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Overview of the structure resonances in the AGS-Booster lattices

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OVERVIEW OF THE STRUCTURE RESONANCES IN THE AGS - BOOSTER LATTICES

Booster Technical Note

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ABSTRACT

WE HAVE INVESTIGATED THE NONLINEAR EFFECTS OF THE STANDARD BOOSTER LATTICE AND SIX PROPOSED ALTERNATE LATTICES FOR THE BOOSTER. IN THIS NOTE, WE WILL GIVE A BRIEF THEORETICAL REVIEW AND AN OVERVIEW OF OUR RESULTS. RESONANCES ARE CROSSED AT INJECTION BECAUSE OF THE SPACE CHARGE TUNE SHIFTS. THE STOP BANDWIDTHS AND RESONANCE STRENGTHS AT THE TUNES NEAR THE RESONANCES ARE CALCULATED AND TABULATED. TUNE DIAGRAMS FOR THE STRUCTURE RESONANCES OF THESE LATTICES ARE INCLUDED.

OVERVIEW

The standard - AGS Booster lattice and its alternates can be classified into two groups. Those with 5 meter straight sections and those without. The selection of an appropriate lattice depends on the choice of the injection and ejection schemes, as well as how the nonlinearities affects the beam dynamics. In this note we have tabulated the structure resonances up to and including the fourth order resonances.

We have calculated the normalized resonance strengths and stop bandwidths for the 7 proposed Booster lattices (two of which have 5 meters straight sections) shown in the tables I -VIII and illustrated in Figs. 1-4. In the following sections each lattice is discussed:

(I) In the standard AGS - Booster lattice (see Table I) we see that three fourth order resonances can be crossed (besides remaining near the $2Qx - 2Qy = 0$ resonance) due to space charge tune shifts. If these resonances can be tuned out then this lattice should not be troublesome. The third order resonance at $Qx = 4$ and/ or $Qy = 4$ is found to be an order of magnitude smaller than the Combined function (V) and Hybrid (VI) lattices.

(II) If the tunes of the standard lattice are split such as $Qx = 3.82$ and $Qy = 4.83$, then we find we must cross a third order resonance at tunes $Qx = 3.333..$ and $Qy = 4.333...$ which is quite strong (see Table II). Besides the third order resonance there is a strong second order resonance at $Qx = 3.$ and $Qy = 4.$ which appears very troublesome.

(III) Splitting the tunes of the standard lattice in the opposite way, for instance $Qx = 4.82$ and $Qy = 3.83$ leads to the same problems as lattice given in (II) (see Table III)

(IV) An alternate booster lattice with a periodicity of 8 short and long straight sections and split tunes of $Qx = 4.82$ and $Qy = 5.83$ is found to have similar troubles as the other split tune lattices considered here (see Table IV).

(V) Using combined function magnets a lattice with five meter straight sections and periodicity of 12 is proposed. We find that at the operating tunes of Qx = 4.82 and Qy = 4.83 the third order resonance (when Qx = 4. and Qy = 4.) can be important depending on the amount of space charge tune shift. Although no fourth order resonance is crossed, the third order resonances in this lattice is an order of magnitude greater than the standard booster given in (I) (see Table V)

(VI) A Hybrid lattice with quadrupoles and combined function magnets was also proposed (very similar to (V)) also has strong third order resonances at Qx = 4. and Qy = 4. (see table VI).

(VII) The 1/3 AGS booster lattice at the operating tunes of Qx = 6.82 and Qy =6.83 doesn't cross any structure resonances of up to (and including) fourth order in space charge tune shift range.

(VIII) Operating the 1/3 AGS booster lattice at the tunes Qx = 5.82 and Qy = 5.83 can cause the beam to cross a third order resonance at Qx = 5.333... and Qy = 5.333... which can be troublesome (see table VIII).

THEORY SECTION

To study the dynamics of an accelerator with chromaticity correcting sextupoles and eddy current sextupoles, we have used the following Hamiltonian [1]:

$$H0= \frac{1}{2} (p_x^{\,2}+ p_y^{\,2}) + (\frac{1}{\rho^2} - K(s))\frac{x^2}{2} + K(s) \frac{y^2}{2} + \frac{S(s)}{6} (x^3 - 3xy^2) \qquad (1)$$

where, x and y are the positions, p_x and p_y are the conjugate momentum, K is the quadrupole focussing strength, S is the sextupole strength, and s is the distance along the orbit (the "time" variable for the Hamiltonian).

