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Collider Accelerator Department

Brookhaven National Laboratory

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CHROMATICITY WINDOW FOR OPERATION OF THE AGS BOOSTER

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This is the summary of presentations given at the February 1987 (A.D.D.) Booster Meeting and the April 1987 (Physics Dept.) Accelerator Physics Seminar. It includes an overview of our analytic formalism (Z. Parsa, S. Tepikian and E. Courant) and some of our results for the Booster. In addition, comparison of our analytic results and those obtained from Tracking Programs PATRICIA (F. Dell) and ORBIT (G. Parzen) is presented (and are in good agreement).

THEORY

Following is a brief overview of our theoretical development with Second Order Perturbation Theory in two dimensions[1]. We can study the behavior of the beam, e.g. emittance growth, perturbation to tune, etc. for a system described by the Hamiltonian:

$$H = \frac{J_x}{\beta_x} + \frac{J_z}{\beta_z} + \sum_{k=3} \sum_{\ell=0}^{k} b_{k\ell}(s)\, \beta_x^{\ell/2}\, \beta_z^{(k-\ell)/2}\, J_x^{\ell/2}\, J_z^{(k-\ell)/2} \cos^{\ell}\phi_x \cos^{k-\ell}\phi_z \tag{1}$$

by using the Hamilton's equations and generating function:

$$G = K_x\phi_x + K_z\phi_z + \sum_k \frac{g_k(K_x, K_z, s)}{\sin\pi(n_{x_k}\nu_x + n_{z_k}\nu_z)} \cos(n_{x_k}\phi_x + n_{z_k}\phi_z + \xi_k). \tag{2}$$

Where $(J_x, J_z, \phi_x, \phi_z)$ and $(K_x, K_z, \Psi_x, \Psi_z)$ are the old and new action-angle variables respectively; $b_{k\ell}(s)$ are the generalized multipole strengths; $g_k(K_x, K_z, s)$ are the generating function resonance strengths (whose magnitude measures the extent to which J_x and J_z deviate from the invariants of the motion); n_x and n_z are integers (defining a given resonance) and Θ is the phase. With the angle variables

$$\Psi_x = \frac{\partial G(K_x, K_z, \phi_x, \phi_z, s)}{\partial K_x} \;;\; \Psi_z = \frac{\partial G(K_x, K_z, \phi_x, \phi_z, s)}{\partial K_z} \tag{3}$$

and the tune defined as

$$\nu_x = \frac{\Psi_x(C) - \Psi_x(o)}{2\pi}, \quad \nu_z = \frac{\Psi_z(C) - \Psi_z(o)}{2\pi} \tag{4}$$

we have obtained expressions for the perturbation to tune:

$$\nu_x = \overset{o}{\nu}_x + 2\ \alpha_{xx}K_x + \alpha_{xz}K_z + \ldots \tag{5}$$

$$\nu_z = \overset{o}{\nu}_z + 2\ \alpha_{xz}K_x + 2\ \alpha_{zz}K_z + \ldots \tag{6}$$

where $\overset{o}{\nu}_x$, $\overset{o}{\nu}_z$ are the unperturbed tunes; (α_{xx}, α_{xz} and α_{zz} coefficients are given in Reference 1). The amplitude dependence of the tune (due to sextupoles in the Booster) can be seen from Fig. 4, which also illustrates that with the perturbed tune and ordinary perturbation theory we obtain similar results to those obtained from superconvergent perturbation theory of the same order. (For further details refer to Reference 2.) Furthermore, the action variables and the emittance are deduced from the generating function given by Eq. (2), e.g.

$$J_x = \frac{\partial G}{\partial \phi_x} = \frac{E_x}{2\pi} ;$$

$$\text{and} \quad J_z = \frac{\partial G}{\partial \phi_z} = \frac{E_z}{2\pi}$$

$$E_x = 2\pi \left[K_x + \sum_k \frac{n_{x_k}\ g_k(K_x, K_z, s)}{\mathrm{Sin}\pi(n_{x_k}\ \nu_x + n_{z_k}\ \nu_z)} \right] \tag{8}$$

$$E_z = 2\pi \left[K_z + \sum_k \frac{n_{z_k}\ g_k\ (K_x, K_z, s)}{\mathrm{Sin}\pi(n_{x_k}\ \nu_x + n_{z_k}\ \nu_z)} \right] \tag{9}$$

