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COMPARISON OF ON AND OFF-DIAGONAL WORKING POINTS FOR THE AGS SEPARATED FUNCTION BOOSTER

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Abstract

Tracking studies for the standard separated function AGS Booster lattice with tunes close to the principal diagonal (ν_x, ν_y) = (4.820,4.830) show evidence of strong coupling. In the current study operation at an off-diagonal tune (4.820,5.830) gives evidence of reduced coupling and consequently reduced beam size. This impacts Booster parameters such as injection schemes and required vertical aperture of the vacuum chamber.

Normal operation

Tracking studies on the Booster for tunes near the principal diagonal give evidence of coupling that produces repeated growth and decay of betatron amplitudes in both transverse planes. This growth enlarges the beam size sufficiently that the maximum vertical size of the vacuum chamber (33 mm) is not sufficient to accomodate a beam having the nominal emittances of $\epsilon_{\circ x} = \epsilon_{\circ y} = 50\pi$ mm mrad.[2] In the current study the working point has been shifted from the standard tune of (4.820,4.830) to (4.820,5.830). The pattern of resonance lines is the same at the two operating points - Fig.1, but the displacement moves the working point away from the $2\nu_x - 2\nu_y = 0$ resonance along the principal diagonal.[3]

In all studies the particles were tracked for 1000 turns, and multipoles due to eddy currents at injection were included.[4] A comparison of the phase plots for the two working points is found in Fig.2(a) and Fig.2(b). It is seen that the radial thickness of the phase plots, a measure of coupling, is less for the off-diagonal working point.

At any time the emittance should be described by the function:

$$\epsilon_z = (z^2 + (\alpha_z z + \beta_z z')^2)/ \beta_z \qquad \text{with } z = x \text{ or } y.$$

The ratio $(\epsilon_z/\epsilon_{\circ z})^{1/2}$ is a measure of the change of beam size from its initial size, and plots of $(\epsilon_y/\epsilon_{\circ y})^{1/2}$ versus $(\epsilon_x/\epsilon_{\circ x})^{1/2}$ give a measure of the particle's behavior in the four dimensions of transverse phase space.[5]

Such plots are shown in Fig.3 for the two working points (4.820,4.830) and (4.820,5.830). The negative slope of the shaded area of Fig.3(a) indicates coupling, while the range of the shaded area is an indication of the size of the coupling.

The values of ϵ_x and ϵ_y can be printed out at each turn during the tracking run. Maximum $\hat{\epsilon}$ and minimum $\check{\epsilon}$ values of ϵ_x and ϵ_y as well as $(\hat{\epsilon} - \epsilon_\circ)/\epsilon_\circ$ and $(\check{\epsilon} - \epsilon_\circ)/\epsilon_\circ$ are tabulated in Table 1 for the first 100 turns of a 1000 turn run. There is a slight indication that $\hat{\epsilon}$ may be growing during the 1000 turn run, but such growth is too small to see on the phase plots. As the beam size varies as $(1 + (\epsilon - \epsilon_\circ/\epsilon_\circ))^{1/2}$, the data of Table 1 indicate a 33% increase in beam size at (4.820,4.830) compared to a 12% increase of beam size due to coupling at (4.820,5.830). Thus operation at the off-diagonal working point seems preferable to that at the on-diagonal working point.

ν_y	$\hat{\epsilon}_x$	$\check{\epsilon}_x$	$(\hat{\epsilon}_x - \epsilon_\circ)/\epsilon_\circ$	$(\check{\epsilon}_x - \epsilon_\circ)/\epsilon_\circ$	$\hat{\epsilon}_y$	$\check{\epsilon}_y$	$(\hat{\epsilon}_y - \epsilon_\circ)/\epsilon_\circ$	$(\check{\epsilon}_y - \epsilon_\circ)/\epsilon_\circ$
4.83	53.3	11.8	0.066	-0.764	90.0	47.0	0.800	-0.060
5.83	61.6	50.6	0.232	0.012	57.9	37.1	0.158	-0.258

Table 1 Relative emittance growth for x and y motion at working points (4.820,4.830) and (4.820,5.830). $\hat{\epsilon}$, $\check{\epsilon}$, and $\epsilon_\circ$ denote maximum, minimum, and initial emittances, respectively in the x and y planes.