First we transform the Hamiltonian to action - angle form:

$$H1= \frac{2\pi}{C} (Q_x J_x + Q_y J_y) + V(J_x, J_y, \phi_x, \phi_y, s) \qquad (2)$$

where, C is the circumference; Jx, Jy are the action variables (directly related to the beam emittances, $2Jy=Ey/\pi$); ϕx, ϕy are the angle variables and Qx, Qy are the betatron tunes.

When the system is operating near a resonance, perturbation theory no longer holds. Although, an approximate invariant can be found for system near a single resonance if the contribution from the other resonances are small. This can lead to the Hamiltonian

$$H2= \frac{2\pi}{C} e K_x + \sum_{\ell=1} W_\ell K_x^{\frac{\ell}{2}+1} + A \cos \Psi_X \qquad (3)$$

where, e is the bandwidth given in eq. (4); Kx and Ky are the new action variables and ψ_x is the transformed angle. W_ℓ are the stablizing coefficients and A is the resonance strength. This Hamiltonian is an invariant of the motion.

$$e = n_x Qx + n_y Qy - p \qquad (4)$$

where the integers n_x, n_y and p define a given resonance.

Following Guignard [5], dynamic properties (such as stop bandwidths, Δe) can be found from invariant of the motion H2. The stop bandwidths (Δe) are defined to be the smallest bandwidth such that the action in Eq. (3) is still bounded. This can be done by first considering the fixed points of Eq. (3) which are defined to be the points at which there is no motion. These fixed points are the solutions to the following equations (note that Ky is also an invariant of the motion and can be treated as a constant).

$$0 = \frac{d\psi_x}{ds} = \frac{\partial H_2}{\partial K_x} \tag{5a}$$

$$= \frac{2\pi}{C} e + \sum_{\ell=1} \left(\frac{\ell}{2} + 1 \right) W_\ell K_x^{\frac{\ell}{2}} + \frac{\partial A}{\partial K_x} \cos\psi_x$$

$$0 = \frac{dK_x}{ds} = - \frac{\partial H_2}{\partial \psi_x} = A \sin\psi_x \tag{5b}$$

There are many solutions [$\psi_x = n\pi$] to eq. (5b) which lead to two cases [$\cos(\psi_x) = \pm 1$] in eq. (5a). We note that for the smallest positive value of Kx there is at most one solution for each of these cases corresponding to stable and /or unstable fixed point(s). The nature of these solutions are determined by the bandwidth (e) and the stabilizing coefficients (W_ℓ).

We find the stop bandwidths for these two cases by substituting $\cos(\psi_x) = \pm 1$ into eq. (3) and obtaining the following two new equations:

$$F_\pm = \frac{2\pi}{C} e K_x + \sum_{\ell=1} W_\ell K_x^{\frac{\ell}{2}+1} \pm A \tag{6}$$

For the given initial conditions, four equations can be deduced from Eq. (6) from which the stop bandwidths and the extreme values of Kx are found. These equations are listed below:

$$
\begin{aligned}
0 &= F_+(K_{x_0}) - F_+(K_x) \\
0 &= F_+(K_{x_0}) - F_-(K_x) \\
0 &= F_-(K_{x_0}) - F_+(K_x) \\
0 &= F_-(K_{x_0}) - F_-(K_x)
\end{aligned}
\qquad (7)
$$

For $e < \Delta e$ Eqs. (7) can be satisfied for any value of action K, where Δe is the stop bandwidth. Hence, the stop bandwidth (derived) for a sum resonance is

$$\Delta e = A \left(\frac{n_x^2}{E_x} + \frac{n_y^2}{E_y} \right) \qquad (8á)$$

and for a difference resonance is

$$\Delta e = A \left(\frac{|n_x|}{E_x} + \frac{|n_y|}{E_y} \right) \qquad (8b)$$

We also note that, Eqs. (8) [5] calculates the necessary distance (δe) between the operating tunes (Qx, Qy) and the resonance line which must be kept to avoid or limit the relative growth of amplitude or beating of a single particle in a given interval $\Lambda = [(Kx/Kxo)^{1/2} - 1]$. Where for a sum resonance

