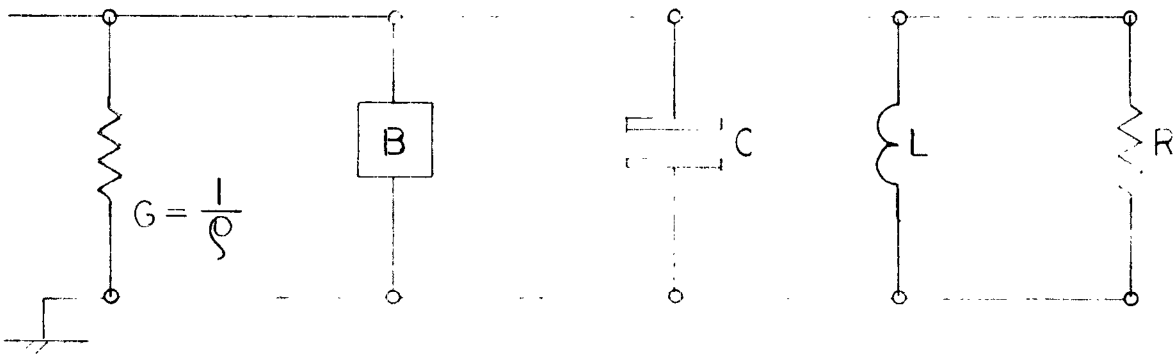
$$\operatorname{Tan}\Psi = Q\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) \cong 2Q\,\frac{\Delta\omega}{\omega_o}$$ we can write:

$$Z = \frac{Re^{-j\Psi}}{\sqrt{1 + \operatorname{Tan}^2\Psi}} = R\,\operatorname{Cos}\Psi\, e^{-j\Psi}$$

At this point we consider the cavity coupled to the power amplifier.

It is at least reasonable to suppose that the amplifier loads the cavity with his own output impedance $Z = 1/(G+jB)$. Where $\rho = \frac{1}{G}$ is the amplifier output resistance. (ρ depends both upon the amplifier scheme and the operating class.)

In this case the scheme of the cavity coupling system, in the neighboring of the operating mode, is again a parallel tuned circuit.



This new circuit has a parallel shunt resistance $R_T = \frac{R\rho}{R+\rho}$ a resonant frequency ω_r and a quality factor Q_L.

Consequently the equivalent parameters of the cavity coupling system are:

$$R_T; \ L = \frac{R}{\omega_r Q_L} \ ; \ C = \frac{Q_L}{\omega_r R_T}$$

The quality factor of the cavity loaded by the amplifier is sometimes referred to as the "cavity loaded quality factor".

A tentative set of parameters for a single unit of the cavity can be as follows: (22.5 kV at the gap).

Frequency	2.4 MHz	4.2 MHz
Power	24 KW	47 KW
Quality Factor	64	56

and the equivalent RLC parameters are:

$\nu_r = 2.4\ 10^6$	$\nu_r = 4.2\ 10^6$
$R = 10.5\ K\Omega$	$R = 5.3\ K\Omega$
$L = 11\ \mu H$	$L = 3.6\ \mu H$
$C = 400\ pf$	$C = 400\ pf$

4) BEAM LOADING

With the reference already chosen for the phases the complex amplitudes of the cavity voltage $\bar{V}$, of the first harmonic of the beam current $\bar{I}_1$, of the amplifier current $\bar{I}_a$ are as follows:

$$\bar{V} = V_o e^{-j\pi/2};\ \bar{I}_1 = -I_b e^{-j\phi};\ \bar{I}_a = GmVge^{j\xi}$$

where Vg is the modulus of the driving voltage and ξ is the appropriate phase angle. From the equivalent circuit we have:

$$-jV_o \left[\frac{1}{R_T} + j\left(\omega C - \frac{1}{\omega L}\right)\right] = -I_b e^{-j\phi} + GmVge^{j\xi}$$

Now because the amplifier should see a real load (even if not matched) we should input the condition: $\bar{I}_a = GmVge^{-j\pi/2}$ and we obtain:

$$\frac{V_o}{R_T} = j\left(\omega C - \frac{1}{\omega L}\right) V_o = -jI_b \operatorname{Cos}\phi - I_b \operatorname{Sin}\phi + GmVg$$

Equating the real and the imaginary parts we obtain:

$$GmVg = \frac{V_o}{R_T} + I_b \, Sin\phi$$

$$\left(\omega C - \frac{1}{\omega L}\right) = \omega \Delta C = -\frac{I_b Cos\phi}{V_o}$$

and the following has to be noted:

a) For $\phi = 0$, the so called storage mode, the beam is in quadrature with the cavity voltage.

This means that the beam does not absorb any real power ($\sin\phi=0$) and full compensation is required.

Because ΔC is negative then the cavity should be tuned high.

b) For $\phi=\pi/2$, not applicable to any synchronous accelerator, the beam is in phase and absorbs real power. On the other hand, no reactive compensation is needed.

c) In the intermediate case of the Booster, ϕ=17 degrees, we have:

V_o = 22.5 KV; I_b = 7.91 AMP.