From these analytic expressions for the emittance we can obtain similar results to those obtained from tracking the particles through the accelerator; thus providing an alternative to tracking programs. For example, the contribution of the $2\nu_x - 2\nu_x = 0$ resonance to the emittance growth, for the Booster, becomes

$$E_x = 2\pi \left[K_x + \frac{2b(\vec{K}, s)}{\sin \pi\delta} \cos\ (2\phi_x - 2\phi_z + \Theta) \right]$$

$$E_z = 2\pi \left[K_z + \frac{2b(\vec{K}, s)}{\sin \pi\delta} \cos\ (2\phi_x - 2\phi_z + \Theta) \right]$$

where the bandwidth $\delta = 2\nu_x - 2\nu_z$. However, to relate our analytic results with those obtained from tracking, one should consider the transient behavior of the beam at injection which determines K_x and K_z and the perturbed tunes in the acclerator.

Transient Behavior

The transient behavior (i.e. initial conditions used) can be described by:

$$E_x(s=0)=E_x\ (K_x,\ K_z,\ \phi_x(s=0),\ \phi_z(s=0),\ s = 0)$$
$$E_z(s=0)=E_z\ (K_x,\ K_z,\ \phi_x(s=0),\ \phi_z(s=0),\ s = 0)$$

Depending on the choice of the initial phase ($\phi_x(s=0)$ and $\phi_z(s=0)$), the resulting invariants K_x and K_z which corresponds to the average emittance ($\langle E_x \rangle$ and $\langle E_z \rangle$), may not equal to the initial emittance, ($E_x(s=0)$ and $E_z(s=0)$). If the coupling resonance $2\nu_x-2\nu_z=0$ is strong enough, then $\langle E_x \rangle$ and $\langle E_z \rangle$ could change greatly with a change in $\phi_x(s=0)$ and $\phi_z(s=0)$. Thus, it will be easy to find a particle such that the perturbed tunes ν_x, ν_z are on a resonance.

In tracking several particles are used, the particle that gives the smallest bandwidth ($2\nu_x-2\nu_z$) will lead to the greatest emittance growth due to the coupling. Thus, many particles will excite the coupling resonance while at the same time many others will not. The number of particles that excite this resonance depends on how large are the coefficients α_{xx}, α_{xz}, α_{zz} (see eq. 5 & 6) as well as how large is the $2\nu_x-2\nu_z$ resonance strength. At chromaticities $c_x=c_z=-5$, both the resonance strength and α's are minimum; they grow quite rapidly as the chromaticities are changed from these values (for the AGS-Booster).

Fig. 1 shows the maximum emittance (in x or z direction) obtained from tracking[3], and the maximum total emittance obtained analytically, as functions of the Booster chromaticity respectively. (We note that one particle was used analytically and four particles were used in tracking.) In region I, variation of the perturbed tunes (due to initial $\phi_x(s=0)$, $\phi_z(s=0)$) are small and do not cross the $2\nu_x - 2\nu_z = 0$ resonance for any particles. In region II and III, variation of the perturbed tune, (due to different initial phases $\phi_x(s=0)$ and $\phi_z(s=0)$), becomes large, allowing the crossing of $2\nu_x-2\nu_z=0$ resonance for some of the particles.

Fig. 2 shows the average emittance as a function of chromaticity of the Booster, assuming that some of the average emittances is $\leq 100\pi$ mm-mrad. In Region I, for $0<c_x=c_z<2$ the strength of this resonance is very strong allowing the average $\langle E_x \rangle + \langle E_z \rangle$ to be $>100\pi$ mm-mrad, leading to a coupling as observed by tracking. However, there exist a chromaticity window between $-8<c_x=c_z<2$ where there is no $2\nu_x-2\nu_z=0$ coupling. The size of this window will increase if you decrease the total emittance which in turn decreases the total average emittance. The observed window is slightly smaller than shown in Fig. 2, because the actual sum of the averaged emittances are greater than 100 as illustrated above. This indicates that to increase the size of the window, either we decrease the total initial emittance and/or increase the number of sextupoles per superperiod.