$$\delta e \geqq \frac{\Delta e}{2} \left(1 + \frac{1}{\Lambda} \; \frac{n_x E_y + n_y E_x}{n_x^2 E_y + n_y^2 E_x} \right) \qquad (9a)$$

and for difference resonance

$$\delta e \geqq \frac{\Delta e}{2} \frac{1}{\Lambda} \qquad (9b)$$

n_x, n_y are the integers (see eq. (4)) defining a given resonance and Ex and Ey are the emittances of the beam at injection [directly related to Kx and Ky].

In the following sections we tabulate our results including the stop bandwidths (defined above) calculated for the alternate lattices [3,4,7,8] using program HARMON [5].

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6. G. Guignard, General treatment of Resonances in Accelerators, Cern 78-11 (Nov. 10, 1978)

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9. The sixth order resonances are important and we will discuss them in subsequent papers using program NONLIN [4] (since HARMON is not equipped to do so). We have used B" = .24 T/m for the eddy current sextupoles.

STANDARD AGS - BOOSTER LATTICE

I. WITH OPERATING TUNES OF [Qx=4.82, Qy=4.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE

RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
3Qx=12	.15164	.022558	4.01	4.11
Qx+2Qy=12	.92329	.076304	4.01	4.11
4Qx=18	.15893	.00041929	4.501	4.511
2Qx+2Qy=18	1.3375	.0017643	4.501	4.511
4Qy=18	.80881	.0021338	4.501	4.511
2Qx-2Qy=0	.18614	******	4.501	4.511

II. WITH SPLIT TUNES [Qx=3.82, Qy=4.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE

RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
2Qx=6	1.4689	5.8755	3.01	4.02
Qx+2Qy=12	.86941	.071851	3.33	4.34
4Qy=18	.62432	.0016471	3.501	4.502
4Qx=12	21.176	.055869	3.01	4.02

III. WITH SPLIT TUNES [Qx=4.82, Qy=3.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE

RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
2Qy=6	4.4052	17.621	4.01	3.02
3Qx=12	.16263	.0241193	4.01	3.02
Qx+2Qy=12	.97337	.080443	4.67	3.68
4Qx=18	.13438	.00035452	4.501	3.511
4Qy=12	.38924	.0010269	4.01	3.02

IV. ALTERNATE SEP.FUNCT. LATTICE WITH P = 8,
SHORT AND LONG STRAIGHT SECTINS,
WITH SPLIT TUNES [Qx=4.82, Qy=5.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE				
RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
2QX=18	2.3842	9.5366	4.01	5.02
QX+2QY=16	6.7644	.55904	4.67	5.68
4QX=16	6.8937	.018187	4.01	5.02

V. ALTERNATE COMB.FUNCT. LATTICE WITH P = 12,
WITH OPERATING TUNES [Qx=4.82, Qy=4.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE				
RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
3Qx=12	1.3185	.19615	4.01	4.11
Qx+2Qy=12	6.6807	.55212	4.01	4.11
2Qx-2Qy=0	.050384	****	4.01	4.11
2Qx-2Qy=0	1.2853	****	4.82	4.83

VI. ALTERNATE HYBRID LATTICE WITH P = 12,
WITH OPERATING TUNES [Qx=4.82, Qy=4.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE				
RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
3Qx=12	1.6643	.24758	4.01	4.11
Qx+2Qy=12	7.8122	.64563	4.01	4.11
2Qx-2Qy=0	.16205	****	4.01	4.11
2Qx-2Qy=0	1.1821	****	4.82	4.83

VII. ALTERNATE 1/3 AGS SEP.FUNCT. LATTICE
WITH P = 8
AND OPERATING TUNES OF [Qx=6.82, Qy=6.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE				
RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
2Qx-2Qy=0	.27135	*****	6.82	6.83

[NO RESONANCES OF UP TO FOURTH ORDER ARE CROSSED DUE TO SPACE CHARGE]