Injection

During proton injection the closed orbit will be displaced inwards (perhaps as much as 75 mm) in a Q5 quadrupole, and bunches having emittances of $\epsilon_x = \epsilon_y = 10\pi$ mm mrad will be injected as the closed orbit is moved outwards towards its equilibrium position. The first bunch will be injected onto the closed orbit, but successive bunches will have increasingly larger displacements from the closed orbit. This displacement increases the effective betatron amplitude and therefore the horizontal emittance of the bunch. Tracking has been done at both tunes (4.820,4.830) and (4.820,5.830) for particles with fixed vertical emittance $\epsilon_y = 10\pi$ mm mrad and with horizontal emittances corresponding to several positions of the closed orbit.

The maximum vertical emittance ϵ_y during the first 100 turns of a 1000 turn tracking run is plotted in Fig.4 for several values of the distance betreen the closed orbit and the injection channel.

The maximum vertical emittance $\hat{\epsilon}_y$ during the first 100 turns of a 1000 turn tracking run is plotted in Fig.4 for several values of the distance between the closed orbit and the injection channel. (The emittance coupling has a period of $\sim$ 28 turns for emittance transfer back and forth from one plane to the other.) For the tune (4.820,4.830), the horizontal emittance is coupled strongly to the vertical emittance and results in the vertical size of the beam increasing rapidly to the point where it exceeds the height of the vacuum chamber. From Fig.4 it is seen that the vertical emittance grows from 10π mm mrad to 50π mm mrad when the distance from the injection channel to the closed orbit is 35 mm. With $\epsilon_x = X^2/\beta_x$ and X = 35mm and β_x = 13.6 m, $\epsilon_x = 90\pi$ mm mrad. Hence with initial emittances of $\epsilon_x = 90\pi$ mm mrad and $\epsilon_y = 10\pi$ mm mrad, the beam becomes round at the (4.820,4.830) working point.

The situation for the tune (4.820,5.830) is much more favorable. In this case the vertical emittance grows slowly as the horizontal emittance is increased, and it remains near its initial value of 10π mm mrad. Even for injection with a 50 mm separation between the closed orbit and the injection channel ($\epsilon_x = 200\ \pi$ mm mrad), the vertical emittance is less than 15π mm mrad. If the beam is ever to become round and the vertical aperture is limited to 50π mm mrad, then $\epsilon_{\circ x} + \epsilon_{\circ y} \leq 100\pi$ mm mrad, and displacement of the closed orbit by 75 mm for injection is not necessary. The requirement that there be a $\pm$ one inch vertical aperture at three inches from the centerline of the vacuum chamber could be relaxed. On the otherhand, if the beam maintains a cross section with the shape of a flattened ellipse, locating the injection channel as far as 75 mm from the equilibrium orbit could have some advantages. However for this scenario to have value, the vertical beam size would have to remain small, and again the requirement of a $\pm$ 1 inch vertical aperture at three inches from the chamber centerline could be relaxed.

Conclusions

1). Tracking of a round beam having $\epsilon_{\circ x} = \epsilon_{\circ y} = 50\pi$ mm mrad indicates that operation at the off-diagonal working point (4.820,5.830) produces less coupling than does operation at (4.820,4.830).

2). Studies for injection with $\epsilon_{cy} = 10\pi$ mm mrad and 11.7 mm $\leq X \leq$ 75 mm (10π mm mrad $\leq \epsilon_{ox} \leq 410\pi$ mm mrad) indicate strong coupling that causes ϵ_y at the on-diagonal working point to increase rapidly to 50π mm mrad when $\epsilon_{ox} = 90\pi$ mm mrad. If the Booster beam is to be round, the requirement that the total emittance $\epsilon_x + \epsilon_y \leq 100\pi$ mm mrad is imposed by the vertical size of the vacuum chamber, and displacement of the closed orbit by 75 mm for proton injection would not be necessary.

3). At both working points a complete period for transferring emittance from one plane to the other and back again is $\sim$28 turns. The beam should be expected to reach its maximum approximately 14 turns after injection, and beam growth during multi turn injection should be included when determining required apertures.