Fig. 3 illustrates the decrease in the total initial emittance from 100 to 50π mm-mrad; and the corresponding increase in the size of the chromaticity window (to $-9<c_x=c_z<3$).

We illustrate some of our results for the AGS-Booster for various chromaticities in the following table. Our results were obtained using canonical perturbation theory with perturbed tunes; which are similar to those produced from superconvergent perturbation theory[2]. Since when the perturbed tunes are on resonance (e.g. $2\nu_x-2\nu_z=0$ resonance) ordinary perturbation theory (with linear tune) would not see the resonance conditions whereas the superconvergent perturbation theory would.

Conclusion

Our investigation of the structure resonances (which are excited due to the eddy currents and chromaticity sextupoles) shows that the 4^{th} order resonances may be crossed in light of the space charge tune shift at injection. Depending on the size of this tune shift the third and 6^{th} order resonances may have to be considered. We have calculated the strengths, bandwidths, fixed points, island width, etc. for these resonances.
Our analytical results agrees quite well with those obtained from tracking programs, if the transient behavior, (initial conditions) is considered. Additionally, we showed the existence of a chromaticity window for the Booster, where there is no $2\nu_x-2\nu_z$ coupling. The size of this window would increase if either we decrease the total initial emittance and/or increase the number of the sextupoles per superperiod. Finally, if the off momentum behavior of the particles in the beam is reasonable we predict a smaller aperture at chromaticity of -5 to -2 for the Booster. (Further analytical and tracking studies are in progress.)