VIII. ALTERNATE 1/3 AGS SEP.FUNCT. LATTICE
WITH P = 8
AND OPERATING TUNES OF [Qx=5.82, Qy=5.83]

RESONANCES (IMPORTANT) DUE TO THE SPACE CHARGE				
RESONANCES	RESONANCE STRENGTH	STOP BANDWIDTH	Qx	Qy
3Qx=16	.42417	.063099	5.34	5.35
Qx+2Qy=16	.92395	.076358	5.34	5.35
2Qx-2Qy=0	.35978	*****	5.34	5.35

PERIODICITY = 6

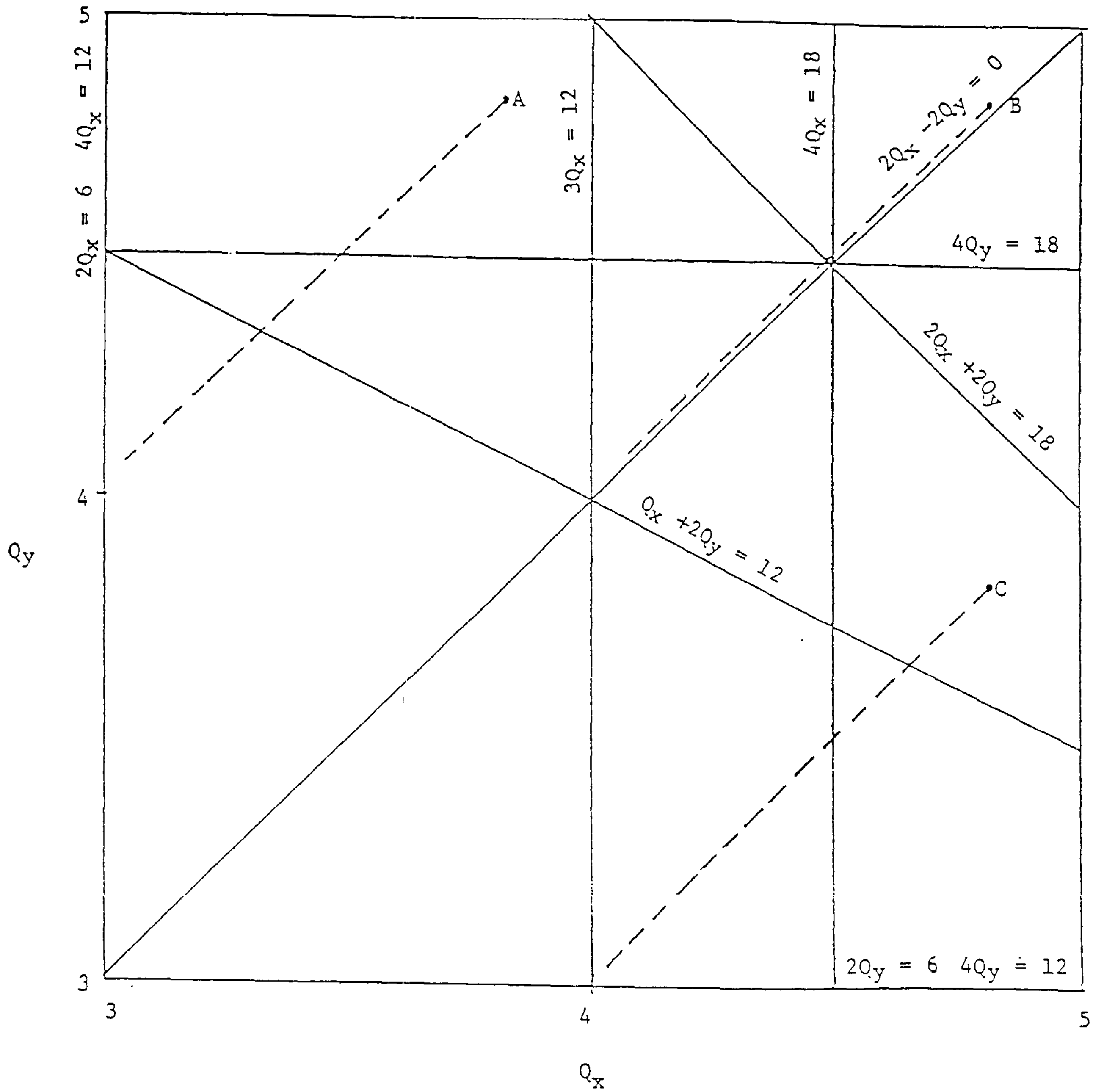

Fig. 1 The standard AGS - Booster lattice at three different operating points: C = (4.82,3.83), B = (4.82,4.83) and A = (3.82,4.83). The dotted lines give the tune shift due to space charge.