4). The lower coupling at the (4.820,5.830) working point should result in improved conditions for injection and should permit injection of a beam having an elliptical cross section with $\epsilon_x + \epsilon_y \geq 100\pi$ mm mrad. This scenario would include having the injection channel farther away from the equilibrium orbit (perhaps as much as 75 mm), but the small vertical emittance would permit relaxing the requirement that the vacuum chamber have a $\pm$ 1 inch vertical aperture at three inches from its centerline.

Acknowledgment

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References

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2). G. Parzen, Booster Note #24.

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4). G. Morgan and S. Kahn, Booster Note #4.

5). T.L. Collins, FNAL, Informal presentations during SSC Reference Designs Study (1984) and Snowmass Summer Study, 1984.

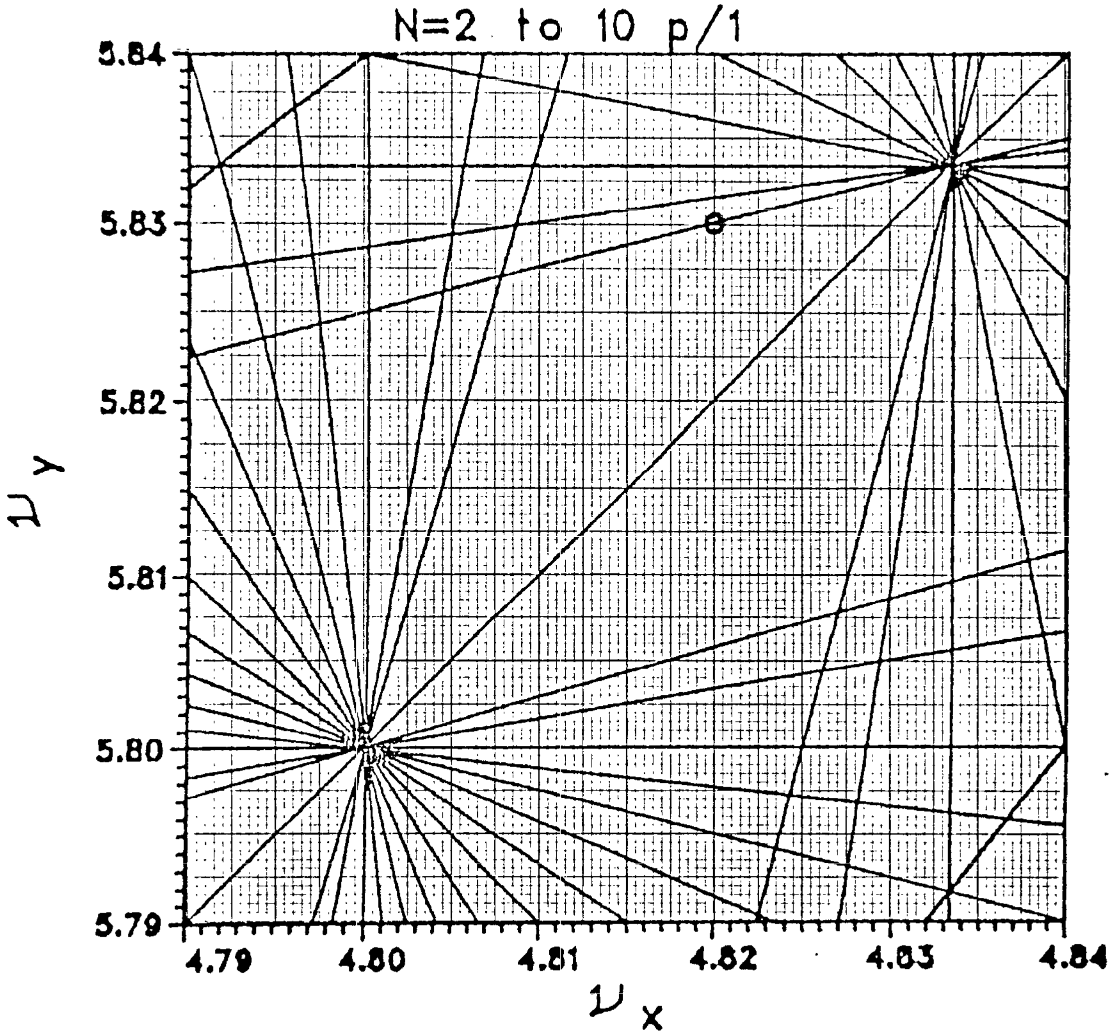

Fig.1 Resonance lines in the region of the operating point $(\nu_x, \nu_y) = (4.820, 5.830)$. The resonance pattern is identical to that at (4.820,4.830). $N\nu_x + M\nu_y = K$ with N,M, and K integers and $|N| + |M| \leq 10$.