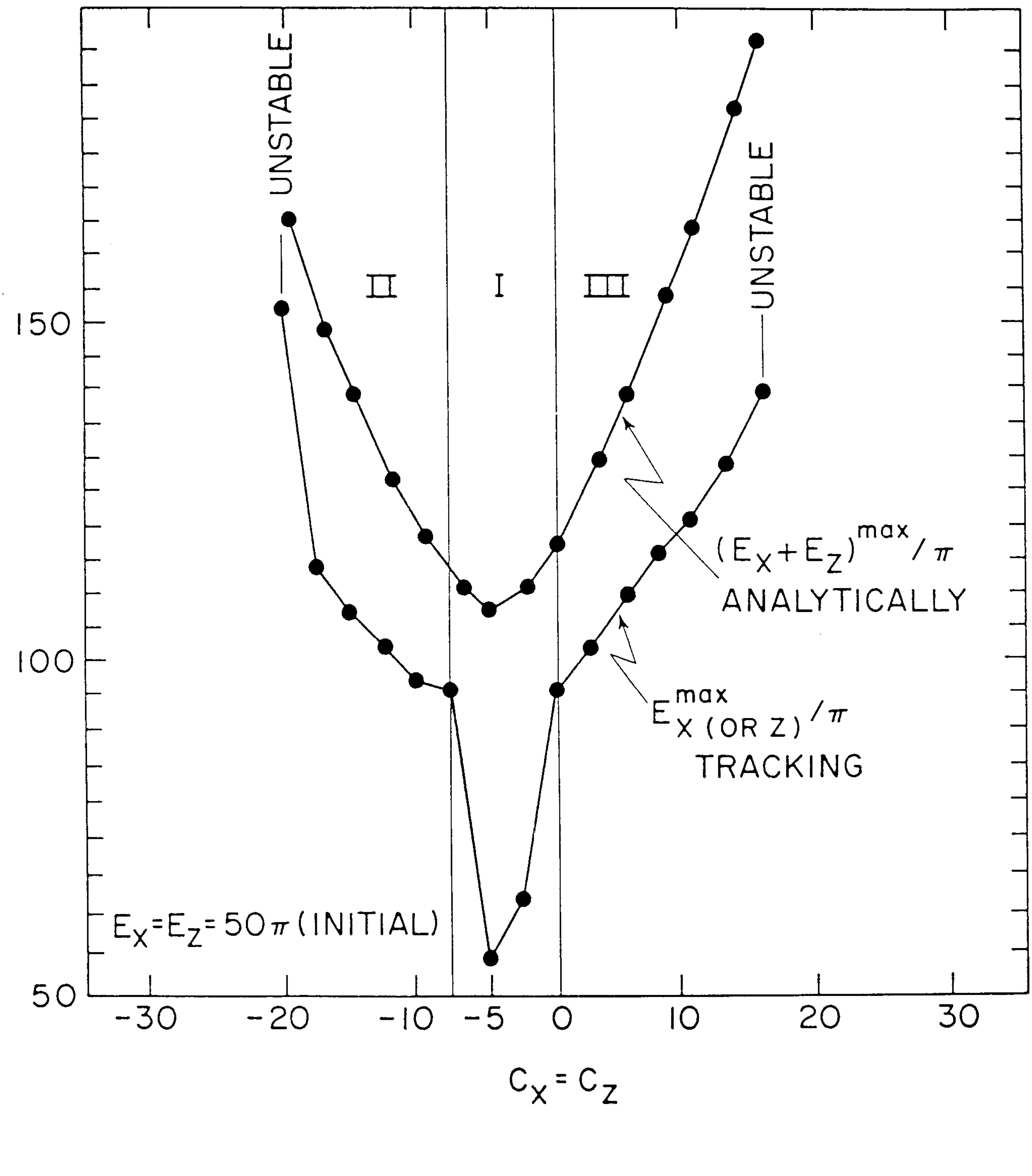
Reference

1. Z. Parsa, S. Tepikian, E. Courant, Second Order Perturbation Theory for Accelerators, BNL-39262
2. Z. Parsa, Proceedings of IEEE March 15-19, 1987, BNL-39450, BNL-39451, BNL-39449
3. Obtained from tracking (with four particles) using program PATRICIA (F. Dell). Similar tracking was done by G. Parzen using program ORBIT. The tracking results obtained by Dell and Parzen agreed qualitatively.
4. Z. Parsa, Booster Parameter List, BNL-39311; and Booster Design Manual

TABLE I

Chrom	Bndwdth 2Vx-2Vz	Max Et/pi mm-mrad	Gen. Fun. Res. Str.	Hamil. Res. Str.
-6.0000000	-.0156880	108.9210000	.0370876	.0215430
-5.5000000	-.0172100	108.2140000	.0370590	.0212390
-5.0000000	-.0186480	108.1590000	.0383278	.0221530
-2.0000000	-.0254620	111.8600000	.0984171	.0762360
0	-.0282800	118.8650000	.1999800	.1658900

Chrom	Stop Bndwdth	Fixed Points	Island Width	Chirikov Criter.
-6.0000000	.0017235	.0010516	6.6580000	.0004030
-5.5000000	.0016991	.0010728	8.0864000	.0003271
-5.0000000	.0017723	.0010686	10.4350000	.0002644
-2.0000000	.0060989	.0004842	15.4580000	.0006143
0	.0132710	.0003068	11.7630000	.0017565



UNSTABLE
UNSTABLE
II
I
III
150
100
50
-30
-20
-10
-5
0
10
20
30
$(E_X+E_Z)^{max}/\pi$
ANALYTICALLY
$E^{max}_{X\,(OR\,Z)}/\pi$
TRACKING
$E_X=E_Z=50\pi$ (INITIAL)
$C_X=C_Z$