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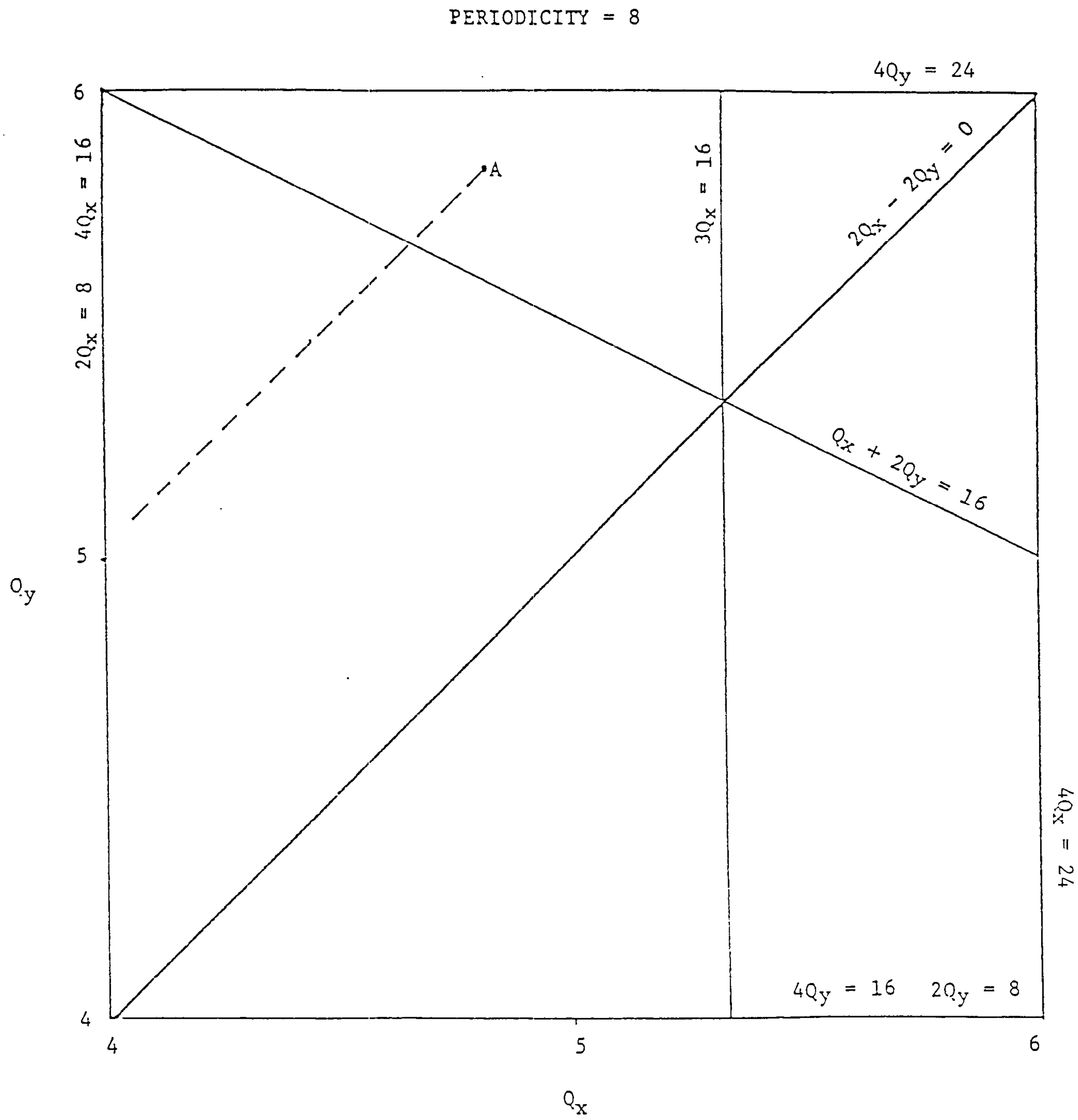

Fig. 2 Tune diagram for the 8 periodicity lattice with long and short straight sections. The point A is the operating tunes of Qx = 4.82 and Qy = 5.83. The tune shift due to space charge is shown in the dotted line.