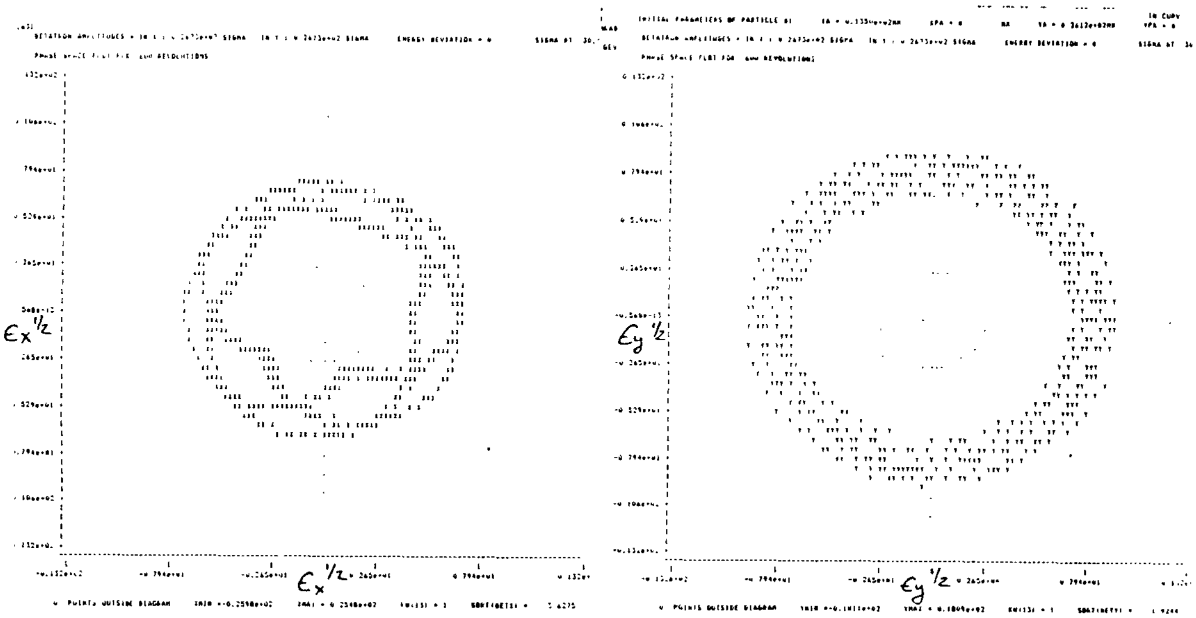

Fig.2(a) Phase plots for X and Y motion at $(\nu_x, \nu_y) = (4.820,4.830)$ showing coupling that decreases the horizontal emittance and increases the vertical emittance. The radial thickness of the plots is an indication of coupling. With eddy current multipoles and $\epsilon_x = \epsilon_y = 50\pi$ mm mrad.