From the above formula we obtain:

$\nu_1 = 2.6 \; 10^6$ Hz $\qquad \Delta C_1 = 22.3$ pf

$\nu_2 = 4.2 \; 10^6$ Hz $\qquad \Delta C_2 = 12.7$ pf

The power absorbed by the beam is easily calculated. $I_1 = I_b \, Cos\,(\omega t-\phi)$ then:

$$W_b = \frac{1}{T}\int_o^T \nu_o \sin \omega t \cdot I_b \, Cos\,(\omega t-\phi)\, dt$$

Expanding the integrand we obtain:

$$Sin\, \omega t \cdot Cos\,(\omega t - \phi) = Sin\, \omega t \, Cos\, \omega t \, Cos\, \phi + Sin^2 \omega t \, Sin\, \phi$$

$$= \tfrac{1}{2}\left\{Cos\, \phi \, Sin\, 2\omega t + (1 - Cos 2\omega t) Sin\, \phi\right\}$$

Substituting and integrating we obtain:

$$W_b = \tfrac{1}{2}\, \nu_o I_b \text{ Sin } \phi$$

In our case: V_o = 22.5 KV, I_b = 7.91 A, Sin ϕ = 0.292 and we obtain W_b = 26 KW per gap.

NOTE: Because we suppose that the cavity voltage is perfectly sinusoidal then it follows that only the first harmonic of the beam current can exchange real power with the cavity and the above formula is perfectly correct. The formula normally used: $W_b = V_o \, I_{ave}$ Sin ϕ is valid only for extremely tight bunches where the amplitude of the first harmonic of the beam current is very near to 2 I_{ave}.

5) ROBINSON INSTABILITY

It is a coherent instability that depends upon the motion of the center of mass of each bunch.

For a beam made of a series of point charges, each equal to q, the fundamental component of the beam current is:

$$I_1 = \frac{2q}{T} \text{ Cos } (\omega t-\phi) = I_b \text{ Cos } (\omega t-\omega\zeta)$$

where $\zeta=\phi/\omega$ is the arrival time of the beam.

Longitudinal stability is obtained if at the arrival of the bunch ($t=\zeta$) the slope of the voltage is positive (or negative above the transition).

For this reason we calculate this slope as function of the phase of the beam. ($\phi=\omega t$)

By superposition we have:

$$\bar{V} = Z\ (\bar{I}_a + \bar{I}_1) = R_T \text{Cos}\ \Psi e^{-j\Psi}\ (\bar{I}_a + \bar{I}_1)$$

expanding:

$$\bar{V} = R_T \text{Cos}\Psi\ e^{-j\Psi}\ \{GmVge^{-j\frac{\pi}{2}} - I_b e^{-j\omega\zeta}\}$$

coming back to the function of time we have

$$V\ (t) = \text{Re}\ \{R_T \text{Cos}\Psi\ e^{-j\Psi}[GmVge^{-j\frac{\pi}{2}} - I_b e^{-j\omega\zeta}]e^{j\omega t}\}$$

where Re $\{\quad\}$ is the operator that takes the real part of a complex quantity and $\omega\zeta = \phi_o$ is the phase of the beam.

After a little algebra we obtain:

$$V\ (t) = R_T \text{Cos}\Psi\ \{GmVg\ \text{Sin}\ (\phi_o - \Psi) - I_b\ \text{Cos}\ (\phi - \phi_o - \Psi)\}$$

For $\phi = \phi_o$ $\quad V\ (\phi_o) = R_T \text{Cos}\Psi\ \{GmVg\ \text{Sin}\ (\phi_o - \Psi) - I_b \text{Cos}\ (\Psi)\};$

$$\dot{V}\ (\phi_o) = \frac{\partial V}{\partial \phi_o} \cdot \frac{d\phi_o}{dt} = R_T \text{Cos}\ \Psi\ GmVg\ \text{Cos}\ (\phi_o - \Psi)\ \frac{d\phi_o}{dt}$$

and the threshold of instability is reached when $\dot{V}\ (\phi_o) = 0$

Consequently the longitudinal stability is lost for:

$$\Psi = \phi_o - \pi/2$$

For compensating the beam loading we already made:

$$\omega C - \frac{1}{\omega L} = - \frac{I_b \text{Cos}\phi}{V_o}$$

And the tangent of the tuning angle is:

$$\text{TAN}\ \Psi = - \frac{R_T I_b \text{Cos}\phi}{Vo}$$

from the above condition it follows:

$$- \frac{R_T I_b \text{Cos}\phi}{Vo} \leqq \frac{\text{Sin}\ [\phi - \pi/2]}{\text{Cos}\ [\phi - \pi/2]}$$

expanding we obtain:

$$R_T \leqq \frac{V_o}{I_b \mathrm{Sin}\phi}$$

where R_T is the total shunt resistance seen by the beam when it crosses the gap.

For each gap of the Booster we have $V_o = 22.5\ 10^3$, $I_b = 7.91$, $\phi = 17°$ and we obtain:

$$R_T \leqq 9.7\ \mathrm{K}\Omega$$

We conclude that the cavity shunt impedance is low enough for taking care of the Robinson instability. This means that any configuration for the driving amplifier is acceptable because at the injection even an output impedance of the driver so high as 30 KΩ is enough for reducing below 9 KΩ the total shunt impedance of each gap.

From the above equation we have:

$$\frac{1}{R_T} = \frac{I_b \mathrm{Sin}\phi}{V_o}$$

Multiplying both sides by $\frac{V_o^2}{2}$ we obtain

$$\frac{V_o^2}{2R_T} = \frac{1}{2}\ V_o I_b \mathrm{Sin}\phi$$

and if R_T is the cavity shunt resistance than it follows that the power delivered to the beam is equal to the one wasted in the cavity.

Fortunately R_T is a parallel resistor. The cavity shunt resistance and the amplifier output resistance both contribute to the value of R_T.

This is the reason why a careful choice of the final amplifier can alleviate the condition set by the Robinson instability.

It is to be noted that the value of the gap capacitance do not play any role when the steady sinusoidal conditions are reached. Vice versa the value of the equivalent capacity of the cavity is determinant under transient conditions.