Figure 1

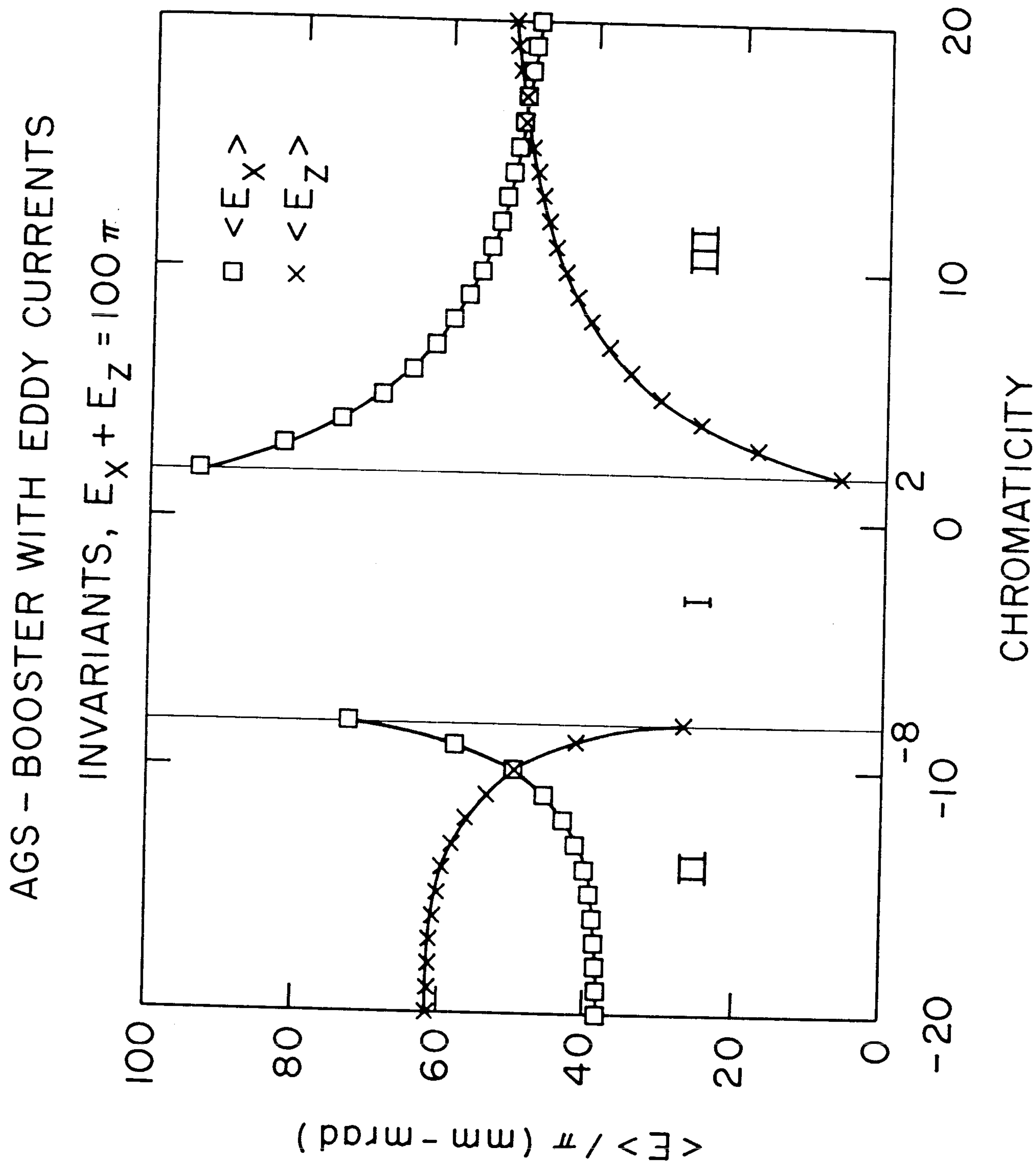

AGS - BOOSTER WITH EDDY CURRENTS
INVARIANTS, $E_X + E_Z = 100\pi$
$\square$ $\langle E_X \rangle$
$\times$ $\langle E_Z \rangle$
I
II
III
$\langle E \rangle / \pi$ (mm - mrad)
CHROMATICITY
100
80
60
40
20
0
-20
-10
-8
0
2
10
20