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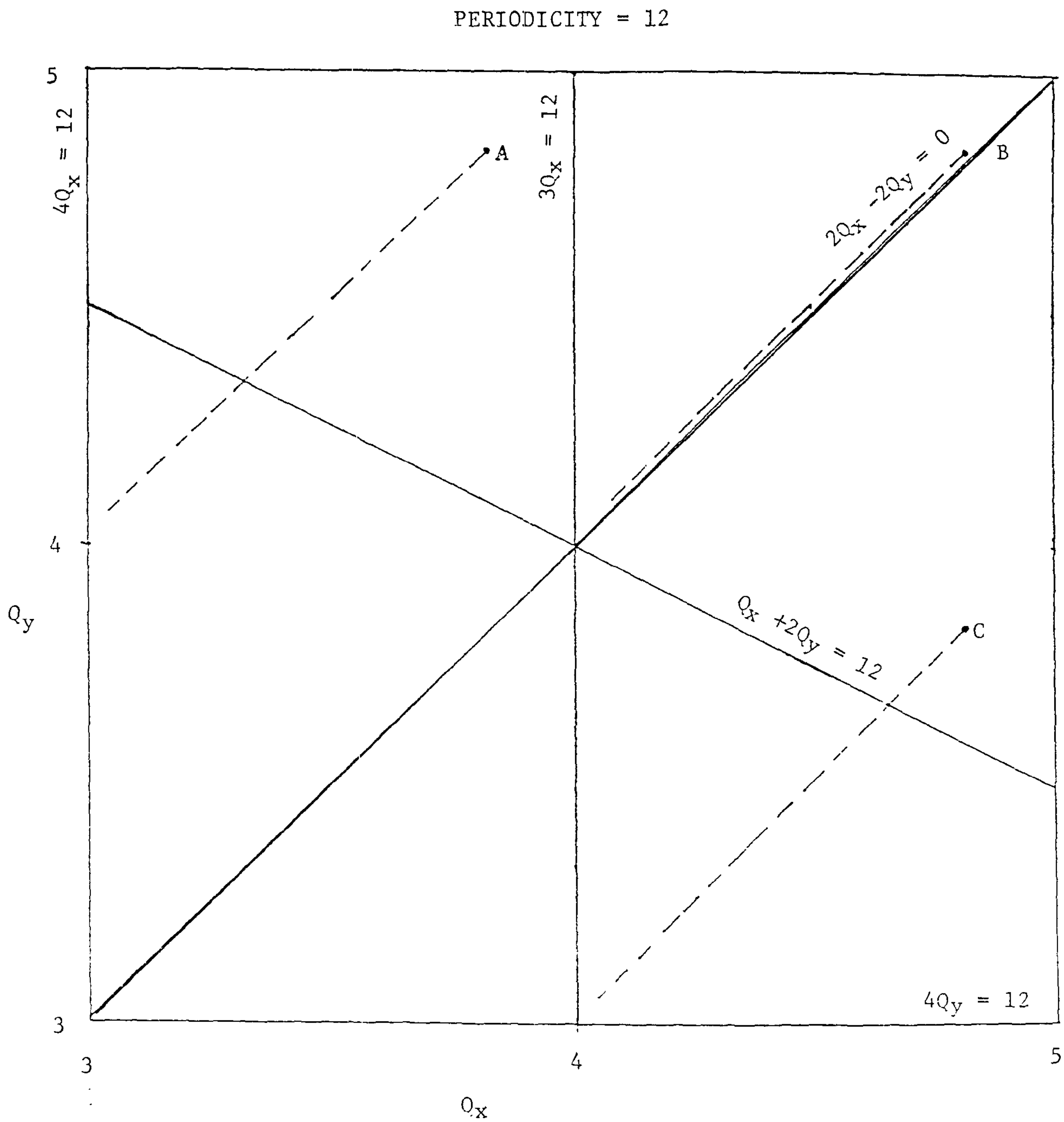

Fig. 3 The Combined function lattice and Hybrid lattice at three different operating tunes: A = (3.82,4.83), B = (4.82,4.83), and C = (4.82,3.83). Note, the split tune cases (A and C), not given in the tables, cross a third order resonance.