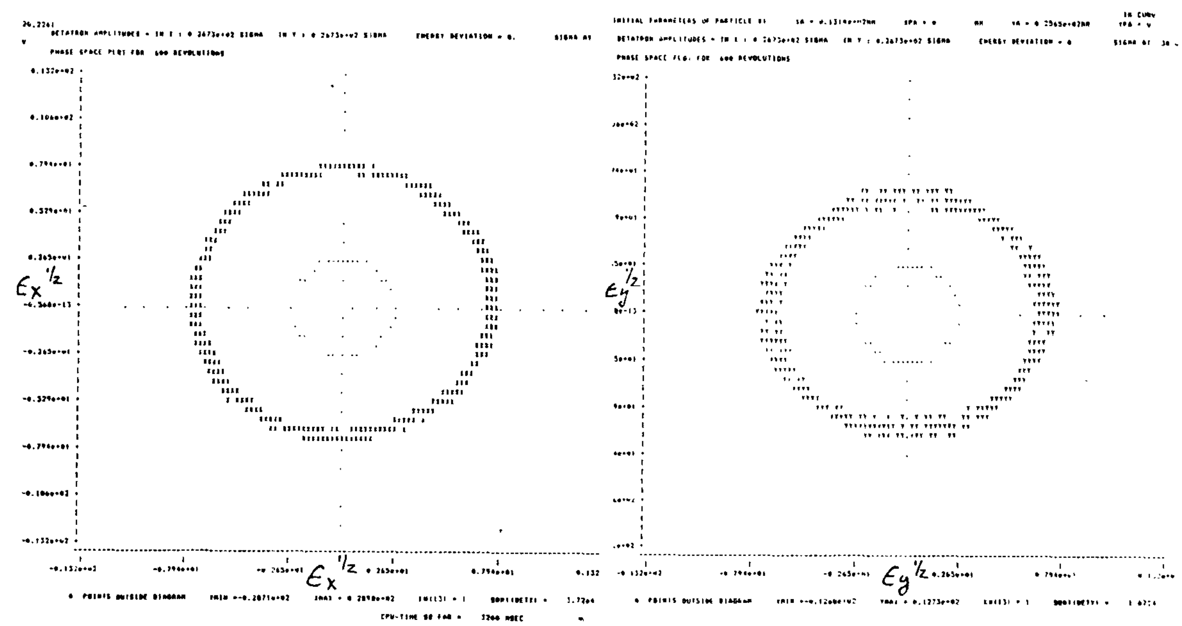

Fig.2(b) Phase plots of X and Y motion at $(\nu_x, \nu_y) = (4.820,5.830)$. Reduced radial thickness indicates less coupling.