Figure 2

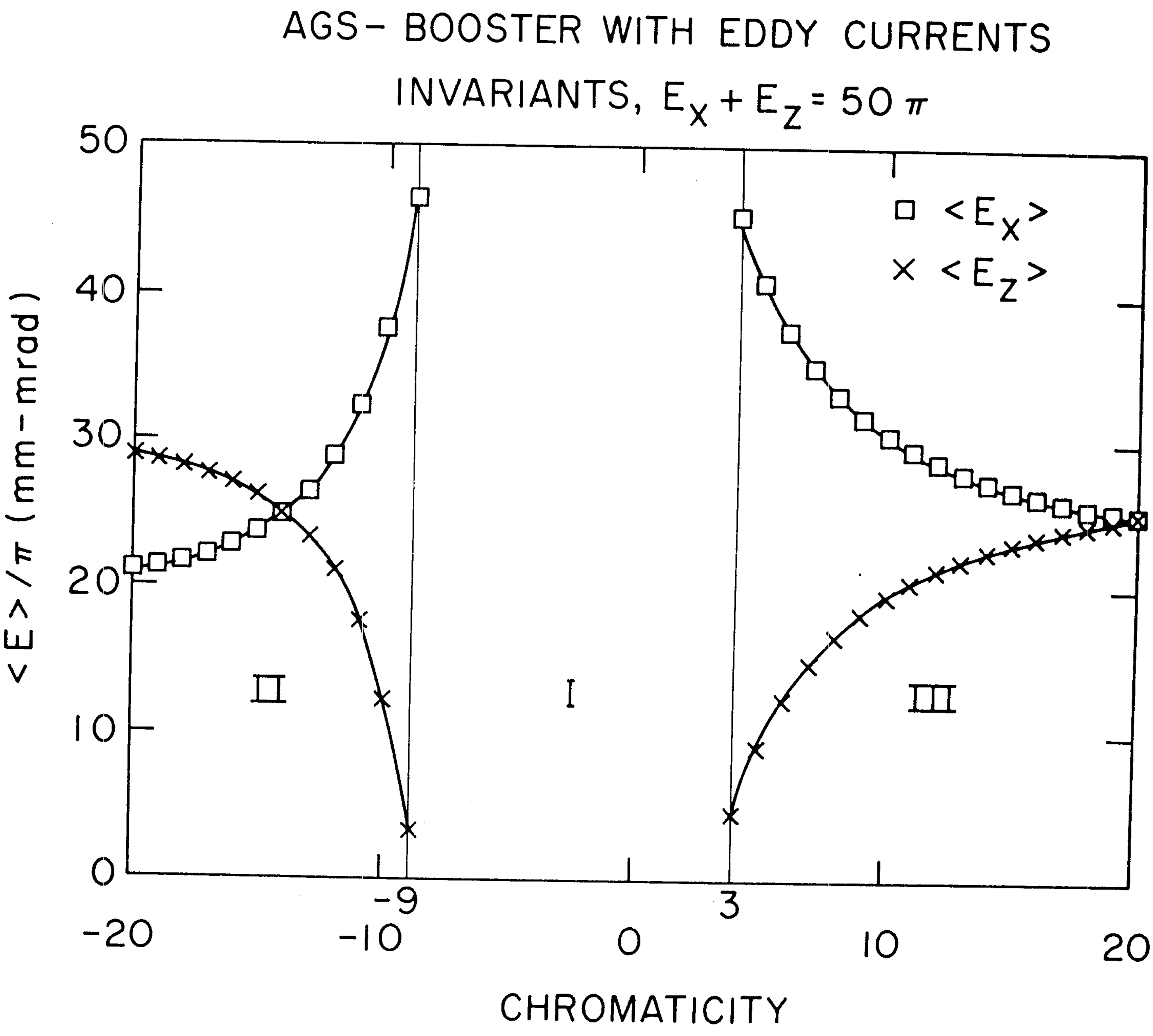

AGS- BOOSTER WITH EDDY CURRENTS
INVARIANTS, $E_X + E_Z = 50\pi$
$\langle E \rangle / \pi$ (mm-mrad)
50
40
30
20
10
0
-20
-10
-9
0
3
10
20
CHROMATICITY
$\langle E_X \rangle$
$\langle E_Z \rangle$
II
I
III

Figure 3