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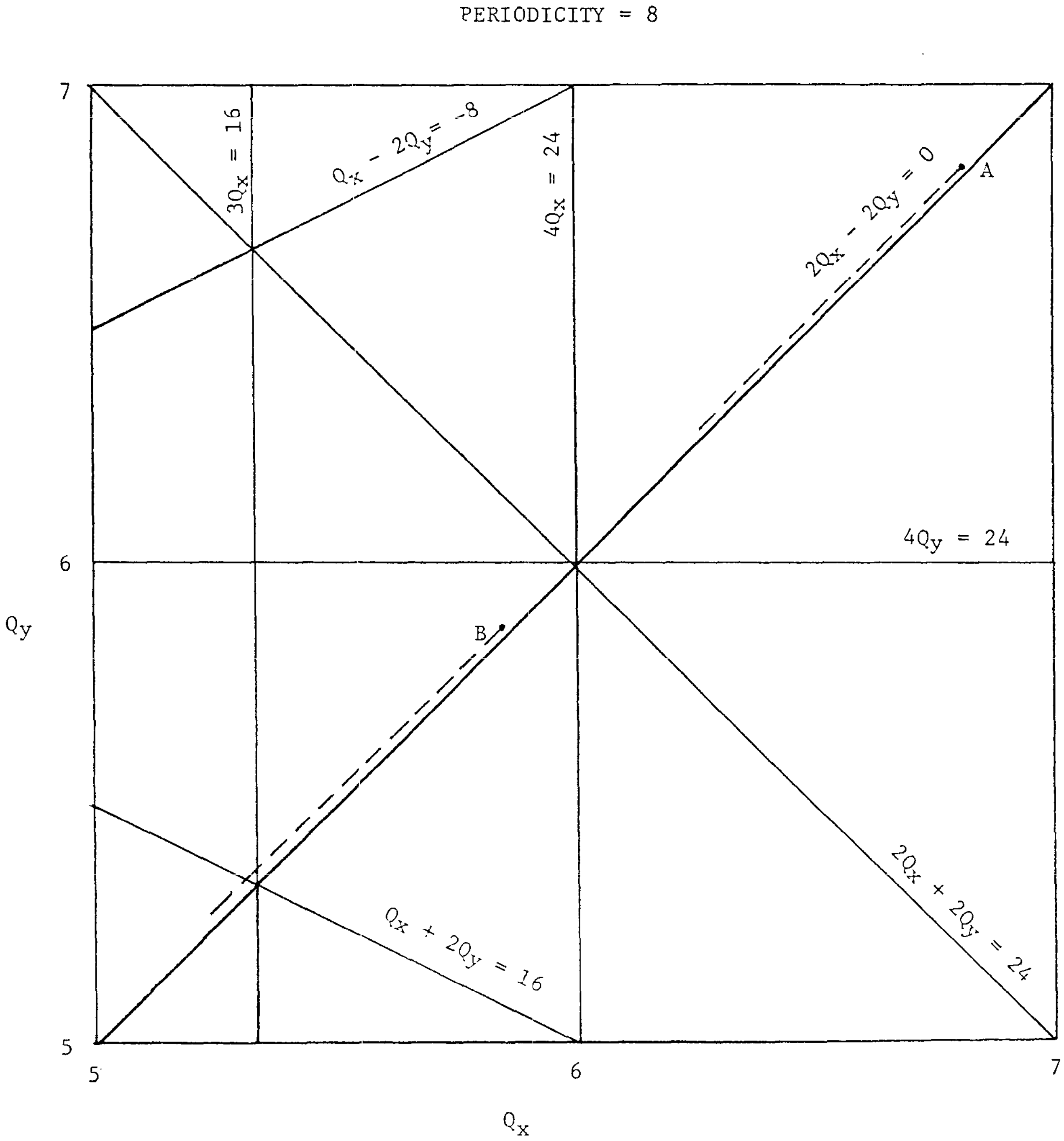

Fig. 4 Tune diagram for 1/3 AGS booster lattice with two operating points: A = (6.82,6.83), B = (5.82,5.83). The dotted line gives the tune shift due to space charge.