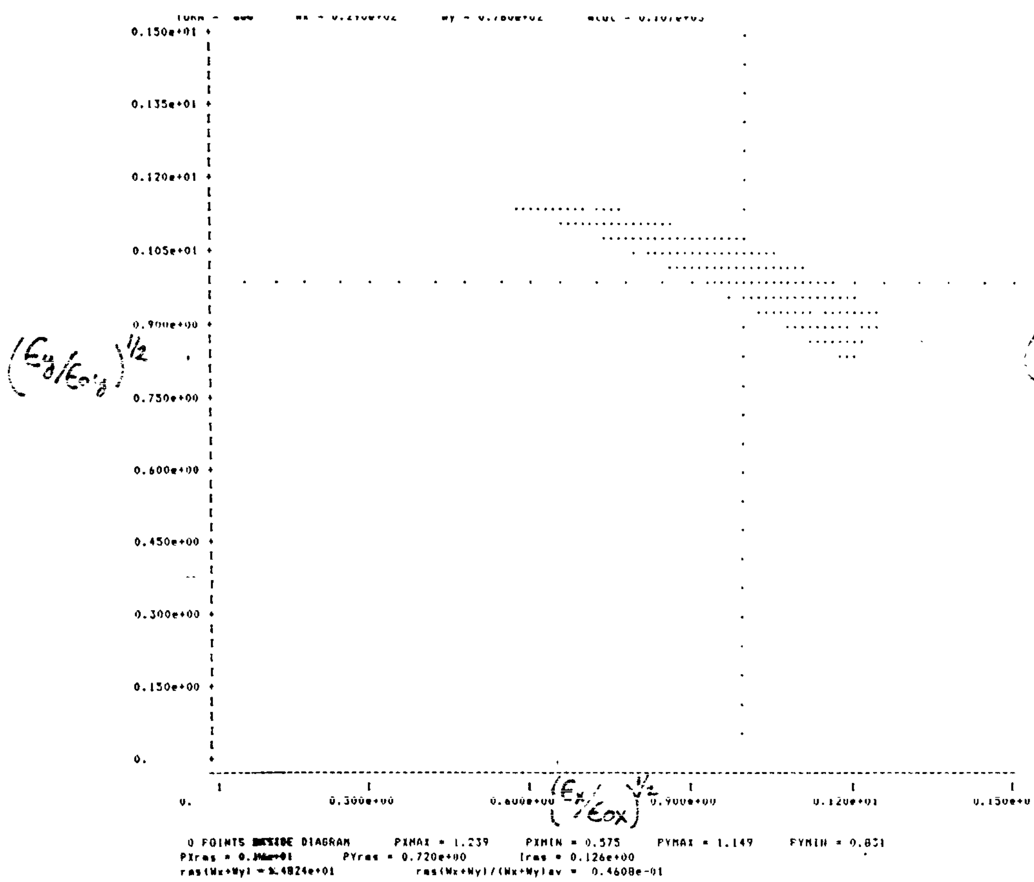

Fig. 3(a) Plot of $(\epsilon_y/\epsilon_{oy})^{1/2}$ as a function of $(\epsilon_x/\epsilon_{ox})^{1/2}$ at $(\nu_x, \nu_y) = (4.820, 4.830)$ with $\epsilon_{ox} = \epsilon_{oy} = 50\pi$ mm mrad. The negative slope of the shaded area indicates coupling.

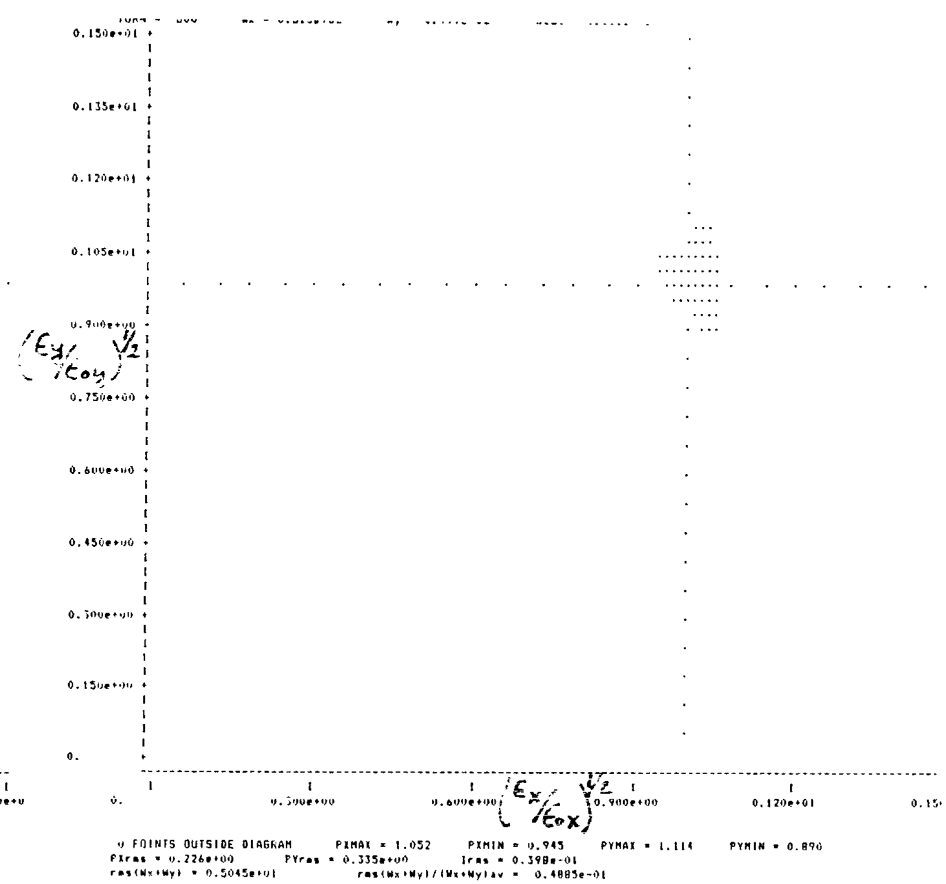

Fig. 3(b) Plot of $(\epsilon_y/\epsilon_{oy})^{1/2}$ as a function of $(\epsilon_x/\epsilon_{ox})^{1/2}$ at $(\nu_x, \nu_y) = (4.820, 5.830)$ with $\epsilon_{ox} = \epsilon_{oy} = 50\pi$ mm mrad. The shaded area is small and indicates little coupling.

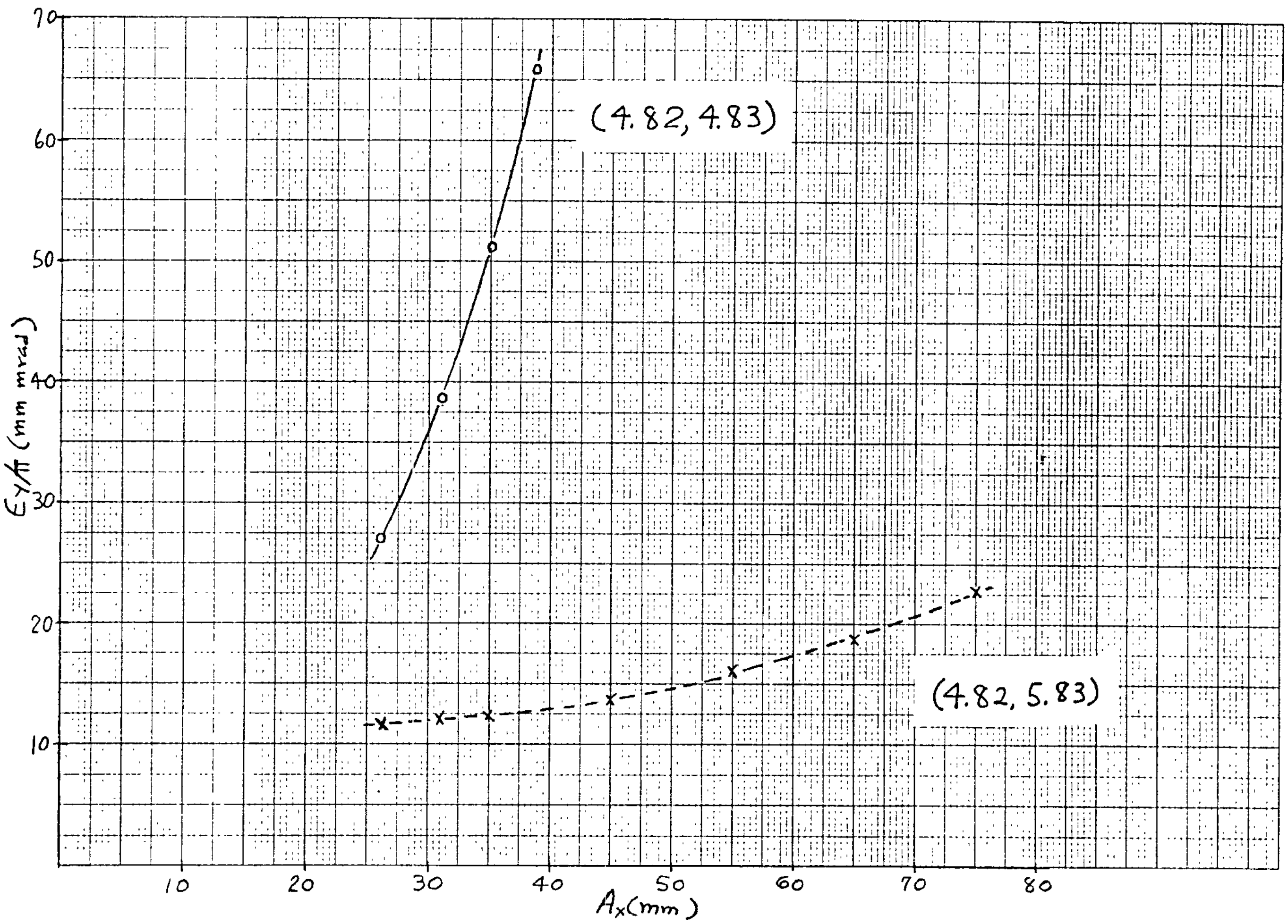

Fig. 4 Transfer of emittance from the horizontal to the vertical plane during injection.at the "on-diagonal" working point (4.820,4.830) and the off-diagonal working point (4.820,5.830). A_x denotes the distance between the injection channel and the closed orbit. $\epsilon_{\circ y} = 10\pi$ mm mrad, $\epsilon_{\circ x} = x^2/\beta_x$.